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Acoustic Characterization of Inhomogenous Layers using Finite Element Method

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***Abstract*— This study presents a finite element approach to estimate acoustic parameters of layers of arbitrary compositions using 2D and 3D models. In this approach the resonance frequency of a layer to be investigated is found by exciting the layer with plane waves and studying the reflected and transmitted sound pressure from the layer. Compressional and shear modes can be excited separately by varying the angle of incidence. A script for generating inhomogeneous layers with randomly distributed inclusions of arbitrary shape and size was developed for this study. A Matlab application was built for processing the result and comparison with analytical calculations. The 2D and 3D models were validated by comparing derived acoustic parameters of known materials with no more than 0.06% deviation from expected values. Estimated parameters for a layer of gold with 10.6% volume fraction of spherical inclusions of voids of 3 µm and 5 µm diameter was found to range from 2540 m/s to 2652 m/s for compressional sound speed and from 1039 m/s to 1067 for shear speed of sound.**

Keywords—Ultrasound, Characterization, Inhomogeneous Layers, FEM, COMSOL Multiphysics.

I. INTRODUCTION

Ultrasound transducer design requires reliable data for acoustic properties of the materials involved. For composite materials these parameters are often not available. Analytical models for estimation of effective elastic parameters exist. The simplest of these models are the Reuss model, where constant stress in the solid is assumed, and the Voigt model, where constant strain is assumed [1]. These models lead to extreme upper and lower bounds on the velocity in the composite. Hashin and Shtrikman [2] presented another approximation where upper and lower bounds are within those formulated by Reuss and Voigt. Berryman [3] has presented three approximations based on single scattering theory, namely the average T-matrix approximation, coherent potential approximation, and the differential effective medium. Devaney and Levine [4] described an approach which is a self-contained formulation of multiple scattering theory. For low volume fractions of inclusions, this model behaves similarly to the other mentioned models. However, for higher volume fraction of inclusions, the multiple scattering effects are dominant. In all the models mentioned, inclusions are assumed to be spherical with a uniform size that is much smaller than the acoustic wavelength.

A two-dimensional finite element modelling approach to estimate effective elastic properties of composite materials consisting of either tungsten particles or piezoceramic particles in an epoxy matrix was presented by Gómez Alvarez-Arenas et. al. [5]. This approach was found to provide tighter upper and lower bounds than those presented by Hashin and Shtrikman and calculated material parameters fit closely to experimental results.

Other experimental results [6] [7] where tungsten particles were enclosed in an epoxy matrix were compared to the previously mentioned analytical models. Both concluded that the Devaney model agreed best with the experimental results as this model gives the best estimate for lower and higher volume fractions of inclusions.

In this paper, we present another finite element approach to characterize inhomogeneous layers. In this approach the resonance frequency of a layer to be investigated is found by exciting the layer with plane waves over a range of frequencies and studying the reflected sound pressure from the layer. Compressional and shear modes can be excited separately by varying the angle of incidence, θ_i. Setting the incoming angle to zero, i.e., normal incidence to the top surface of the layer to be investigated, excites only the compressional mode in the layer. If the thickness of the layer is known, it is possible to find the compressional sound speed by

$$c_l = 2T\Delta f \tag{1}$$

At resonance, all acoustic energy will be transmitted through the layer, resulting in a minimum in the frequency spectra of the reflection coefficient, Δf.

Estimation of the shear speed of sound can be found by setting the incident angle such that the refracted angle of the compressional wave in the layer reaches 90°. This angle is known as the first critical angle (θ_{cr1})and exists for all solid materials with a compressional speed of sound (c_l) larger than that of the surrounding media (c_m) and can be found by

$$\theta_{cr1} = sin^{-1}(\frac{c_m}{c_l}) \tag{2}$$

A second critical angle (θ_{cr2}) exists for solid materials with shear speed of sound(c_s) larger than that of the surrounding media. At the second critical angle the refracted angle of the

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shear wave reaches 90° and the shear modes of the layer no longer exist beyond this angle,

$$\theta_{cr2} = sin^{-1}(\frac{c_m}{c_s}) \tag{3}$$

If the incident angle of the plane wave is between the first and second critical the shear wave velocity can be calculated as

$$c_s = \frac{2TC_m\Delta f}{\sqrt{4T^2\Delta f^2 sin^2\theta_i + C_m^2}} \tag{4}$$

The characteristic acoustic impedance of the layer can be found by

$$Z = \rho c_l \tag{5}$$

For a composite layer the density is found from the volume fraction VF of the filler material

$$\rho^* = \rho_1(1 - VF) + \rho_2 VF \tag{6}$$

II. METHOD

A. *Finite Element Model*

A FEM model was made using COMSOL Multiphysics 5.6 (COMSOL AB, Stockholm, Sweden). The modelled structure, illustrated in Fig. 1, consists of a water column (domains 1, 2, 4 and 5) separated by a solid layer to be investigated (domain 3). An incident plane wave (domain 2) is swept over a range of frequencies at normal and oblique incidence. The reflection coefficient is found by the ratio of the scattered pressure field to incident pressure field at the boundary shown in red. This boundary is separated from the layer to be investigated by a short distance into the water domain to avoid the influence of evanescent waves.

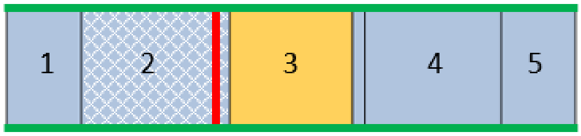


Figure 1 – The structure consists of a water column, shown in blue, separated by a solid layer, shown in yellow. Background Pressure Field is set in domain 2. Periodic Floquet Boundary Conditions are set on the sides of the column. The reflection coefficient is calculated at the boundary marked in red as the ratio of scattered pressure to incident pressure.

Periodic Floquet boundary conditions are used on the sides of the column for continuity of the plane incident wave at oblique incidences. The Floquet boundary condition requires a wave vector defined in each direction. k_i and k_k is used for the x- and y-plane in the case of 2D, while k_k is used to define the z-plane in the case of 3D

$$k_i = \frac{2\pi}{\lambda}\cos(\theta_i) \quad , \quad k_j = 0 \quad , \quad k_k = \frac{2\pi}{\lambda}\sin(\theta_i) \tag{3}$$

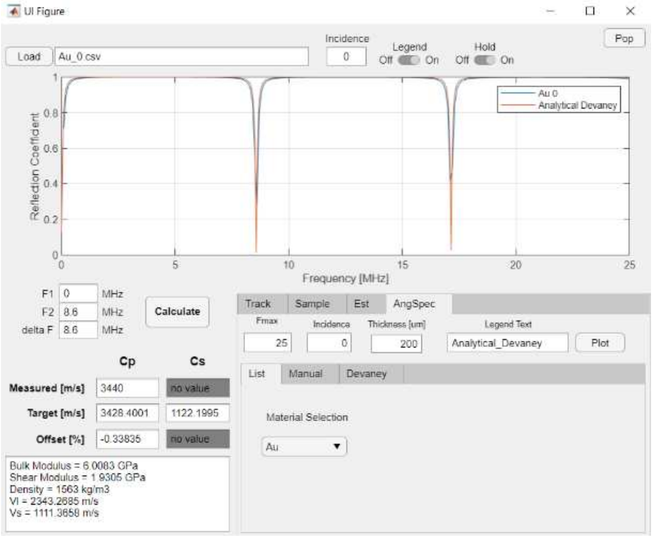
Accuracy of the estimated material parameters is dependent on the frequency step size and mesh density. A fine mesh resolution was needed to represent the inclusions as spheres. For the remaining layers the mesh size was set to be much smaller than the wavelength.

The model was validated by estimating parameters for known homogenous layers of gold, silver and tungsten and comparing the derived values to the input values used to describe the material in the model.

B. *Post Processing*

Material parameters, such as compressional and shear speed of sound and characteristic acoustic impedance are found through post-processing in Matlab [MathWorks, Natick, MA, USA]. A user interface, shown in Fig. 2, was developed using the Matlab App Designer feature.

The input values needed to estimate material parameters and plot the frequency spectra of the reflection coefficient include layer thickness, angle of incidence, speed of sound in surrounding media as well as the calculated reflection coefficient for each frequency point. These input values can be set manually or imported from the exported results table from the finite element model. An option to compare the resulting plot and calculated parameters to an analytical approach is included. The input values for the speed of sound of the layer needed for the analytical approach can be input manually or calculated using the Devaney model if the layer is a composite.



Au_0.csv
Incidence 0
F1 0 MHz
F2 8.6 MHz
delta F 8.6 MHz
Fmax 25
Incidenca 0
Thickness [um] 200
Legend Text Analytical_Devaney

	Cp	Cs
Measured [m/s]	3440	no value
Target [m/s]	3428.4001	1122.1995
Offset [%]	-0.33835	no value

Bulk Modulus = 6.0083 GPa
Shear Modulus = 1.9305 GPa
Density = 1563 kg/m3
Vl = 2343.2685 m/s
Vs = 1111.3658 m/s

Material Selection Au

Figure 2 – Graphical user interface for post processing.

C. *Characterization of Inhomogeneous Layers*

Inhomogeneous layers were represented by a layer with multiple inclusions. The technique for generating randomly distributed inclusions was based on a COMSOL blogpost where the Application Builder feature was used [8]. The script was developed to generate geometry files of inhomogeneous layers with a specified volume fractions of inclusions with shapes and sizes that can be specified too.

The 3D-implementation of the model was used to estimate compressional and shear speed of sound of 20 µm thick layers of gold with 10.6% volume fraction of spherical voids randomly distributed within the layer, as shown in Fig. 3. Void diameters were fixed at 3 µm and 5 µm. Three layers of each void size were characterized to study the variation due to randomized distribution of the inclusions.

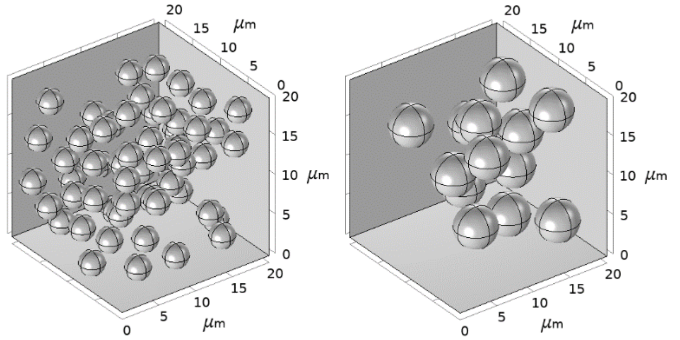


Figure 3 - 20 µm thick layers with 10.6% volume fraction of spherical voids randomly distributed within the layer. Void diameters were fixed at 3 µm (left) and 5 µm (right).

III. RESULTS

A. *Model Validation*

The 2D and 3D implementations of the model were validated by estimating the speed of sound of known homogeneous layers of gold, silver, and tungsten. The simulated and theoretical reflection coefficients for a 200 µm thick gold are shown in Fig. 4, both for normal and oblique incidence. Table 1 lists the derived values for compressional and shear speed of sound as well as the values used as input parameters to the model which are considered as the target value. The values estimated by the finite element approach were found to be accurate with less than 0.06% deviation from the expected value.

B. *Characterization of Inhomogenous Layers*

Table 2 lists the derived values for the compressional and shear speed of sound for gold layers with 10.6% volume fraction of voids with diameters 3 µm and 5 µm. Effective characteristic acoustic impedance was calculated according to equations 5 and 6. Three layers of each kind were characterized with some spread in the derived values for the sound speed.

IV. DISCUSSION

A finite element modelling approach was developed in COMSOL Multiphysics for estimating effective elastic material parameters derived from the responding reflection coefficient from plane waves at normal and oblique incidence. The material (or layer) to be characterized may be either homogeneous or inhomogeneous where a technique for generating materials with randomly distributed inclusions was presented.

A. *Model Validation*

Both the 2D and 3D implementations of the model yielded maximum deviation from target values of 0.02% for compressional speed of sound and 0.06% for shear speed of sound. As these deviations are small, the characterization approach is considered valid.

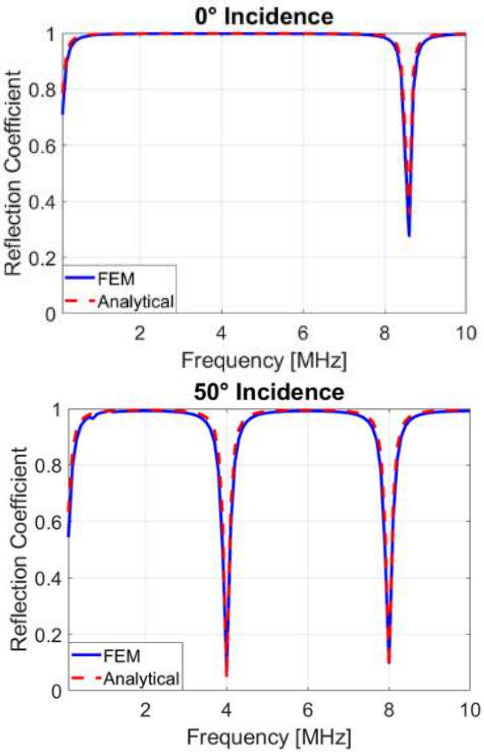


Figure 4 – Frequency spectra of reflection coefficient for a 200 µm thick layer of gold at normal and 50° incidence.

TABLE 1 – DERIVED PARAMETERS FOR HOMOGENEOUS LAYERS

	Derived Value 2D [m/s]	Derived Value 3D [m/s]	Target Value [m/s]
		Gold	
c_l	3428	3428	3428
c_s	1122	1122	1122
		Silver	
c_l	3740	3740	3739
c_s	1699	1699	1698
		Tungsten	
c_l	5212	5212	5211
c_s	2879	2879	2880

TABLE 2 – DERIVED EFFECTIVE PARAMETERS FOR GOLD LAYER WITH 10.6% VOLUME FRACTION OF VOIDS

Void Diameter [µm]	Compressional Speed of Sound c_l [m/s]	Shear Speed of Sound c_s [m/s]	Characteristic Acoustic Impedance Z [MRayl]
3	2636 ± 16	1056 ± 11	45.5 + 0.3
5	2564 ± 24	1046 ± 7	44.2 + 0.4

B. *Characterization of Inhomogenous Layers*

The 3D implementation of the model was used to estimate the material parameters of a gold layer with 10.6% volume fraction of void inclusions, where the void diameter was fixed at 3 µm or 5 µm. The inhomogeneous layers were found to have a reduced longitudinal and compressional speed of sound by approximately 24% and 6%, respectively. The characteristic acoustic impedance was found to be reduced by approximately 30% relative to a solid layer of gold. This reduction in characteristic acoustic impedance is as expected since introducing voids in the layer will effectively lower both the density and the elastic moduli of that layer.

C. Void Size

The sizes of the voids described in this study correspond to approximately 1/13 and 1/8 of the acoustic wavelength for the 3 µm and 5 µm diameter voids, respectively. Hence, the voids in this study can be described as small relative to the wavelength. Three layers of each void size were characterized. Void sizes and concentration were kept constant, thereby only varying the positions of the voids within the layer. Derived values for the speed of sound yielded a spread of approximately 1%. This variation is likely due to the connectivity between the inclusions.

A lower sound speed and characteristic acoustic impedance was found for the layers of 5 µm voids compared to the layers of 3 µm voids for a 10.6% volume fraction of voids. This suggests that larger inclusions will lower the effective elastic properties of the layer more compared smaller voids for an identical volume fraction. In both cases the voids are much smaller than the acoustic wavelength, so reflection is unlikely the cause of this variation.

D. Comparison to Existing Approaches

The Devaney model, which has been found to be the most accurate model for estimating effective elastic properties of a composite material [6] [7], cannot predict effects from geometric variations as all inclusions are assumed to be spheres of fixed size much smaller than the acoustic wavelength in the composite material. The findings from this modelling approach does however suggest the distribution will influence the elastic properties of the composite material.

Like the work of Gómez Alvarez-Arenas et. al. [5] we also use finite element modelling to predict the elastic properties of complex composite materials. However, our work includes three-dimensional composites and elastic parameters were extracted based on frequency domain simulations, whereas Gómez Alvarez-Arenas et. al. performed a two-dimensional study in time domain. Advantages of using 3D models include a higher level of accuracy, while computation times significantly benefit from the use of the Floquet boundary conditions combined with the frequency domain approach.

E. Potential Areas of Application

Our previous work [9] on the use of solid-liquid interdiffusion (SLID) bonding [10] in ultrasound transducers reported on the existence of voids in the metallurgical bondline. Such layers are challenging to characterize experimentally as the bondlines are thin and most often sandwiched in-between two layers. The characterization approach described in this paper may be employed to better understand effects induced by voids in a bondline of any characteristic.

V. CONCLUSION

A finite element method approach to characterize complex multi-phase composites has been developed. Two- and three-dimensional implementations of the model have been described in detail. The modelling approach was validated by estimating compressional and shear speed of sound for known materials. The model was then used to characterize the characteristic acoustic impedance and compressional and shear speed of sound of a gold layer with 10.6% volume fraction of voids. This modelling approach may be utilized for describing effects on the material parameters induced by geometric variation of the inclusions in a composite material.

ACKNOWLEDGMENT

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