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Bhanu Bhakta Kharel

Comparing Methods for System Identification on the Quadruple Tank Process

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Abstract:

System Identification in control engineering has been the field of interest for the research and development to find the optimal model of dynamic system. Both MIMO and SISO systems can be modeled using different system identification techniques with satisfactory result using input and output data and easy to implement the control strategies.

System identification of a quadruple Tank which is a MIMO system with two user inputs and two outputs and adjustable zero. Using first principle, non-linear mathematical model is developed and it is linearized. Input and output data are taken from the real system using LabVIEW and Stability, observability and controllability are analyzed. Both minimum and non-minimum phase with the change in valve parameters are analyzed.

Using input and output data from the real process system identification model is developed by DSR, PEM and N4SID methods. All these identification analysis are done using MATLAB software while the input and output data taken using model developed in LabVIEW. Model developed using all three methods are compared against each other both by simulation and experimentally. Model developed are validated individually using new set of data and the quality of methods are again compared against each other using indices such as MAE, RMSE.

Finally, DSR method of identification is found out to be the best method among all three methods and it is suggested to proceed for the implementation of control strategies and further analysis

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Preface

Master thesis on '' Comparing Methods for System Identification on the Quadruple Tank Process''
is done and submitted to Telemark University College (TUC) in Porsgrunn, Norway. For the
completion of the MSE degree, thesis report is prepared based on the experiments and simulation
completed under the supervision of David Di Ruscio

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Porsgrunn

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Abbreviation

SID	System Identification
SIM	System Identification Method
SISO	Single input single output
MIMO	Multiple input multiple output
DAQ	Data Acquisition
OLS	Ordinary Least Square
PEM	Prediction error model
ISE	Integral Square Error
IAE	Integrated absolute error
MAE	Mean absolute error
RMSE	Root mean square error
PRBS	Pseudo Random Binary Signal
MPC	Model Predictive Control
LQ	Linear Quadratic

1 Introduction

Identifying the system properties with computational efficiency and help the user to choose the correct system order are considered as the advantages using system identification methods. The process of constructing a mathematical model of a dynamic system. Models are used to extract the essentials from complicated evidence and to quantify the Implications aiming to increase the understanding of the mechanism by making the complicated system simpler.

Precise representation of a real world system dynamics developed via analysis and simulation can be defined as a model, though there may be multiple models/methods for a single physical system depending on the problem we want to solve.

1.1 System Identification Historical approach

The rule of system identification tries to build mathematical models of systems with certain purpose, guided by measurements and some other criterion depending on the process. These models are different from others like mental models or graphical models (Pajonk, 2009) as we can evaluate them using a computer they tend to be highly complex. Thus creating such a model can be a very challenging task. We have seen that many different types of systems exist and that there is essentially no limit in complexity for both the relations inside the system and the entities that interact in and out of the system. So the mathematical models that we create to describe certain aspects of a system can become arbitrarily complex too.

The problem of system identification is also pervasive in science and engineering, thus many different applications have resulted in a multitude of different approaches, model types and methods to solve the problem. Some of the methods are specific to the respective application or purpose while some have their broader use.

As it is the case with many modern methods, also the birth of system identification can be noticed back quite a long time. As an example the famous Gauss-Newton-method was developed by Carl Friedrich Gauss aimed to solve the system of equations arising in his non-linear least squares method for regression (Gevers, 2006). With the help of this method he found values of parameters in a model of the trajectory for the dwarf planet Ceres - which clearly is a system identification problem. This happened between 1795 and 1802, so more than 200 years ago. The method of least square Gauss developed is well known and, of course, still in use. It took another 150 years before the advent of electronic computers, and with it came the rise of (computational) system identification. According to (Gevers, 2006) the modern discipline of system identification started around 1960 as part of control theory. It was part of model-based control design, which was very much en vogue at that time due to the development of the Kalman filter.

System identification went from deterministic methods to maximum likelihood methods and finally to stochastic methods until the 1970. A various and confusing amount of approaches were developed until that time, so that structuring them became more and more necessary. The inevitable clean up period of 1975 to 1985 resulted in the first edition which was released in 1987 together with a MATLAB toolbox for system identification.

Then important steps were made in closed-loop identification, subspace based identification and non-linear identification. According to (Gevers, 2006) which contains a lot more detailed historical account of system identification, this is the current state of development.

1.2 Important uses of identification

There are several use cases for system identification procedures in industry and science. System identification is known for its important aspects listed below (Pajonk, 2009).

System Analysis We want to obtain more insight in a certain system.

Prediction with the current state of a system, to be able to predict the behavior of a certain system. The popular example is weather forecast, to predict.

Simulation Simulate the behavior of a system with given input.

Optimization Optimize a certain aspect of the true system (operate directly on it might not be possible) to find this optimum. Reasons may be cost, safety, security or time constraints.

Control Develop an advanced controller for a real process which involves a model of that process.

Fault Detection Detect false behavior of a true system by comparing the model output with the output of the true system.

1.3 Objectives

The main aim of system identification is to determine a mathematical model of a physical/dynamic system which is the Quadruple Tank Laboratory Process from observed data. The main objectives of this thesis is to develop a mathematical model, get input and output data from LabVIEW connecting it to the real process. Using the input and output data from the real process we will identify the model. Model developed using different system identification methods will be compared. Key steps that are involved in system identification process can be listed as:

(1) Develop an approximate analytical model of the structure, stability, observability and controllability.

(2) Establish levels of structural dynamic response which are likely to occur using the analytical model and characteristics of anticipated excitation sources.

(3) Determine the instrumentation requirements needed for quadruple tank (MIMO) process model to sense the process with prescribed accuracy.

(4) Perform experiments and record data, input and output data from real system.

(5) Apply system identification techniques to identify the dynamic characteristics such as system matrices, modal parameters using different identification methods like PEM, DSR and N4SID and

(6) Analysis the analytical models based on identified results using MATLAB.

1.4 Identification Process

Simplification as the initial step and followed by Parameterization we identify and consider the quantities that we need to describe the system as clearly as possible. Forward modelling as the next step where it depends on the available data and the use of the model. Finally inverse modelling using different methods basically using computer technology.

Forward modelling is simply done by deriving the model from first principle or using general purpose model and adopt it to our system (Pajonk, 2009). White box model is an example of it where the initial knowledge dominates the model but still something left after for the inverse modelling step (Pajonk, 2009). Thus the model do not depend at all on data and this might invalidate the inverse modelling step which is the central part of system identification. Black box model choose a generic model structure with number of parameters and follows different steps taking the measurements into account (Pajonk, 2009). Static black box models are used for the simpler linear equations to complex models that use Neural Networks and Neuro-Fuzzy models. Grey Box Model is introduced with the combination of White Box and Black Box Models (Pajonk, 2009). White Box Modelling is time consuming in comparison to Black Box and detailed domain knowledge is needed which helps the developer to understand the true system which is more reliable. Black box models, on the other hand, tend to be easily derived, even without explicit domain knowledge, by simply incorporating the measurements into a generic model structure (Pajonk, 2009). A big advantage is that we can use these models even if we do not have better understanding of the true system.

In inverse modelling actual model parameters values will be determined taking into considerations that the measurements of true system involves uncertainties and modelling issues (Pajonk, 2009). The main separation here will be between the deterministic and stochastic methods. Figure 1-1 below shows the schematic diagram of the system identification process.
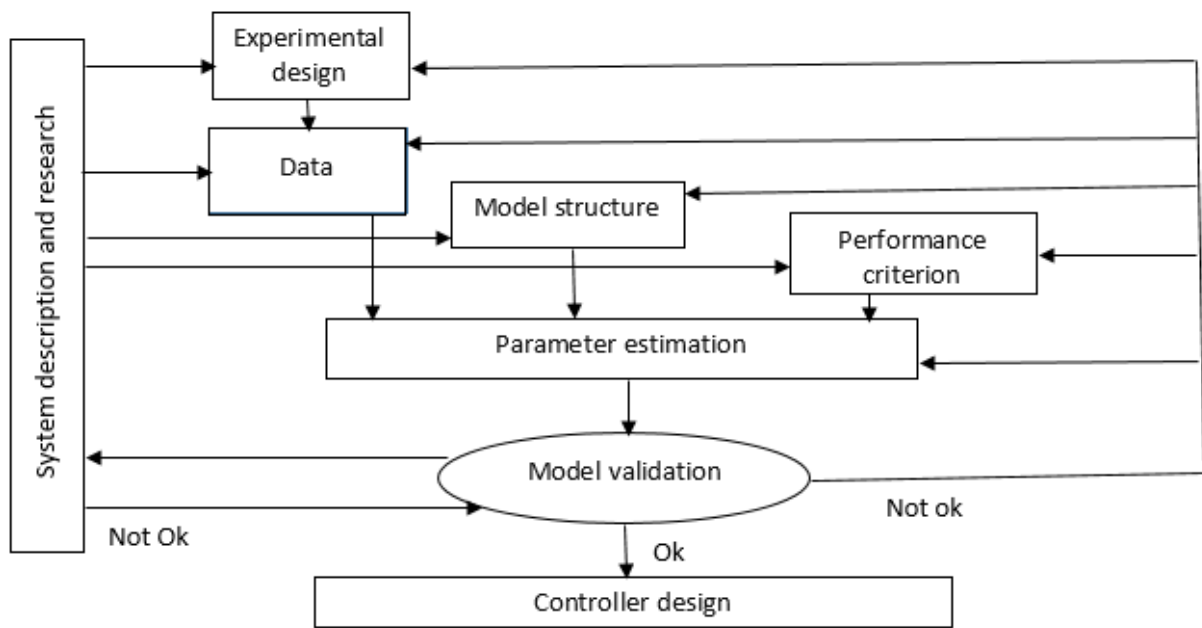
Figure 1-1: System identification process schematic diagram

Practically the above given flow chart describes the complete identification process in following steps (Pelckmans, 2012a) :

1) System description and research based on the criterion of the model, the purpose of the model. Properties that need to be focused on during the identification experiments and decide the identified model is satisfactory at the end.

2) Initial data is taken by understanding the effects of crucial importance. Knowing the challenges present in the task pinpoint the phenomenon displaying the graph of data.

3) Performance analysis with some initial experiments in order to compare the outcome. Possible analysis of correlation and random effects appear during the experiment. Collect some ideas of the form of disturbances.

4) Experimental design enumerates key challenges for identification and guide where to focus on during the experiment. Get as much information as possible that can be extracted from the observations of the experiment. Keeping the system in the ''operation mode'' during the experiment make sure the dynamics are sufficiently excited.

5) Model structure, what is the good model structure for the system is the key point to know. The parameters that can explain the behavior of the system. Model structure can be refined and the parameter estimation to compensate the effects that couldn't be expressed, example: order of dynamic model.

6) Validate model by observing if it gives the satisfactory results, explaining the important effects. Finally implement the model and get the work done.

1.5 Identification Methods

From the observed input-output data of the quadruple tank process we build a mathematical model which is the System Identification (SID) of the Process. The known inputs, outputs of the dynamic system can be measured and collected where these outputs are generally affected by the process error like process noise, disturbances and measurement noise. Due to this reason the model developed from the system identification process is considered more efficient as it includes all possible disturbances of the system. Thus, Identified model can then be used to implement various control strategies like Model Predictive Control (MPC), LQ Control etc.

Different methods for the system identification has been used and modified depending on the process structure. Several subspace identification methods, such as PEM, CCA, N4SID, MOESP and DSR are in use based on performance quality criteria and requirements of the process, in order to select the best-reduced model. We are going to use these three methods, DSR, PEM and N4SID and compare the results of our process control and select the best one for the further analysis. The selected model is validated with a data set not used in the identification procedure called validation data set to describe the complex dynamics of the process. This model is asymptotically stable and it can be used for control, optimization, prediction and monitoring purposes.

Subspace Identification Method (SIM) and Prediction Method (PEM) (Ljung, 1976) are the most used system identification methods where it optimizes the difference between predicted output and model output (Ljung, 2002). PEM, the idea is that rather than a plain least squares approach, or a statistical maximum likelihood approach there is a third important principle in use for estimating the parameters of a dynamic model based on recorded observations (Pelckmans, 2012b). This approach considers the predictions accuracy computed for the observations, rather than the model mismatch are the possibility of the corresponding statistical model. This approach perhaps is most tightly connected to systems theory as it explicitly exploits the dynamical structure of the studied system.

A subspace system identification method, based on observed input and output data, which estimates the system order as well as the entire matrices in the Kalman filter including the Kalman filter gain matrix, K, and the square root of the innovations covariance matrix, F, was presented in (Ruscio, 1995). This algorithm is implemented in the DSR function in the D-SR Toolbox for MATLAB. The DSR estimate of the innovation square root covariance matrix F is consistent both for open loop as well as for closed loop data. The DSR method was compared with other algorithms and found to give the best model in comparison to the other methods, based on validation, and on a real world waste and water example in (Sotomayor, 2003). The DSR e method presented in David Di Ruscio (2003) and used in the thesis by (Nilsen, 2006) is a simple subspace system identification method for closed and open loop systems. DSR e is a two stage method where the

innovations process is identified consistently in the first step. The second step is simply a deterministic system identification problem.

Numerical Algorithms for subspace state space system identification (N4SID) are viewed and used as another alternative in the system identification history. This seems to be good method especially for higher-order multivariable systems, for which it is not trivial to find a useful parameterization among all (Peter Van Overschee, 1994). This parameterization is needed for the start of classical identification algorithms which signifies that a-priori knowledge of the order, controllability and observability indices is required.

Using N4SID method of system identification most of this a-priori parameterization can be avoided and only the system order is needed where it can be obtained by inspection of dominant singular values of a matrix calculated during identification (Peter Van Overschee, 1992). State space matrices are not calculated in canonical forms rather as full state space matrices in optimally conditioned basis which means there is no problem in identification. Another advantage of using N4SID can be their non-iterative process without non-linear optimization. This is because this method is free from the disadvantage of iterative algorithms like, local minima of the objective criterion sensitivity to initial estimators.

2 Model development

In this section the four tank process (Quadruple Tank), a model developed by Johansson et al., 1990 and constructed in Telemark University College is described in detail. Mathematical non-linear model for the quadruple tank process is derived using first principle and linearized around some nominal values.

2.1 Characteristics of Experimental (Quadruple) tank

Certain characteristics and parameters of the quadruple tank were calculated during the experiment process. Among them correlation of the discharge flow through the tank and the correlation of the flow rate of pump were studied to minimize the error.

Tank discharge flow correlation was simply calculated by taking the level of the appropriate tank to the steady level. Measuring the water level and total volume of water discharged in known time we estimate the flow rate. Measurement from each tank is taken and using the relation shown in Equation (2-1) parameters are calculated using Ordinary Least Square (OLS) method.

$$q^{out}(t) = C^1 h_1(t) + C^0 \qquad (2\text{-}1)$$

$$Y = X\theta$$

Where,

$C^0 = flow\ in$

$C^1 = flow\ out$

$$Y = \begin{bmatrix} q_1^{out} \\ q_2^{out} \\ . \\ q_n^{out} \end{bmatrix}, \quad X = \begin{bmatrix} h_1 & 1 \\ h_2 & 1 \\ . & . \\ h_3 & 1 \end{bmatrix}, \quad \theta = \begin{bmatrix} C^1 \\ C^0 \end{bmatrix} \qquad \forall\ i = 1,2,3 \dots N\ (i = number\ of\ mearurements)$$

Unknown parameter vector θ can be calculated as

$$\theta = (X^T X)^{-1} X^T Y$$

Parameters for each of the tank are calculated and are listed in Table 2-1 below.

Table 2-1: Rate of inflow and out flow for each tank by experiment

4-Tanks	$C^1(cm^{2.5}/s)$	$C^0(cm^3/s)$
Tank 1	6.5123	8.7126
Tank 2	4.5143	10.9821
Tank 3	4.1312	14.8761
Tank 4	4.4206	7.7298

Flow rate by the pump is considered as an important characteristics for the efficiency of the model. This is done simply by giving different input (Voltage) at different known time stamps in the laboratory model developed in LabVIEW and the level in the tank is measured.

Relation in Equation (2-2) shows that the flow generated by each pump is directly proportional to the voltage applied.

$$F_1 = k_1 u_1 \qquad\qquad (2\text{-}2)$$

Parameters obtained from the experiment are used to calculate the correlation with the help of the expression in Equation (2-2). MATLAB script polyfit() was used to find the solution and the code is attached in Appendix A.

Table 2-2 below shows the pump characteristics obtained from the experiment.

Table 2-2: Characteristics of pump1 and pump 2

Characteristics of pump	cm^3/s	cm^3/s
Pump 1	k1 =9.3894	c1 = -9.8774
Pump 2	k2 =8.9727	c2 = -8.0986

2.2 System Description

A quadruple tank process (Johansson, 2000) was designed and constructed to give the multivariable control concept for the academic students. It was used mainly for constructing transfer functions of multivariable systems and linearizing the nonlinear dynamics and selecting control structure based on multivariable process (Johansson, 2000).

Identification of the real process begins with the mathematical model development of the system. We develop the model in LabVIEW connecting to the DAQ system. The real 4-Tank process is

than connected to the LabVIEW model developed in the personal computer. Every process of the experiment was completed in PLS Laboratory of Telemark University College. Figure 2-1 below shows the connection and the experimental rigs used.

Figure 2-1: Real 4-Tank system connected to the computer system

A full physical system of the quadruple tank process is illustrated in Figure 2-1. Our task is to control the level of the liquid in the lower two tanks (tank 1 and tank 2) with two pumps available to pump the water from the basement. The process inputs are u_1 and u_2, input voltages to the pump 1 and pump 2 and the outputs are the level of liquid at tank 1 and tank 2.

Process is designed in such a way that the liquid through the pump 1 goes to both tank 1 and tank 4 and that of pump 2 goes to tank 2 and tank 3 using three way valve (Samson 5824-10). The ratio can be manipulated by the operator using voltage signal of 0-5V. Lower tanks (tank 1 and tank 2) also receive the gravity flow i.e. flow from tank 3 and tank 4 respectively. Output y_1 and y_2 from the process which is the level of the tank 1 and tank 2 is measured using the level sensors (BD SENSORS LMK 351, 0-40 mbar, 4-20 mA) which gives the signal in voltage.

As each of the pumps affects both of the outputs this system exhibits multivariable dynamics with adjustable multivariable zero where the position can be changed by the valve settings of the experiment. The process with the schematic diagram is given in Figure 2-2 below.

This process has been designed and studied in control courses in many universities. The research and investigation of this system has also yielded plenty of conference and journal papers. A number of these reports, can be (S. Dormido, 2004), (R.Suja Mani Malar, 2009).

2.3 Development of mathematical model

As shown in the Figure 2-2, the schematic diagram of the 4 tank process and its operating procedure is described. With the aim to control the lower two tanks of the process using the controller input signals u_1 and u_2 (voltage signals to the pump) which gives the process outputs as y_1 and y_2, voltage signal from the level sensors.
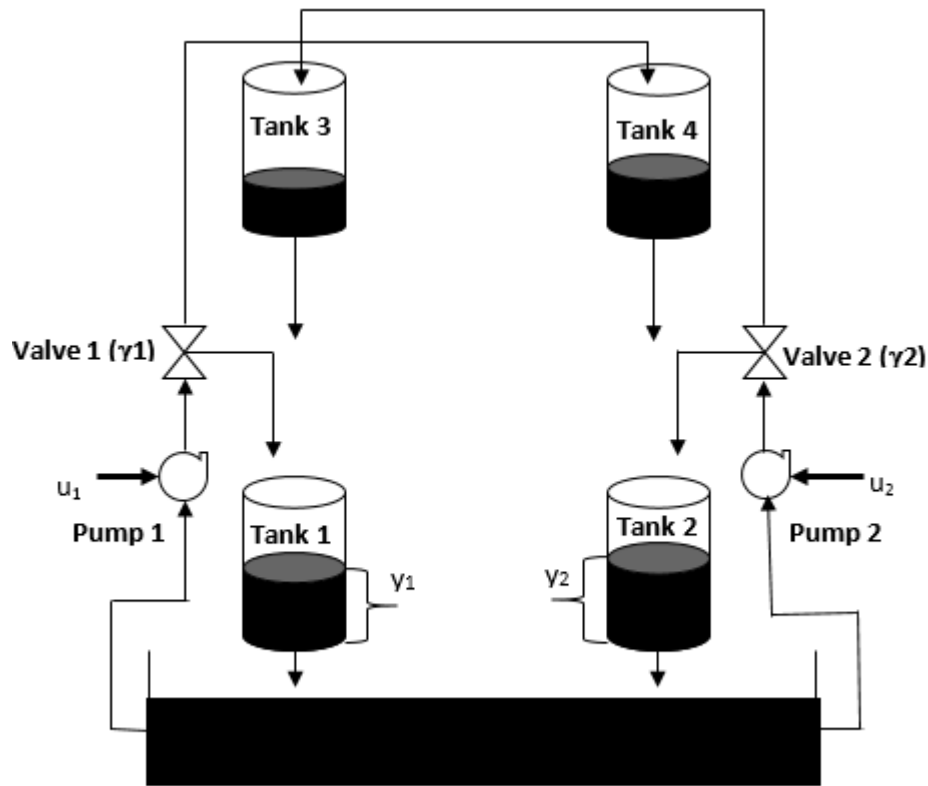
Figure 2-2: Schematic diagram of the 4 tank process

Sensors read the level of the tank 1 and tank 2 only out of 4 tanks and we are to control them but the input signal given is divided in two by a valve in a pump. Input u1 is divided to tank 1 and tank 4 in a ratio of γ_1 and u2 gives the input to tank 2 and tank 3 in a ratio of γ_2. Moreover, the input to the lower tanks tank 1 and tank 2 also adds the outflow from the upper tanks, tank 3 and tank 4 respectively. Thus, the experiment can be carried out in two different phases, the minimum phase and the non-minimum phase. This state can be chosen by adjusting the position of the valves (Johansson, 2000). Minimum phase where the majority of input goes to the lower two tanks ($\gamma_1 +$

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$\gamma_2 \geq 1$) and the non-minimum phase where majority goes to upper tanks ($\gamma_1 + \gamma_2 \leq 1$). Non-minimum phase is relatively complex to control in real life. This thesis will mainly focus on the minimum phase of the system though the non-minimum phase will also be studied partly later in chapter 3.

2.3.1 Nonlinear system

Dynamic model of the process can be developed by starting with the simple mass balance equation for the cylindrical tank as shown in Equation (2-3). This will lead to the nonlinear model for the quadruple tank process.

$$\frac{dm(t)}{dt} = \dot{m}_{in} - \dot{m}_{out} \tag{2-3}$$

Where,

$\dfrac{dm(t)}{dt}$ = rate of change mass inside the tank

$\dot{m}_{in}$ = mass flow of water into the tank

$\dot{m}_{out}$ = mass flow of water out of the tank.

Assuming the constant density of liquid the equation above can be expressed in volumetric flow as in (2-4).

$$\rho\frac{dV(t)}{dt} = \rho(\dot{q}_i - \dot{q}_o) \tag{2-4}$$

Using $dV(t) = Adh(t)$, (2-4) can be written as,

$$\frac{dh(t)}{dt}\frac{1}{A}(\dot{q}_i - \dot{q}_o) \tag{2-5}$$

Where, A is the cross section area of each tank.

Using Bernoulli's law, expression for the flow out can be expressed as in Equation (2-6).

$$q_i^{out} = a_i\sqrt{2gh_i}, \qquad i \; \forall \; 1,2 \tag{2-6}$$

Where, 'a' is area of outlet hole and the 'g' is acceleration due to gravity.

Flow generated by the pump is then divided in the ratio given by the relation, $\gamma \in [\gamma_1, \gamma_2]$ where, $0 \leq \gamma \leq 1$. Characteristics of the process and some assumptions made are shown in Table 2-3.

Table 2-3: Assumptions made for the easy mathematical model development

Assumptions	Functions
γ_1 and γ_2	split ratio for valve 1 and valve 2
F_1 and F_2	flow through Pump 1 and Pump 2
K_1 and K_2	Pump characteristics
c_1 and c_2	Pump characteristics

Now the flow through the each outlet of the pipe can be shown as in Equation (2-7)

$$\begin{cases} q_1^{in} = \gamma_1 F_1 = \gamma_1(K_1 u_1 + c_1) \\ q_2^{in} = \gamma_2 F_2 = \gamma_2(K_2 u_2 + c_2) \\ q_3^{in} = (1 - \gamma_2)F_2 = (1 - \gamma_2)(K_2 u_2 + c_2) \\ q_4^{in} = (1 - \gamma_1)F_1 = (1 - \gamma_1)(K_1 u_1 + c_1) \end{cases} \tag{2-7}$$

Using Equation (2-6) and Equation (2-7) in Equation (2-5) the final nonlinear model for the quadruple tank (Tank 1, Tank 2, Tank 3 and Tank 4) can be expressed as in Equation (2-8).

$$\begin{cases} \dfrac{dh_1}{dt} = \dfrac{\gamma_1(K_1 u_1 + c_1)}{A_1} + \dfrac{a_3\sqrt{2gh_3}}{A_1} - \dfrac{a_1\sqrt{2gh_1}}{A_1} \\ \dfrac{dh_2}{dt} = \dfrac{\gamma_2(K_2 u_2 + c_2)}{A_2} + \dfrac{a_4\sqrt{2gh_4}}{A_2} - \dfrac{a_2\sqrt{2gh_2}}{A_2} \\ \dfrac{dh_3}{dt} = \dfrac{(1 - \gamma_2)(K_2 u_2 + c_2)}{A_3} - \dfrac{a_3\sqrt{2gh_3}}{A_3} \\ \dfrac{dh_4}{dt} = \dfrac{(1 - \gamma_1)(K_1 u_1 + c_1)}{A_4} - \dfrac{a_4\sqrt{2gh_4}}{A_4} \end{cases} \tag{2-8}$$

2.3.2 Steady state analysis of the model

Applying the steady state condition to the nonlinear system, all time varying variables settled to some constant value. It gives that $\dot{h}_i = 0 \quad i \, \forall \, 1, 2, 3, 4$ for each tank. Now we lead to the four equations for the six steady state values $h_1^s, h_2^s, h_3^s, h_4^s, u_1^s \ and \ u_2^s$. We are to control the level of the tank 1 and tank 2, thus selecting h_1^s and h_2^s and solving the Equation (2-8) in section 2.3.1 gives the Equation (2-9).

$$\begin{cases} 0 == \dfrac{\gamma_1(K_1 u_1^s + c_1)}{A_1} + \dfrac{a_3\sqrt{2gh_3^s}}{A_1} - \dfrac{a_1\sqrt{2gh_1^s}}{A_1} \\[2mm] 0 = \dfrac{\gamma_2(K_2 u_2^s + c_2)}{A_2} + \dfrac{a_4\sqrt{2gh_4^s}}{A_2} - \dfrac{a_2\sqrt{2gh_2^s}}{A_2} \\[2mm] 0 = \dfrac{dh_3}{dt} = \dfrac{(1-\gamma_2)K_2 u_2^s}{A_3} - \dfrac{a_3\sqrt{2gh_3^s}}{A_3} \\[2mm] 0 = \dfrac{(1-\gamma_1)K_1 u_1^s}{A_4} - \dfrac{a_4\sqrt{2gh_4^s}}{A_4} \end{cases}$$

<div align="right">(2-9)</div>

Thus the steady state equation for the tank 3 and tank 4 can be presented as in Equation (2-10) and Equation (2-11).

$$(1-\gamma_2)K_2 u_2^s = a_3\sqrt{2gh_3^s} \tag{2-10}$$

$$(1-\gamma_1)K_1 u_1^s = a_4\sqrt{2gh_4^s} \tag{2-11}$$

System of two linear equations can be developed using the steady state equation for tank 1 and tank 2 and the above derived equation for tank 3 and tank 4. It can be expressed in the form as in Equation (2-12).

$$\begin{bmatrix} a_1\sqrt{2gh_1^s} \\ a_2\sqrt{2gh_2^s} \end{bmatrix} = \begin{bmatrix} \gamma_1 K_1 & (1-\gamma_2)K_2 \\ (1-\gamma_1)K_1 & \gamma_2 K_2 \end{bmatrix} \begin{bmatrix} u_1^s \\ u_2^s \end{bmatrix} \text{ Or}$$

$$\begin{bmatrix} u_1^s \\ u_2^s \end{bmatrix} = \begin{bmatrix} \gamma_1 K_1 & (1-\gamma_2)K_2 \\ (1-\gamma_1)K_1 & \gamma_2 K_2 \end{bmatrix}^{-1} \begin{bmatrix} a_1\sqrt{2gh_1^s} \\ a_2\sqrt{2gh_2^s} \end{bmatrix} \tag{2-12}$$

Following the steady state values for h_3^s and h_4^s are,

$$h_3^s = \left(\frac{(1-\gamma_2)K_2 u_2^s}{a_3\sqrt{2g}}\right)^2 \text{ and } h_4^s = \left(\frac{(1-\gamma_1)K_1 u_1^s}{a_4\sqrt{2g}}\right)^2$$

Note: for $\gamma_1 + \gamma_2 = 1$ steady state voltage cannot be computed with the above given expression since matrix in equation is not invertible and the determinant is equal to 0. We cannot choose h_1 and h_2 independently.

2.3.3 Linearization of the model

We have discussed the nonlinear dynamics of the model (quadruple tank) in section 2.2.1 above as the basic for the control problem. Further analysis like frequency response, stability analysis etc. depends on the linear model (Haugen, 2010) which describes the behavior of the system around nominal or operating values.

Nonlinear model mentioned in above section can be presented in the form Equation (2-13) and Equation (2-14).

$$\dot{x} = f(x, u) \qquad \qquad \text{(2-13)}$$

$$y = g(x, u) \qquad \qquad \text{(2-14)}$$

Where

$x \in \mathbb{R}^n$ is state vector

$u \in \mathbb{R}^r$ control input vector

$y \in \mathbb{R}^m$ output vector

For our quadruple tank process we have four state variables, two control variables and two output variables which can be represented as:

    State Variables $(x) = [h_1, h_2, h_3, h_4]^T$

    Input variables $(u) = [u_1, u_2]^T$

    Output variables $(y) = [y_1, y_2]^T$

Now the linearization of the non-linear model above will be done by taking the first two linear terms of the Taylor series expansion, linear model is obtained as shown in Equation (2-15).

$$f(x, u) = f(x_0, u_0) + \frac{\partial f}{\partial h^T}\Big|_{(x_0, u_0)} \cdot (x - x_0) + \frac{\partial f}{\partial u^T}\Big|_{(x_0, u_0)} \cdot (u - u_0) \qquad \text{(2-15)}$$

Considering the initial values x_0 and u_0 are known.

Deviation variables and matrices can be defined as:

$$(x - x_0) = \Delta x, \qquad (u - u_0) = \Delta u$$

State matrices and Input matrices

$$\frac{\partial f}{\partial x^T}\Big|_{(x_0, u_0)} = A_c,$$

$$\frac{\partial f}{\partial u^T}\Big|_{(x_0, u_0)} = B_c$$

Here A_c and B_c are state and input matrix respectively, now the linearized State Space model can be written as in Equation (2-16).

$$\Delta \dot{x} = A_c \Delta x + B_c \Delta u + v \qquad (2\text{-}16)$$

But, v here is usually zero since x_0 and u_0 are constant value.

$$v = f(x_0, u_0) - \dot{x}_0$$

Thus,

$$\dot{x}_0 = f(x_0, u_0) = 0$$

And finally linearized state space equation can be written as,

$$\Delta \dot{x} = A_c \Delta x + B_c \Delta u \qquad (2\text{-}17)$$

Output of our process are the level of tank 1 and tank 2 and the equation of it can be obtained by Taylor series expansion which is given as,

$$\Delta y = D \Delta x \qquad (2\text{-}18)$$

Now linearized state space equation in matrix form Ac, Bc and D can be given as shown below. Complete linearization process is attached in Appendix B.

$$Ac = \begin{pmatrix} -\dfrac{1}{T_1} & 0 & \dfrac{A_3}{A_1 \cdot T_3} & 0 \\[2ex] 0 & -\dfrac{1}{T_2} & 0 & \dfrac{A_4}{A_2 \cdot T_4} \\[2ex] 0 & 0 & -\dfrac{1}{T_3} & 0 \\[2ex] 0 & 0 & 0 & -\dfrac{1}{T_4} \end{pmatrix}$$

$$Bc = \begin{pmatrix} \dfrac{\gamma_1 \cdot k_1}{A_1} & 0 \\[2ex] 0 & \dfrac{\gamma_2 \cdot k_2}{A_2} \\[2ex] 0 & \dfrac{(1-\gamma_2) \cdot k_2}{A_3} \\[2ex] \dfrac{(1-\gamma_1) \cdot k_1}{A_4} & 0 \end{pmatrix}$$

$$D = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

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And the time constants are defined as

$$T_i = \frac{A_i}{a_i} \cdot \sqrt{2 \cdot \frac{h_i}{g}} \qquad \forall i = 1,2,3,4$$

2.4 Stability analysis

Study of system stability is important for the analysis of control strategy. However, we will briefly discuss about the Operating point, controllability and the observability of the model and validate it in order to make the analysis easier for the upcoming problems.

2.4.1 Zero location and Operating point

Changing the valve position of the tank, for both minimum phase and non-minimum phase the multiple zero dynamics can be studied has been presented by (Johansson, 1999). It is clearly mentioned that the system is non-minimum phase for $0 < \gamma_1 + \gamma_2 < 1$ and minimum phase for $1 < \gamma_1 + \gamma_2 < 2$. Which implies the system $y(s) = H_P(s)u(s)$ which has zeros in the right half plane is called non-minimum phase.

Minimum phase is quite easier to control as the flow (majority) goes to the lower tank in comparison to the non-minimum phase where the valve position are set to pump the flow in the upper tank than in lower one which is hard to control in real.

We have selected minimum phase setting for the system identification of the model. Due to the inequality of the flow in the upper tank, though experiment was done and data were taken for both phases. Setting the valve position and input and input voltage experiment was done till the process state variable becomes steady and the nominal values of linearized system are collected as listed below.

Table 2-4: Nominal values of the linearized system

Valve position (γ)	$\gamma_1 = 0.7$	$\gamma_2 = 0.7$
Input (u)	$u_1 = 3.2$	$u_2 = 3.3$
level (h)	$h_1 = 13.36$ & $h_2 = 2.3$	$h_3 = 8.53$ & $h_4 = 5.63$

2.4.2 Controllability and Observability of the model

According to the (Ruscio, 1995b), The pair (A, B) is said to be controllable if and only if the controllability matrix C_n Equation (2-19) holds the rank n.

$$C_n = [B \quad AB \quad A^2B \quad . \quad . \quad A^{n-1}B] \quad \in \mathbb{R}^{n \times n.r} \qquad (2\text{-}19)$$

$$\text{Rank}(C_n) = n$$

This is valid for both continuous and discrete time models and it is calculated $(C_n) = 4$ for our system.

Checking the observability of the system is very important as it is possible to compute the state vector elements x(t), by using the known system input vector u(t) and the system output vector y(t). From the theorem described by (Ruscio, 1995b). The pair (A, D) is said to be observable if and only if the observability matrix O_n in Equation (2-20) holds the rank n.

$$O_n = \begin{bmatrix} D \\ DA \\ . \\ . \\ DA^{n-1} \end{bmatrix} \in \mathbb{R}^{n.m \times n} \qquad (2\text{-}20)$$

$$\text{Rank}(O_n) = n$$

This is valid for both continuous and discrete time models. Calculation done and it shows that $(O_n) = 4$ for our system.

3 System identification of the process using different methods

In this section of the report we will deal with different identification process applied on quadruple tank system. Individual methods like using PEM and DSR were studied in Telemark University College by some master students under the supervision of David Di Ruscio in their Master's thesis and even in Master's project. Here we will discuss on three different identification methods, DSR, PEM and N4SID. We will then be able to compare the results of the process for the implementation of various control strategies like MPC, LQ etc. Implementation of control strategies will not be the part of this report though will be the suggestion for future work.

3.1 Experimental design

Experimental Model of quadruple tank process was designed in LabVIEW so that the data of the real process can be implemented to design the model. Our experiment is open loop experiment where we don't have any feedback and no any control to get the minimum correlation between input and output. Open loop experimental design of our system is shown in Figure 3-1.
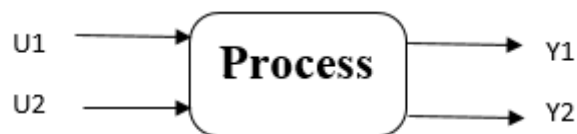
Figure 3-1: Experimental design

Real process data (input data and output data) from four tank process is collected using model designed in LabVIEW program. NI-DAQ devices are connected to the tank for input and output signals from sensors connected to the process. Flow chart of the complete process to extract input and output data is shown in Figure 3-2.
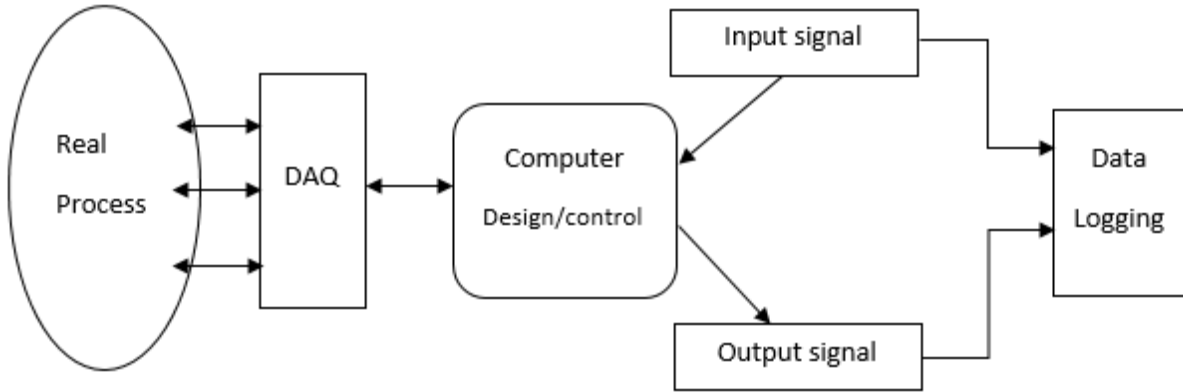
Figure 3-2: figure showing the flow diagram of input experimental design

Computer Design includes designing of LabVIEW model for reading of input signals, output signals, scaling, converting the voltage signal to corresponding level measurements. User Interface designed in LabVIEW is shown in Figure 3-3.
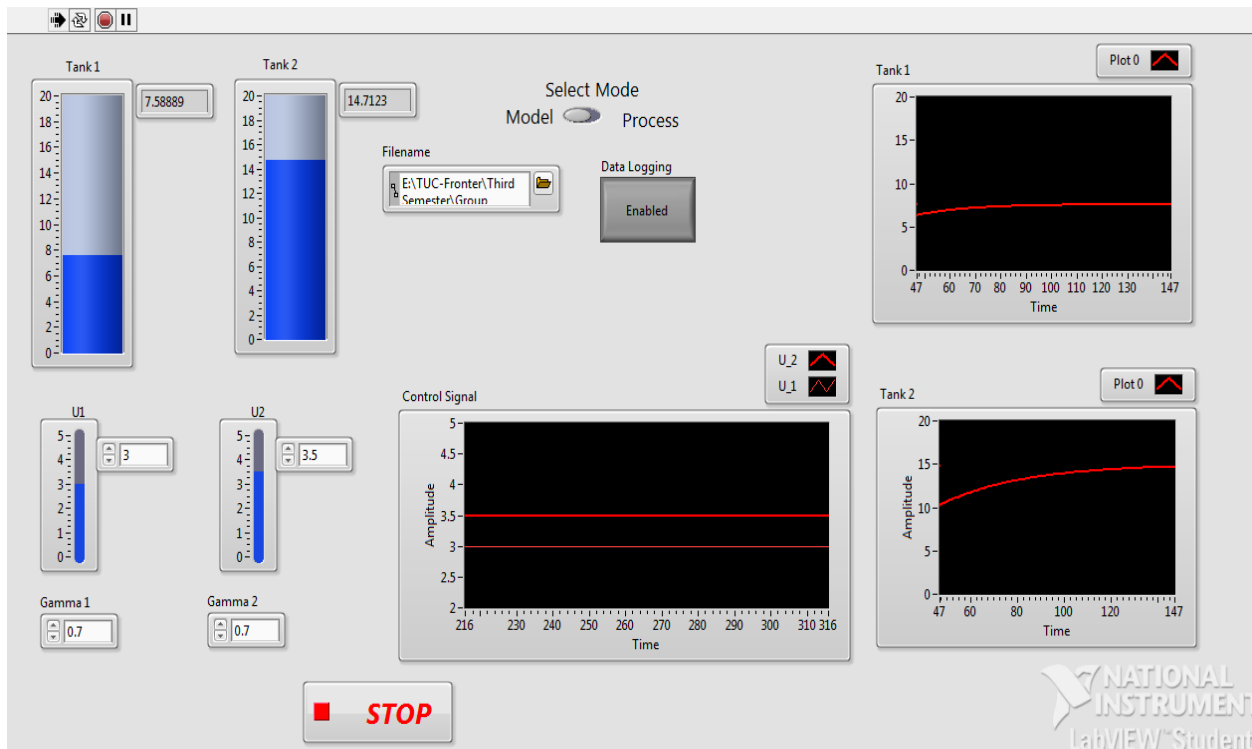
Figure 3-3: User Interface of LabVIEW design

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3.2 Data collection

As mentioned in earlier section data is collected using the model developed in LabVIEW. It is important that the model depends on better experiment with various data and it is required that input signal excite the process with different inputs knowing the structure of process.

In quadruple tank process Input signal is in voltage ranging from 0-5 Volts, both u1 and u2 input for Tank 1 and Tank 2 respectively range the same. After testing the model developed in LabVIEW it was implemented on four tank process. In order not to let the tank overflow the maximum input went up to 75 (5V=100) in maximum considering both input pump on working stage. Minimum input for the pump to pump the water in the tank is 50 but we have minimum input used for our experiment is 54 for the operation. Different input signal for each tank is selected at the mean time in order to capture different states of the process. Figure 3-4 showing the graph of input signal is attached below and the MATLAB code is attached in Appendix C.
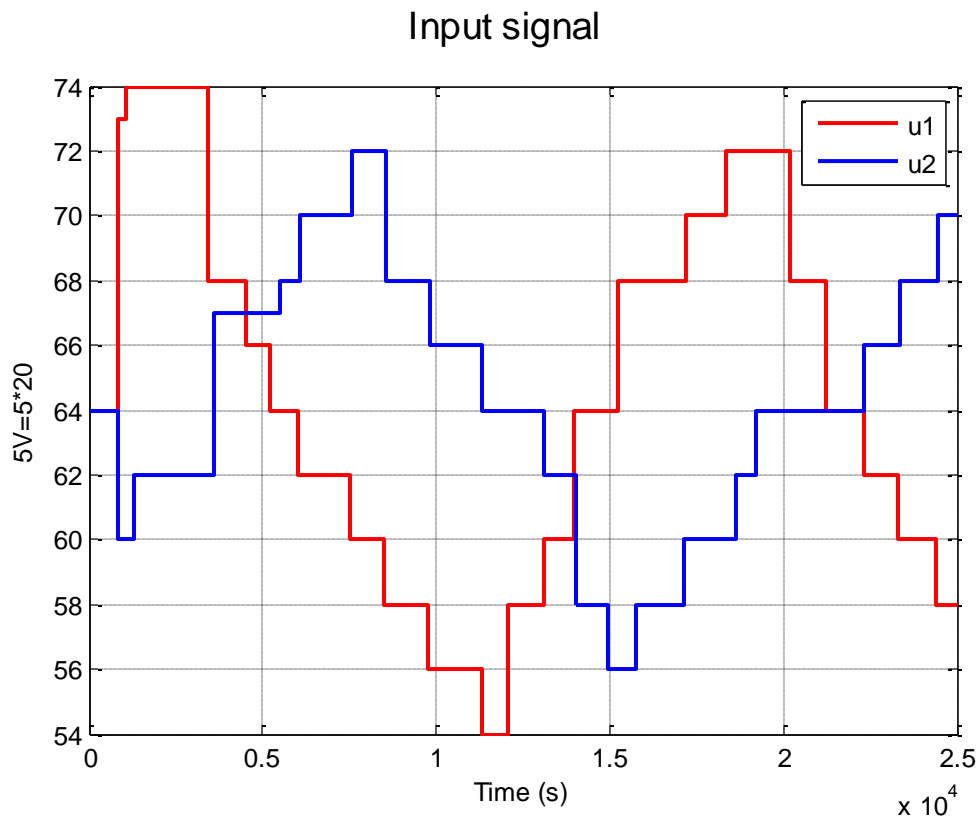
Figure 3-4: Figure showing the input signal of the process

It is still important that an operator give certain time for specific input in order for the system to reach the steady state for that input. Using Pseudo-Random Binary Signals (PRBS), inbuilt MATLAB function we generate random binary signal to modify according to the requirements.

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These signals are periodic, deterministic signals including the properties of white noise with ad advantage that they are easy to implement in real, suitable for system identification experiment. The period of the PRBS is as large as possible and equals $M = 2^n - 1$ if the coefficients are chosen correctly (Aarts, 2011/2012). The binary signal *s(t)* can be transformed into a signal *u(t)* with amplitude *c* and mean *m* with $u(t) = m + c(-1 + 2 s(t))$ (Aarts, 2011/2012).

 PRBS signal generated from MATLAB is shown in Figure 3-5 while similar input signal is used to generate input data.
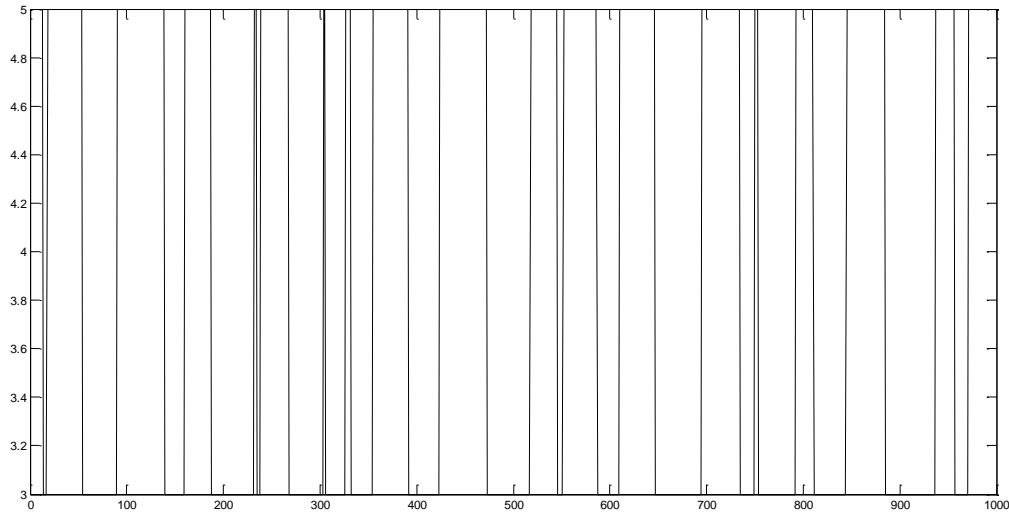
Figure 3-5: An example of PRBS signal

Input signal is adjusted according to the output in order not to let the level above and below the limit (0-20 cm), since the level in lower tank depends on the input given and the outflow from the upper tank. Here the split ratio (γ_1 and γ_2) plays an important role for the input we choose. Thus non-minimum phase is considered complicated in real process, though it is studied briefly in this thesis.

Now the constant trends like mean values and linear terms will be removed from the raw measurements. Trends can be nonzero constants or mean values and low frequency noises (Ruscio, 1995b). For the scaling and centering of raw data an inbuilt function is used, this improves the performance of the output model. Technical approach is to remove the mean from the individual samples for removing the trends.

Again the plot of the input and output signal from the real process is as shown in Figure 3-6 and the MATLAB code is attached in the Appendix C.
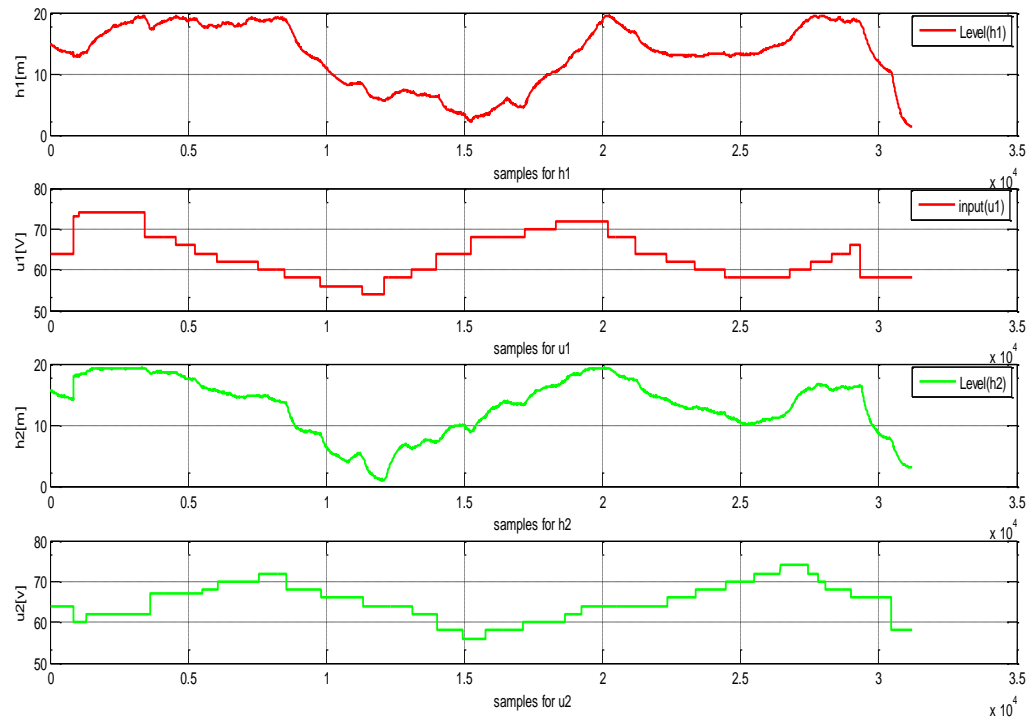
Figure 3-6: Input and Output signal from the real process

Scaling for the plot is done just by adding the mean values to the trended data. In the figure upper two red lines represents the output and input from Tank 1 while the lower two green lines represents the level and input at Tank 2 respectively. This set of data represents total of 35,000 samples which will later be divided in experimental set to develop model and validation set to validate the model.

3.3 System identification using simulated data

In order to validate model we first check the model using simulated data. Figure 3-7 below shows the simulated output and the identified model from the simulated data using nonlinear mathematical model of the quadruple tank. Identification is done using DSR method giving PRBS input signal with red color plot is for input 1 and black color shows the input 2 with the model order 4 and horizon L=4 and J=L. MATLAB code used for the process is attached in Appendix D.
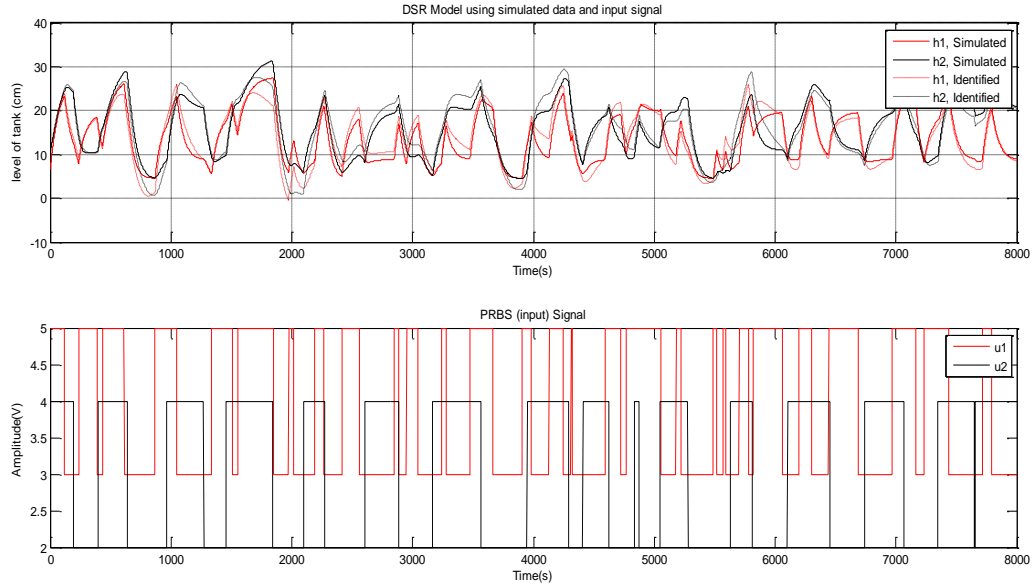
Figure 3-7: Identification of simulated data

The result from the identification model of simulated data is satisfactory as shown in Table 3-1.

Solid red line shows the simulated plot for level at tank 1 while the dashed red line is for identified model for level at tank 1, similarly solid black is for simulated output for tank 2 and dash black line for identified model of level at tank 2.

Giving the order (n) =4 and horizon of 4, MAE for level at Tank 1 and Tank 2 are 1.1584 and 1.1946 respectively, where the Root Mean Square Error for level at Tank 1 is 0.1154 and Tank 2 is 0.1094 which seems good enough. Zeros are 0.9983 and 0.9849 keeps the system stable as they are within the unity circle. Observing the result from the simulated model we decided to proceed with the model we have developed.

Table 3-1: Result from simulated identified model using DSR method.

Order (n)	Mean Absolute Error (MAE)	Root Mean Square Error (RMSE)	Poles	Zeros	Remarks
4	h1= 1.1584 h2= 1.1946	h1= 0.1154 h2= 0.1094	0.9624 0.9938 + 0.0065i 0.9938 - 0.0065i 0.9931	0.9983 0.9849	Minimum phase with Split ratio $\gamma_1 = 0.7\ and$ $\gamma_2 = 0.7$

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Discrete state space model using DSR algorithm for the minimum phase can be given as in Equation (3-1).

$$\begin{cases} x_{k+1} = Ax_k + Bu_k + v \\ y_k = Dx_k + Eu_k + w \end{cases} \hspace{3cm} (3\text{-}1)$$

Where, u_k and y_k are measured input and output and x_k is state.

A, B, D and E matrices are:

$$A = \begin{bmatrix} 0.9893 & -0.0173 & -0.8397 & -0.2983 \\ 0.0023 & 0.9825 & 0.2921 & -0.8223 \\ 0.0001 & 0.0004 & 0.9949 & 0.0345 \\ -0.0000 & -0.0002 & 0.0015 & 0.9763 \end{bmatrix}$$

$$B = \begin{bmatrix} -0.1868 & -0.1853 \\ 0.2095 & -0.1486 \\ -0.0041 & 0.0046 \\ 0.0015 & -0.0018 \end{bmatrix}$$

$$D = \begin{bmatrix} -0.3316 & 0.3815 & -0.5843 & 0.3364 \\ -0.3848 & -0.3437 & -0.3272 & -0.5778 \end{bmatrix}$$

$$E = \begin{bmatrix} -0.0001 & -0.0014 \\ -0.0003 & -0.0002 \end{bmatrix}$$

3.4 System Identification using real data

In this section of the report data collected from the real system will be used to develop the model. From the total samples 25,000 samples will be used to develop model and 12,000 samples will be used to validate the models. Both minimum and non-minimum phase will be experimented only by DSR method and compare to each other.

3.4.1 DSR method of identification

In DSR there are four parameters g, n, L and J that can be chosen by the user (Ruscio, 2003). If the structure parameter g is 0, usually default in DSR, data matrix E is identified. E is zero matrix if the value of g is zero. Parameter n specifies the model order and L is the number of block rows in extended observability matrix. Order is chosen in the interval of $1 \leq n \leq L.m$, where m is the number of outputs. J finally is the number of tie instants in the past horizon used to define the instrument variable matrix to remove noise. Minimum error is detected by using the MATLAB code for the values of J and L from 2 to 10, running the ''for loop'' in MATLAB executes the best MAE result using best suitable values of L and J.

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```
optimal_L =

       3


optimal_J =

       2
```

Figure 3-8: Figure showing the optimal values of J and L executed in MATLAB

Quadruple tank process is an open loop case so we should choose the L parameter as small as possible to reduce the variance of the estimates especially if the input signals are poorly excited.

Identification of real data using DSR method is carried out after the successful completion of identification of simulated data. Executing the system with different order value (n) and also with different numbers of block rows in extended observability matrix (L), error between the model of real data and the DSR model were evaluated. Optimal values of L and n used and the evaluation of model is shown in Table 3-2 below.

Table 3-2: Table showing the errors given by DSR method using optimal value of L and n for minimum phase.

L	J	Order (n)	Mean Absolute Error (MAE)	Root Mean Square Error (RMSE)	Remarks
3	2	3	h1= 0.6987 h2=0.7173	h1=0.1988 h2=0.3441	Minimum phase with Split ratio $\gamma_1 = 0.7$ $\gamma_2 = 0.7$

After executing the MATLAB code with different values of L, n and J, result with *L=3, n=3* and *J=3* gives the best suitable (minimum) error, MAE for level in Tank 1 is 0.6987 and for Tank 2 is 0.7173 which is relatively a good model as shown in Table 3-2. Also the RMSE value for level in Tank 1 is 0.1988 and Tank 2 is 0.3441 is satisfactory result too. This is the case for the minimum phase, non-minimum phase is followed later in this section. DSR method of identification is compared with other methods in chapter 4.

The resultant graph of the real process output data and the identified DSR model for both level in Tank 1 and Tank 2 is plotted and shown in Figure 3-9 using code in MATLAB and the code are attached in Appendix E of the report.
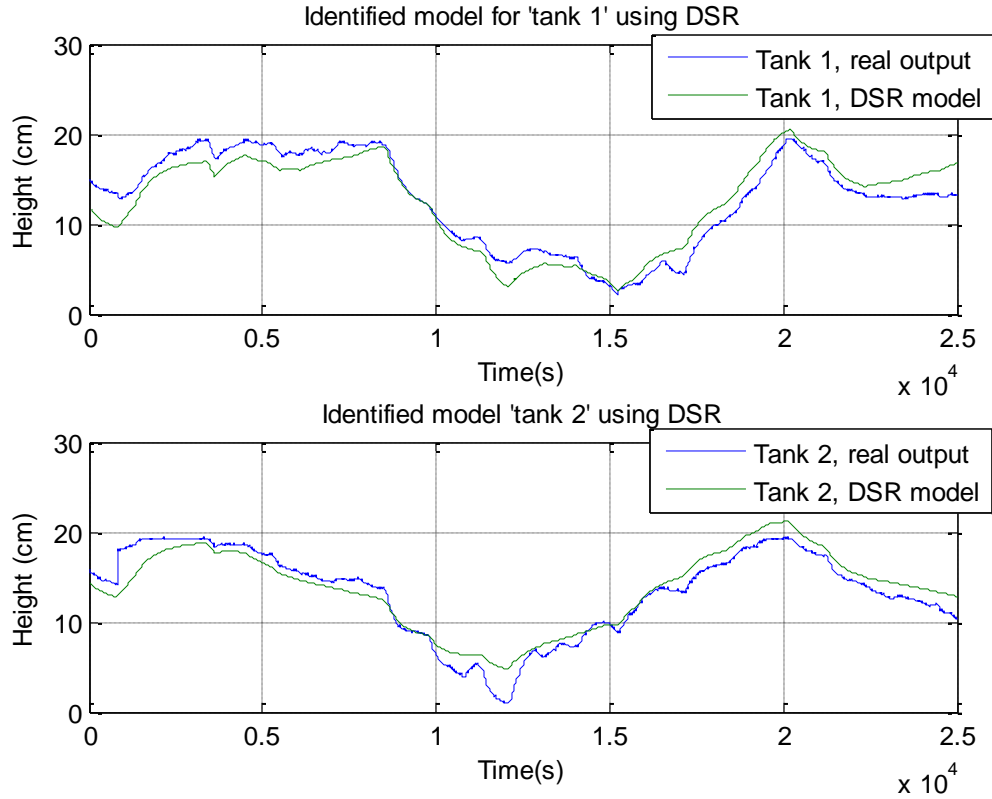
Figure 3-9: Graph of the process output and predicted output using DSR method for minimum phase.

Discrete state space model using DSR algorithm for the minimum phase can be given as in Equation (3-1).

$$x_{k+1} = Ax_k + Bu_k + v$$
$$y_k = Dx_k + Eu_k + w$$

Where, u_k and y_k are measured actual input and output and x_k is state. Equation (3-1) may arise from linearizing non-linear models around some nominal steady state and input variables or from system identification based on trend variables (Ruscio, 2012). Thus in our case external noise variables *v* and *w* are known. Moreover, in these case noise variables are considered insensitive but the system and the measurements may be influenced by drifts in which the noise variables will be varying slowly and also unknown.

A, B, D and E matrices are:

$$A = \begin{bmatrix} 0.9985 & 0.0001 & 1.1399 \\ 0.0001 & 0.9998 & 0.3541 \\ -0.0000 & -0.0000 & 0.9518 \end{bmatrix}$$

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$$B = \begin{bmatrix} -0.0029 & -0.0041 \\ 0.0004 & -0.0004 \\ -0.0000 & -0.0000 \end{bmatrix}$$

$$D = \begin{bmatrix} -0.4827 & -0.3178 & 0.6330 \\ -0.3183 & 0.4822 & 0.3123 \end{bmatrix}$$

$$E = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Similar experiment is done for the non-minimum phase of the experiment and compare the performance. Figure 3-10 shows the plot of the identified model using DSR for the non-minimum phase and the real data.

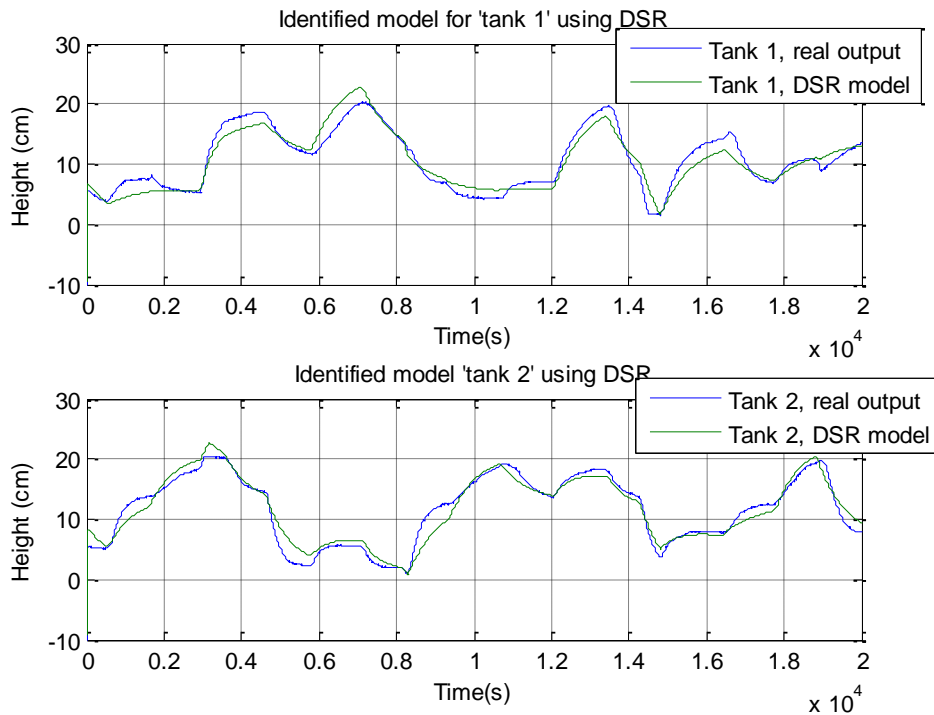*Figure 3-10: Graph of the process output and predicted output using DSR method for Non-minimum phase.*

New set of process data are collected for the non-minimum phase 20,000 number of samples are used to develop the model.

Comparison between minimum and non-minimum phase using DSR method from the result from both phases are compared and shown in the Table 3-3 below.

Table 3-3: Minimum and Non-minimum phase comparison using DSR method

Phase	J	L	n	MAE	RMSE	Poles	Zeros	Remarks
Minimum phase	2	3	3	h1= 0.6987 h2= 0.7173	h1=0.1988 h2=0.3441	0.9522 0.9982 0.9996	0.9531	Minimum phase with Split ratio $\gamma_1 = 0.7 \; and$ $\gamma_2 = 0.7$
Non-minimum phase	10	10	3	h1 =1.3662 h2 =1.0629	h1=0.2535 h2=0.0291	0.9143 0.9986 0.9988	1.6559	Non-minimum phase with $\gamma_1 = 0.4 \; and$ $\gamma_2 = 0.4$

As mentioned in Section 2.3 system is said to be in minimum phase if $\gamma_1 + \gamma_2 \geq 1$ and non-minimum phase if $\gamma_1 + \gamma_2 \leq 1$. Both cases were studied experimentally setting the values of $\gamma_1 = 0.7, \gamma_2 = 0.7$ and $\gamma_1 = 0.4, \gamma_2 = 0.4$ respectively. Comparing the identified model using DSR method as shown in Table 3-3 for Minimum and non-minimum phase we can simply analyze that MAE is less in minimum phase and even comparing the zeros, non-minimum phase is unstable as the zeros value is 1.6559 which lies out of the unity circle. This shows non-minimum phase is quiet unrealistic and difficult to control as it makes the process slower.

3.4.2 PEM method of Identification

Prediction Error Method (PEM) is widely used in comparing the measurement data vector y(t) with the predicted output of the dynamic model $\tilde{y}$(t) using the last *t-1* measurements data vector and the difference between the measurement and prediction is called prediction error (Kimon P. Randal, 2009).

State space model structure of our system as described in Equation (3-1) is given as:

$$x_{k+1} = Ax_k + Bu_k$$
$$y_k = Dx_k + Eu_k$$

Where,

u_k and y_k are measured input and output and x_k is state.

In order to determine the states (x_k) of the system we will follow the steps below:

- Determine the ''n'' order from a Singular Value Decomposition (SVD) of the "correct" matrix. Getting the estimator for the states.

- After the states(x_k), input (u_k) and output (y_k) are known we will compute matrices A, B, D and E

  Note: All these computations are done in MATLAB

  Using 25,000 samples from the process, input and output data to develop an identification model by PEM method in MATLAB. After testing the model with different order (n) the best suitable PEM model was executed at n=3 where MAE for level in tank 1 and tank 2 are 0.8639 and 1.1233 respectively while the RMSE of the level at tank 1 is 0.2862 and tank 2 is 0.3409 as listed in Table 3-4. PEM identification model for our system with zero of 0.9999 describes the system is stable (minimally). Since the identified system is stable, we can relate the pole-zero plot to the frequency response of our real system. A stable system that produces a bounded output and input to our real system is possible using PEM identification method.

Table 3-4: Table showing the properties of the model developed using PEM method

Order (n)	Mean Absolute Error (MAE)	Root Mean Square Error (RMSE)	Poles	Zeros	Remarks
3	h1= 0.8639 h2=1.1233	h1=0.3328 h2=0.0213	0.9835 0.9982 0.9998	0.9447	Minimum phase with Split ratio $\gamma_1 = 0.7 \; and$ $\gamma_2 = 0.7$

Characteristics of the PEM model are given in Table 3-4 above and the graphical representation of the real output and the identified model using PEM method is as shown in Figure 3-11. Respective MATLAB code used to develop model is attached in Appendix F.

Figure 3-11: Real process data vs predicted output using PEM method.

Discrete state space model using PEM algorithm for the minimum phase as described in Equation (3-1) is:

$$x_{k+1} = Ax_k + Bu_k$$

$$y_k = Dx_k + Eu_k$$

Where, A, B, C and E matrices are:

$$A = \begin{bmatrix} 0.99897 & 0.00026081 & -0.0016741 \\ 0.00054099 & 0.99972 & 0.00034344 \\ 0.0061327 & -0.00022647 & 0.98283 \end{bmatrix}$$

$$B = \begin{bmatrix} 2.0788e-006 & 2.4858e-006 \\ -1.1762e-006 & -2.0783e-007 \\ -4.7784e-005 & -5.795e-005 \end{bmatrix}$$

$$C = \begin{bmatrix} 282.22 & 31.552 & -0.17078 \\ 143.4 & -173.16 & -0.030524 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

3.4.3 N4SID method of Identification

This method of system identification estimates a state-space model using a sub-space method. Model developed by N4SID method mainly depends on input and out data from the real process and the order of the model which is the dimension of the state vector. It focus on the properties like prediction, simulation, filter and stability of the model (Peter Van Overschee, 1992). It is important that the model developed is stable.

Input and output data from the real quadruple tank process were used to implement the N4SID method of identification. Selecting the different values of system order (n) to execute the MATLAB code, n=3 gives the minimum error but the system with zero 1.2504 is unstable. MAE for level in tank is 0.7167 and tank 2 is 0.7533 which is relative a good result while RMSE between the real process output and the identified model for tank 1 and tank 2 are 0.7167 and 0.7533 respectively. Poles are still within the unity circle (less than 1) though it is very close to 1 but zeros are greater than 1 implies that the system is unstable. Parameters from the identified model this method are listed in Table 3-5. Loss function is 1.08472e-007 and FPE 1.08654e-007, Sampling interval: 1, N4Horizon =Auto and DisturbanceModel =None.

Table 3-5: Error, Poles and Zeros by Identified N4SID method.

Order (n)	Mean Absolute Error (MAE)	Root Mean Square Error (RMSE)	Poles	Zeros	Remarks
3	h1= 0.7167 h2= 0.7533	h1= 0.7167 h2=0.7533	0.8996 0.9985 0.9997	1.2504	Minimum phase with Split ratio $\gamma_1 = 0.7 \ and$ $\gamma_2 = 0.7$

Graphical representation of the model executed using MATLAB shows the plot between the real process input and the identified model using inbuilt N4SID method of system identification as shown in the Figure 3-12 below. MATLAB script of the process is attached in Appendix G.
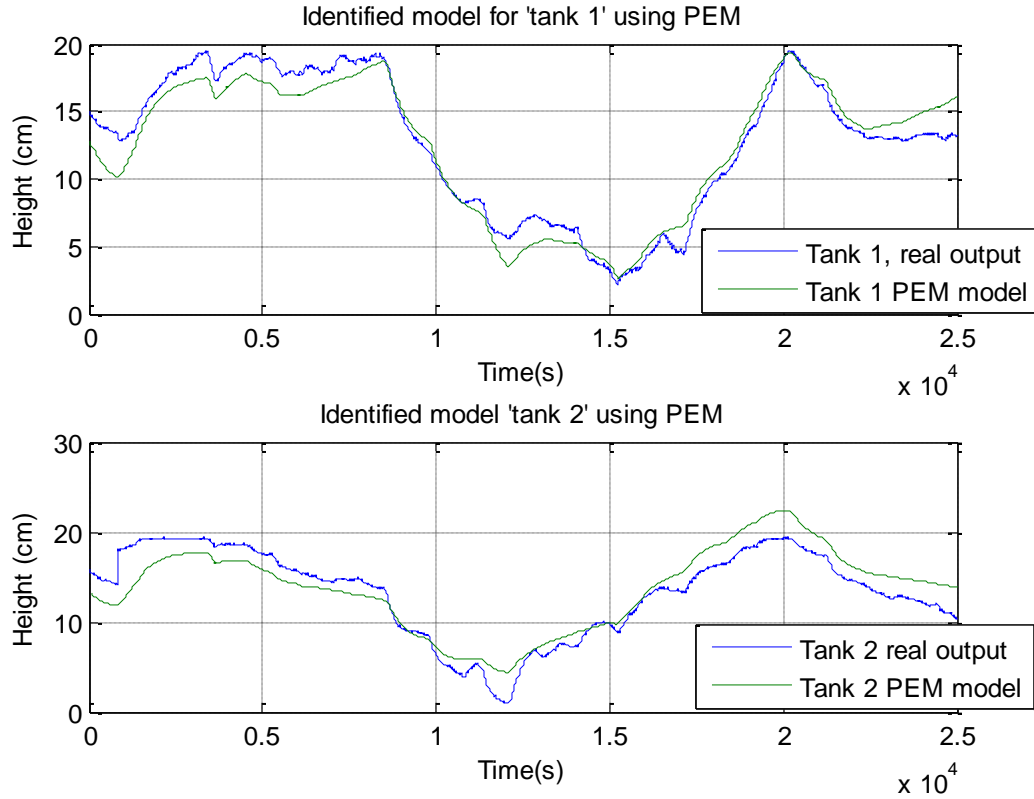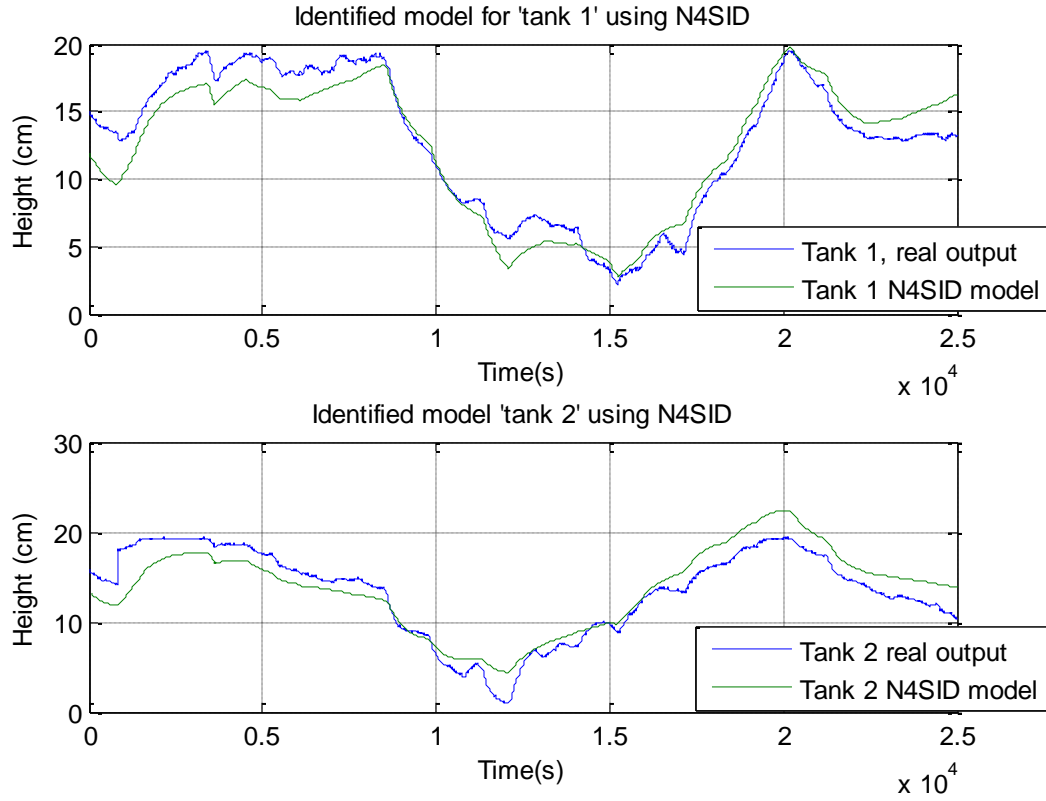
Figure 3-12: Real process data vs predicted output using N4SID method.

Discrete state space model using N4SID algorithm for the minimum phase as described in Equation (3-1).

$$x_{k+1} = Ax_k + Bu_k$$

$$y_k = Dx_k + Eu_k$$

Where, A, B, C and E matrices are:

$$A = \begin{bmatrix} 0.99864 & 0.00027749 & -0.0039519 \\ 0.00045002 & 0.9996 & -0.00054868 \\ 0.001993 & -4.7288e-006 & 0.89956 \end{bmatrix}$$

$$B = \begin{bmatrix} 4.0103e-007 & -1.6177e-006 \\ -2.7082e-006 & -1.5783e-006 \\ -0.00014776 & -0.00022617 \end{bmatrix}$$

$$C = \begin{bmatrix} 282.22 & 31.552 & -0.17078 \\ 143.4 & -173.16 & -0.030524 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Where D is a 2×2 zero matrix.

4 Results from different methods

We have used and studied the different methods of identification and their properties in chapter 3. In this section of the report, we will try to compare the different methods used to identify the model of Quadruple Tank using different properties of the model.

Technically a good model depends on the requirements or the goal, dose it fulfill the most important dynamic behavior of the system or is the model ready to design a controller? According to our study a good model set is required for a small bias in the model so as to give flexible model structure of the system for further analysis.

4.1 Comparing different models

Quality of system performance is achieved through a performance index. The performance of the model selected depends on the process under consideration and is chosen such that emphasis is placed on specific aspects of system performance. Furthermore, performance index is used as a quantitative measure to depict the performance of the dynamic system. An 'optimum system' can be compared between different methods using this technique designed to meet the required specification. For a controlled system, we can use one of the four indices to depict the system performance like Integral Square Error (ISE), Integrated Absolute Error (IAE), Mean Absolute Error (MAE) and Mean Square Error (MSE). They can be formulated as (Ahmad M. El-Nagar, 2014):

$$ISE = \int_0^N (e(t))^2 \, dt$$

$$IAE = \int_0^N |e(t)| dt$$

$$MAE = \frac{1}{N} \int_0^N |e(t)| dt$$

$$MSE = \frac{1}{N} \int_0^N (e(t))^2 dt$$

Where,

N is the total number of samples, Error (e) is the difference between measured and the predicted output.

$$e = \sum_1^N (y - \hat{y})^2$$

In this section we are going to evaluate the different methods of identification used in chapter 3 with respect to Mean Absolute Error (MAE), and Root Mean Square Error (RMSE) values. Table 4-1 below shows the differences for the minimum phase comparing with different errors mentioned above.

Table 4-1: Comparing PEM, DSR and N4SID methods based on MAE, RMSE, ISE, and IAE indices.

Errors Methods	MAE	RMSE
DSR	h1= 0.6987 h2= 0.7173	h1= 0.1988 h2= 0.3441
PEM	h1= 0.8639 h2= 1.1233	h1= 0.2862 h2= 0.3409
N4SID	h1= 0.7167 h2= 0.7533	h1= 0.7167 h2= 0.7533

On comparing the different error values calculated using different identification methods as shown in Table 4-1, DSR method gives minimum error both in MAE and RMSE comparing to both PEM and N4SID methods. It gives the Mean Absolute Error for level at tank 1, h1=0.6987 and tank 2, h2= 0.7173 which symbolize relatively a good model by DSR method though other methods are also not too bad. While the RMSE of level at tank 1 is h1= 0.1988 and tank 2 is h2= 0.3441. In fact DSR and N4SID are giving more or less the same MAE model error, while N4SID seems not suitable to proceed as the RMSE error is too high. Also if we see Table 3-5, zeros by N4SID method is 1.2504 which symbolize the system is not stable. From this error evaluation of different methods it can be clearly seen that DSR methods can be used for the further analysis.

4.2 Model Validation

We will now use the new set of data from the real process, originally a set of data was divided into experimental data and validation set data to validate against the model developed by PEM, DSR and N4SID methods. To check the quality of methods we will use indices as the IAE indices, MAE and RMSE etc.

From the model developed using different methods, we will use the properties of newly developed model and validate new set of data in order to check the quality of the model and validate the respective model. Validation graph for PEM and N4SID are shown in Figure 4-2 and validation graph for DSR is shown in Figure 4-3 respectively and the respective MATLAB code are attached in Appendix H. Figure 4-1 shows the plot of input and output for the validation set of data.
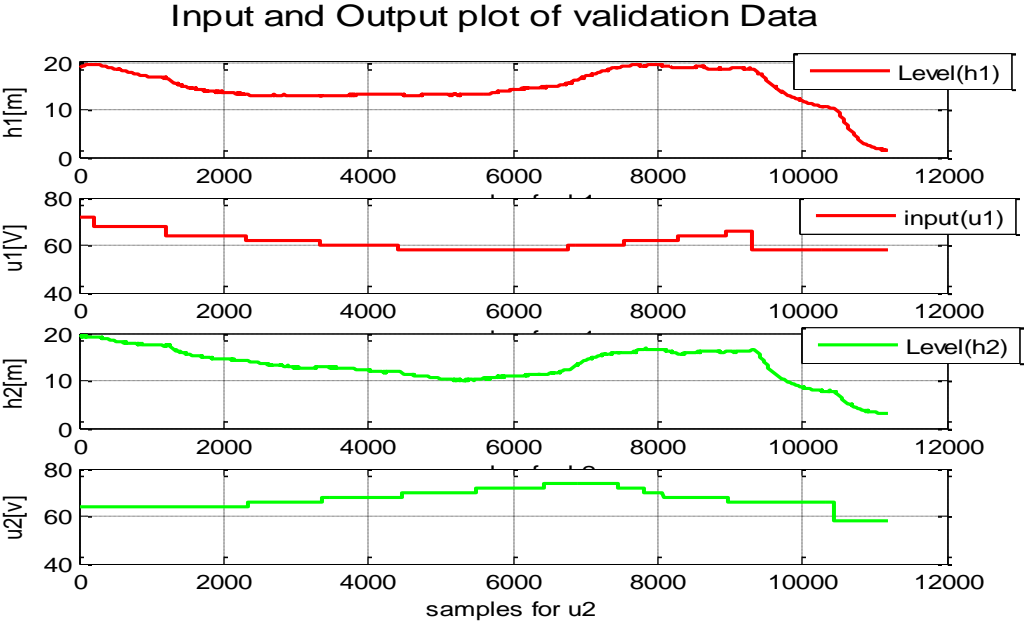

Figure 4-1: Input vs Output for the validation data set
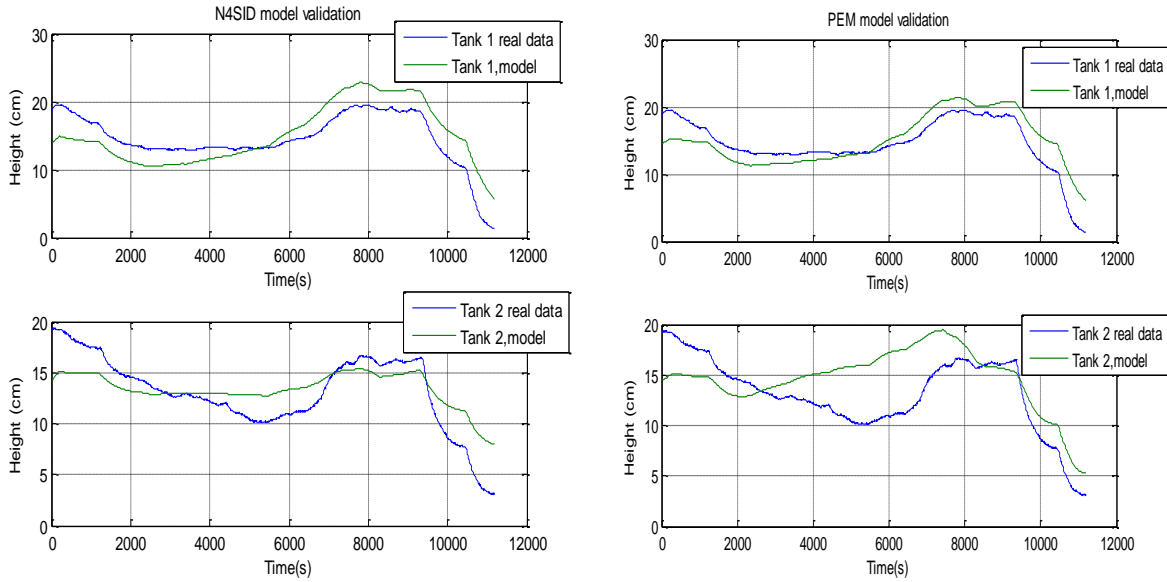
Figure 4-2: Validating the model developed using N4SID method (left) and PEM method (right)
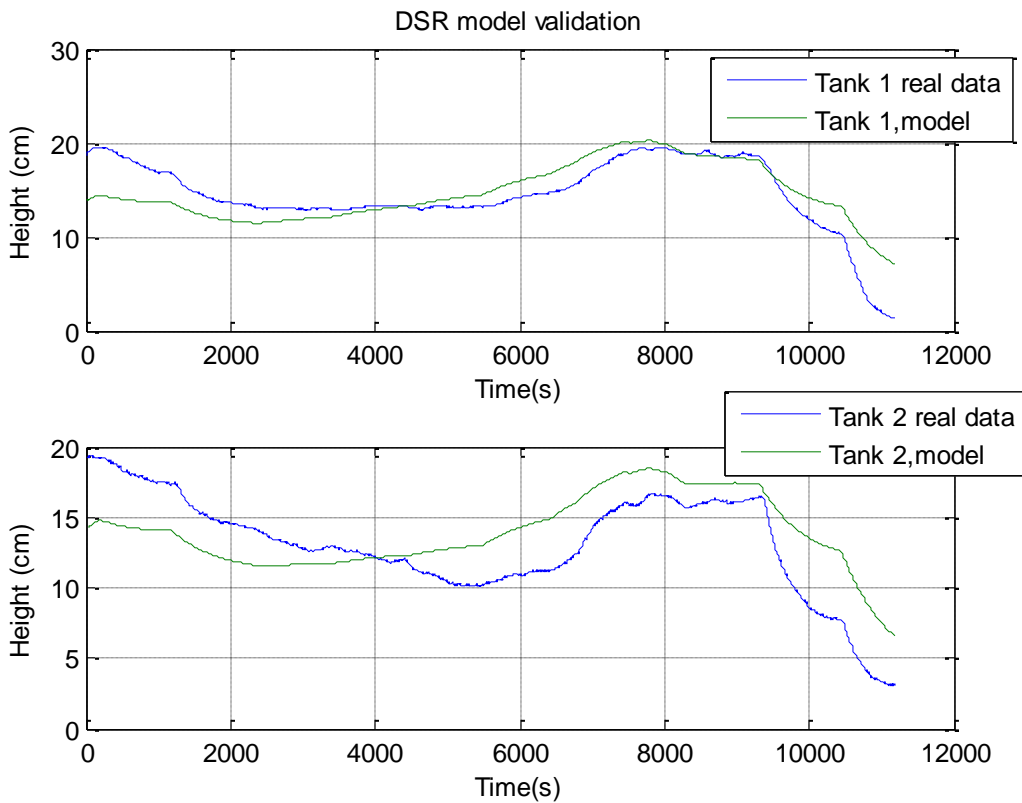
Figure 4-3: DSR model validation using validation data set

Quality of the model developed using different method is evaluated comparing the different residuals criterion using the validation data. Table 4-2 below shows the residual analysis of different System Identification methods.

Table 4-2: Table showing the residual analysis of different methods

Errors / Methods	MAE	RMSE	IAE
DSR	h1= 1.2845 h2= 1.2969	h1= 0.1068 h2= 0.3363	h1= 1.6103e+005 h2= 1.1946e+005
PEM	h1= 1.2187 h2= 2.3910	h1= 0.4311 h2= 0.1511	h1= 1.3645e+004 h2= 2.6770e+004
N4SID	h1= 1.8202 h2= 1.6100	h1= 0.5007 h2= 0.3365	h1= 2.0379e+004 h2= 1.8025e+004

From the result obtained by validating different identification methods used, we have seen as in Table 4-2 MAE from DSR model (h1= 1.2845, h2= 1.2969) seems to be better than both the PEM and N4SID methods but none of the methods give too bad result except the PEM model for level at Tank 2, may be because of some process error. RMSE from the validation of DSR model (h1= 0.1068, h2= 0.3363) seems the best among all three methods. Error for the model of tank 2 is slightly higher than that of tank 1 may be due to the pump characteristics as the pumping rate from pump 2 was fluctuating. Thus, from this observation it is seen that DSR model can be used for the control strategy as it is slightly better than both the PEM method and N4SID method.

5 Summary and conclusion

The main idea of this report is to compare the different system identification methods on the basis of the collected input and output data from the experiment on quadruple tank process. Starting with the general introduction of system identification and their types, individual identification methods are used to develop a model and are compared, residual analysis to select a better model for the implementation of control strategy.

General conclusion from the report can listed as:

1. General introduction of system identification and their historical approaches were listed. It mainly gives the brief introduction of the identification process, objectives and the importance of identification and their types mainly to the new users.

2. Model development or the process modeling describes the system briefly. Characteristics of the quadruple tank were studied along with the process parameters based on the experimental data. Non-linear mathematical model based on (Johansson, 2000) was developed. Model was linearized and the steady sate of the model was studied. Model stability, zero location, operating point controllability and observability were studied using linearized continuous model.

3. Experimental design of the model and data acquisition was done and input and output data were collected successfully using LabVIEW. Multiple zeros which can be adjusted by changing the physical position of the valve was studied. In order to check the model and to validate it, MATLAB code was generated for the simulation and model was developed using DSR method which gives satisfactory result to proceed. Real input and output data were collected from the process and different identification methods (DSR, N4SID and PEM) methods were used to develop the model and their properties were studied. DSR method as the best method in this individual identification method comparison.

4. Model identified using DSR, PEM and N4SID from experimental data set were compared to each other based on computational error analysis, stability analysis etc. Using the new set of validation data and validating different models, RMSE and MAE for PEM, DSR and N4SID were compared. DSR method with MAE (h1= 1.2845, h2= 1.2969) and RMSE (h1= 0.1068, h2= 0.3363) shows that this method is better than both PEM and N4SID. While MAE is more or less the same for all.

Finally, the conclusion from the report and the result obtained from the identified models of all three methods, DSR, PEM and N4SID fits real data somehow in their own way. Both the identification methods PEM and DSR fit the real data comparatively better than the N4SID method of identification. From the simulated result (Table 4-1) to the result from validation (Table 4-2) PEM and DSR methods gives more or less the similar results but DSR is marginally

better in both RMSE and MAE. Thus model developed using DSR method can be further implemented in control strategies like MPC, LQ optimal control etc. or any further analysis.

However, the result has shown that the model identified is more accurate as the practical circumstances from the physical system like water flow, split ratio by valves, level accuracy etc. Thus, the controller applied on identified model will be more accurate than other methods.

6 Future recommendations

Planned task for this project are completed successfully. However, research and development in science can never end. This report could have been made even more effective but of some limitations. Thus, my experience after the completion of this projects recommends some task to improve the result for those who wish to work in the field of system identification and with quadruple tank process.

- Quality of the pump can be improved for the stable performance. Moreover, calibration of the experimental rig must be done before the experiment as it is not used regularly.
- Two tank process can be studied by adjusting both input, one to the lower tank and the other to the upper tank only. Controlling the two lower tanks individually at different times can be something new to study. For this the pump inflow and the outflow should be reduced slightly for the better study, though the existing system should also work well.
- System identification of the quadruple tank process can be improved and even better results can be achieved, so different existing methods like first principle, PO-MOESP, CVA can be used to compare result with the result in this report. Combination of two different methods or the new method can also be studied.
- Finally, advanced control strategies like MPC, LQ controller etc. can be implemented to the best identified model suggested from this report

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Appendix

Task description of Master's thesis

Telemark University College
Faculty of Technology

FMH606 Master's Thesis

Title: Comparing Methods for System Identification on the Quadruple Tank Process

TUC supervisor: David Di Ruscio

External partner: None

Task description:

1. Give a short theoretical overview and investigation of different methods for system identification, e.g. as the DSR method, the N4SID method and the PEM method. It is important that MATLAB software is available.
2. Perform an input experiment on the quadruple tank process and collect the input and output data. Use and compare the chosen methods for system identification again each other.
3. Validate the models on independent data and check the quality of the methods. Use indices as the prediction error, IAE indices computational time etc.

Task background:

Methods for System Identification of linear state space models are grate tools for simple and cost effective building of mathematical process models. The discipline are well understood and matured. However, it is important from time to time to demonstrate the effectiveness of such methods for the identification of simple models to be used for e.g. controller tuning or model based control. This task is to compare some important and well known methods, both by simulations and experimentally on the quadruple tank process.

Student category:

SCE students

Practical arrangements: This work is to be done at TUC and with the laboratory.

Signatures:

Student (date and signature):

Supervisor (date and signature):

Appendix A

Pump flow rates

```
%pump characteristics
r=3; % radius of tanks are given
%for tank 1

voltage1=[5 5 4 3.5 3];
time1=[14.41 11.92 12.95 18.27 17.99];
height1=[20 15 12 15 12];
vdott1=(pi*r^2.*height1)./time1;
p=polyfit(voltage1,vdott1,1);
k1=p(1)
c1=p(2)

% for tank 2
voltage2=[ 5 4.5 4 3.5 4];
time2=[15.11 9.13 11.98 18.19 15.20];
height2=[20 10 12 15 15];
vdott2=(pi*r^2.*height2)./time2;
q=polyfit(voltage2,vdott2,1);
k2=q(1)
c2=q(2)
plot(voltage1,vdott1)
```

Appendix B

Linearization

For linearized model in matrix form

A_c matrix can be given as

$$A_c = \begin{pmatrix} \dfrac{\partial f_1}{\partial h_1} & \dfrac{\partial f_1}{\partial h_2} & \dfrac{\partial f_1}{\partial h_3} & \dfrac{\partial f_1}{\partial h_4} \\[2mm] \dfrac{\partial f_2}{\partial h_1} & \dfrac{\partial f_2}{\partial h_2} & \dfrac{\partial f_2}{\partial h_3} & \dfrac{\partial f_2}{\partial h_4} \\[2mm] \dfrac{\partial f_3}{\partial h_1} & \dfrac{\partial f_3}{\partial h_2} & \dfrac{\partial f_3}{\partial h_3} & \dfrac{\partial f_3}{\partial h_4} \\[2mm] \dfrac{\partial f_4}{\partial h_1} & \dfrac{\partial f_4}{\partial h_2} & \dfrac{\partial f_4}{\partial h_3} & \dfrac{\partial f_4}{\partial h_4} \end{pmatrix}$$

$$\frac{\partial f_1}{\partial h_1} = \frac{\partial}{\partial h_1}\left(-\frac{a_1}{A_1} \cdot \sqrt{2 \cdot g}\right) \cdot h_1^{\frac{1}{2}}$$

$$-\frac{a_1}{A_1} \cdot \sqrt{2 \cdot g} \cdot \frac{1}{2 \cdot \sqrt{h_1}} = -\left(\frac{a_1}{A_1}\right) \cdot \sqrt{\frac{g}{2 \cdot h_1}}$$

$$\frac{\partial f_1}{\partial h_1} = -\frac{1}{T_1}$$

$$\frac{\partial f_1}{\partial h_2} = 0$$

$$\frac{\partial f_1}{\partial h_3} = \frac{\partial}{\partial h_3}\left[\left(\frac{a_3}{A_1} \cdot \sqrt{2 \cdot g}\right) \cdot h_3^{\frac{1}{2}}\right]$$

$$\frac{a_3}{A_1} \cdot \sqrt{2 \cdot g} \cdot \frac{1}{2 \cdot \sqrt{h_3}} = \frac{a_3}{A_1} \cdot \sqrt{\frac{g}{2 \cdot h_3}} = a_3 \cdot \frac{A_3}{A_3 \cdot A_1} \cdot \sqrt{\frac{g}{2 \cdot h_3}}$$

$$\frac{\partial f_1}{\partial h_3} = \frac{A_3}{A_1 \cdot T_3}$$

$$\frac{\partial f_1}{\partial h_4} = 0$$

Similarly,

$$\frac{\partial f_2}{\partial h_1} = 0$$

$$\frac{\partial f_2}{\partial h_2} = \frac{\partial}{\partial h_2} \cdot \left[\left(-\frac{a_2}{A_2} \cdot \sqrt{2 \cdot g}\right) \cdot h_2^{\frac{1}{2}}\right]$$

$$\frac{\partial f_2}{\partial h_2} = -\frac{a_2}{A_2} \cdot \sqrt{2 \cdot g} \cdot \frac{1}{2 \cdot \sqrt{h_2}} = -\frac{a_2}{A_2} \cdot \sqrt{\frac{g}{2 \cdot h_2}}$$

$$\frac{\partial f_2}{\partial h_2} = -\frac{1}{T_2}$$

$$\frac{\partial f_2}{\partial h_3} = 0$$

$$\frac{\partial f_2}{\partial h_4} = \frac{\partial}{\partial h_4} \cdot \left[\left(\frac{a_4}{A_4} \cdot \sqrt{2 \cdot g}\right) \cdot h_4^{\frac{1}{2}}\right]$$

$$\frac{\partial f_2}{\partial h_4} = \frac{a_4}{A_2} \cdot \sqrt{2 \cdot g} \cdot \frac{1}{2 \cdot \sqrt{h_4}} = a_4 \cdot \frac{A_4}{A_4 \cdot A_2} \cdot \sqrt{\frac{g}{2 \cdot h_4}}$$

$$\frac{\partial f_2}{\partial h_4} = \frac{A_4}{A_2 \cdot T_4}$$

$$\frac{\partial f_3}{\partial h_3} = -\frac{1}{T_3}$$

$$\frac{\partial f_4}{\partial h_4} = -\frac{1}{T_4}$$

Rest of the terms for h_3 and h_4 are zero, which gives the matrix in the form of :

$$Ac = \begin{pmatrix} -\dfrac{1}{T_1} & 0 & \dfrac{A_3}{A_1 \cdot T_3} & 0 \\ 0 & -\dfrac{1}{T_2} & 0 & \dfrac{A_4}{A_2 \cdot T_4} \\ 0 & 0 & -\dfrac{1}{T_3} & 0 \\ 0 & 0 & 0 & -\dfrac{1}{T_4} \end{pmatrix}$$

B_c matrix can be given as

$$Bc = \begin{pmatrix} \dfrac{\partial f_1}{\partial u_1} & \dfrac{\partial f_1}{\partial u_2} \\ \dfrac{\partial f_2}{\partial u_1} & \dfrac{\partial f_2}{\partial u_2} \\ \dfrac{\partial f_3}{\partial u_1} & \dfrac{\partial f_3}{\partial u_2} \\ \dfrac{\partial f_4}{\partial u_1} & \dfrac{\partial f_4}{\partial u_2} \end{pmatrix}$$

Where,

$$\frac{\partial f_1}{\partial u_1} = \gamma_1 \cdot \frac{k_1}{A_1}$$

$$\frac{\partial f_4}{\partial u_1} = (1 - \gamma_1) \cdot \frac{k_1}{A_4}$$

$$\frac{\partial f_2}{\partial u_2} = \gamma_2 \cdot \frac{k_2}{A_2}$$

$$\frac{\partial f_3}{\partial u_2} = (1 - \gamma_2) \cdot \frac{k_2}{A_3}$$

And the rest are zero.

$$\frac{\partial f_2}{\partial u_1} = \frac{\partial f_3}{\partial u_1} = \frac{\partial f_1}{\partial u_2} = \frac{\partial f_4}{\partial u_2} = 0$$

Appendix C

Input signal

```
clc
[data,txt]=xlsread('data2.xlsx');
%[data2,txt]=xlsread('tan2min.xlsx');
u1=(data(:,1)); %input for tank1(h1)
u2=(data(:,2)); %input for tank2(h2)
y1=(data(:,3)); % h1
y2=(data(:,4)); %h2
figure(1) ;clf
subplot(411);plot(y1,'r','LineWidth',2);grid                    on;
legend('Level(h1)');ylabel('h1[m]');xlabel('samples for h1')
subplot(412);plot(u1,'r','LineWidth',2);grid                    on;
legend('input(u1)');ylabel('u1[V]');xlabel('samples for u1')
subplot(413);plot(y2,'g','LineWidth',2);grid                    on;
legend('Level(h2)');ylabel('h2[m]');xlabel('samples for h2')
subplot(414);plot(u2,'g','LineWidth',2);grid                    on;
legend('input(u2)');ylabel('u2[v]');xlabel('samples for u2')
suptitle('Input and Output plot of validation Data')
figure (2)
plot([u1, u2])
legend ('input u1', 'input u2')
ylabel('5V=5*20'); xlabel ('Time (s)')
grid on
suptitle ('Input signal')
```

Appendix D

System identification using simulated data

```
t_min=0;
```

```matlab
t_max=6000;
%%%parameters for simulation
delta_t= 0.5;
N= (t_max-t_min)/delta_t;
D=[1 0 0 0 ; 0 1 0 0];

gamma1=input('Please give the GAMMA1 value:');
gamma2=input('Please give the GAMMA2 value:');
gamma=[gamma1 gamma2];

%% state matrix X where, h1=X(1), h2=X(2)....to initialize.
%when tank is empty
X=zeros(4,1);
a=round(N/3);
X_data1=zeros(a,4);
Y_data1=zeros(a,2);
U_data1=zeros(a,2);
U=[3 3]'; % initial input before system reaches steady state
for k=1:a % 3333
    Y=D*X;
    Y_data1(k,1:2)=Y';
    U_data1(k,1:2)=U';
    X_data1(k,1:4)=X';
    X= X+delta_t*nonlinearmodel(X,U,gamma);
end
%
plot(1:a,X_data1(:,1),'r',1:a,X_data1(:,2),'b',1:a,X_data1(:,3),'c',1:a,X_dat
a1(:,4),'k')
%  legend('Tank 1','Tank 2','Tank 3','Tank 4')
%  xlabel('Time(s)');
%  ylabel('Height (cm)')
 %figure
%% real data collection
b=N-a;
X_data2=zeros(b,4);
Y_data2=zeros(b,2);
U_data2=zeros(b,2);
U_1= prbs1(b,500,800) + 4;
U_2= prbs1(b,600,1000) + 3;
for l=1:b %% 3334-10000
    Y=D*X;
    Y_data2(l,1:2)=Y';
    U_new=[U_1(l) U_2(l)]';
    U_data2(l,1:2)=U_new';
    X_data2(l,1:4)=X';
    X=X+delta_t*nonlinearmodel(X,U_new,gamma);
end

figure(1)
subplot(2,1,1)
%title('Simulated   and   Identified   (DSR)   Output   from   Mathematical
model(nonlinear) ')
plot(1:b,Y_data2(:,1),'r',1:b,Y_data2(:,2),'k')
% hold on
% plot(t(a+1:N),X_data2(:,3),'c',t(a+1:N),X_data2(:,4),'k')
```

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```
xlabel('Time(s)');
ylabel('Height (cm)')
grid on
hold on
figure(2)
%% system identification using dsr method
% sid from simulated data
[A_sim,B_sim,D_sim,E_sim,CF,F,x0]= dsr(Y_data2,U_data2,4)
Y_dsr_sim=dsrsim(A_sim,B_sim,D_sim,E_sim,U_data2,x0);
figure(1)
title('System Identification using Simulated Data')

%title('Output identified model(DSR)using simulated data and input signal')
subplot(2,1,1)
plot(1:length(Y_dsr_sim),Y_dsr_sim(:,1),':r',1:length(Y_dsr_sim),Y_dsr_sim(:,
2),':k')
legend('h1, Simulated','h2, Simulated','h1, Identified','h2, Identified')
xlabel('Time(s)');
ylabel('level of tank (cm)')
grid on
hold off
subplot(2,1,2)
plot(1:b,U_data2(:,1),'r',1:b,U_data2(:,2),'k')
title('PRBS (input) Signal')
legend('u1', 'u2')
xlabel('Time(s)');
ylabel('Amplitude(V)')



% validation
% error11=norm(Y_dsr_sim(:,1)-Y_data(:,1));
% error21=norm(Y_dsr_sim(:,2)-Y_data(:,2));
% error_criterion_dsr=max(error11,error21)/N
MAE_DSR_h1= 1/N*(sum(abs(Y_dsr_sim(:,1)-Y_data2(:,1))))
MAE_DSR_h2= 1/N*(sum(abs(Y_dsr_sim(:,2)-Y_data2(:,2))))

RMSE_pem_h1= 1/N*sqrt((sum((Y_dsr_sim(:,1)-Y_data2(:,1))).^2))
RMSE_pem_h2= 1/N*sqrt((sum((Y_dsr_sim(:,2)-Y_data2(:,2))).^2))

% poles and zeros
sys_zeros=tzero(A_sim,B_sim,D,zeros(2))
pole=eig(A_sim)
```

Appendix E

Using DSR method

```
%system order 3 (6iw3-iydz-zljs)
RealData= importdata('data2.xlsx');
u1_data=RealData.data(1:25000,1);
u2_data=RealData.data(1:25000,2);
h1_data=RealData.data(1:25000,3);
```

```matlab
h2_data=RealData.data(1:25000,4);
Y_data=detrend([h1_data h2_data]);
U_data=detrend([u1_data u2_data]);

 a=mean(h1_data);
 b=mean(h2_data);


% Y_data_trended=[normc(h1_data) normc(h2_data)];
% U_data_trended=[normc(u1_data) normc(u2_data)];
N=length(Y_data);

MAE_h1=zeros(9,9);
MAE_h2=zeros(9,9);
for J=2:10
    for L=2:10
%% sid from real data
[A_dsr,B_dsr,D_dsr,E_dsr,CF,F,x0]=dsr(Y_data,U_data,L,0,J,1,3);    %non-minimum
L=15, Minimum L=3
%% simulation of dsr model

Y_dsr_sim=dsrsim(A_dsr,B_dsr,D_dsr,E_dsr,U_data,x0);
error_h1=(Y_dsr_sim(:,1)-Y_data(:,1));
error_h2=(Y_dsr_sim(:,2)-Y_data(:,2));
MAE_dsr_h1= 1/N*(sum(abs(error_h1)));
MAE_dsr_h2= 1/N*(sum(abs(error_h2)));
MAE_h1(J-1,L-1)= MAE_dsr_h1;
MAE_h2(J-1,L-1)=MAE_dsr_h2;
    end
end
o=(MAE_h1+MAE_h2)/2;
p=min(o);
[minimum_mean_error,L]=min(p);
q=o(:,L);
[d,J]=min(q);
optimal_L=L+1
optimal_J=J+1

%% sid from real data
[A_dsr,B_dsr,D_dsr,E_dsr,CF,F,x0]=dsr(Y_data,U_data,optimal_L,0,optimal_J,1,3
) %non-minimum L=15, Minimum L=8
%% simulation of dsr model

Y_dsr_sim=dsrsim(A_dsr,B_dsr,D_dsr,E_dsr,U_data,x0);

error_h1=(Y_dsr_sim(:,1)-Y_data(:,1));
error_h2=(Y_dsr_sim(:,2)-Y_data(:,2));

IAE_dsr_h1= (sum(abs(error_h1)))
IAE_dsr_h2= (sum(abs(error_h2)))

MAE_dsr_h1= 1/N*(sum(abs(error_h1)))
MAE_dsr_h2= 1/N*(sum(abs(error_h2)))

ISE_dsr_h1= ((sum(error_h1).^2))
ISE_dsr_h2= ((sum(error_h2).^2))
```

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```matlab
RMSE_dsr_h1= 1/N*sqrt((sum(error_h1).^2))
RMSE_dsr_h2= 1/N*sqrt((sum(error_h2).^2))


%poles and zeros
sys_zeros_dsr=tzero(A_dsr,B_dsr,D_dsr,zeros(2))
pole_dsr=eig(A_dsr)

%ploting
figure
subplot(211)
plot([h1_data Y_dsr_sim(:,1)+a]),grid
% ,legend('Tank 1 real data','b','Tank 1,DSR','r');
legend('Tank 1, real output','Tank 1, DSR model')
xlabel('Time(s)');
ylabel('Height (cm)')
title('Identified model for ''tank 1'' using DSR')
subplot(212)
plot([h2_data Y_dsr_sim(:,2)+b]),grid
% legend('Tank 2 real data','b', 'Tank 2 DSR','r');
legend('Tank 2, real output','Tank 2, DSR model')
title('Identified model ''tank 2'' using DSR')
xlabel('Time(s)');
ylabel('Height (cm)')
% THE END
```

Appendix F

PEM system Identification

```matlab
%system order 3
RealData= importdata('data2.xlsx');
u1_data=RealData.data(1:25000,1);
u2_data=RealData.data(1:25000,2);
h1_data=RealData.data(1:25000,3);
h2_data=RealData.data(1:25000,4);
Y_data=detrend([h1_data h2_data]);
U_data=detrend([u1_data u2_data]);


% Y_data_trended=[dtrend(h1_data) dtrend(h2_data)];
% U_data_trended=[dtrend(u1_data) dtrend(u2_data)];
N=length(Y_data);

 a=mean(h1_data);
 b=mean(h2_data);

%pem for system identification
Data=iddata(Y_data,U_data,1);
pem_model=pem(Data,3)
A_pem=pem_model.A;
```

```matlab
B_pem=pem_model.B;
D_pem=pem_model.C;
E_pem=pem_model.D;
x0_pem=pem_model.x0;


Y_pem_sim=dsrsim(A_pem,B_pem,D_pem,E_pem,U_data,x0_pem);


error_h1= Y_pem_sim(:,1)-Y_data(:,1);
error_h2= Y_pem_sim(:,2)-Y_data(:,2);


IAE_pem_h1= (sum(abs(error_h1)))
IAE_pem_h2= (sum(abs(error_h2)))


MAE_pem_h1= 1/N*(sum(abs(error_h1)))
MAE_pem_h2= 1/N*(sum(abs(error_h2)))


ISE_pem_h1= ((sum(error_h1).^2))
ISE_pem_h2= ((sum(error_h2).^2))


RMSE_pem_h1= 1/N*sqrt((sum(error_h1).^2))
RMSE_pem_h2= 1/N*sqrt((sum(error_h2).^2))


sys_zeros_pem=tzero(A_pem,B_pem,D_pem,zeros(2))
pole_pem=eig(A_pem)

%plot
figure
subplot(211)
plot([h1_data Y_pem_sim(:,1)+a]),grid
% ,legend('Tank 1 real data','b','Tank 1,DSR','r');
legend('Tank 1, real output','Tank 1 PEM model')
xlabel('Time(s)');
ylabel('Height (cm)')
title('Identified model for ''tank 1'' using PEM')
subplot(212)
plot([h2_data Y_pem_sim(:,2)+b]),grid
% legend('Tank 2 real data','b', 'Tank 2 DSR','r');
legend('Tank 2 real output','Tank 2 PEM model')
title('Identified model ''tank 2'' using PEM')

xlabel('Time(s)');
ylabel('Height (cm)')

% THE END
```

Appendix G

N4SID Method of Identification

```matlab
%system order 3 for minimum phase
RealData= importdata('data2.xlsx');
u1_data=RealData.data(:,1);
u2_data=RealData.data(:,2);
h1_data=RealData.data(:,3);
h2_data=RealData.data(:,4);
Y_data=[h1_data h2_data];
U_data=[u1_data u2_data];

Y_data_trended=[dtrend(h1_data) dtrend(h2_data)];
U_data_trended=[dtrend(u1_data) dtrend(u2_data)];
N=length(Y_data);

%n4sid for system identification
Data=iddata(Y_data,U_data,1);
n4sid_model=n4sid(Data,3)
A_n4sid=n4sid_model.A;
B_n4sid=n4sid_model.B;
D_n4sid=n4sid_model.C;
E_n4sid=n4sid_model.D;
x0_n4sid=n4sid_model.x0;

Y_n4sid_sim=dsrsim(A_n4sid,B_n4sid,D_n4sid,E_n4sid,U_data,x0_n4sid);
N4Horizon = 'Auto'

%ERROR calculations
error_h1=abs(Y_n4sid_sim(:,1)-Y_data(:,1));
error_h2=abs(Y_n4sid_sim(:,2)-Y_data(:,2));

IAE_n4sid_h1= (sum(abs(error_h1)))
IAE_n4sid_h2= (sum(abs(error_h2)))

MAE_n4sid_h1= 1/N*(sum(abs(error_h1)))
MAE_n4sid_h2= 1/N*(sum(abs(error_h2)))

ISE_n4sid_h1= ((sum(error_h1).^2))
ISE_n4sid_h2= ((sum(error_h2).^2))

RMSE_n4sid_h1= 1/N*(sum(i.*error_h1(i,:).^2))
RMSE_n4sid_h2= 1/N*(sum(i.*error_h2(i,:).^2))

 %poles and zeros
sys_zeros_n4sid=tzero(A_n4sid,B_n4sid,D_n4sid,zeros(2))
pole_n4sid=eig(A_n4sid)

subplot(2,1,1)
plot(1:length(Y_n4sid_sim),Y_data(:,1),'b',1:length(Y_n4sid_sim),Y_n4sid_sim(
:,1),'r')
legend('Tank 1 real data','Tank 1,n4sid')
grid on
subplot(2,1,2)
plot(1:length(Y_n4sid_sim),Y_data(:,2),'b',1:length(Y_n4sid_sim),Y_n4sid_sim(
:,2),'r')
legend('Tank 2 real data', 'Tank 2 n4sid')
grid on
```

```
%
plot(1:length(Y_n4sid_sim),Y_n4sid_sim(:,1),'r',1:length(Y_n4sid_sim),Y_n4sid
_sim(:,2),'b')
% title('Identified model using n4sid')
% legend('Tank 1, Identified n4sid','Tank 2, Identified n4sid')
% xlabel('Time(s)');
% ylabel('Height (cm)')
% grid on
```

Appendix H

Validation code for DSR

```
data_valid=xlsread('validation_data.xlsx');
U1_valid=(data_valid(:,1));
U2_valid=(data_valid(:,3));
Y1_measured=(data_valid(:,2));
Y2_measured=(data_valid(:,4));
U=([U1_valid U2_valid]);
Y=([Y1_measured Y2_measured]);
A_dsr =[0.9998 0.0013 1.2154; -0.0001 0.9997 -0.1155; -0.0000 0.0000 0.9946];
B_dsr=1.0e-003*[0.1535   -0.1076; -0.1538    0.0955; 0.0011   -0.0014];
D_dsr =[-0.4546    0.3565    0.5941;   -0.3559   -0.4543    0.3837];
E_dsr =[-0.0085    0.0042; 0.0976   -0.0532];
%x0 =[-34.6565; -1.4233; 0.0044];
x0 =[-34.6565; -1.4233; 0.0044];
Y_valid=dsrsim(A_dsr,B_dsr,D_dsr,E_dsr,U,x0);
N1=length(Y1_measured);

%Computational error
 error_h1=(Y1_measured-Y_valid(:,1));
 error_h2=(Y2_measured-Y_valid(:,2));


MAE_dsr_h1= 1/N1*(sum(abs(error_h1)))
MAE_dsr_h2= 1/N1*(sum(abs(error_h2)))

RMSE_dsr_h1= 1/N1*sqrt((sum(error_h1).^2))
RMSE_dsr_h2= 1/N1*sqrt((sum(error_h2).^2))

IAE_dsr_h1= (sum(abs(error_h1)))
IAE_dsr_h2= (sum(abs(error_h2)))


subplot(211)
plot([Y1_measured Y_valid(:,1)]),grid,legend('measured','valid');
legend('Tank 1 real data','Tank 1,model')
xlabel('Time(s)');
ylabel('Height (cm)')
title('DSR model validation')
subplot(212)
plot([Y2_measured Y_valid(:,2)]),grid,legend('measured','valid');
legend('Tank 2 real data','Tank 2,model')
```

```
xlabel('Time(s)');
ylabel('Height (cm)')
%
plot(1:length(Y_dsr_sim),Y_dsr_sim(:,1),':b',1:length(Y_dsr_sim),Y_dsr_sim(:,
2),':g')
% legend('h1, Sim','h2, Sim','h1, Ident','h2, Ident')
% xlabel('Time(s)');
% ylabel('Height (cm)')
```

Validation code for PEM

```
data_valid=xlsread('validation_data.xlsx');
U1_valid=(data_valid(:,1));
U2_valid=(data_valid(:,3));
Y1_measured=(data_valid(:,2));
Y2_measured=(data_valid(:,4));
U=([U1_valid U2_valid]);
Y=([Y1_measured Y2_measured]);
A =[0.99998 0.00015016 -0.00035968; -0.00066789 0.99971 -7.0167e-005; 0.0014535
-0.00018314 0.99791];
B=[1.5076e-006  1.9606e-006; 7.7852e-007  1.4637e-007;  1.0013e-005  1.0256e-
005];
C=[808.05 -53.599 -0.30415; 606.35  227.85 -0.11962];
D=[0 0; 0 0];

%x0 =[-34.6565; -1.4233; 0.0044];
x0_pem =[ 0.019891; 0.016888; 0.64549];
Y_pem_sim=dsrsim(A,B,C,D,U,x0_pem);

N1=length(Y1_measured);

%Computational indices error
 error_h1=(Y1_measured-Y_pem_sim(:,1));
 error_h2=(Y2_measured-Y_pem_sim(:,2));

MAE_pem_h1= 1/N1*(sum(abs(error_h1)))
MAE_pem_h2= 1/N1*(sum(abs(error_h2)))

RMSE_pem_h1= 1/N1*sqrt((sum(error_h1).^2))
RMSE_pem_h2= 1/N1*sqrt((sum(error_h2).^2))

IAE_pem_h1= (sum(abs(error_h1)))
IAE_pem_h2= (sum(abs(error_h2)))


subplot(211)
plot([Y1_measured Y_pem_sim(:,1)]),grid,legend('measured','valid');
legend('Tank 1 real data','Tank 1,model')
xlabel('Time(s)');
ylabel('Height (cm)')
title('PEM model validation')
subplot(212)
plot([Y2_measured Y_pem_sim(:,2)]),grid,legend('measured','valid');
legend('Tank 2 real data','Tank 2,model')
```

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```matlab
xlabel('Time(s)');
ylabel('Height (cm)')


%
plot(1:length(Y_dsr_sim),Y_dsr_sim(:,1),':b',1:length(Y_dsr_sim),Y_dsr_sim(:,
2),':g')
% legend('h1, Sim','h2, Sim','h1, Ident','h2, Ident')
% xlabel('Time(s)');
% ylabel('Height (cm)')
```

Validation code for N4SID

```matlab
data_valid=xlsread('validation_data.xlsx');
U1_valid=(data_valid(:,1));
U2_valid=(data_valid(:,3));
Y1_measured=(data_valid(:,2));
Y2_measured=(data_valid(:,4));
U=([U1_valid U2_valid]);
Y=([Y1_measured Y2_measured]);
A = [1.0001    0.0003055  -0.0014701; -0.00055351      0.99968 -1.4336e-005;
0.00037891  -0.0003587       0.98989];


B=[4.6869e-007    1.026e-006; 5.2155e-007 -2.9432e-007; 5.7725e-006   5.1173e-
006];


C= [808.05  -53.599  -0.30415; 606.35 227.85      -0.11962];


D=[0 0; 0 0];


%x0 =[-34.6565; -1.4233; 0.0044];
x0_n4sid =[ 0.019692; 0.01712;  0.073826];
Y_n4sid_sim=dsrsim(A,B,C,D,U,x0_n4sid);


N1=length(Y1_measured);

%Computational indices error
 error_h1=(Y1_measured-Y_n4sid_sim(:,1));
 error_h2=(Y2_measured-Y_n4sid_sim(:,2));

MAE_n4sid_h1= 1/N1*(sum(abs(error_h1)))
MAE_n4sid_h2= 1/N1*(sum(abs(error_h2)))

RMSE_n4sid_h1= 1/N1*sqrt((sum(error_h1).^2))
RMSE_n4sid_h2= 1/N1*sqrt((sum(error_h2).^2))

IAE_n4sid_h1= (sum(abs(error_h1)))
IAE_n4sid_h2= (sum(abs(error_h2)))


subplot(211)
plot([Y1_measured Y_n4sid_sim(:,1)]),grid,legend('measured','valid');
legend('Tank 1 real data','Tank 1,model')
```

```matlab
xlabel('Time(s)');
ylabel('Height (cm)')
title('N4SID model validation')
subplot(212)
plot([Y2_measured Y_n4sid_sim(:,2)]),grid,legend('measured','valid');
legend('Tank 2 real data','Tank 2,model')
xlabel('Time(s)');
ylabel('Height (cm)')


%
plot(1:length(Y_dsr_sim),Y_dsr_sim(:,1),':b',1:length(Y_dsr_sim),Y_dsr_sim(:,
2),':g')
% legend('h1, Sim','h2, Sim','h1, Ident','h2, Ident')
% xlabel('Time(s)');
% ylabel('Height (cm)')
```