Belief Propagation Detection For Large-Scale MIMO Systems With Low-Resolution ADCs

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***Abstract*—We derive the belief propagation (BP) detector for large-scale multiple-input multiple-out communication systems where the low-resolution analog-to-digital converters (ADCs) are used to save the power and hardware costs. By modeling the quantization noise as an additive noise element, we derive a new expression for the extrinsic information passed from the observation node to the symbol node on the Tanner graph. Furthermore, we study the performance of the BP-based MIMO detector under different low-resolution ADCs and multiple-input multiple-output (MIMO) configurations. The study results show that one can achieve almost the same performance of high-resolution MIMO systems with the 5-bit ADC in the worst scenario where the number of transmit antennas M and the number of receive antennas N are equal. In the favorable situation that $N > M$, for example $M = 10$ and $N = 30$, the 3-bit ADC system has the performance that approaches the performance of the high-resolution ADC system.**

***Index Terms*—Large-scale MIMO, low-resolution ADCs, belief propagation MIMO detector, message-passing.**

I. Introduction

Wireless networks currently have rapid growth in traffic from smart devices. For that reason, increasing network capacity is a crucial requirement to guarantee service quality. Large-scale/ massive MIMO techniques will play a pivotal role to achieve spectral and energy-efficient networks. The great feature of the LS-MIMO systems is that the base station with a large number of antennas can concentrate transmitted signal energy into short-range areas to bring significant enhancements in terms of system capacity [1].

In the standard multiple-antenna system, each radio frequency (RF) port at the receiver is equipped with two high-resolution Q-bit ADC (typically Q greater than 10). Nevertheless, using high-resolution ADCs in the massive MIMO scenario, which has hundreds or thousands of active antenna elements, could result in immense power dissipation and hardware costs. Using the low-resolution ADC with a few quantization bits is a practical solution to keep the power consumption and hardware costs within controllable limits [1], [2].

In addition to the problem of the ADC power consumption and hardware costs, LS-MIMO systems also face challenges in the signal detection since the detectors, designed for the traditional MIMO systems, become unsuitable and unscalable to LS-MIMO systems. In [3], authors derived a modified minimum mean square error (MMSE) receiver for low-resolution ADC MIMO systems. The quantization noise effect is taken into account to design the linear receiver, which performs better than the conventional MMSE receiver at the high signal to noise ratio (SNR) regime. Fukada et al. [4] proposed a low complexity detector based on the belief propagation (BP) technique for the massive MIMO system. The proposed BP-based MIMO detector requires the second-order calculation and has a fairly low complexity in comparison to the MMSE detector. From the performance perspective, the BP-based MIMO detector has significant bit error rate (BER) improvement in contrast to the MMSE detector.

Even though the BP-based MIMO detector has both performance and complexity advantages over the MMSE detector, the BP-based MIMO detector was designed for high-resolution MIMO systems. Therefore, in this paper, our first contribution is to derive a BP-based MIMO detector where the quantization noise of the low-resolution ADC is modeled as additive noise and, it is included in the derivation of the extrinsic message expression.

The next contribution of this paper is that we address the question - How many bits one should use in the ADCs to avoid significant performance loss when using the BP-based detector in the low-resolution MIMO systems. Our study results prove that 5-bit ADC is enough to achieve almost the same performance of the high-resolution MIMO systems in the worst MIMO scenario where $N = M$. In the favorable communication scenario where $N > M$, one can use 4-bit ADC to reduce the complexity and power consumption while sacrificing a very marginal performance loss.

The rest of this paper is organized as follows. Section II describes the system model and the scalar uniform quantizer of the Q-bit ADC. Then, Section IV derive the BP-based MIMO detector based on the Tanner graph. The simulation results for different low-resolution ADCs and MIMO configurations are presented in Section IV. Section V summarizes the contribution of the paper.

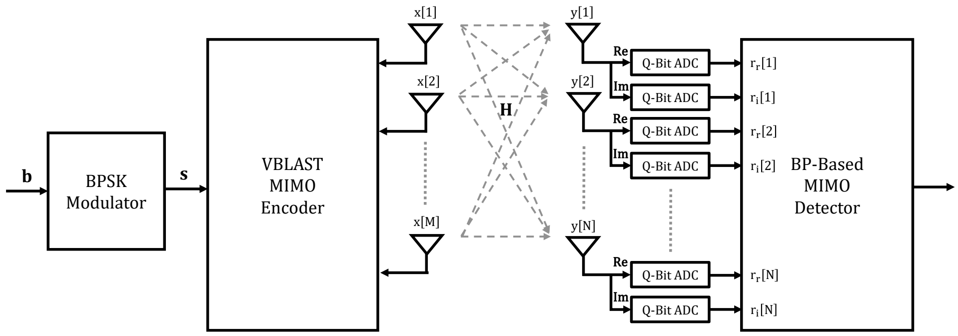

Fig. 1. The MIMO communication model.

II. System Model

Consider an uncoded wireless MIMO communications system with M transmit antennas and N receive antennas, as shown in Fig. 1. The information bit b is next modulated by the binary phase-shift keying (BPSK) modulator where the modulated symbol x takes values in $\{+1, -1\}$. In one channel use, M modulated symbols are transmitted over M transmit antennas using spatial multiplexing (V-BLAST) scheme, [5].

The received signal model is given by

$$\mathbf{y} = \sqrt{\frac{1}{M}}\mathbf{H}\mathbf{x} + \mathbf{w}. \tag{1}$$

Here, $\mathbf{x} = [x[1], x[2], \cdots, x[M]]^T$ is the transmitted symbol with its elements belong to the BPSK modulation alphabet. $\mathbf{H} \in \mathbb{C}^{N\times M}$ is channel matrix. The entries $h[n][m]$ in the n-th row and m-th column of $\mathbf{H}$. Those elements are modeled as i.i.d complex Gaussian with zero mean and unit variance $\mathcal{CN}(0, 1)$. In this paper, we assume that the perfect channel state information (CSI) is available at the receiver only. The vector $\mathbf{w} = [w[1], w[2], \cdots, w[N]]^T \in C^{N\times 1}$ is complex additive white Gaussian noise vector whose entries follow i.i.d complex Gaussian with zero mean and N_0 variance $\mathcal{CN}(0, N_0)$. Finally, $\mathbf{y} = [y[1], y[2], \cdots, y[N]]^T \in \mathbb{C}^{N\times 1}$ is the received symbol.

In the large-scale MIMO communication scenarios, the radio frequency (RF) signal at each receiving antenna is quantized by a low-resolution ADC. In this paper, we consider Q-bit uniform scalar quantizer with $Q = 2, 3, 4, 5$. In previous research works, 1-bit ADC is investigated. But, we do not include the investigation of the 1-bit ADC in this work.

In practice, the received signal is a continuous random variable with infinite support. Therefore, the received signal is first truncated to have finite support in the range $[-L_s, L_s]$. The value of L_s is selected such that the distortion due to the truncation process is negligible. In this work, we apply the *three-sigma* rule as in [6] to find the truncation limit for the received signal $y[n], n = 1, 2, \cdots, N$ as below

$$\begin{aligned} L_s[n] &= 3 \times \sigma_{y_{Re}[n]} = 3 \times \sigma_{y_{Im}[n]} \\ &\approx 3 \times (0.5 + 0.5 \times N_0)^{\frac{1}{2}} = L_s. \end{aligned} \tag{2}$$

The righthand side of (2) is constant. Hence, we have a common truncation limit for all received signals across receive antennas.

Let $\overline{\mathbf{y}}$ be the truncated version of the received signal $\mathbf{y}$, the truncated signals are given by

$$\overline{y_l}[n] = \begin{cases} -L_s, & y_l[n] < -L_s; \\ y_l[n], & -L_s \le y_l[n] \le L_s; \\ L_s, & y_l[n] > L_s. \end{cases} \tag{3}$$

with $l \in \{Re, Im\}$.

The ADC equipped at each antenna quantizes the real and imaginary components of the truncated signals separately by a uniform scalar quantizer [7]. Let $\mathbf{r}$ be the quantized version of the truncated received signal $\bar{\mathbf{y}}$, we can mathematically present the quantized signal as

$$r_l[n] = I^Q_{n_q} \text{ if } \overline{y_l}[n] \in (g^Q_{n_q-1}, g^Q_{n_q}], l \in \{Re, Im\}, \tag{4}$$

where the representation point $I^Q_{n_q}$, the midpoint of each interval, is expressed

$$I^Q_{n_q} = \frac{g^Q_{n_q} + g^Q_{n_q-1}}{2}, n_q = 1, 2, \cdots, 2^Q. \tag{5}$$

The boundary points are given by

$$g^Q_q = -L_s + q\Delta, q = 0, 1, \cdots, 2^Q. \tag{6}$$

In (6), the parameter Δ is the quantization step size which is calculated by

$$\Delta = \frac{2L_s}{2^Q}. \tag{7}$$

Using the additive quantization noise model (AQNM) which is conventionally assumed in the study of the MIMO systems with low-resolution ADCs [8], we can model for the quantized signal as below

$$\mathbf{r} = \varphi\overline{\mathbf{y}} + \mathbf{w}_q \approx \varphi\mathbf{y} + \mathbf{w}_q, \tag{8}$$

with $\varphi = 1-\rho$ and ρ is the inverse of the signal-to-quantization noise ratio, and $\mathbf{w}_q$ is the additive Gaussian quantization noise vector that is uncorrelated with $\mathbf{y}$.

For the uniform scalar quantizer, we approximate the quantization noise as $\Delta^2/12$, [9]. As a result, we obtain the parameters ρ and φ as follows

$$\rho \approx \frac{3}{2^{2b}} \rightarrow \varphi \approx 1 - \frac{3}{2^{2b}}. \tag{9}$$

For given channel realization matrix $\mathbf{H}$, the variance of $w_q[n], n = 1, 2, \cdots, N$ is given by

$$\sigma^2_{n_q}[n] = \varphi(1-\varphi)\left(\frac{1}{M}\sum_{m=1}^{M}|h[n][m]|^2+N_0\right). \quad (10)$$

The quantized received signal $\mathbf{r}$ is fed to the BP-based MIMO detector, which is elaborated in details in the next section.

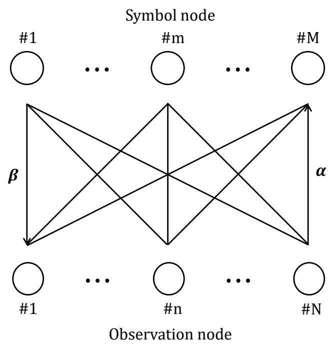

Fig. 2. The Tanner graph for the BP-based MIMO detection.

III. BP-Based Detection With Soft Interference Cancellation

When the number of antennas is in order of hundred elements, the conventional MIMO detection algorithms such as zero-forcing, minimum mean square error spatial filtering, and sphere decoding are highly complex, and they may become impractical. The message passing algorithm is an excellent choice to deal with the complexity issue [4].

Fig. 2 shows the Tanner graph for MIMO communication systems. In this graph, there are two types of nodes; observation nodes (receive antennas) and symbol nodes (transmit antennas). The symbol nodes and observation nodes are fully connected as the characteristics of the radio propagation in the wireless MIMO channel. Nevertheless, the connection strengths are randomly varied from time to time, according to the channel gain between the observation node and the symbol node. In this section, we study a BP-based MIMO detector using this Tanner graph where reliability messages $\alpha[n][m]$ and $\beta[m][n]$ are iteratively transferred between the n-th observation nodes and the m-th symbol nodes.

A. Message Update At Observation Nodes

At the n-th observation node, the transmitted symbols are detected, and the extrinsic information (message) is passed to the symbol nodes. In the following, we present the detection algorithm using a message update rule at the n-th observation node.

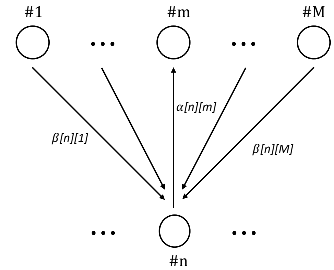

Fig. 3. Message update at the observation nodes.

The quantized received signal on the n-th receive antenna/ observation node can be rewritten as

$$\begin{aligned} r[n] &= \frac{\varphi}{\sqrt{M}}\sum_{m=1}^{M} h[n][m]x[m] + \varphi w[n] + w_q[n] \quad (11)\\ &= \frac{\varphi}{\sqrt{M}} h[n][m]x[m] + \underbrace{\frac{\varphi}{\sqrt{M}}\sum_{k=1,k\neq m}^{M} h[n][k]x[k]}_{Interference} \\ &\quad + \varphi w[n] + \underbrace{w_q[n]}_{QuantizationNoise}, \end{aligned}$$

where $x[m]$ is the symbol sent on the m-th antenna/ symbol node, $h[n][m]$ is the channel gain from nth transmit antenna to mth antenna, $w[n]$ and $w_q[n]$ the additive white Gaussian noise and the quantization noise, respectively.

As shown in (11), the quantized received signal includes inter-substream interference. In this work, we employ the parallel interference cancellation technique to cancel the inter-substream interference. To do so, we first estimate the soft symbol of $x[n][m]$ based on the extrinsic LLR transferred from the m variable node to the n-th observation node. Denote $\hat{x}[n][m]$ the soft symbol obtained by using the LLR passed from the m-th symbol node to the n-th observation node. For the BPSK modulation, the soft symbol estimate is given by

$$\hat{x}[n][m] = \tanh\left(\frac{\beta[n][m]}{2}\right), \quad (12)$$

where $\beta[n][m]$ is the extrinsic information passed from the m-th symbol node to the n-th observation node.

The soft symbol estimate in (12) is now used to cancel the interference from the quantized received signal at the n-th antenna for the m-th transmit symbol as below

$$\hat{r}[n][m] = r[n] - \frac{\varphi}{\sqrt{M}}\sum_{k=1,k\neq m}^{M} h[n][k]\hat{x}[n][k], \quad (13)$$

where $\hat{r}[n][m]$ is the quantized received signal of the transmitted symbol $x[m]$ at the received antenna n after the interference cancellation.

In general, the soft symbol estimate is an imperfect replica of the transmitted symbol. Therefore, the residual interference remains in the signal $\hat{r}[n][m]$. We approximate this residual interference as additive Gaussian noise. Let $z[n][m]$ be the residual interference plus noise component which is given by

$$z[n][m] = \frac{\varphi}{\sqrt{M}} \sum_{k=1,k\neq m}^{M} h[n][k](x[n][k] - \hat{x}[n][k]) \quad (14)$$
$$+\varphi w[n] + w_q[n].$$

Based on (14), we can rewrite $\hat{r}[n][m]$ in (13) as below:

$$\hat{r}[n][m] = \frac{\varphi}{\sqrt{M}} h[n][m]x[m] + z[n][m]. \quad (15)$$

The power of the residual interference plus noise component is calculated by the following equation

$$\Psi[n][m] = \frac{\varphi^2}{M} \sum_{k=1,k\neq m}^{M} |h[n][k]|^2(1 - |\hat{x}[n][k]|^2) \quad (16)$$
$$+\varphi^2 N_0 + \varphi(1-\varphi)\left(\frac{1}{M}\sum_{m=1}^{M}|h[n][m]|^2 + N_0\right),$$

where φ is given in (9).

The message passed from the nth observation node to the mth symbol node is the log-likelihood ratio (LLR) and given by

$$\alpha[n][m] = \ln \frac{\Pr(\hat{r}[n][m]|\mathbf{H}, x[m] = +1)}{\Pr(\hat{r}[n][m]|\mathbf{H}, x[m] = -1)} \quad (17)$$
$$= \frac{4\varphi}{\sqrt{M}\Psi[n][m]} \Re(h^*[n][m]\hat{r}[n][m]).$$

The processing at the n-th observation node is finished by passing the massage $\alpha[n][m]$ to the m-th symbol node.

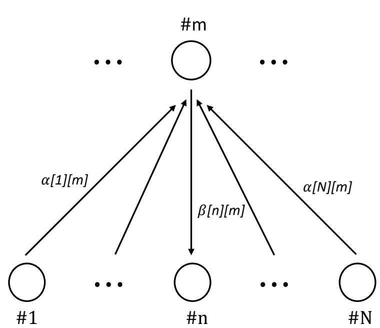

Fig. 4. The message update at the symbol nodes.

B. Message Update At Symbol Nodes

The main task of the m-th symbol node is to calculate a posteriori probability of the symbol $x[m]$ with the messages received from the observation nodes. The posteriori LLR $\Gamma[m]$ of the symbol $x[m]$ can be obtained by summing up all the extrinsic LLRs received from the observation nodes as below

$$\Gamma[m] = \sum_{n\in N} \alpha[n][m]. \quad (18)$$

The next step is to calculate the extrinsic information to transfer from the m-th symbol node to the n-th observation node using the posterior LLR, $\Gamma[m]$. The extrinsic information should be included the information provided by the observation nodes except the n-th node to avoid message duplication as illustrated in Fig. 4. The message from the m-th symbol node to the n-th observation node is given by

$$\beta[m][n] = \Gamma[m] - \alpha[n][m]. \quad (19)$$

As we see from Fig. 2, the Tanner graph for BP-based detection has a lot of short loops which can degrade the convergence performance of the iterative algorithm. When the reliability does not converge correctly, the LLR values often oscillate [10]. The oscillation phenomenon can be damped by using damping factor ε as below

$$\beta^{(t)}[m][n] = \varepsilon\beta^{(t-1)}[m][n] + (1-\varepsilon)(\Gamma[m] - \alpha[n][m]), \quad (20)$$

where t is the iteration index.

The messages are iteratively transferred between the symbol nodes and the check nodes. After each iteration, the reliability of the symbol is improved. At the end of the iteration process, the symbol $\hat{x}[m]$ is estimated as

$$\hat{x}[m] = \text{sign}(\Gamma[m]). \quad (21)$$

IV. Simulation Results

In this part, we carry out experiments with computer simulation to assess the performance of the proposed BP-based detector for some typical low-resolution ADCs and MIMO configurations. The modulation scheme is BPSK, and a Rayleigh fading channel is employed. The channel state information is assumed to be perfectly known at the receiver. The maximum number of iteration is set to 10.

First, we investigate the 10×10 MIMO configuration with a 3-bit ADC converter is applied. We observe from Fig. 5 that the proposed BP-based detector gives better performance than the unquantized MMSE detector, in good agreement with the result reported in [4]. Concerning the impact of iteration processing, the performance of the proposed detector is improved in the first 5 iterations. Afterward, the performance improvement is marginal. In comparison with the unquantized/high-resolution system, the 3-bit ADC system has a big gap. In particular, at the BER $= 10^{-3}$, the difference between two curves is about 2.5 dB.

To investigate the connection between the performance and the resolution of the ADC, Fig. 6 presents the BER performance of different low-resolution ADC systems. One can see that there is a big gap between the 2-bit ADC system and 3$-$bit ADC system. Increase the resolution of the ADC one more bit, i.e., 4-bit ADC, the performance is further improved. However, the performance improvement is diminished when increasing the resolution from 4-bit ADC to 5-bit ADC. It is also seen from Fig. 6 that the performance difference between 5-bit ADC system and the high-resolution/unquantized system is tiny. For instance, at BER = 10^{-3}, the performance of 5-bit ADC system and the unquantized system is roughly the same. When moving further down to BER = 10^{-4}, the gap is around 1 dB. This study result suggests that one can use 4-bit ADC or

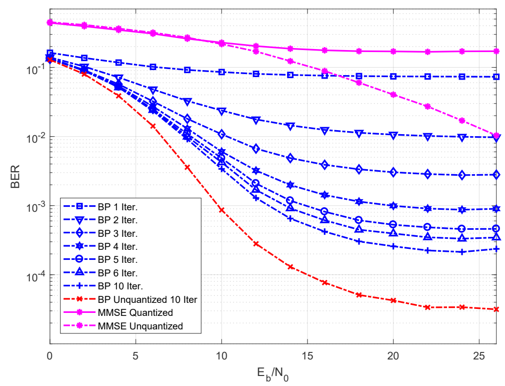

Fig. 5. The BER performance: LS-MIMO 10×10, 3-bit ADC, ε = 0.2

5-bit ADC to decrease the complexity and power dissipation of the RF processing in exchange for negligible performance loss.

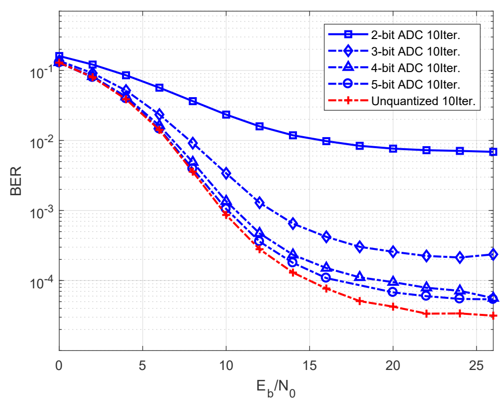

Fig. 6. The BER performance: LS-MIMO 10×10, 2-bit ADC to 5-bit ADC, ε = 0.2

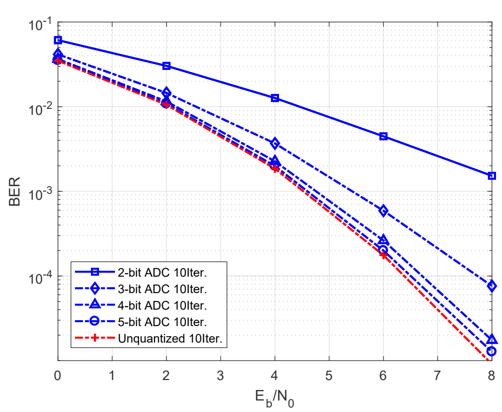

Fig. 7. The BER performance: LS-MIMO 10×20, 2-bit ADC to 5-bit ADC, ε = 0.2

When the number of transmit antennas M is fixed to 10, and the number of receive antennas N is 10, 20, and 30, the performance gap between 2-bit ADC system and 3-bit ADC system is significantly decreased as observed from Fig. 6 - Fig. 8. In the specific 10×30 MIMO configuration, the performance of 3-bit ADC is very close to the performance of the unquantized MIMO system. This result is meaningful for massive MIMO communication systems where the base station is equipped with hundreds or thousands antennas because we can use low-resolution ADC to reduce the complexity and power consumption without significant performance loss.

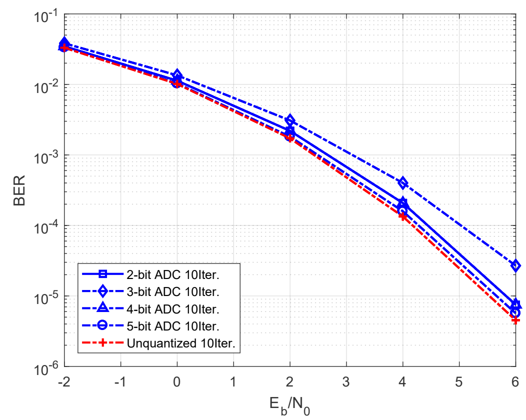

Fig. 8. The BER performance: LS-MIMO 10×30, 2-bit ADC to 5-bit ADC, ϵ = 0.2

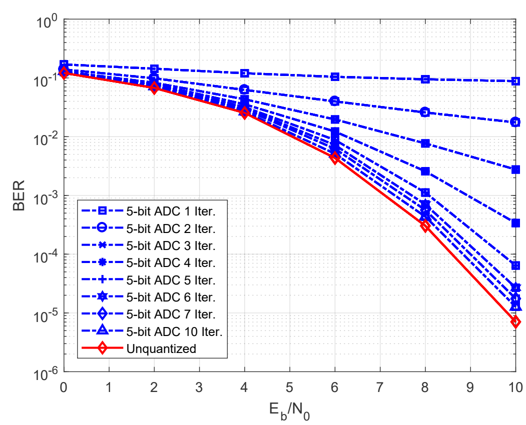

Fig. 9. The BER performance of the massive MIMO system vs. number iterations , ε = 0.2

Finally, we present the experiment results of low-resolution ADCs for massive MIMO scenario in Fig. 9 and Fig. 10 where the number of the transmit antennas is set to 100, and the number of the receive antennas is 100, 150, 200, and 300. The iteration behaviour of the proposed BP-based detector is shown in Fig. 9. The observation is the same with the large-scale MIMO case: That is the iteration is very effective in the first five iterations in terms of performance improvement. The study results of various massive MIMO configurations are shown in Fig. 10 where we use 4-bit ADC and 5-bit ADC. The experiment results show that the performance gap of the low-resolution (4-bit and 5-bit) and the high-resolution is also negligible. This observation confirms that the low-resolution ADC massive MIMO system suffers a small performance loss in exchange for the complexity of the RF processing.

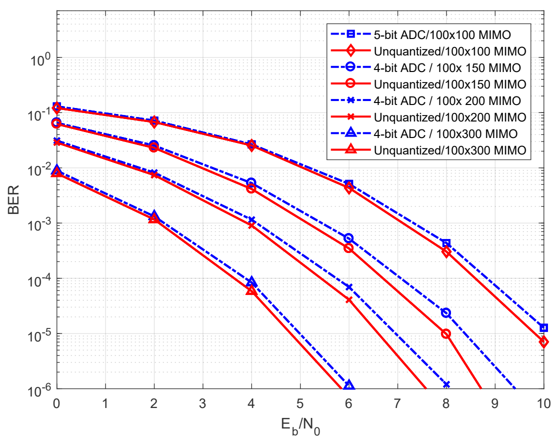

Fig. 10. The BER performance of the massive MIMO system: High-resolution vs. Low-resolution, $\varepsilon = 0.2$

V. Conclusion

We derive and study the BP-based detector for the low-resolution ADC MIMO systems. By utilizing the additive quantization noise model, we can achieve the new expression for message update at the observation nodes to build the BP-based detector for the low-resolution ADC MIMO systems. Our investigation indicates that one can use 4-bit ADC or 5-bit ADC in the massive MIMO systems to cut down the power consumption and hardware costs for the RF module while experiencing a slight performance loss.

VI. Acknowledgement

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