TELEMARKSFORSKING

NOTODDEN

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# FROM THE TIMSS 1999 VIDEO STUDY OF MATHEMATICS IN SEVEN COUNTRIES

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## *SUMMARY*

*The TIMSS 1999 Video Study of Mathematics in Seven Countries was a large international study where more than 600 classrooms from seven countries were video-taped. The study had a focus on teaching practices, and the aim was to improve teaching and learning. This report puts more focus on a few perspectives of this study, from a Norwegian view.*

*The use of history in mathematics is new in the Norwegian curriculum, L97, and Smestad's article has a focus on how history is used by teachers in other countries. Bekken and Mosvold focus on how history can be used in the mathematics classroom in another and more indirect way. They present these ideas in an example from the TIMSS 1999 Video Study and discuss how teachers can become more reflective practitioners. Another issue that has been emphasised in L97 is the connection with everyday life, and Mosvold presents examples on how this has been done in two countries where few real-life connections were made and one connection where the teachers made many connections with real or everyday life. The article of Cestari, Santagata and Hood discusses video studies on another level, and they focus on how teachers can use videos to reflect on their own teaching.*

*There is now a development towards a new curriculum in Norway, and in this process it should be of vital importance to reflect on the practices of teachers in other countries. A study of teaching in other countries can reveal one's own practice in a new and more powerful way, and one can also discover new approaches and get new ideas.*

## *PREFACE*

In the Spring term of 2003 the writers of these articles were situated in Los Angeles, CA., for various periods of time. The studies presented here were conducted while the authors were in residence at LessonLab, Santa Monica, as members of the TIMSS 1999 Video Study of Mathematics in seven countries (TIMSS stands for the Third International Mathematics and Science Study). Thanks are due to James W. Stigler and Ronald Gallimore for opening the doors to the LessonLab and letting us participate in this unique study. The four articles are all based videos from the TIMSS 1999 Video Study, but they have different perspectives.

Bekken & Mosvold adopt a genetic approach to the teaching of mathematics in their article. The genetic approach is not a new idea. It builds on the theories of didactical thinkers of the past like Bacon, Comenius, Branford and Klein, and it represents a way of using history in an indirect way in reflections on the teaching of mathematics. Bekken and Mosvold present these ideas taking an example from the TIMSS 1999 Video Study as a starting point, and discuss how teachers could become reflective practitioners by having access to a data base of such videos.

The article of Cestari, Santagata and Hood focuses on how teachers are encouraged to reflect on their own teaching practice while watching videos of mathematics lessons. Three teachers are presented in the article as having participated in a course called: TIMSS Video Studies: Exploration of Algebra Teaching. Videos from the public release collection of the TIMSS 1999 Video Study were studied in-depth. In the final phase of this course, which was developed by Hiebert and Stigler, the teachers were asked to reflect on how this course could influence their own teaching. Cestari, Santagata and Hood focus their article on these reflections, and they discuss how videos can be used to analyze and enhance teaching.

The connections with history of mathematics and with everyday life situations are emphasized in the current Norwegian curriculum for schools. Mosvold discusses how teachers in Japan, Hong Kong and the Netherlands make real-life connections in their teaching. Japan and Hong Kong had the lowest percentage of real-life connections of the seven countries in the TIMSS 1999 Video Study, but the pupils from these countries were also among the highest achieving. The Netherlands had the by far highest percentage of real-life connections. Mosvold presents several examples on how teachers in these three countries make connections with real life situations in their mathematics teaching.

The fourth article, by Smestad, focuses on how connections with history of mathematics were represented in the TIMSS 1999 Video Study. His study looks at the more direct ways of using history in the mathematics classroom found in the videos. He discusses the amount of references to history as well as how history was implied and the attitudes of teachers.

The first two articles focus on how teachers can become more reflective practitioners, one from a genetic perspective and the other from an analytic perspective. The last two articles focus on the connections with history and with real life, which are considered new in Norwegian curricula. All articles implicate how video studies can be used to make teachers become reflective practitioners in these areas. These articles are preliminary versions all to be published elsewhere independently.

# 1. The TIMSS 1999 Video Study. Helping teachers to become reflective practitioners

## Reflections on a Japanese 8th grade lesson: Equations and inequalities - a genetic approach?

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*Reidar Mosvold, Telemark Educational Research, Notodden*

Abstract

The goal of this talk is to give some reflections on a problem situation, originally put in an ANNEX, from a Japanese 8th grade lesson included in the TIMSS 1999 Video Study. After viewing some excerpts from the lesson we present the following questions for teachers' development from the Lesson Lab course Explorations of Algebra Teaching: - Why did the teacher in this lesson have the students present their strategies in the order that he did? - What are the advantages of having students share their alternative solutions? - Wouldn't it be a more effective approach to algebra just to present the final equations and inequalities statements? - and to forget the lower level attempts made by some students?

As theoretical basis we present some reasoning behind the use of videos for professional development work for teachers, and we review the roots of our genetic viewpoint.

Both authors were supported by Telemark Educational Research, for which we are most grateful.

We acknowledge the kind assistance with this paper from Angel Chui, Ronald Gallimore, Rossella Santagata, and James Stigler of Lesson Lab Inc, Santa Monica, California.

### 1.1 Equations and inequalities: The problem situation

The TIMSS 1999 video study included 28 public release lessons, 4 from each of the 7 participating countries. The Japanese public release lesson no. 3 forms the basis for our reflections here. The teacher presents the following problem situation to his class:

> *It has been one month since Ichiro's mother entered the hospital. He has decided to give a prayer with his small brother at a local temple every morning so that she will be well soon. There are 18 ten yen coins in Ichiro's wallet and just 22 five yen coins in his smaller brother's wallet.*
>
> *They have decided every time to take one coin from each of them and put in the offertory box and continue the prayer up until either wallet becomes empty. One day after they were done with their prayer when they looked into each other's wallet the smaller brother's amount of money was larger than Ichiro's. The problem now is:*
>
> *How many days has it been since they started the praying?*

Now we should watch some part of the video (00.02.14-00.04.57 & 00.18.31-00.31.39).

A set of four CDs including 28 public release lesson videos from 7 countries can be ordered from http://www.lessonlab.com at LessonLab, Inc., 3330 Ocean Park Blvd., Santa Monica, CA 90405, USA

We can see the flow of the lesson in a one page Lesson Graph of the Lesson Lab course. See the attached pdf-file on page 15.

The students' presentations can be viewed as falling in one of three categories:

1) a count down procedure, either using hands-on material or by creating a table:

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *Day number:* | *1* | *2* | *3* | *4* | *...* | *11* | *12* | *13* | *14* | *15* | *16* | *17* | *18* |
| *Ichiro's money:* | *170* | *160* | *150* | *140* | *…* | *70* | *60* | *50* | *40* | *30* | *20* | *10* | *0* |
| *Brother's money:* | *105* | *100* | *95* | *90* | *…* | *55* | *50* | *45* | *40* | *35* | *30* | *25* | *20* |

2) a recipe for calculation (a la in the Aljabr of Al-Khwarizmi 825 ):

*Take the total difference 180-110 which is equal to 70 and divide with the difference 10 – 5 in daily contribution, which is 5. This is 70 : 5 which makes it 14 days before they have equal amounts, so after 15 days the brother has more money.*

3) using a symbolic representation ( a la in the Bijaganita of Bhaskara 1150):

*Let x denote the number of days, and let Ichiro's amount of money be yI and let the brother's amount be yB . Then we have the linear relations*

$$yI = 180 - 10x \quad \text{and} \quad yB = 110 - 5x$$

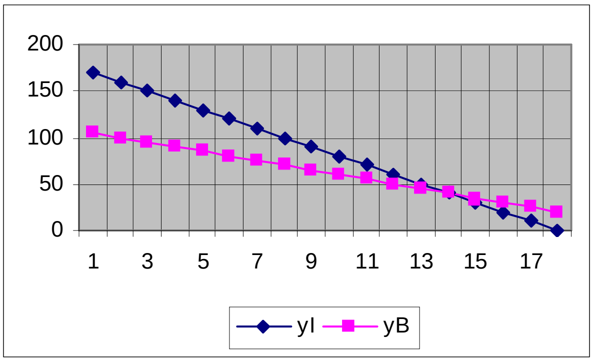
*They have equal amounts when* $180 - 10x = 110 - 5x$ *, i.e. when* $x = 14$
*and the brother has more when* $180 - 10x < 110 - 5x$ *, i.e. when* $x > 14$

To solve the equations/inequalities with the symbolic representation in 3) the students actually apply the methods of 1) or 2).

Later in their lives the students probably also will have access to one more approach:

4) A visual geometric representation (a la in the Geometrie of Descartes 1637):

*In a rectangular coordinate system we may represent the linear relations between x and yI , and between x and yB as points on two lines. From the graphs we may read out the solutions for when the line for Ichiro is below the line for his brother as being when x > 14.*




Lesson Lab's professional development course: Explorations of Algebra Teaching, includes this Japanese lesson in their material. Here the following questions are posed for the participating teachers:

1) Why did the teacher in this lesson have the students present their strategies in the order that he did? - How might the order assist students' learning?

2) What are the advantages of having students share their alternative solutions?

3) Wouldn't it be a more effective approach to algebra just to present the final equations and inequalities statements?

4) - and to forget the lower level attempts made by some students ?

## 1.2 From the genetic viewpoint our reflections were:

*The task of the educator is to make the child's spirit pass again where its forefathers have gone, moving rapidly through certain stages but suppressing none of them. In this regard, the history of science must be our guide. (Henri Poincare 1889)*

In his book on arithmetic and algebra of 1150 the Indian mathematician Bhaskara presented many problems of this type, and he solved them both arithmetically and symbolically using "ya" for our "x" and "ka" for our "y". He also has the following philosophical didactical commentary:

*The clever and intelligent can possibly solve these problems only using arithmetical reasoning, but the grand operation is to introduce symbols for the unknowns and just follow general methods*

The Scottish mathematician Colin Maclaurin in 1748 defines algebra as:

*Algebra is a general method of calculation with certain signs and symbols, which have been conceived and found useful exactly for the purpose of solving certain types of problems.*

The origin, however without any symbolization, lies in the concept Aljabr of the Baghdad scholar Al - Khwarizmi of the 800s , with earlier roots in the writings of the Indian astronomer Brahmagupta from the 600s, or maybe even in ancient China.

## 1.3 Our theoretical background: The genetic viewpoint

*The task of the educator is to make the child's spirit pass again where its forefathers have gone, moving rapidly through certain stages but suppressing none of them. In this regard, the history of science must be our guide. (Henri Poincare 1889)*

Genesis ideas have always played a role in educational theory. In reality we are today talking about a web of genesis principles: historical, psychological, natural, logical, cognitive, social, cultural, contextual, situated … development of mathematical ideas, methods and concepts.

Schubring (1978) traces the theory almost five centuries back in time. The historical genetic method aims to lead pupils from basic to complex knowledge, in much the same manner as mankind has progressed in the history of mathematics. The aim of the psychological genetic method is to let pupils rediscover, or reinvent, mathematics by using their own aptitude.

### *1.3.1 Early versions: Bacon, Comenius and Lindner*

Francis Bacon (1561-1626) introduced the natural method of teaching. Comenius and Ratke based their work on Bacon's studies, and jointly these three are regarded as predecessors of genetic principles. Bacon developed a theory or a method for discovering new knowledge, which is referred to as the inductive method. He called it a natural method, as it had the very nature of things as its origin (op.cit.17ff)

The method goes from the specific to the general. We might argue that this is exactly the manner in which children learn. First they come across specific cases of various phenomena, later they appreciate the existence of general concepts, which the specific cases form part of. The very idea that there is a connection between the way children acquire knowledge and the way knowledge has come about, is fundamental. Bacon felt that the teacher's task should be to lead his pupils on to the roads of science, in the same way as he himself had arrived there (Bacon, 1994, p. 125).

When Bacon's method is to be applied in teaching, everyday problems, the so-called specific cases, should be the starting point, only later should mathematics be made abstract. Symbolic expressions should not be the start; the symbolization should be worked out along the way. The cognitive subject, as Bacon called it, had to be in activity in relation to the cognitive object. Hence the pupils had to be active in order to acquire knowledge; which is a thought well known in the view of learning as found in the theories of constructivism and reinvention, cf. van Amerom 2002.

Johan Komensky (1592-1670) is commonly known as Comenius. He was a Czech philosopher, educationist and poet, and is widely acknowledged as one of the founders of general educational science through his major work Didactica Magna completed in 1657. The basis of his educational science was that all humans are co-creative beings. Von Raumer establishes in his three-volume work Geschichte der Pädagogik that Comenius considered Bacon's studies to be the framework for his own work, a view also shared by Schubring (von Raumer vol. II, 1843, p. 63 and Schubring, 1978, p. 19).

Friedrich Wilhelm Lindner (1779-1851) only published two shorter works on his method: de methodo historico-genetico in utroque genere institutionis abhibenda cum altiori tum inferiori (published 1808 in Leipzig) and de finibus et praesidiis artis paedagogicae secundum principia doctrinae christianae (published 1826). He was led to his methods by Bacon's Organon. Lindner strongly criticized the schools' time tables as too tied to a cycle of class-break-class. According to him, the genetic method required stamina, and too frequent changes of subject would only breed distraction (op.cit. p. 59).

### *1.3.2 Benchara Branford*

In 1908 when Branford published his A study of Mathematical Education, he represented something new in the English speaking culture. His book points out the relation between the development of mathematical skills in the individual and the development of mathematics historically.

Branford had behind him years of experience as a teacher, and he had his own understanding of the teacher's role. It should be to structure the teaching according to the lines suggested by the development of knowledge in mankind. Hence, a teacher should be aware of the history (Branford, 1924, p. 244).

Branford provides numerous examples from his lessons. We should start with the ideas that pupils take with them from their everyday lives into schools. We should treat our pupils as brave young pioneers, and their assertions should be met with respect and the mild criticism that is due discoverers of such concepts (op.cit. p. 11).

According to Branford children are born with several mental ideas. These ideas can be hard to discern at first. Children have innate ideas about several mathematical concepts, but they are not, and will never be perfect as long as their meanings are contextual. Towards the end of his study Branford discusses the relationship between teaching principles and practice (op.cit. p. 345):

*All principles, I take it, represents but partial aspects of reality. Nothing, perhaps, is more fatal to progress and to success in teaching than the attitude of the doctrinary belief in the universal validity of any abstract principle or system of principles, and consequent insistent adherence to it in practice. Principles thus viewed and applied are life-killing mechanisms.*

### *1.3.3 Felix Klein and the Genetic Principle*

Felix Klein (1849-1925) developed mathematics in a sequence of common sense, with constant references to history. Towards the end of his career Klein was mostly occupied with educational issues. In his book Elementary Mathematics – from an Advanced Standpoint, originally published in German in the early 1900s, Klein starts by presenting how to teach pupils numbers, the very basis of all arithmetic. Speaking on this, he says (Klein, 1945, p. 6):

*The manner of instruction as it is carried on in this field can perhaps best be described by the words intuitive and genetic, i.e. the entire structure is gradually erected on the basis of familiar, concrete things, in marked contrast to the logical and systematic method.*

It is a common argument that mathematics can and should be taught deductively; by starting with certain facts and by manner of logic proceeding from there. On this, Klein (1945, p. 15) comments:

*In fact, mathematics has grown like a tree, which does not start at its tiniest rootlets and grow merely upward, but rather sends its roots deeper and deeper at the same time and rate that its branches and leaves are spreading upwards … mathematics began its development from a certain point corresponding to normal human understanding, and has progressed, from that point, according to the demands of science itself and of the then prevailing interests, in the one direction toward new knowledge, and in the other through the study of fundamental principles.*

The understanding of foundational principles is constantly changing, according to Klein, and there is no end, and hence no initial starting point that could provide an absolute fundament.

*Instruction should guide it slowly to higher things, and finally to abstract formulations; and in doing this it should follow the same road along which the human race has striven from its naïve original state to higher forms of knowledge (Klein, 1945, p. 268).*

Klein states that it is necessary often to repeat this principle because it is very common to start the teaching with the most general concepts. Furthermore he says:

*An essential obstacle to the spreading of such a natural and truly scientific method of instruction is the lack of historical knowledge, which so often makes itself felt.*

Towards the end of his book Klein sums up his view, op. cit. p. 236:

*If you lack orientation, if you are not well informed concerning the intuitive elements of mathematics as well as the vital relations with neighbouring fields, if, above all, you do not know the historical development, your footing will be very insecure.*

By his organization of the student solutions the Japanese teacher is following the historical and the psychological genesis and development, and several ideas quoted above. The lesson follows the path described by Toeplitz as the indirect genetic method, cf. Mosvold (2003) p. 92. To be able to follow such an approach a teacher must know the history of mathematical ideas, as well as being able to reflect on the cognitive development of these youngsters.

*　　*　　*　　*　　*

From the recent book about the “Math Wars in the US” entitled *California Dreaming* we sense that the recent mathematics education debate has involved:
- the “skills” people
- the “concepts” people
- the “real life applications” people

with quite different views on goals and issues.

Learning mathematics is a process of practice and memorization. The Teaching Gap of Stigler & Hiebert (1999) gives a recent description of this script of teaching that emphasizes terms and procedures – the skills dimension of mathematics – and that often excludes exploration of mathematical ideas related to those skills. Stigler and Hiebert’s analysis documents clearly how this script is enacted in US mathematics classrooms, when compared to the rich discussions in other countries (Wilson 2003, p. 149 & p.7).

### 1.4 The scripts

Based on data from earlier video studies Santagata & Stigler (2000) argue that mathematics teaching is a cultural activity, varying more across cultures than within. Teaching practices are determined by deeply held beliefs that are not easily targeted by teacher education programs. They could then identify the following practices:

*The Japanese script:*
1) reviewing previous lesson through lecture, discussion or student presentation
2) presenting the problem for the day
3) individual student generation of solutions to be discussed with classmates
4) students presenting/discussing solution methods on blackboard with summary
5) highlighting/summarizing main points by teacher’s lecture

*The US script:*
1) reviewing previous material (homework or warm-up activities)
2) demonstration of problem solution for the day with student guidance
3) individual or group seatwork practicing
4) assigning homework & correcting seatwork

*The Italian script:*
1) reviewing previous material (student on blackboard - homework/lessons)
2) presenting the topic of the day – concepts, problems, procedures
3) students applications/practice on the blackboard
4) assigning homework

Italian students, like Japanese students, are asked to verbalize what they are writing on the blackboard and are subjected to teachers’ questions and comments. In Japan, students are asked to share their solutions arrived at during seatwork by writing and explaining on the blackboard, whereas in Italy, students are asked to do their work and explanations directly on the board with no previous independent seatwork.

## 1.5 The new video study. Goals and results in brief

(from COMET 2003 & report of NCES 2003)
Why study teaching in other countries? - and why do it using videos?
Because it:

- reveals one's own practices more clearly
- discovers new alternatives
- stimulates discussions about choices of teaching strategies
- deepens educators' understanding of teaching
- enables the study of complex processes
- enables coding from multiple perspectives
- stores data in a form that allows repeated analysis
- facilitates communication of results

The goals of the 1999 study were to describe teaching of 8th grade mathematics, to compare practices across countries, and to build a library of public-release videos that can be used to promote cross-national research and discussions on teaching of mathematics. Analyzing 638 lessons from 7 countries, some brief comparisons that are made are:

- In the Netherlands students were more likely to encounter problems including real-life connections.
- Lessons in Japan included more problems making connections to concepts and facts.
- Lessons in Hong Kong included a larger percentage of problems targeted at using formulas/procedures.
- Lessons in Australia and the US were least likely to emphasize mathematical connections or relationships
- Review of previously taught lessons played a larger role in the US and in Czechia.
- Calculators were used in more lessons in the Netherlands.
- Computers were used in only a few lessons across all countries.

The estimated median time spent in mathematical work pr. year varies from 116 hrs. in Japan, 107 hrs. in the US, to 84 hrs. in the Netherlands. Japanese lessons differed from all the other countries on 17 (15%) of the analyses done for the NCES 2003 report, while the Netherlands differed on 10 (9%) of the analysis. Japanese teachers frequently posed problems that were new for their students and then asked them to develop solutions on their own. After allowing time to work on the problem, they engaged the students in presenting and discussing alternative solution methods, and then summarized the mathematically most important points of the lesson.

About 2/3 of the lesson time were devoted to independent problems, an average of three problems pr. lesson, and on the average 15 minutes on each problem, see NCES 2003, fig.3.4, 3.5 & 3.6. On introduction of new content see fig.3.8.

The definition of proof included rather informal demonstrations giving some form of mathematical reasoning. This aspect was evident to a substantial degree only in Japan. Here 26% of the problems included proofs and 39% of the lessons contained at least one proof. Japanese lessons contained more problems that were mathematically related, more that were thematically related, and fewer repetitions, and the problems also had a higher procedural complexity than in the other countries, see fig.4.1 & 4.6 in the NCES report.

Based on these video studies some people prematurely could conclude:

| | | |
|---|---|---|
| *high mathematics achievement is possible* | ***only if*** | *we are adopting teaching practices like Japan* |

while a more reasonable conclusion could be if and not necessarily only if. Even this is debatable: To what extent is it really possible to adopt teaching practices from another culture into our classrooms?

## 1.6 Closing the teaching gap

(from Hiebert, Gallimore & Stigler 2002 and Gallimore & Stigler 2003)
Standards set the course, and assessments provide the benchmarks, but it is the teaching that must be improved to push us along a path of success. Many believe improved teaching and learning will follow from structural reforms. Reforms, however, have limited effect unless intended changes are implemented in the classrooms, and that implementation depends on widespread and robust professional development.

Anthropology teaches us that classroom changes lag behind only in the margins of cultural practices. One of the major barriers is the narrow range of instructional practices teachers have observed as students prior to entering the profession. Classroom change will require a rich, broad, and validated professional knowledge base that includes alternative practices, as well as an environment that both encourages and supports continual improvement of teaching and learning practices.

John Dewey noted that one of the saddest things about education is that

> *…the successes of excellent teachers tend to be born and die with them*

His laboratory school planted the seeds of a school-based, teacher-engaged system of building professional knowledge. Dewey was soon succeeded by Judd and Thorndike, whose views rather shaped education and educational research. The tendency to look for quick solutions has made education a graveyard of good ideas condemned by the pressure for fast results. Educational research has too little influence on improving classroom teaching and learning. Teachers rarely draw from a shared knowledge base to improve their practice. They do not routinely locate cases in research archives to help them interpret students' conceptions and learning trajectories.

Learning can be facilitated by seeing ideas and concepts in a variety of contexts and styles. Lesson Lab proposes digital libraries with lesson videos coded in ways which makes it possible to retrieve a variety of themes and approaches, created with the intent of public examination, with the goal of making it shareable among teachers, open for discussion, verification, and refutation or modification. Other professions have created ways to share knowledge through case literature. Teaching, unfortunately, has yet to develop a professional knowledge system.

Knowledge for teaching is most useful when it is represented through theories with examples. Theories ensure that the knowledge rises above ad-hoc technique. Examples keep the theories grounded in practice. Three major barriers that impede bringing quality professional development to scale are:

1) Lack of a knowledge base to support teacher learning
2) Lack of tradition among teachers for analyzing and learning from practice
3) Lack of time for collaborative work
4) The tendency to look for quick solutions

There are programs that use video studies to give teachers the opportunity to learn from *Best Practices* by studying examples of effective teaching. Lesson Lab's approach, however, is to include a variety of examples, and also to reflect upon problematic classroom situations as well as examples of more effective practices. Their professional development model *Learning from Practice* believes that the improvement of analysis, planning and reflection hold the greatest potential for improving teaching practices. Cultural routines that underlie teaching can here more easily be brought to awareness, evaluated, and changed through international comparisons. Analysis, planning and reflection should not be based on ad-hoc skills, but rather on disciplined application of educational theories.

The Japanese lesson studies turn practitioner knowledge into professional knowledge. Groups of teachers meet regularly to collaboratively plan, implement, evaluate, and revise lessons. Changes are based on specific problems evidenced by students as the lesson progresses, and often researchers are solicited to serve as consultants

## 1.7 Lesson studies

(mostly from Fernandez & al 2003)
Subject knowledge in the case of mathematics is rarely a problem in Japan, even for those teachers qualifying to teach in elementary school, because of the importance placed on mathematics in schooling. Thus teacher education can concentrate more on teaching methods and professional development issues. In the Japanese system, a whole class approach is common and involves a high level of pupil participation and interaction. A high level of teacher professionalism is expected. Discussion of learning points for both teachers and pupils is encouraged; learning difficulties are identified and discussed. Considerable attention is given to the construction of lesson plans (Jaworski & Gellert, 2003, p.838).

The Japanese "Lesson study" has a history in elementary and middle schools with origins in the early 1900s. Strong claims have been made about the potential of lesson studies, as a form of professional development in which teachers collaboratively plan and examine actual lessons. To benefit, however, from such a study teachers need to be able to apply *critical lenses* to their examination of lessons.

Fernandez et al (2003) reviews a collaborative effort to introduce lesson studies in the US. The Japanese teachers brought to this collaboration a number of critical perspectives, and a constant concern with how to sequence and connect children's learning experiences. In fact they conveyed the importance of thinking about students' entire learning experience even before they began planning the lessons. They were preparing themselves through *the curriculum developers lens* with an eye towards skillfully orchestrating children's learning both across and within the lessons.

*If there is knowledge that is very useful for solving these problems, perhaps you need to spend more time early in this lesson talking about such knowledge*

Thus, this teacher conveyed that it is important to examine what prior experiences make students choose a strategy, and what this means for the design of a lesson. This teacher's rationale for his proposed order was clearly focused on considering how to develop a strong understanding of the conceptual content targeted in this lesson.

*I always look to see the solution method the majority of students use. I believe this method is what they have learned from their mathematics education up to that point*

This Japanese teacher was trying to use the lesson to build a principle about teaching that could be generally applied to his classroom, and which he felt other teachers should consider. The US teachers rarely referred to any broader principles or theories.

Another perspective the Japanese teachers conveyed was to examine all aspects of a lesson through the eyes of their students. They emphasized the importance of teachers adopting *the student lens* by attempting to anticipate students' behavior and determine how to use this knowledge to build students' understanding. To anticipate solutions to mathematical problems and explain how these solutions would be used to deepen students' understanding, became an important part of the lesson study.

Implementing lesson studies in other countries, we cannot overlook the substantial challenges that must be overcome to make this practice purposeful and powerful. There is currently a call for teachers to be more reflective in their practices. These reflections will require development of critical lenses.

## 1.8 Reflective practitioners

(mostly from Hatton & Smith 1995)
Lerman (1994) defines reflection as " developing the skills of sharpening attention to what is going on in the classroom, noticing and recording significant events and 'working' on them in order to learn as much as possible about children's learning and the role of the teacher"

The terms *reflection/critical reflection* have increasingly appeared in descriptions of approaches to teacher education in recent decades. Schon (1983) talks about *reflection-on-action* and *reflection-in-action*. Most kinds of reflection involve looking back upon action with a view to evaluate the effectiveness after an attempt at implementation.

Schon's *reflection-in-action* involves simultaneous reflecting and doing, implying that the professional has reached a stage of competence where s/he is able to think consciously about what is taking place and modify actions instantaneously. The *reflection-on-action and reflection-in-action* involve a professional practice base upon knowledge. Such tacit knowledge is derived from the construction and reconstruction of experiences.

Four broad strategies that are claimed to promote teachers' reflection are:

- Ethnographic studies of students, teachers, classrooms, and schools
- Microteaching and other supervised practicum experiences
- Structured curriculum tasks
- Action research

Barriers that hinder the achievement of reflective practices are.

1) Reflection is not generally associated with working as a teacher. Teaching is often seen to be about action, while reflecting is perceived as a more academic pursuit.
2) To foster effective reflection, time and opportunity is needed for development. The identification of a suitable knowledge base from an historical point of view of some major perspectives which have guided approaches to teaching and learning of mathematics is missing.
3) Feelings of vulnerability which follows from exposing one's perceptions and beliefs to others support a case for collaborative approaches within which teachers can work together as critical friends.
4) A critically reflective approach demands an ideology of teacher education not only involving models of best practices, but also recognizing conflicts between institutional ideals.

In spite of all the barriers listed as 1) – 4) and as a) – d) above, and in spite of all attention and care that has to be given to cultural barriers mentioned in connection with the efforts to introduce lesson studies in the US, our answer to the main question above is: This approach is worth trying out in our Scandinavian setting by starting to develop a video base of mathematics classroom practices, and collaborative groups of teachers locally doing lesson study type professional development work partly based on this video base with university or college groups of researchers as consultants.

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# 2. Teachers Watching Videos of Mathematics Lessons and Reflecting on their own Practice: The Analytical Perspective1

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## 2.1 Introduction and Aim of the Study

Recent developments of digital technologies have led to increased use of videos for teacher education. Videos are considered by many teachers and researchers as an effective tool for linking theoretical issues to classroom practices. Despite this growing interest, there is little empirical research on the effects of the use of videos on the improvement of teacher knowledge and practices, and on the improvement of students' learning.

In this paper, we will describe a pilot study in which U.S. teachers completed an evaluative reflection task after having participated in a video- and Internet-based algebra course. In this course teachers watched and analyzed a series of lessons from the countries videotaped as part of the Third International Mathematics and Science Study (TIMSS 1999; Hiebert et al., 2003). The aim is to describe the kind of reasoning about teaching practices that the exposure to video material, as well as the engagement in analysis tasks, may elicit. In the following sections we will describe the historical development of the use of videos for teacher learning and the TIMSS studies and Algebra Course. We will then proceed to the analysis of teachers' responses to a final reflection task. Ways teacher integrate the discussion of mathematical content and teaching strategies will be the focus of our analysis. In the concluding section we will summarize our findings and present three different perspectives teachers may take when asked to reflect on practice.

## 2.2 Teachers Learning from Video: An Historical Perspective

The use of videos for the professional development of teachers has its roots in the 1960's. Particularly influential at that time was the work of Bandura and Walters in the psychological field of social learning. In their book "Social Learning and Personality Development" (1963), the authors introduced two fundamental mechanisms in social learning: modeling and imitation. This theoretical model was incorporated in teacher training programs at Stanford University in what became a very popular approach, labeled "microteaching" (Allen, 1966; Allen & Ryan, 1969). Beginning teachers were required as part of their teacher education program to take a three-step course. During this course they observed (in most cases, on video) a model teaching episode in which a specific skill was demonstrated. They then tried out the new technique and received feedback on their performance.

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Numerous studies were carried out through the mid 1970's on various aspects of microteaching and on its effects on teacher acquisition of new techniques (e.g., Acheson & Zigler, 1971; Allen & Clark, 1967; Limbacher, 1971; Ward, 1970). Overall, these studies supported the use of microteaching in teacher professional development. Microteaching was found to facilitate the acquisition of specific instructional techniques and required less time than more traditional training formats. In addition, ample evidence was found for the positive effects of microteaching on students' attitudes and learning (for a review of these studies see Turney et al., 1973).

Although the popularity of the microteaching approach decreased as teacher development started to focus more on subject matter content and complex behaviors (mirroring the transition from "behaviorism" to "cognitivism" in psychological research), the basic idea of microteaching—learning by observing effective practices—has remained popular through the 1980's and 1990's and it is still used today. For example, drawing on Carver & Scheier's (1981) control-theory approach to human behavior, Gallimore, Dalton, & Tharp (1986) found that when teachers were presented with video models of new practices, given time to apply them into their classrooms, and then given feedback, they appropriated the new standards of behavior and matched them to their own behavior through self-regulatory activity driven by the desire to bring their behavior into conformity. This approach was found to be most effective when the discrepancy between the new standard and the individuals' current level of performance was moderate.

More recently, educational researchers have proposed an alternative approach to the use of video for teacher professional development. This approach is centered on the idea that teaching is cyclical (Hiebert et al, 2002; Ball & Cohen, 1999). Teachers plan, teach, and reflect on practice in a continuing cycle. All teachers engage in these processes to some degree.

According to this approach, the reflection phase holds great potential for teacher learning because it is more deliberate and leisurely than is implementation, and it allows for cultural routines that underlie teaching to be more easily brought to awareness, evaluated, and changed. Furthermore, during the reflection phase teachers can isolate problems and evaluate alternatives. This process directly influences their planning and, consequently, their teaching (Schon, 1983).

Developing analysis of practice can allow teachers to more skillfully "see" the subject matter in lessons, discriminate ways that learners comprehend subject matter, identify problematic features, assess student responses, detect, diagnose, and develop instructional responses to student errors, etc (Berthoff, 1987; Burnaford, Fischer, & Hobson, 1996; Cochran-Smith & Lytle, 1993; 1999). If teachers learn robust ways of analyzing practice they will become more knowledgeable in how to integrate content and teaching strategies, they will thus increase their pedagogical content knowledge (Shulman, 1986). The project described here is based on this second approach. In the following section we summarize the TIMSS Video Studies results and we introduce the TIMSS Algebra Course.

## 2.3 Development of TIMSS: Studies and Course

The Third International Mathematics and Science Study (TIMSS), a comprehensive international study of schools from 41 nations in 30 languages at three different levels (fourth, eighth, and twelfth) to compare achievement in mathematics and science, was conducted in 1994-95. The study included not only testing of the students at each level, but also involved analyses of students, teachers, schools, curricula, instruction, and policy in order to understand the educational context in which teaching and learning took place. TIMSS also included an extensive videotape survey of eighth-grade mathematics lessons in the United States, Japan and Germany: the TIMSS 1995 Video Study (Stigler et al. 1999). This was the first attempt to collect videotaped observations of classroom instruction from nationally representative samples of schools and classes.

As Japan was the only country in the TIMSS 1995 Video Study with a relatively high TIMSS eighth-grade mathematics score, a possible unintended and unwarranted inference was that it would be necessary to use Japanese style teaching methods to produce high levels of mathematics achievement (Stigler el al, 2003). When TIMSS was conducted again in 1998-99, an expanded video study, the TIMSS 1999 Video Study2, was designed to investigate this issue. Seven countries - Australia, Czech Republic, Hong Kong SAR (Special Administrative Region), Japan, Netherlands, Switzerland, and the United States - participated in the study that included both mathematics and science lessons. The TIMSS 1999 Video Study report (Hiebert et al., 2003) was released in 2003 along with a collection of 28 public release lessons, four from each of the seven countries (see www.lessonlab.com).

The *TIMSS Video Studies: Exploration of Algebra Teaching* course is based around the study's findings and resources. While the findings are extensive and complex, some of the conclusions that can be drawn are quite simple and significant: no single method of teaching mathematics is required for students to achieve well; and there is much to be learned by examining a variety of teaching methods and searching for ways to engage students in serious mathematical work. Using these as its foundation, the course aims to enable participants to identify teaching strategies that sustain or inhibit students' engagement in serious mathematical work; reflect on their own practice; and, learn about the TIMSS 1999 Video Study. The Course was developed by Hiebert and Stigler, directors of the TIMSS Video Studies, and colleagues. Eight lessons from the public release collection are included.

The course has five components:

- ***Introduction*** covers course goals, software navigation and an overview of TIMSS 1995 and 1999.
- ***Initial Explorations*** includes *Getting Your Feet Wet* activities, which explore, first as an individual reflection followed by a discussion in a public virtual forum, the opening segments of lessons from Australia, the Czech Republic, Hong Kong and the Netherlands.

[2] TIMSS 1999 Video Study was funded by the National Center for Education Statistics (NCES), the U.S. Department of Education's Fund for the Improvement of Education, and the National Science Foundation (NSF). It was conducted under the auspices of the International Association for the Evaluation of Educational Achievement (IEA), based in Amsterdam. For more information: http://www.lessonlab.com

- ***TIMSS 1999 Video Study Up Close*** component gives details of the methodology of the video study along with details of some of the major findings that are pertinent to this course. As the concept of 'Making Connections' problems (mathematically challenging problems) is a major focus of the course, two examples are included to illustrate how often, these are introduced but then not sustained within mathematics classrooms.
- ***Case 1: Japan, Case 2: Hong Kong SAR, and Case 3: Switzerland*** are in-depth studies that provide the opportunity for participants to explore, reflect, and analyze mathematics problems and associated lessons. Each follows a similar format and provides the opportunity for individual work and public discussions.
- ***Reflections*** tasks have participants reflect on what they have learnt and how it has, or could, influence their own teaching.

The course is designed to guide and encourage participants to explore and think deeply before accessing expert comment, providing many opportunities for them to construct their own knowledge. In the first task of the last course component, for example, participants reflect on how a missed teaching opportunity could be changed to maintain the original intent of a challenging problem. In the second task, they take what they see in the video cases and apply it to their own teaching.

The Course can be delivered totally online, or in a combination of face-to-face and online sessions. The online version can be facilitated by a leader who has an editable group homepage and moderates the discussion forum using Email facilities. The online version can also be taken without any facilitation.

Four pilots were conducted during the design phase of the course. These covered the range of four delivery options – the first was totally face-to-face with optional online exploration between sessions; the second had face-to-face sessions to start and finish, with online work between; the third was totally online and non-facilitated; and the fourth was totally online with facilitation.

Evaluations of each pilot resulted in changes to the course. For example, during the first pilot, at the meeting following the Japan Case Study, several participants talked of lessons they had tried using the strategies employed by the Japanese teacher – one teacher had even videotaped his lesson. The teachers had not been prompted to do this but had been stimulated by what they had seen. They were surprised at the response from their students. This resulted in the inclusion of a new task in the Reflections section with participants being asked to reflect on their own teaching and to share experiences. In the following section, we analyze responses to this Reflection task for three of the teachers who participated in pilot 4.

## 2.4 Teachers Reflecting on Their Own Teaching

Fourteen teachers from different schools in the U.S. completed pilot 4. After an extensive reading of all contributions, we have selected 3 cases, whose reflections were described in a particular detailed way to illustrate the kind of elaboration the course materials had elicited in relation to the teachers' own practices. We focused on two aspects: (1) the ideas they select from the videos, and (2) the ways they construct *pedagogical content knowledge* moving from what they see in the videotaped lessons to reflections on their own practices. The analysis we provide is not exhaustive and does not represent all possible interpretations of these reflections; its main goal is to capture the process of meaning making teachers engage in when asked to connect what they have learned from the course to their practices. In this way, we can characterize this work as a case study from where data cannot be generalized. According to Shulman (1986) the act of teaching requires the mastering of content knowledge, pedagogical knowledge and the integration of these two kinds of knowledge.

The *Reflecting on your teaching* task includes three questions:

***1. How can I change my lessons to increase students' mathematical thinking?***

After reflecting on what you have learned from exploring the lessons in this Course, what changes could you try in your classroom to increase student Mathematical thinking?

Include the strategies you would use to maintain the level of complexity of problems you pose.

***2. Applying the changes.***

How, exactly, will you go about making the changes you describe above?

Think of a lesson you have coming up – how would you apply these changes?

***3. Implementing the changes.***

If you have the opportunity, try the changes you describe in question 2 in your class room. Describe what happened. Was it as you expected?

This Reflection task provides an opportunity to analyze teachers' attempts to make this integration. We now proceed at presenting the reflections of the three teachers we have selected.

### *2.4.1 Karin*

Karin graduated from college with a minor in Mathematics. Her professional experience includes 28 years of teaching at the middle-school level, and 2 years in high school. She now teaches 9th- and 10th-grade algebra and geometry. Following is her response to the first question:

*1. I think I need to create more problems to pose that will lend themselves to a variety of entries. Using more manipulatives for big ideas. Allow more time for student presentations. Take more time-don't rush through...I guess I am looking at more time for deeper understanding utilizing interesting problems. Give more thoughtful attention to how ideas are developed rather than throwing them together...Recognize the craft and use it!*

The reflective mode of this answer is revealed by the way in which Karin initially reacts to the question: she begins with "I think…" The series of procedures enumerated shows attention to and understanding of the nature of mathematical knowledge construction that students need to make: the increasing diversity in problem formulations in order to open more possibilities for solutions; the passage from the concrete to the abstract; the importance of sufficient time for reflection; and, the emphasis on the development of the thinking process. Karin's concern with various aspects of the learning process shows her attunement to the students' needs.

Her answer to the second question is:

*2. I am looking at inequalities and thinking "why not try to duplicate with the problem given?" See what happens? Teach the lesson and see where it leads... I know I don't have the chalkboard with the magnets - what can I do? I've been thinking of this a long while, wishing. Now is the time to move forward and do something... a felt board or magnet board... Perhaps use overhead money, nickels and dimes... Hmm - I've moved away from some things, and need to remind myself of the options.*

This second question is approached in a pragmatic way: Karin considers the use of concrete materials. However, the reflective mode emerges again, linked this time, to the way a real-life problem posed in the Japanese lesson[3] is formulated and treated. Conjectures, as well as clear propositions for improvement in her classroom, are formulated. Following is Karin's response to the third question:

*3. Yes, I tried the changes. I know I am trying some of the things I saw on the video, to deepen understanding of what's going on. I tried using the inequality lesson and was amazed at how much the kids became engaged - the story really caught their attention from the start. When I allowed them to work the problem, they wanted verification if they were right - I wouldn't give it - just encouraged them to explain why they thought they had an answer. I actually saw the first two methods and the equality one. The first was done by a student who is extremely low performing. When I finally got her up, the other kids were surprised at her response and gave her their attention. When we shifted to practice - she actually wanted to try... It was interesting because this happened all day with various classes. The unexpected was the involvement of low end and the opportunity to allow them to shine in front of their peers.*

These comments are directly linked to ways a problem on inequalities can be worked on in the classroom. In this particular *story - problem* introduced in the Japanese lesson, there is a strong emotional component: the illness of the mother and the religious practices children use in order to handle their own anxieties. Karin underlines how "the story really caught their [her students'] attention from the start". She also shares her discourse strategies, and her attempt to move from a widely used strategy in classroom discourse, in which teachers respond to students' answer with an evaluation (the initiation-response-feedback pattern; Mehan, 1979, Cazden, 1988, Cestari, 1997)— to a more argumentative one, in which students' are asked to explain their answers (Lampert, 1990). Here is a moment in which cognition touches instruction, i.e., the importance to open possibilities for multi-representations and to allow multiple solution methods to the same problem.

Finally, Karin concludes her response by reporting that a low performing student in her class was able to solve the problem. Here history touches instruction: according to Bekken & Mosvold (2003, in this volume), in the Japanese lesson Karin refers to, the students deploy a variety of solution methods, which reflect the order in which these methods emerged in the discipline of mathematics. The Japanese teacher, in the videotaped lesson, is able to recognize the increasing level of sophistication of the students' solutions, and respects their historical order when calling students to present them at the blackboard. What catches our attention here is the fact that accepting different types of solution methods allows even low performing students to find their ways to solve the problem. It seems that taking into account the historical progression of the discipline in the lessons may facilitate the inclusion of low performing students. The last comment by the tea-

[3] Case 1: Japan

cher is eloquent in this sense: “The unexpected was the involvement of low end and the opportunity to allow them to shine in front of their peers.”

Karin’s analysis of the videotaped lessons has clearly stimulated a process of reflection on her teaching practices. Although she has many years of teaching experience, the videotaped lessons, and the accompanying course materials (expert commentary and tasks) provided examples of valuable practice to be experimented in her classroom. Her response reveals a particular attention to the students’ learning process. In her last comment, Karin discusses the inclusion of all students, facilitated by working with different solutions to the same problem. Being sensitive to individual differences in the classroom is a difficult task for most teachers. The introduction of historical perspectives may be seen as a way to help teachers integrate knowledge of specific mathematics teaching strategies and of students’ understanding of the subject matter.

### *2.4.2 Patricia*

Our second participant, Patricia majored in Mathematics in college. She then received a graduate degree in Econometrics. She has been teaching 7 years in middle school and 3 years in high school and has written mathematics’ textbooks. Following is her answer to the first question:

*1. I think that the main thing is to make sure students are actively engaged in the learning process. If they are just sitting and listening to the teacher impart information they are not learning, they are listening. Lessons need to be designed to help students discover the concepts and ideas that the teacher is trying to impart. By setting up a progression of work starting from familiar ideas and leading students to new concepts the teacher helps them learn the new ideas.*

In this comment two main issues are introduced: The necessity of engaging students in active participation during classroom activities, and the planning of lessons to attain this specific aim. Patricia reflects on the effects of her actions on the students’ learning process. She also discusses the idea of going from the simple to the complex, from the familiar to the construction of new concepts. These are the main ideas that Patricia has selected from the course materials. Her response to the second question reads as follows:

*2. Mainly, I think that you can take almost any lesson and make it more student centered by reversing the order you intend to do things in. Traditionally, we show them the new idea, do an example and then have them do it. Start instead with the students doing a problem. Make sure it is a problem they can solve or at least attempt with the skills they already have. Use this problem to lead them into the new ideas you are trying to present and finally formalize at the end.*

Here Patricia describes in concrete terms how students may be actively engaged in learning activities. She contrasts a lesson in which the teacher shows step by step to the students how a problem must be solved, and then has students practice on similar problems, to a lesson in which a problem is posed at the beginning for students to solve, and the teacher formalizes the procedure at the end. This comment mirrors the differences described by Stigler & Hiebert (1999) between the U.S. and the Japanese scripts for mathematics teaching. In the following response Patricia reflects on the implementation of the changes in her classroom:

*3. Yes, I have tried the changes. I am fortunate to have been involved in many studies and courses involving math reform in the US. I have tried many of these strategies previously and find them to be very successful. My methods of teaching have changed dramatically as a result.*

Patricia recognizes the suggestions given in the TIMSS course as reflecting US mathematics teaching reform. Patricia is clearly actively engaged in applying reformed teaching in her classroom. Participating in the course helped her to reinforce beliefs she already holds and has provided her with additional illustrations of practices that she finds effective.

### *2.4.3 Liv*

The third participant, Liv, has a college degree in mathematics and a graduate degree is in Education. She has taught for14 years in high school, grades 8 to12. Following is her response to the first question:

*1. In general, I believe that I need to work on the organization and focus of my lessons. Too often I get side - tracked or lack a focus on one main concept. I think I try to cram too much into one class period and perhaps "go with the flow" a little too much.*

*I'm frustrated in my school by the attendance problems and the need to always be helping kids catch up on missed work and by the seemingly constant interruptions that I deal with every class period. The videos we saw seem to reflect a much calmer more focused session than my typical class seems to be. So... Strategies... hmmmm*

*1. State the focus topic of each class.*
*2. Summarize and reflect on what was covered at the end of class.*
*3. Stay on track and not allow myself to be sidetracked from the topic at hand.*

Liv’s main concern is with keeping the lesson focused on one specific topic. She recognizes this as something she needs to work on and she describes her plan in detail: she will state the topic of the lesson at the beginning, she will summarize what was covered at the end, and she will try to focus on the chosen. It is interesting to observe how Liv is able to elaborate on what she has learned from the course to make it meaningful for her own practice.

In her answer to question 2, Liv describes how she implemented her plan:

*2. Tomorrow's lesson is on plotting data and making predictions from the graph. We will be letting a birthday candle burn down a bit while it is on a scale and record the weight of the candle as it decreases over time. I will try to stress that we are making the graph of this relationship specifically so that we can use it to find the answer to the question "how long will it take for the candle to go out?" or "how long will a candle like this burn?" In the past, I think I got caught up in the procedures and lost sight of the real purpose, namely, using the graph to draw a line of best fit and using that line to make a prediction. I will try to stress that the purpose of the lesson is to see how such a mathematical model can be useful and resist the urge to develop the equation of the line of best fit, since I think the students I have are not quite ready for that yet. I use a curriculum that is well designed and need to trust a bit more in its scope and sequence. Additionally, watching the videos, especially the Swiss lesson, makes me think that I moved through the introduction to variables too quickly. I will go back and revisit that concept at a later date and use the algebra tiles that we already use more like the teacher did in the Swiss lesson. At least that might be a good way to help students with their confusion over the difference between 2x and x squared. I will revisit the lessons that I now think I didn't focus on sufficiently or clearly enough.*

This answer displays two concerns. Liv reflects on the fact that it is important for students to understand the purpose of the proposed activity before they move to the mastering of the procedures. In the background lies an analysis of the Swiss lesson: this analysis stimulates a comparison with her lessons and the formulation of more efficient teaching strategies for the introduction of the line of best fit's equation. Liv's second concern is with the teaching of the concept of variable. Here again she reflects on the necessity to give students more time to fully understand mathematical concepts.

*3. Yes, I tried the changes. I tried to focus in on the main concept of the lesson, which was to use a graph to make a prediction. I also tried to make the lesson more organized and more cohesive. As I said in my previous response, I think if there is one thing that I would like to emulate in the lessons we watched it would be the organization and clear focus of the lessons. I think that it helped to think about these things today as I tried to present gathering and graphing data to make a linear model.*

*I made a couple of management changes to this lesson as well in order to increase my focus. In the past I have had several different groups gather their own data. Just in terms of logistics, this makes things more fragmented. Instead, this time I had the entire class gather one set of data. I also did not allow the kids to struggle as much with the scaling concepts and conversion steps that are required to plot the data effectively. My thought was that I needed to hone in on the one main idea and not let the other concepts or struggles (even if they were mathematical concepts that we are covering) intrude on the main idea. So I helped with the little steps more, and tried to keep guiding the students to the main end goal.*

*I will see tomorrow when the graphs are completed and the assignment is turned in, if more of my students were able to make sense of the process. However, even as we did the experiment, my sense is that the process was less confusing for most students. In general, focusing on the main objective made everything go a bit smoother.*

Liv describes in detail what happened in her classroom. From the analysis of the videos and the reflections on what could be changed, Liv has moved to implement the changes and now analyzes her own teaching. Focusing on one main learning goal and making all students collect the same set of data facilitated her task. The concern with the students' process of meaning making is also apparent from her answer. In order to teach is necessary to understand if the students are learning. This attention to students' understanding is a fundamental prerequisite to the acquisition of pedagogical content knowledge.

## 2.5 Final Remarks

Two aspects emerged from the analysis of these three teachers' responses to the reflection task: 1. Reflections focused on a wide range of topics concerning everyday teaching practices; 2. Mathematical content, teaching strategies and students' learning - essential elements of pedagogical content knowledge - were integrated in the teachers' elaborations.

Teachers seemed to take three different perspectives when reflecting on their own practices. Their reflections are sometimes based on *intra-lesson analyses*. Teachers describe and reflect on teaching as it occurs in their classrooms, as well as on ways their students approached specific mathematical tasks.

Other times, they conduct *inter-lesson analyses*: they compare activities they proposed and implemented in their classrooms with what they observed in the videotaped lessons. Watching teachers in other countries dealing with topics also included in the U.S. curriculum has stimulated a process of comparison. Certain commonly used activities are now questioned by Santagata and Stigler (2000).

A third perspective sees teachers stepping back and assuming an *analytical position*, similar to that of researchers analyzing teaching practices. Teachers here distance themselves from their everyday didactical activities and look at teaching phenomena from different angles. This perspective creates the potential for acquiring ways to monitor and evaluate one's own teaching, stimulating alternative teaching activities. It also opens up for teaching and learning in socially different contexts including multicultural classrooms (see below Interactional Model where intra and inter- class dimensions are included).

In conclusion, videos have been used widely in teacher education since the first Microteaching clinic in the late sixties. What has distinguished different approaches is the relationship between the subject and the object, between the teacher and the video. The approach proposed in this exploratory work reflects an attempt to develop analytical tools for teachers, which they can apply for the improvement of their own practices. Many questions remain open. Among others, we need to investigate effective ways to guide teachers' analyses, we need to better understand the needs of teachers with different levels of experience and knowledge, and ultimately, we need to study the effects of teachers' analytical abilities on the improvement of their teaching practices and of their students' learning.

## 2.6 Literature


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# 3. Real-life Connections in the TIMSS 1999 Video Study

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Several curriculum reforms over the world have incorporated ideas of connecting mathematics with real or everyday life. In this article we take a closer look at how teachers actually implement these ideas. We discuss theoretical ideas of connecting mathematics and real life with practical experiences of teaching, exemplified with a sample of lessons from the TIMSS 1999 Video Study. This study showed that Dutch classrooms contained far more real-life connections than the others, and that high-achieving countries like Japan and Hong Kong had few real-life connections. In this article we look closer into how the teachers of these three countries actually implied the ideas of real-life connections, and we discuss the findings in comparison with theoretical ideas from the tradition of Realistic Mathematics Education and others.

## 3.1 Real life connections

The idea of connecting school mathematics with everyday life, daily life or real life is widespread. Research has addressed the issue. Theories in general pedagogy as well as in mathematics education seem to support the idea of connecting the mathematics in school with something the pupils know and are familiar with in order to enhance learning. Curricula and frameworks around the world, for instance the most recent Norwegian curriculum – called L97 – try to implement these ideas to some variable extent. Even if the theme is emphasized in frameworks as well as research, many teachers have problems implementing these ideas.

Mathematics in everyday life has become one of five main areas in mathematics in L97, and it has thereby been given much emphasis.

*The work with mathematics in the compulsory school is intended to arouse interest and convey insight, and to be useful and satisfying to all pupils, in their study of the discipline, their works with other subjects, and life in general*

…

*The syllabus seeks to create close links between school mathematics and mathematics in the outside world. Day-to-day experiences, play and experiments help to build up its concepts and terminology (RMERC, 1999, p. 165).*

The area of mathematics in everyday life is somewhat different from the other areas of mathematics, and it has a different aim, in that it is supposed to establish the subject in a social and cultural context. It is also more oriented towards users (RMERC, 1999, p. 168). Since this issue is strongly emphasized in the Norwegian

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framework, it is interesting for us to see how the idea of connecting mathematics with real or everyday life is carried out in other countries. Studies of teaching practices in different countries can reveal one's own practices more clearly. One can discover new alternatives, it can stimulate discussion about choices within each country and it can deepen educators' understanding of teaching (Hiebert et al., 2003, pp. 3-4). With that in mind, this article goes into some classroom situations from the TIMSS 1999 Video Study where real-life situations were implemented or used in different ways.

## 3.2 What theory says

Before we study the videos, we will take a brief look at what theory says about the issue of real-life connections, or connecting school mathematics with everyday life. This will give a brief reference to the theoretical basis of our study. To begin with, it would be reasonable to ask why one should use real-life connections, or why one should connect with everyday life.

*By closely observing student activities, experiences, interests, and daily endeavors, one may be able to capture situations whose everydayness makes them potentially powerful departure points for establishing bridges to academic mathematics. Such bridging between the everyday and the academic may then consist of integrating the genuine, meaningful, and engaging origin of the problem (children's experiences) with guidance for developing and using mathematical tools (possibly ad hoc at the beginning) to help students make deeper sense of the problems […]. The bridges also provide ways to return to the everyday situations with more powerful knowledge about handling and approaching them (Arcavi, 2002, p. 16).*

The issue of motivation often comes up in this discussion, and although others have emphasized different aspects, we let Arcavi's words stand as a reasonable answer to the question of why. Another question that is reasonable to ask is how this connection could be or should be carried out.

The Dutch tradition of Realistic Mathematics Education (RME), which originates in the thoughts of Hans Freudenthal, provides one answer to this. In RME, an important idea is that the pupils should be actively involved in the reconstruction or re-invention of the mathematical ideas. Context problems, as they are often called, would normally serve as a qualitative introduction to certain mathematical concepts. The pupils are guided by the teacher through a process of reinvention, and in this process organization and mathematization are important activities. These ideas are strongly connected with ideas of constructivism and activity theory (see Freudenthal, 1991; Gravemeijer, 1994; Gravemeijer and Doorman, 1999; Jaworski, 1995; van Amerom, 2002).

## 3.3 The TIMSS video studies

In *The Learning Gap* (Stevenson & Stigler, 1992), the results of the SIMS study are discussed. A major idea is to study teachers and teaching practices in different countries in order to improve teaching. In 1995 another large international study was conducted. The TIMSS student assessment was comparing the students' knowledge and skills in mathematics and science, by country. This study was followed by a video study, which was the first study to use video technology to in-

vestigate and compare classroom teaching on a country wide basis (Hiebert et al., 2003, p. 9).
As a supplement to the next TIMSS, the TIMSS 1999, another video study was conducted, now in a much larger scale than before. This study recorded more than 600 lessons from 8th grade classrooms in 7 countries: Australia, Czech Republic, Hong Kong SAR, Japan, Netherlands, Switzerland and United States. The Japanese videos were collected in the 1995 study and re-analyzed. All videos were transcribed, the transcriptions translated into English, coded and analyzed. In this article, we have chosen to use the transcripts as they appear in the data collection from Lesson Lab, and not make any adjustments or corrections of grammatical or other kind. In 1995 as well as in 1999, Japan and Hong Kong were among the highest achieving countries in the student assessment part of TIMSS. When we call them high achieving in the following, this is what we mean. In this article though, we will focus almost exclusively on the TIMSS 1999 Video Study.

When it came to how the mathematical problems were presented and worked on, the coding team explored several aspects, including (Hiebert et al., 2003, pp. 83-84):

- ***The context in which problems were presented and solved:*** Whether the problems were connected with real-life situations, whether representations were used to present the information, whether physical materials were used, and whether the problems were applications (i.e., embedded in verbal or graphic situations.
- ***Specific features of how problems were worked on during the lesson:*** Whether a solution to the problem was stated publicly, whether alternative solution methods were presented, whether students had a choice in the solution method they used, and whether teachers summarized the important points after problems were solved.
- ***The kind of mathematical processes that were used to solve problems:*** What kinds of process were made visible for students during the lesson and what kinds were used by students when working on their own.

The issue of real-life situations is addressed like this (Hiebert et al., 2003, p. 84):

*The appropriate relationship of mathematics to real life has been discussed for a long time (Davis and Hersh, 1981; Stanic and Kilpatrick, 1988). Some psychologists and mathematics educators have argued that emphasizing the connections between mathematics and real-life situations can distract students from the important ideas and relationships within mathematics (Brownell, 1935; Prawat, 1991). Others have claimed some significant benefits of presenting mathematical problems in the context of real-life situations, including that such problems connect better with students' intuitions about mathematics, they are useful for showing the relevance of mathematics, and they are more interesting for students (Burkhardt, 1981; Lesh and Lamon, 1992; Streefland, 1991).*

When comparing average percentage of problems per eight-grade mathematics lesson that were set up with the use of real-life connections, there were some interesting differences. In Netherlands, 42 percent of the lessons were set up using real-life connections, whereas only 40 percent using mathematical language and symbols only. This was the most special result in the study, where the other six countries differed between 9 and 27 percent real-life connections. It is also interesting to see that in Japan only 9 percent of the lessons, and in Hong Kong only 15 percent had real-life connections.

*In all the countries, if teachers made real-life connections, they did so at the initial presentation of the problem rather than only while solving the problem. A small percentage of eighth-grade mathematics lessons were taught by teachers who introduced a real-life connection to help solve the problem if such a connection had not been made while presenting the problem (Hiebert et al., 2003, p. 85).*

A larger percentage of applications were discovered in the Japanese classrooms (74%), than in Netherlands (51%) and Hong Kong (40%). These applications might or might not be presented in real-life settings (Hiebert et al., 2003, p. 91).

Another interesting point is connected with the mathematical processes. In Japanese classrooms 54% of the problems were classified as having to do with making connections. In Hong Kong this was only the case in 13%, and 24% in Netherlands (Hiebert et al., 2003, p. 99, figure 5.8). Hong Kong had a high percentage of “using procedures”, i.e. involving problem that was typically solved by applying a procedure or a set of procedures. In Japan this was the case in only 41% of the problems, and in Netherlands 57% (Hiebert et al., 2003, pp. 98-99).

Viewing the statistics only, one might assume that the use of real-life connections will not have any positive effect learning. This is an example showing how difficult it is to draw conclusions based on quantitative results alone. We will now go into some actual lessons from this study, in order to shed more light on the issue.

## 3.4 Choice of material

We watched more than 30 videos from the collection, or about 10% of the data material from these three countries. We only chose videos with at least one real-life connection, as coded by the coding team at Lesson Lab, since our incentive was more to study teaching practices of teachers that actually did make real-life connections, rather than studying to what extent teachers did or did not connect with real life in general. Of these 30 videos, we chose about 20 videos which contained a large amount of real-life connections to collect transcripts from. Finally, 9 of these were chosen for further analysis here. In the last two stages of the selection process, we did not only regard the number of real-life connections, but we also chose lessons where different methods of teaching and classroom organization were used. When lessons with equal or similar content and/or structure were found, only one was selected for further analysis. We ended up with three Dutch lessons, three Japanese lessons and three lessons from Hong Kong. The reason for focusing on these three countries was that they were all extremes when real-life connections were concerned. Netherlands had the highest percentage of real-life connections among the participants of the Video Study, while Japan and Hong Kong had the lowest percentages of real-life connections. An important question that we wish to answer is: How do the teachers actually connect their mathematics teaching with real life?

## 3.5 Defining the concepts

Before discussing real-life connections it would be appropriate to discuss what lies within the concept of "real life". In research in mathematics education we come across a variety of concepts like everyday life, daily life, real life, real world, realistic as well as contextual, situated and other concepts that are directly or indirectly related (cf. Boaler, 1997; Brenner and Moschkovich, 2002; Lave and Wenger, 1991; Wistedt, 1992). A proper question might be: "What do you mean with real life?"

According to the Norwegian curriculum, real-life connections are connections between the mathematics taught in school and the outside world. The conception of the outside world is not trivial. The everyday life of the pupils is often limited, and if one would focus only on issues contained in the everyday life of pupils, mathematics would become limited. Then, there is also the aspect of different pupils having different experiences of the outside world. We therefore do not wish to limit real or everyday life to the pupils' conception of the outside world. If adopting a view of real life as everything that might be encountered in the outside world, this would imply that the real-life connections in school mathematics in many cases are not part of the pupils' everyday life, and therefore do not automatically provide more meaning to them. A goal for school mathematics should not only be to reflect the pupils' everyday life, but also to prepare them for their future vocational life and life in society. Having introduced this goal, real-life connections could provide meaning although not being directly meaningful to the pupils, in that they are connected with the everyday life of the pupils. We do not thereby wish to suggest that making real-life connections is the proper or 'best' way of teaching mathematics. In some instances, direct connections with real life can make it harder for the pupils to understand because of culturally related issues or other (cf. Bransford et al., 2000, p. 72). But in a curriculum where the connections with real or everyday life are emphasized, like the Norwegian curriculum for years 1-10, connections with the pupils' present and future everyday life would often be included, as well as with vocational life, life in society, games, etc. When a real-life connection, i.e. some kind of reference to issues in the outside world, is made, we will discuss if this is authentic or not. Fake real-life connections often seem to serve more as a wrapping of a mathematical theory rather than authentic real-life connections.

This article is based on the TIMSS 1999 Video Study, so we will therefore have a closer look at the definitions of concepts made in this study. All the lessons of the Video Study were coded, and the coding team made a distinction between real life connections/applications, and whether they were set up as a problem or not. The coding team chose not to make a distinction between real-life connections and real-life applications, although these are two different issues. Two categories were defined: real-life connections or applications in problems, and real-life connections in non-problem situations. The definition of the real life connection/application – non-problem (RLNP) was presented like this:

*The teacher and/or the students explicitly connect or apply mathematical content to real life/the real world/experiences beyond the classroom. For example, connecting the content to books, games, science fiction, etc. This code can occur only during Non-Problem (NP) segments.*

As we can see here, they compare real life to real world or experiences beyond the classroom. This is a quite vague description, which was clarified somewhat with examples on how these connections could be made. In our analysis of lessons, we marked a sequence RLNP whenever it made a reference to issues in the outside world, and where this reference was not connected with a problem the pupils worked on.

The by far most frequent occurring of the two was simply called real life connections, and appeared in actual problems in class. There was made a distinction between situations where the real life connection appeared in the problem statement or set-up, or if the real life connection was brought up during the discussion or work with the problems. The definition of these kinds of real life connections, called RLC, was:

*Code whether the problem is connected to a situation in real life. Real life situations are those that students might encounter outside of the mathematics classroom. These might be actual situations that students could experience or imagine experiencing in their daily life, or game situations in which students might have participated.*

Real life is then whatever situation a student might encounter outside of the mathematics classroom, actual situations or imagined situations that the students might experience. A situation was coded RLC whenever a reference was made to the outside world, directly or indirectly, in a problem the pupils worked with or discussed.

We have adopted this distinction between RLC and RLNP as it helps us answering two initial questions: are there any connections to real life? Are these connections related to a problem or not? When a sequence is coded as a real-life connection, whether in a non-problem sequence or not, certain contexts will be presented. In our analysis, we will discuss some of these contexts, to see if they are pseudo-contexts or not.

We have extended this coding scheme, including the first two categories in what will be called level 1. Level 2 will go further into the kind of connections, if they are textbook tasks, pupil initiatives, etc. The third and final level of analysis will focus on how these connections are carried out, or methods of work. A coding scheme could then look like this:

| | |
|---|---|
| **Level 1:**<br>- RLC (Real life connections in problem situations)<br>- RLNP (Real life connections in non-problem situations)<br><br>**Level 2:**<br>- TT (Textbook tasks)<br>- OT (Open tasks)<br>- TELX (Teacher's everyday life examples) | **Level 3:**<br>- GW (Group work)<br>- IW (Individual work)<br>- TAWC (Teacher addresses whole class)<br>- P (Projects)<br>- R/GR (Reinvention/guided reinvention)<br>- OA (Other activities) |

| | |
|---|---|
| - PI (Pupils' initiatives)<br><br>- OS (Other sources, like books, games, science fiction, etc.) | |

This coding scheme provided the base for our selection of episodes below, and they also present a basic idea behind our analysis. Having these foundations laid, we will go into the actual lessons and see how they were conducted.

## 3.6 The lessons

### *3.6.1 The Dutch lessons*

The Dutch lessons had a very high percentage of real life connections in the TIMSS 1999 Video Study, much more than any of the other participating countries. The lessons would often include a large number of problems connected with real life. We looked at some of these lessons. From the videos, a pattern seemed to emerge. In most of the lessons we looked at, the teacher reviewed problems from the textbook together with the class. It seemed as if the pupils had already worked on the problems before, and the pupils were asked questions related to the answers of the problems. When working on problems, they mainly worked individually, but they might also be seated in groups. What struck us was that the teachers were very focused on the textbook, and the problems from the textbook seemed to be almost exclusively collected from real life settings. Most of the real life connections could be coded RLC, TT, TAWC, i.e. real life connections in problems, textbook tasks presented by the teacher addressing the whole class. This was the case in most of the lessons we viewed.

An example of this can be found in the lesson M-NL-021, where the teacher goes through problems like this in the entire lesson:

Teacher: Now another possibility with percentages. I have an item in the store. At present it costs three hundred ninety-eight guilders. Next week, that same item will cost only three hundred twenty guilders. With what percentage has that item been reduced in price, Grietje?

*Student: Um, seventy-eight guilders was subtracted.*

*T: Seventy-eight guilders was subtracted, yes.*

*S: Eight, uhm divide it by the old amount times one hundred.*

*T: So – yes. By which – by which number?*

*S: Three hundred and ninety-eight and then times one hundred.*

*T: By three hundred and ninety-eight and then times one hundred. And that gives you the solution.*

As we can see, the teacher reads the problem from the book, and asks a pupil to give the solution. The pupils have maybe already been working on the problems. Some of the problems are larger and more complex, containing figures and tables. In this lesson, many of the contexts seem to be collected from statistical material, like in a problem on the wine imports to Netherlands in 1985, introducing picture diagram, bar diagram and line diagram. Other problems focus on temperatures, amounts of umbrellas sold on a rainy celebration day, coffee consumption in Holland etc. The contexts of these textbook tasks have an authentic appearance, and the numbers and figures presented seem realistic.

One of the other lessons we viewed, M-NL-031, was different when methods of work were concerned. In this lesson they worked on probability. The teacher divided the class into different groups. One of the groups should flip coins and write down the results, another group should roll dice and yet another group should look outside the window and write down how many men and women that passed. The groups worked five minutes on each task, and then moved to the next station. The pupils should use these data and calculate the chance (the fraction and the percentage). The real-life connections in this lesson were different from the previous in that they didn't work with textbook tasks only, but with other sources, sources which provided a set of data that the students gathered themselves. They also worked in groups, and during their work they encountered several real-life applications and connections in non-problem settings.

The final Dutch lesson that was selected for this article (M-NL-050) focused on exponential growth, mainly on a problem concerning the growth of duckweed:

*T:* *Uhm… A piece of five centimeters by five centimeters of duckweed in the pond, it's really annoying duckweed. It doubles. But the owner of the pond doesn't have the time to clean it. He takes…*

*S:* *Sick?*

*T:* *No, he takes three months of vacation. Now, the question is… the pond, with an area of four and a half square meters. Will it be completely covered in three months or not?*

*S:* *Yes.*

*S:* *( )*

*T:* *Shh. This is the spot that has duckweed at this moment. It doubles each week, no, and the pond is in total four and a half square meters, and the time that he's gone on vacation is three months. So the question now is whether the pond has grown over or not.*

The pupils were then asked to use their calculators. After the pupils have worked with it for a while, the teacher asked them what they have come up with:

*T:* *Who says it's full after three months?*

*S:* *No idea why, but it's full.*

*T:* *Uhm, who doesn't?*

*S:* *( )*

*T:* *And, uhm, who says "I don't know"?*

*S:* *Ha ha.*

*T:* *Uhm, so there are six. I have six unknown, no one for not full, and, uhm, so there are twenty-five for full. Uhm, Paul, how did you come up with full? What did you try, what did you do?*

*S:* *I don't know.*

Then, the teacher tried to figure out how the pupils have thought and what they had calculated. They eventually came up with a formula for calculating the growth during the twelve weeks. At the end of the twelfth week, they found out it was two to the twelfth. Then they had to convert square meters into square centimeters. After a discussion on this, the teacher summed it all up:

*T:* *Uhm, so you must make sure that, in the end, you are comparing. So, or the an swer that you came up with… that'll be twenty-five thousand times four, so that is somewhere close to hundred thousand, and so it's full. This is something that will be explained in Biology. In economics, well, then you will get the following: that the doubling of bacteria, then you get something like this ( ).*

The context presented in this lesson is also authentic, and duckweed could be encountered as problematic in real life. Now, many pupils are raised in cities, and they might never have experienced duckweed as a problem in ponds. In real life, the issue would also be to clean the pond rather than calculating on the growth. In that manner, it would seem as a wrapping of mathematical theories and considerations.

From the statistical analysis of the Video Study, as well as from reading about Realistic Mathematics Education from the Freudenthal Institute, we get the impression that real-life connections are important in Dutch schools. This impression has been supported from our sample of videos. The RME tradition strongly supports the idea of guided reinvention and thereby an integral amount of student activity should be included in the work on real-life connected problems or realistic problems as they are often called in this tradition. This was not so evident in the sample we have seen, and here it seemed to be more teacher talk in connection with a review of textbook problems than a process of guided reinvention of mathematical concepts. In many of the Dutch lessons we have seen, the teaching was rather traditional – with real-life connected textbook problems.

### *3.6.2 The Japanese lessons*

What was most striking about the Japanese lessons was their structure. They were well structured, and as we learned already from *The Learning Gap*, mathematics lessons in Japan would often follow exactly the same pattern in corresponding lessons all over the country. We saw examples of this with different schools and different teachers where the lessons were almost exactly the same. A Japanese lesson would often focus on one problem only, and this would often be a rich problem and a "making connections" - problem.

An example of such a lesson is M-JP-022. In this lesson, the teacher starts off with a short introduction to the concept "center of gravity". Here he gives a comment that in sports, like baseball or soccer, center of gravity is important. This comment was marked as RLNP-situation in the Video Study. Then, he shows how to find the center of gravity in a book, balancing a textbook on a pencil. All along he discusses with the pupils, letting them think it all out where the center of gravity is, leading them into ever more precise mathematical formulations.

Next he challenges them to find the center of gravity in a triangle, and this becomes the main focus for the entire lesson. First the object is simply to find the center of gravity by balancing a paper triangle on a pencil. Then, as the teacher states, it is time to look at this more mathematically:

*T:* *Okay this time open your notebooks. Uh let's try drawing one triangle.*

*(pupils are drawing in their notebooks)*

*T:* *Okay. If it were a cardboard you can actually tell saying it's generally around here where it is using a pencils and such. Okay it's written in your notebooks. It's written on the blackboard. You can't exactly cut them out right? You can't exactly cut them out. And without cutting them out … I want you to look for like just now where the balancing point is, … that's today's lecture. Using this cardboard from just now … in many ways. I will give you just one hint. It'll be difficult to say at once here, so on what kind of a line does it lie? … On what kind of a line does the point lie? Please think about that.*

So, first they find the center of gravity by testing on a cardboard, then the next challenge is to find this center (mathematically) without cutting out the triangles. The pupils get time to think and discuss, and they play around with pencil and triangle. Then the teacher forms groups of six, and the pupils discuss further in groups. The teacher walks around and comments on the work. He asks them to draw lines or points on the cardboard and try it out to see if it balances. Some pupils discover that their solutions are wrong. The teacher interrupts the work by presenting to the class one false solution that a pupil tried:

*T:* *Okay. It's okay. Just for a second, sorry Shinohara. Shinohara just tried with the bisectors of angles right? The bisectors of angles. And … when you try it like this*

*S:* *( )*

*T:* *Unfortunately it doesn't balance. Um … at the bisector of the angle please look up front for a second those of you facing the back. Group one girls, look … look for a second. Let's see … if you go like this at the bisector of an angle, Shinohara.*

*S:* *Yes?*

*T:* *Look over here. If you are asked whether it balances?*

*S:* *Um*

*T:* *Uh huh. This side ended up little … heavy right? It ended up heavy. That's why even if you go like this it doesn't balance. So the areas are the same … unless the areas are the same … it's no good, is it?*

The pupils continue trying out their theories on the cardboards. From time to time, the teacher interrupts by showing some of the pupils' solutions on the blackboard. The pupils get plenty of time to think and try things out, and the teacher mainly uses the pupils' ideas and answers in a reconstruction of the theory. Eventually they reach a proof, and the teacher sums it all up in a sentence. In the end he reviews the essence of the lesson again.

Such an approach can be seen in many lessons. The pupils get lots of time to work with one problem at a time, and very often, the pupils reinvent the theory. Sometimes the pupils would also present their solutions and methods on the blackboards, and the class would discuss which method to prefer. Quite often the mathematical content of a lesson would be purely mathematical, as this lesson was, except for the tiny comment on center of gravity in sports. We do not know if this lesson was the introduction to the topic, so we cannot claim that the pupils were really discovering or reinventing the methods and theories connected with centre of gravity. The pupils seemed to be enthusiastic about the activity though, and they get the opportunity to see the link between theory and practice, and also to discuss their choices of methods and solutions. Even though much of the teaching is arranged as the teacher discussing with the whole class, the pupils are active.

In M-JP-035, the approach is a bit different. They are working on congruence and similarity, and the teacher has given the pupils a homework assignment:

T: *Okay. Ah…then up to now … up to the previous lesson we were learning about congruent geometric figures, … but today we'll study something different. As I was saying in the last class … I said we'll think about geometric figures with the same shape but different sizes, and I was asking you to bring such objects to the class if you find any at home.*

Not all the pupils brought things, but some brought angle rulers, some protractors and erasers, and one brought origami paper. The teacher has also brought some things, and she uses this to introduce the topic:

T: *Okay. Then, next I'm going to talk … all right? What similarity means is that the figure whose size is expanded or reduced is similar to the original figure. Then, well a few minutes ago I introduced the objects you have brought to the class. I, too, have brought something. What I have brought is … some of you may have this bottle at home. Do you know what this is? Yasumoto, do you know?*

S: *( )*

T: *What? You don't know what kind of bottle this is? Taka-kun do you know?*

S: *A liquor bottle.*

T: *A liquor bottle. A ha ha … that's right. It's a whisky bottle. Whisky … a whisky is a liquor which … we all like. Cause we even call it Ui-suki (we like).*

S: *A ha ha.*

T: *A ha ha. Did you get it? Then, … about these whisky bottles … look at these. They have the same shape don't they. They do, but have different sizes. Well, I have borrowed more bottles from a bottle collector. This.*

S: *A ha ha.*

T: *This.*

S: *A ha ha.*

T: *See … then I wondered if there were more different sizes so I went to a liquor store yesterday. And, they did have one which contains one point five liter of … one point five liter of whisky, but it was too expensive so I didn't buy it. As you can see that these whisky bottles … have the same shape … but they come in various sizes. All of these bottles are called similar figures.*

The teacher starts with connecting to real life through the examples of things the pupils have brought, and then goes on to present some things she has brought herself, kind of teacher's everyday life examples. She has also brought a couple of squid airplanes, with different sizes. And she has brought a toy dog. She shows how to draw this dog in a larger scale, using rubber bands. Then, she goes into more specific mathematics, asking the pupils to draw geometrical figures like quadrilaterals and triangles in larger scales. At the end of the lesson, she leads the pupils into finding out that the angles are equal in these expanded figures, and that they are therefore similar. She also introduces a symbol for similarity.

In the last lesson M-JP-034 from Japan that we looked into, they also work with similarity. This teacher gives lots of examples from real life, and he asks the pupils to give examples also. Some of the examples he comes with are the desks in the classroom, negatives of a film, fluorescent light and different sizes of batteries. All along, there is a dialogue with the class. It seems as if real-life connections are merely used in the introduction of a new topic.

As we could see very clearly in some of the Japanese lessons, the teacher would start off with one or a couple of real life examples and gradually move towards the mathematical concepts. The aim would often be to use the real-life situations more like motivational examples, not really to solve real life problems.

### *3.6.3 The Hong Kong lessons*

Like the Japanese lessons, the Hong Kong lessons also contained a low percentage of real life connections, according to the TIMSS 1999 Video Study (Hiebert et al. 2003, p. 85). We will look into some of the lessons that did contain such connections, and see how the teachers carried this out.

The first example is from M-HK-019, where the teacher gives an example introducing a new chapter:

*T:* *Okay, you will find there are two supermarkets – the last supermarket in Hong Kong, okay? Okay, one is Park N Shop and the other is Wellcome, okay? I think all of you should know. You know these two supermarkets, okay? And then – now, and you should know that in these few month, okay? This two supermarket, okay, want to attract more customer. Do you agree? Therefore, they reduce the price of th- of the- of the- uh, uh, of the products. Okay? And they want to attract more customers. Do you agree? Okay, and then- now, here- there is a person called Peter, okay? He come into this two supermarket and he want to buy a Coca Cola, okay? And then now, yes, I give you the price of the two shop. The different price of the two shop. For Park N Shop, okay? For the price of Cola, okay? Okay? It show the price- the price is what? One point nine dollars per- uh, for one can, okay? For one can. One point nine dollars for one can. And for the Wellcome shop. For the Wellcome, okay? It showed for the price of the Cola, okay? Uh, twelve dollars, okay, for six can.*

Having given this example, the pupils are asked what price is the cheapest. And they use this to introduce the concept of rate. Another example is a man that walks four kilometers in two hours. This lesson involves quite a lot of teacher talk, and not so much time for pupil activities as did the Japanese lessons. There are several other real life examples in this lesson, all of them concerning ratio between two quantities. Most of the time the teacher explains, but sometimes the pupils are drawn into the discussion. To a large extent, this lesson is like a lecture.

The next lesson we will look at is M-HK-020, and in many ways, this is like some of the Japanese lessons. For the entire lesson, they work within one problem setting, with many different examples, with the aim of approaching a mathematical theory concerning equations with two unknowns. The teacher very much wants the pupils to discover this for themselves, and he starts off giving an example:

*T:* *Okay. Ask you a question. Birds… have how many legs?*

*S:* *Four.*

*S:* *Two.*

*T:* *How many?*

*S:* *Two.*

*T:* *Two. Birds have two legs.*

*(…)*

*T:* *Legs. Okay. Birds have two legs, how about rabbit?*

*S:* *Four.*

*T:* *Four*

Then he asks the pupils: if there are two birds, how many legs in total? Then he asks if there are one bird and one rabbit, how many legs, and then two birds and two rabbits. Then it evolves:

*T:* *Something harder. How about this? One bird plus one- two rabbits?*

*S:* *Ten.*

*T:* *How many legs?*

*S:* *Ten legs.*

*T:* *Ten legs. Okay. It's coming. What if I don't tell you how many birds or rabbits, but tell you that…*

*S:* *How many legs.*

*T:* *There are a total of twenty-eight legs- twenty eight legs. Well, there aren't enough hints. I need to tell you also there are how many…*

*S:* *Heads.*

*T:* *Heads. How about that? Nine heads.*

The pupils solve this and other similar examples, using their own methods (normally some kind of trial and error). When the examples get too difficult, the need of a stronger method of equations arrives. The pupils reinvent the setting up of the equations, using X for birds and Y for rabbits. The teacher gives them time to struggle with these equations, and he doesn't tell them the solution or answer at once. The only problem, whether planned or not, is that he doesn't reach the point of it all, because the lesson ends. He makes the following remark in the end:

*T:* *Okay. Next time, we'll continue to talk about what methods we can use to find it - find X and Y. Okay. Is there a systematic method. We systematically found two formulae. Is there a systemic way to find X and Y. Next time, we'll talk about it. But everyone is very sharp, flipping through your book asking "Sir, is this the method, sir, is this the method". You should be right. The book has many methods.*

We here get an example that shows us that such methods of work might be quite time-consuming, and the planning of the lesson in detail is important.

In the last example from the Hong Kong lessons, M-HK-080, we see a class working on proportions. The young teacher gives quite a lot of examples and connections to real life, some in a problem setting, but most not. He starts off with an open question:

*T:* *I have discovered one thing…*

*S:* *A dinosaur's footprint.*

*T:* *In ancient times – yes, a dinosaur's footprint. Yes, it really is this one – this one. I want to give you a question now. The footprint is this size. I want to ask you to guess how tall the dinosaur is. I help you – the only thing I can help you is measuring the length of this.*

He then leads them into a discussion on how to guess a dinosaur's size, by knowing the length of its footprint only. He then asks how this would be if it were a human footprint, and he shows how this is connected to proportions.

The teacher has also brought a couple of maps, and he asks two students to find the scale. They then discuss distances on the map compared to distances in reality, etc. All the time, the pupils get some tasks, things to calculate and figure out. He then hands out some brochures of housing projects, and the pupils are asked to figure out some things about the map contained in them. After working for a while with two-dimensional expansions, he introduces some Russian dolls, and thereby presents them with the concept of three-dimensional expansion. So, for the entire lesson, the pupil activities are connected with some real world items like maps, dolls or dinosaur footprints. They are both RLC and RLNP, but they are exclusively the everyday life examples of the teacher, and they are presented by the teacher addressing the whole class.

### 3.7 Summarizing

We have now presented nine lessons from the TIMSS 1999 Video Study, and we have brought to your attention some episodes and points from these lessons. Our initial question was how these teachers actually connect mathematics with real life.

As we said above, there was a pattern in the Dutch classrooms that the teacher would spend much time reviewing textbook problems. The first Dutch lesson, M-NL-021, is a typical example of this. Almost all the real-life connections were RLC-TT-TAWC, i.e. real-life connections in problem situations, where the problems were textbook tasks and the teacher was addressing the whole class. The one exception was when the teacher made a remark concerning one of the problems.

The idea of guided reinvention, which is emphasized in the Dutch tradition of Realistic Mathematics Education (RME), was not so visible in the lessons we saw, neither was the idea of mathematization. One of the lessons, M-NL-031, contained a more extensive activity where the pupils worked in groups, but although being based on a more open task, it didn't seem to represent the ideas mentioned above. In the last lesson we focused on from the Dutch classrooms, M-NL-050, the main focus was on a real-life connected problem. The problem was concerning growth of duckweed, and it seemed to be a textbook task presented by the teacher addressing the whole class. This problem was discussed and worked on for the main part of the lesson, and here we could see elements of reinvention.

In the total collection of Japanese videos there were not so many real-life connections, but in the lessons we have looked at the teachers would often use a structure similar to the approach in RME, like in the first lesson we refer to. Here, the teacher made the problem realistic to the pupils through his introduction, and the pupils were then guided through a process of reinvention of the theory. In the next, we saw examples where quite a lot of connections were made to real life, some of them being by things the pupils had brought, or other pupil initiatives, and some where real-life connections made by the teacher presenting her everyday life examples. The teacher would normally address the whole class. In conclusion, some Japanese classes involved a method of work strongly related to the ideas of RME, and although this seemed to be exceptions, the teachers would sometimes make explicit real-life connections in their lessons.

In Hong Kong, the main emphasis seemed to be on procedures, but the teachers would in some cases give quite a lot of real-life connections in their classes. Some of the RLC-problems seemed like teacher's everyday life examples, and some were textbook problems. The main method of work was that the teacher lectured or discussed with the class, but on some occasions the pupils would also work individually with problems. The RLNP-situations were mainly comments and references to the problems discussed. On one occasion, the teacher included a pupil and his daily life in a problem, making it a problem of finding out the walking speed of this pupil when going to school. The second Hong Kong lesson, M-HK-020, was interesting. For the entire lesson they worked on one problem or within one context only. The problem they worked on was concerning rabbits and birds, and the number of their heads and legs. In this lesson the pupils were guided through a process of reinvention of early algebra, but unfortunately the lesson ended before they had reached any conclusions. Anyway, we could discover clear links to the ideas of RME in this class. In the last lesson, M-HK-080, the teacher gave many examples from his everyday life, and he had also brought some physical objects like maps and figures to make it more real to the pupils. The teacher was addressing the whole class in a discussion-style, and on some occasions pupils were picked out to do some activities in front of the class.

## 3.8 Final discussion

Based on our previous knowledge about the role of RME, we expected that the Dutch classrooms would contain activities where the pupils were mathematizing and reinventing mathematical theories through realistic or real-life connected problems. In the lessons we have seen, they were working with real-life connected problems, but often in a traditional way. Some of the lessons from Japan and Hong Kong had adopted the ideas of reinvention and mathematizing to a larger extent than what was visible in the Dutch videos, although they did not contain so many coded real-life connections. In the Japanese lessons the pupils' ideas and solution methods were taken into account, and the pupils would often take an active part in the discussion of which methods to use. The pupils in these classrooms seemed much more involved and active than what we could see in the Dutch videos.

Many mathematics teachers, at least in Norway, are dependent on the textbooks. The Dutch textbooks seemed to focus a lot on real-life connected problems, so we would assume that Dutch classrooms – through the textbooks – would involve many real-life connections in problems. An important observation was that most of the Dutch classrooms did not seem to apply the ideas of Realistic Mathematics Education. Both the Hong Kong teachers and especially the Japanese teachers were to a larger extent using other sources than textbooks in their lessons. One difference was perhaps that the Hong Kong teachers seemed to be more concerned about teaching the procedures, while Japanese teachers seemed more concerned about organizing activities where the pupils could discover these procedures for themselves (Hiebert et al., 2003, p. 116).

We started off by pointing at the Norwegian curriculum, which is strongly influenced by the NCTM Standards as well as the Dutch tradition of Realistic Mathematics Education. As we have also mentioned, L97 has a strong focus on the connection of mathematics with everyday life, but it doesn't say all that much on how this is going to be carried out. Teachers are often left on their own, trying to figure it out for themselves, and the curriculum leaves us with an impression that real-life connections are trivial. In this article we have seen how teachers in different country carry out the connections with real life in their teaching of mathematics. Often, the connections are artificial and serve as wrappings more than authentic connections. These observations imply that real-life connections are not trivial, and much more emphasis should be given to how they are carried out and presented in mathematics classrooms.

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# 4. History of mathematics in the TIMSS 1999 Video Study

*Bjørn Smestad, Oslo University College, Norway (A preliminary report)*

The TIMSS 1999 Video Study of 8th grade mathematics classrooms included up to 100 lessons from each of seven countries: Australia, Czech Republic, Hong Kong SAR, Japan, Netherlands, Switzerland and United States. The first results from this study were published March 2003 in the report Teaching Mathematics in Seven Countries by NCES (2003). The study was conducted at LessonLab, Santa Monica, California, directed by James Hiebert, Ronald Gallimore and James W. Stigler. In this article we look at what is connected to the history of mathematics in these lessons.4

## 4.1 The Norwegian context

In 1997, history of mathematics was included in the national curriculum for 1st-10th grade in Norway. A study of Norwegian textbooks (Smestad 2002) showed that the treatment of history of mathematics was problematic, and that textbook writers struggled to include history of mathematics in a meaningful way. A small classroom study (reported in Alseth et al (2003)) suggested that history of mathematics does not play an important role in Norwegian classrooms either. In this connection, it was interesting to look at the TIMSS Video Study material to see how history of mathematics was treated in other countries.

## 4.2 Method

All the 638 lessons have been transcribed and coded by the team at LessonLab. One of the code items used was "historical background", defined in the Math Coding Manual (page 58) as

*The teacher and/or the students connect mathematical content to its historical background (e.g. Pythagoras as the originator of a mathematical theorem).*

We were given the opportunity to watch all the videos where this code item applied, and also transcripts of the relevant passages. Our analysis afterwards has been based on these transcripts. While the Video Study is designed to show differences and similarities between countries, the material is too small to say anything about that when it comes to historical background (as it is too infrequent to give statistical significance). We will therefore refrain from discussing particular countries, and instead we view the material as one sample.

[4] Thanks are due to LessonLab, in particular Angel Chui and Rossella Santagata, for their kind assistance during the study of these videos. I also wish to thank Otto B. Bekken, who made this visit possible, who took part in the viewing and analysis of the videos, and who has commented on drafts of this article. This study was conducted in April 2003 while we were in residence at UCLA and LessonLab as a member of the TIMSS 1999 Video Study of Mathematics in Seven Countries.

### 4.3 Quantity

The first question to ask is to what degree history of mathematics was included in the lessons. The analysis shows that history of mathematics does not play a major part in these lessons. Only about 3 % of the lessons (21 of 638 lessons) included some reference to the history of mathematics at all. The parts devoted to history of mathematics have a total duration of about 69 minutes.5 If we exclude the two longest, the remaining 19 lessons only include a total of about 18 minutes of "historical background".

There are nine instances where Pythagoras is discussed, three with Thales, two with the (ancient Egyptian) method of making right angles with a rope with 13 knots, two mention pyramids, and one instance where each of the following is mentioned: Euler, Goldbach, Plato, Euclid, Descartes, Venn, Henri Perigal, Leonardo, James Garfield, Tower of Hanoi, beautiful rectangles, Egyptian multiplication, Canadian multiplication, and $\pi$.

### 4.4 Analysis

In the analysis below, we have also included some instances found in the videos from U.S. classrooms collected for the TIMSS 1996 Video Study (Old TIMSS).

### 4.5 On the theorem of Pythagoras

About half of the examples concern the theorem of Pythagoras. It therefore seems fitting to use these examples to show how historical themes are used in the mathematics lessons.
One example is extreme: it lasts for most of a lesson (43 and a half minute), and thereby contributes almost two thirds of all the time devoted to history of mathematics in this material. This is a traditional lecture, with the teacher speaking most of the time (and using Power Point), giving three historical proofs of Pythagoras' theorem (attributed to Henri Perigal, Leonardo da Vinci and James Garfield). The teacher also adds some more historical information at the end. It is impossible to say whether this teacher often included history of mathematics in this way. However, the example does show that teachers from time to time give more comprehensive accounts than the other examples in this sample suggest.6

On the other extreme there are four examples where only the name of Pythagoras is mentioned, for instance:

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[5] There is one very lengthy example in this material, where almost the entire lesson was used for the history of the theorem of Pythagoras. Since it is impossible to say with any accuracy how frequent such lessons are, any estimate for the average time spent on history of mathematics in mathematics lessons in general will also be inaccurate (that is, any confidence interval based on this material will be quite large).

[6] The teachers were asked to teach as usual and to carry out the lesson they would have taught had the video camera not been present. Most teachers considered their lesson to be typical of their teaching, NCES (2003) p. 7 and p. 34. This particular teacher's answers suggest that this lesson was fairly typical of his teaching, but he was not asked whether the amount of history of mathematics included was typical.

*Remember what I told you, that the Pythagorean theorem for the first time was created by Pythagoras, but that it had been used a long time before that.*

and

*This relationship comes from a Greek mathematician. (…) We call him Pythagoras. His full name we have forgotten. It is called Pythagoras' Theorem.*

In between these extremes, there are four examples giving some pieces of biographical information, and there are two more examples giving some information on the mathematics of Pythagoras as well.7 Of biographical information, this is an example:

*Why is this called Pythagoras' theorem? Since there was a person whose last name is Pythagoras, and he invented this. That person is called Pythagoras, and it was about 540 B. C.*

In the two examples where the mathematics of Pythagoras is also mentioned, the students are told that Pythagoras used numbers *"to explain why things happen in nature"*, *"came up with some rules that stated that music is related to mathematics"*, and that he *"worked on magic numbers"*.

The examples not regarding Pythagoras follow a similar pattern: there is one long sequence on Euler (about 12 minutes long), one small occurrence where both mathematics and biographical information is included, three instances where only the name and some biographical information is given, and four examples of only the name of a mathematician being given.

What we see from this part of the analysis, is that with only few exceptions, what is mentioned about the history of mathematics is anecdotal: giving only names and some biographical information.

## 4.6 Different kinds of mathematical knowledge

To analyze the contents of the historical connections, I use a division of knowledge into five categories: facts, skills/concepts, strategies, attitudes, and others. For instance, giving information on Pythagoras may help students remember the name of the theorem – this name belongs to the mathematical facts. It may also influence the students' attitudes. On the other hand, working on alternative algorithms may increase the students' understanding of their own algorithm, and thereby increasing their mathematical skills.[5]

### *4.6.1 Facts*

We have already indicated how Pythagoras is treated in the lessons. There are three lessons in which Thales is mentioned in much the same way (in connection with the theorem of Thales), while Venn and Plato are mentioned in one instance each (in connection with Venn diagrams and Platonic solids, respectively). There is also one example where the definition of Cartesian coordinates is introduced with a story about Descartes in bed watching a fly on the ceiling and thinking

[7] Because I have included the U.S. videos from "Old TIMSS" in this analysis, the number of examples does not add up to nine, which is the number of examples related to Pythagoras in TIMSS 1999 Video Study.

about how to describe its movements. In all of these, the historical information may help students remember the names of mathematical objects. In addition, the anecdote on Descartes may help students remember the definition.

Giving historical proofs of Pythagoras' theorem, on the other hand, may help students understand the content of the theorem (and not just its name). This is the only example in the material where historical proofs are given.

### 4.6.2 Skills/concepts

History of mathematics may show students a multitude of algorithms, and thereby making it possible to see their own algorithm in a new light. There is only one example of this in the material, where the students are working on what is often called Egyptian multiplication: multiplication by successive doubling.
History of mathematics may also show the students how different concepts have developed (and even show the connection between concepts). The anecdote on Descartes and the fly may be put under this heading – although the factual basis for the anecdote is questionable.

### 4.6.3 Strategies

Strategies for solving mathematical problems are not discussed in connection with history of mathematics.

### 4.6.4 Attitudes

It seems to be far easier to use history of mathematics to improve the students' attitudes towards mathematics than it is to use it to improve their skills.8 The TIMSS material also suggests this.

One way of influencing students' attitudes towards mathematics, is to explain the role of mathematics in society. This can of course be done by focusing on the situation today, but it can also be done with reference to the history of mathematics. There are only two examples of this, and they regard magic numbers and art. The role of mathematics in the development of technology, for instance, is not touched.9

History of mathematics is also a treasure trove when it comes to showing that difficulties are a natural part of any development. Discussing the difficulties of intelligent mathematicians may be a good alternative to focusing on the students' difficulties (and the difficulties are often similar!) In this material there is only one

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[8] In Smestad (2002), Norwegian textbooks for elementary school are analyzed. The analysis showed that a lot of what was written on history of mathematics might influence the pupils' attitudes, and that history of mathematics seldom was used to give insight into the facts, skills, concepts and strategies directly.

[9] NCES (2003) figure 5.1 shows that problems with real-life connections are not uncommon, but further analysis is needed to say if the problems given are suited to improve students' attitude towards mathematics. Anyway, they are not connected to the history of mathematics.

example with any connection to this: a statement that the value of $\pi$ has been a problem for mathematicians from ancient times.

Working on history of mathematics will almost automatically make students aware that mathematics is the result of the work of generations – except if the history is presented in a way that makes students feel that mathematics has not changed at all for the last two thousand years. Be it the development of Cartesian coordinates or Euler's work on polyhedra, students will get a glimpse of mathematics in development. Most of the examples in the material work in this regard.

History of mathematics may also provide glimpses from the lives of mathematicians, and thereby making the subject more interesting. If the students get an understanding of the motivation behind some work on mathematics, that is even better. There is at least one good example of giving a human touch, when one teacher tells about Euler and his blindness. Most of the examples, however, seem to be collected pieces of biographical information (place and date of birth, date of death and so on), which are probably not very illuminating for the students. Moreover, the motivations of the mathematicians are never discussed.

### *4.6.5 Others*

Including history of mathematics in the mathematics teaching may also give other benefits. For instance, it may provide an opportunity for writing essays and using different kinds of source material. There is only one example of this kind alluded to in our material, where the students apparently have written a paper on one mathematician each. History of mathematics may also provide opportunities for cross-curricular work, but there are no examples of this in the material. It may be the case that teachers avoided this because the video taped lessons were supposed to be mathematics lessons. It is difficult to draw any conclusion from this. History of mathematics may also increase the respect of other cultures (also contemporary, foreign cultures). Egypt's pyramids are mentioned (but only in passing), Egyptian multiplication is also worked on. One teacher says about the Pythagorean theorem that

*Now, this was long ago which means that the math that we're doing today is still as important as it was five hundred years before the birth of Christ. So this shows you that this kind of thing that we're doing has been around a long time, and it still remains important. It also shows you a bunch of smart people back then too, okay?*

On the other hand, another teacher says, "the Babylonians are accredited with the fact of knowing what a right triangle is" – not very impressive. All in all, not much is done which may increase the respect of different cultures.

### *4.6.6 Preliminary conclusions*

Although I have noted a few exceptions, there is a similar pattern here as I have found in other places earlier: the history of mathematics included, often consists of not too useful pieces of biographical information, while information more connected to the mathematics as such often is ignored.

## 4.7 Is the history mentioned only in isolated instances?

It is interesting to see whether the teachers that mention the history of mathematics do so often or only in isolated instances. As the material in this study consists of isolated lessons, it is difficult to say much about this. However, in a few places we get some hints.

If a teacher mentions history of mathematics only in an isolated instance, you would perhaps not expect to be able to recognize that from the transcript. However, in one instance a teacher says, when talking about Euler, *"Which one is the other mathematician we dealt with? Oh, practically the only one… Pythagoras."* This suggests that history of mathematics may not be frequent in this teacher's lessons.

However, there are more examples of the opposite. One teacher mentions the *"mathematics report"* where students were supposed to write about a mathematician. Another mentions having talked about Sophie Germain earlier, and talks of *"those silly mathematicians I always give you"*. One teacher says that the class had looked at some historical examples in the last few weeks, and another reminds the class what he told them in an earlier lesson.

In one instance we see that the class will be working on (or at least reading about) history of mathematics later: *"We have the historical comments in the textbook. You will read them later on."*

My impression from this is that there are a few teachers who include history of mathematics as part of their teaching, but it seems that most teachers only make historical connections "in passing".

## 4.8 Errors

In Smestad (2002) I pointed out that there were many errors in the Norwegian elementary school textbooks. I have looked for errors in the TIMSS material as well, and found a few. However, the material is too small to be able to give any indication on what kind of errors are "typical". Therefore I do not comment on those errors in any detail here.

## 4.9 Teacher words vs. student words

A result I found interesting in the TIMSS 1999 Video Study was that teachers utter about ten times as many words as all the students combined during the "public interaction" part of the lessons. In the material related to history of mathematics, I have calculated a ratio of about 15 to 1.[10] This suggests that the history of mathematics is often lectured, with little discussion with the students. This is also the impression we get from reading the transcripts – the part that the students play is often only to read aloud from the textbook or to answer simple yes/no-questions (to show that they have been listening).

[10] I had to exclude the lesson with most history of mathematics from this calculation, as I did not have a complete transcript of this.

## 4.10 Conclusion

It seems that the history of mathematics does not constitute an important part of teaching in the 8th grade in these seven countries. Few lessons include history of mathematics, the history of mathematics is often lectured (with the students listening) and the information included is often biographical information of little connection to the mathematics taught. The rich ideas presented in the recent ICMI study by Fauvel & van Maanen (2000) have not yet reached these classrooms to any large extent.

## 4.11 References


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