An assessment of swinger techniques for the playground swing oscillatory motion

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Much attention has been devoted to how playground swing amplitudes are built up by swinger techniques, i.e. body actions. However, very little attention has been given to the requirements that such swinger techniques place on the swinger himself. The purpose of this study was to find out whether different swinger techniques yield significantly different maximum torques, endurance and coordinative skills, and also to identify preferable techniques. We modeled the seated swinger as a rigid dumbbell and compared three different techniques. A series of computer simulations was run with each technique, testing performance with different body rotational speeds, delayed onset of body rotation, and different body mass distributions, as swing amplitudes were brought up towards $90°$. One technique was found extremely sensitive to the timing of body actions, limiting swing amplitudes to $50°$ and $8°$ when body action was delayed by $0.03s$ and $0.3s$, respectively. The two other more robust techniques reached $90°$ even with the largest of these delays, although more time (and endurance) was needed. However, these two methods also differed with respect to maximum torque and endurance, and none was preferable in both these aspects, being dependent on the swinger goals and abilities.

Wordcount: 191

1. Introduction

The playground swing has a place in most people's childhood. It allows a swing that is initiated,

maintained and controlled by the swinger, but doing it on the playground is one thing, understanding

the physics involved is yet another. A little child is able to initiate the swing from standstill and reach

considerable heights without knowing any mechanical law. At the same time, the mechanics of swing initiation and amplitude build-up is intriguing enough to justify a whole series of scientific contributions over the last few decades, see e.g. McMullan (1972), Curry (1976), Case and Swanson (1990), Case (1996), Wirkus et al. (1998), Piccoli and Kulkarni (2005), Post et al. (2007), and references therein.

Conservation of angular momentum about the point of suspension explains how it is possible get moving without a helping hand. When a still-standing swinger initiates body rotation, that angular momentum triggers a simultaneous (oppositely directed) rotation of the whole system's center of mass about the suspension point. After swing initiation, a similar transfer of angular momentum occurs, allowing swing amplitudes to grow with the right back and forth body rotation of the swinger. Body rotations will generally also change the effective distance between the point of suspension and the system's center of mass. This offers another important contributor to amplitude build-up. Put simply, for a single up and down swing, if the center of mass of the swinger is closer to the point of suspension when going up than when going down, the torque of gravity has a net increasing effect on the system energy. With the right rhythm, the swinger may exploit this effect in every period of the swing. This mechanism is usually referred to as *parametric* (Case, 1980), since the essence is a (repeated) change of the parameter value that represents the effective length of the pendulum. Still, with reasonable assumptions on the system and swinger motion, Case and Swanson (1990) showed that for smaller swing amplitudes ($< 40°$), system dynamics is dominated by *driving* terms, also stemming from the swinger's body rotation, rather than the parametric terms. This also applies when the swing is executed by body rotations in a standing position, as shown by Case (1996). In the standing position, crouching and standing up with the right rhythm provides a pure parametric technique for pumping the swing. Wirkus et al. (1998) ran computer simulations to compare this standing approach with a pure rotational approach for a seated swinger, modeling the swinger's center of mass to be located on the line along the ropes of the swing. They concluded that seated pumping was better at low amplitudes, but that the standing technique was more effective at higher amplitudes.

Much attention has been devoted to the understanding of the influence of swinger techniques

on swing amplitudes. For the swinger, however, it is also of interest to know what kind of demands those different techniques place on the swinger. Do different techniques require substantially different abilities from the swinger, or are they essentially similar? Swingers surely differ in strength, endurance and coordinative skills, and, in particular with larger amplitudes, such factors might limit the choice of technique for some swingers.

In this work, we investigate whether different swinger techniques yield significantly different maximum torques, endurance and coordinative skills, and seek to identify whether any technique is preferable over others.

2. Methods

We model the swinger as a rigid dumbbell with three point-masses m_1, m_2 and m_3, letting L_2 and L_3 represent the lengths of legs and torso, respectively (Fig. 1). The ropes of the playground swing were taken to be massless and rigid with length L_1. A specified rotational motion of the dumbbell then corresponds to a certain swinger technique that will drive the system.

2.1 Mathematical model

The Lagrangian of the swing system illustrated in Figure 1 reads

$$
\begin{aligned}
L = ML_1 g \cos\theta_1 - Ng \cos(\theta_1 + \theta_2) + \left(\tfrac{1}{2}I_1 + \tfrac{1}{2}I_2 - L_1 N \cos\theta_2\right)\dot\theta_1^2 \\
+ \left(I_2 - L_1 N \cos\theta_2\right)\dot\theta_1\dot\theta_2 + \tfrac{1}{2}I_2\dot\theta_2^2
\end{aligned}
\qquad ,
\tag{1}
$$

where θ_1 and θ_2 are the angles indicated in Figure 1, $M = m_1 + m_2 + m_3$, $I_1 = ML_1^2$,

$I_2 = m_2 L_2^2 + m_3 L_3^2$ and $N = m_3 L_3 - m_2 L_2$, a notation adopted from Case and Swanson (1990).

From the Lagrangian, we may derive the swinger torque τ in the standard way as

$$\tau = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right) - \frac{\partial L}{\partial \theta_2}, \tag{2}$$

which gives

$$\tau = -gN\sin(\theta_1 + \theta_2) - L_1 N \sin\theta_2\, \dot{\theta}_1^2 + (I_2 - L_1 N \cos\theta_2)\ddot{\theta}_1 + I_2 \ddot{\theta}_2. \tag{3}$$

In a similar way, assuming zero friction at the point of suspension, the angular acceleration of the ropes may be found from

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) - \frac{\partial L}{\partial \theta_1} = 0, \tag{4}$$

which allows the angular acceleration to be expressed as

$$\ddot{\theta}_1 = \frac{-MgL_1\sin\theta_1 + Ng\sin(\theta_1 + \theta_2) - L_1 N \sin\theta_2\, \dot{\theta}_2^2 - 2L_1 N \sin\theta_2\, \dot{\theta}_1\dot{\theta}_2 - (I_2 - L_1 N \cos\theta_2)\ddot{\theta}_2}{I_1 + I_2 - 2L_1 N \cos\theta_2}$$

$$\tag{5}$$

Equations (3) and (5) are valid for any amplitude, and express how swinger torque and swing amplitude develop as a function of system parameters and swinger motion (through $\theta_2(t)$ and

its derivatives). By choosing different functions for $\theta_2(t)$, different swinger techniques may be analyzed for a chosen set of system parameters.

It should be noted that the swing control problem studied here is quite different from the one that involves another person pushing the swinger. In (5), the control is provided by the swinger alone through the known function $\theta_2(t)$ and its time derivatives. When time derivatives of the controls enter the equations, it is usually referred to as impulsive control (Bressan, 1990) See, e.g., Bressan (2007) for a review of the fundamental theory and an application of the basic ideas to the playground swing.

2.2 Swinger techniques and comparison

The three swinger techniques studied herein, are based on swinger strategies suggested by Wirkus et al. (1998) and Case and Swanson (1990). A dumbbell modeling approach was used in both of these works.

Wirkus et al. (1998) let the swinger rotate (relative to the ropes) only at each top point of the swing, staying static elsewhere in the swing. With reference to Fig. 1, the technique might be explained as follows. Assume the swinger is in the maximum backward leaning position ($\theta_2 = \theta_{2,\max}$), moving upwards, i.e., $\theta_1, \theta_2 > 0, \dot{\theta}_1 > 0$ and $\dot{\theta}_2 = 0$. As the top point of the swing is reached ($\dot{\theta}_1 = 0$), the torso is quickly lifted up to the ropes, giving $\theta_2 = 0$. During the backswing ($\dot{\theta}_1 < 0$), body position relative to the ropes is kept unchanged ($\theta_2 = 0$, $\dot{\theta}_2 = 0$). As the other top point of the swing is reached ($\theta_1 < 0$, $\dot{\theta}_1 = 0$), the swinger quickly leans backwards, back to the position $\theta_2 = \theta_{2,\max}$, and swings forward to repeat the motion pattern just described. Conservation of angular momentum then makes the amplitude grow with each period of the swing. In their work, Wirkus et al. (1998) used a simplified model, letting $L_2 = L_3$ and $m_2 = m_3$. This gives $N = 0$ and has the effect of excluding parametric terms from the dynamics, since the swinger's center of mass

become positioned at m_1. In our analysis, we treat $N = 0$ to be only a special case, and investigate

swinger demands for this technique (referred to as "W" in the following) with $\theta_{2,\text{max}} = 40°$.

Case and Swanson (1990) let the swinger move according to $\theta_2 = \theta_{2,\text{max}} \cos(\omega t)$, with

$\theta_{2,\text{max}} = 40°$ and $\omega = \omega_0$, i.e., the natural frequency of the system. Small-angle approximations

were introduced in the trigonometric expressions of the dynamic model, limiting their analysis to

$\theta_1 < 40°$. Furthermore, no phase correction was included in the swinger motion scheme. This

prohibits θ_1 amplitudes larger than about $40°$, since the increasing pendulum period eventually

brings the swinger (applying a constant ω) out of resonance when such amplitudes are reached. Here,

we avoid small-angle approximations and introduce a simple phase correction scheme to allow larger

amplitudes to be reached. The phase correction is made at each forward ($\dot{\theta}_1 > 0$) bottom passing,

updating ω based on the recorded amplitude from the previous swing period. This recorded amplitude

is used to calculate the time period T that a corresponding *passive* pendulum would have if released

from such amplitude. The approximate period T may be found (Lima 2008) from

$$T = T_0 \frac{\ln\left(\frac{1}{a}\right)}{1 - a}, \qquad (6)$$

where $T_0 = \dfrac{2\pi}{\omega_0}$ and $a = \cos(\frac{1}{2}\theta_{1,\text{max}})$, with $\theta_{1,\text{max}}$ being the maximum angular position reached

during the swing in the previous period. This will provide the swinger with an ω that is close enough

to the optimal frequency for swing amplitudes to continue growing (as demonstrated by our numerical

experiments). The Case and Swanson (1990) technique, without small-angle approximations and with

phase correction included, will be referred to as "C1" in the following. As a rationale for our third and

final technique, we note that during resonance with C1, body rotation ($\dot{\theta}_2 \neq 0$) lasts for the entire up

(down) swing, i.e., from the bottom (top) position all the way to the top (bottom) point of the swing.

However, the parametric effect would be larger if body rotation between $\theta_2 = 0$ and $\theta_2 = \pm\theta_{2,\text{max}}$

was done quicker, both at the top and the bottom position. Consequently, the swinger should be waiting in an upright position for most of each upswing and waiting at $\theta_2 = \pm\theta_{2,\max}$ for most of each downswing. We let this strategy be our third technique, and will refer to it as "C2" in the following. For W and C2, we chose a basic swinger body rotation 4 times higher than for C1, and introduced a factor $k \geq 1$ to further regulate the speed of body rotation. The three techniques may then be summarized as

W: $\quad \ddot{\theta}_2 = -\dfrac{\theta_{2,\max}}{2}(k4\omega_0)^2 \cos(k4\omega_0 t), \quad \theta_2(0) = \theta_{2,\max}, \quad \dot{\theta}_2(0) = 0, \quad t = \left[0, \dfrac{1}{2}\dfrac{T_0}{k}\right]$

$\quad\quad \ddot{\theta}_2 = \dfrac{\theta_{2,\max}}{2}(k4\omega_0)^2 \cos(k4\omega_0 t) \quad \theta_2(0) = 0, \quad\quad \dot{\theta}_2(0) = 0, \quad t = \left[0, \dfrac{1}{2}\dfrac{T_0}{k}\right]$

during each change of θ_2 from $\theta_{2,\max}$ to 0, and from 0 back to $\theta_{2,\max}$, respectively. Note that each phase of θ_2 change is followed by a phase of latency, when the swinger is waiting for the next top-point of the swing.

C1: $\quad \ddot{\theta}_2 = -\theta_{2,\max}\omega^2 \cos(\omega t), \quad\quad\quad \theta_2(0) = \theta_{2,\max}, \quad \dot{\theta}_2(0) = 0, \quad t = [0, T]$

during each full cycle of θ_2 change, changing from $\theta_{2,\max}$ to $-\theta_{2,\max}$ and back. Note that each such period is followed by a brief phase of latency, when the swinger waits (phase adjustment) for the next bottom position before repeating the motion. Note also that $\omega = \dfrac{2\pi}{T}$, i.e. a frequency that drops as the period T grows.

C2: $\quad \ddot{\theta}_2 = -\dfrac{\theta_{2,\max}}{2}(k4\omega_0)^2 \cos(k4\omega_0 t), \quad \theta_2(0) = \theta_{2,\max}, \quad \dot{\theta}_2(0) = 0, \quad t = \left[0, \dfrac{1}{2}\dfrac{T_0}{k}\right]$

$\quad\quad \ddot{\theta}_2 = -\dfrac{\theta_{2,\max}}{2}(k4\omega_0)^2 \cos(k4\omega_0 t), \quad \theta_2(0) = 0, \quad\quad \dot{\theta}_2(0) = 0, \quad t = \left[0, \dfrac{1}{2}\dfrac{T_0}{k}\right]$

$$\ddot{\theta}_2 = \frac{\theta_{2,\max}}{2}(k4\omega_0)^2 \cos(k4\omega_0 t) \quad \theta_2(0) = \theta_{2,\max}, \quad \dot{\theta}_2(0) = 0, \quad t = \left[0, \frac{1}{2}\frac{T_0}{k}\right]$$

$$\ddot{\theta}_2 = \frac{\theta_{2,\max}}{2}(k4\omega_0)^2 \cos(k4\omega_0 t) \quad \theta_2(0) = 0, \quad \dot{\theta}_2(0) = 0, \quad t = \left[0, \frac{1}{2}\frac{T_0}{k}\right]$$

during each change of θ_2 from $\theta_{2,\max}$ to 0, from 0 to $-\theta_{2,\max}$, from $-\theta_{2,\max}$ to 0, and, finally, from 0 to $\theta_{2,\max}$, respectively. Note that each such phase of θ_2 change is followed by a phase of latency, when the swinger is waiting for the next top-point or bottom position.

The development of system configuration is found by solving (5) together with a $\ddot{\theta}_2$ equation that follows from technique (W, C1 or C2) and phase of that technique. Swinger torque may then be found from (3).

Comparison of swinger demands, as dictated by each of these techniques, will particularly emphasize differences in maximum torque (force) required, the endurance needed, and the coordinative skills. For a chosen set of system parameter values, the maximum torque needed by the swinger with C1 will vary with amplitude, whereas with W and C2, it will also vary with the rotational speed of the body. The endurance needed will depend on how long time it takes to reach the desired amplitude (defined as $90°$ in the present study), while coordination requirements will be strongly related to how critical the timing of body actions is for the buildup of swing amplitudes. System parameter values will of course have an impact on the results, so we also include simulations with different values of N. Each swing session was initiated from rest by use of C1. After one period, the swinger continued with W, C1 or C2, according to the method of choice. When $N \neq 0$, the body center of mass will be under the point of suspension for some non-null θ_1. We define the bottom position to be that for which the center of mass is at its lowest position, corresponding to a θ_1 close to $1°$ for the values of N investigated here.

2.3 Numerical experiments

In each numerical experiment (Ei, $i = 0,1,...,6$), all three techniques were simulated with the same set of parameter values. Between experiments, we changed one of k, d and N (see Tables I, II and III), where d is a time parameter (given as % of T_0) used to delay each onset of θ_2 - change by a fixed time lag. For the system parameters, we chose a total mass $M = 61kg$ with $m_1 = m_3 = 0.4M$ and $m_2 = 0.2M$. For the length parameters we used $L_1 = 2.5m$ and let $L_2 = L_3 = 0.4m$, which gave $I_1 = 381.25kgm^2$, $I_2 = 5.86kgm^2$ and $N = N_a = 4.88kgm$. This value of N is the same as used by Case and Swanson (1990) for an adult swinger. We used T_0 as found from a small-angle approximation as

$$T_0 = 2\pi\sqrt{\frac{I_0}{K_0}} \ ,$$

where $K_0 = ML_1g - Ng\left(1 - \frac{1}{4}\theta_{2,\max}^2\right)$ and $I_0 = I_1 + I_2 - 2L_1N\left(1 - \frac{1}{4}\theta_{2,\max}^2\right)$ (Case and Swanson, 1990). With $N = N_a$, we get $T_0 \approx 3.15s$, which we keep fixed throughout. The parameter N was changed by re-distributing mass between m_2 and m_3 only, leaving inertias, total mass and length parameters unchanged. Notice that C1 does not depend on k.

All simulations were carried out with scripts written in Matlab, solving the differential equations with the Runge-Kutta method of order 4 and 5, which is available in the built-in 'ode45'. The solution was provided every milli-second.

3. Results and discussion

In the reference experiment (E0), swing amplitudes grew linearly with time for W and C1, but exponentially for C2 (Figure 2). The final amplitude of $90°$ was reached by W in $271s$, while C1 and

C2 were about twice as fast, needing only 55% and 46% of the time, respectively. The periodic maximum torque required with W (Figure 3, top) remained close to $\pm 150 Nm$ for the whole interval, with a slight asymmetry in positive and negative values as caused by the kinematic differences in body movements between the top points, i.e. for positive and negative θ_1. Both C1 and C2 displayed an exponential growth in their maximum torques (Figure 3, middle and bottom). C1 torques increased from $\pm 50 Nm$ to $\pm 100 Nm$, i.e. substantially lower than W, while C2 torques were higher than W, ranging from $\pm 150 Nm$ to $\pm 225 Nm$.

Increasing the speed of body rotation (E1 and E2) for W and C2 demanded much higher swinger torques. As k was increased to 1.5 and 2, W torques were amplified by factors of about 2 and 3, respectively. For C2, the corresponding factors were approximately 2 and 4. However, the extra effort did not pay off much when measured in the time spent on reaching the final swing amplitude of $90°$. Increasing k to 1.5 and 2 reduced the total swing time for W and C2 with $< 6\%$ and $< 4\%$, respectively. Amplitudes evolved with time as in E0.

Delaying the onset of body rotation (E3 and E4) illustrated that C1 was extremely sensitive to the timing of body actions compared to the other two techniques. A delay of just 1% (of T_0) deteriorated resonance substantially and caused amplitudes to reach only $50°$ (Figure 4, top). Maximum torque requirements became upwards limited to $60 Nm$ (Figure 4, bottom). A further delay of 10% made C1 amplitudes only reach about $8°$. Very little response was noticed for W and C2 when timing was delayed by 1%. Total swing time increased by $< 4\%$ and maximum torque developed in essentially the same way as before. With 10% delay, total swing time for W and C2 increased by approximately 65%. C2 now displayed a more linear growth of amplitudes but with a torque development as before (Figure 5). For W, the maximum torque developed much as before, but on the negative side ($\theta_1 < 0$), demands grew slightly ($< 20\%$) since backward rotation now was initiated after the downswing had started.

Changing body mass distribution (E5 and E6) first and foremost affected torque requirements, leaving amplitude progress essentially unchanged for all three techniques, whether N was halved or

set to zero. Swinger torques with W became more symmetric and fully symmetric, respectively, as N was changed to $N = \frac{1}{2} N_a$ and $N = 0$. This owed to the change in location of body center of mass, equaling the work required at the top point on either side. Otherwise W torques developed as before. As N was halved (set to zero), C1 torques dropped by a factor of 0.6 (0.3), while C2 torques only changed noticeably with $N = 0$. Torques then diminished by a factor of 0.8.

4. Conclusion

It was investigated here in this work whether distinct swinger techniques require significantly different maximum torques, endurance and coordinative skills, and whether any preferable technique could be identified. A dumbbell modeling approach was chosen, as previously used also by Case and Swanson (1990) and Wirkus et al. (1998). One technique (C1) demanded considerably lower maximum torques than the others. It was also nearly as quick to build up swing amplitudes as the fastest technique (C2). However, it was extremely sensitive to the timing of body actions compared to the other two alternatives.

The model did not include air resistance and changes in swinger body configuration. In reality, the swinger holds on to the ropes with arms that bend and stretch to produce the torque that we compute with our model. Often, the knees also bend and stretch during the swing. These factors will generally differ somewhat between the techniques and mass distribution will be affected. However, the influence on system dynamics is expected to be small. Similarly, the chosen range of swinger body rotation ($\pm \theta_{2,\max}$) is assumed to be realistic. Other choices would have affected the dynamics, but not the relative performances of the techniques. For example, halving $\theta_{2,\max}$ increased the time to reach

$\theta_1 = \dfrac{\pi}{2}$ for all three methods, but, C2 ($289 s$) was still faster than C1 ($302 s$), which in turn was faster than W ($533 s$). Likewise, air resistance will have an effect at larger amplitudes, but the effect is expected to be comparatively small and is assumed to affect all techniques in about the same way.

Our findings imply that even if C1 performs well on maximum torque and buildup of swing amplitudes, it is a much more demanding technique than W and C2 if larger amplitudes (above $40°$ - $50°$) are strived for. This owes to the very strict timing requirements of body actions. With swing amplitudes being upwards bounded to $50°$ when body rotation is delayed by only 0.03 s ($\approx 1\%$ of T_0), it seems virtually impossible to reach higher amplitudes with this technique. The two alternative techniques, W and C2, are much more robust in this respect, since $90°$ amplitudes could be reached even with a delay ten times larger. The endurance requirements of C2 are substantially lower than for W. On the other hand, higher maximum torques are needed. What technique to choose, will therefore depend on the goal and abilities of the swinger. If the aim is to reach large amplitudes as quickly as possible, C2 is the method of choice if the resources are there. A swinger with less competitive attitude may keep smaller amplitudes going with any of the techniques, or combination of techniques.

In conclusion, different swinger techniques do place very different demands on the swinger. Of the three techniques studied, two methods (W and C2) were identified as preferable over the third alternative. Among these two, the choice should be done according to swinger goals and abilities.

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Table I Parameter values while changing k, i.e. the speed of body rotation.

Experiment	k	d (%)	N (kgm)
E0	1	0	N_a
E1	1.5	0	N_a
E2	2	0	N_a

Table II Parameter values while changing d, i.e. the delay of action (in % of T_0).

Experiment	k	d (%)	N (kgm)
E3	1	1	N_a
E4	1	10	N_a

Table III Parameter values while changing N, i.e. the body mass distribution.

Experiment	k	d (%)	N (kgm)
E5	1	0	$\frac{1}{2} N_a$
E6	1	0	0

Figurer...
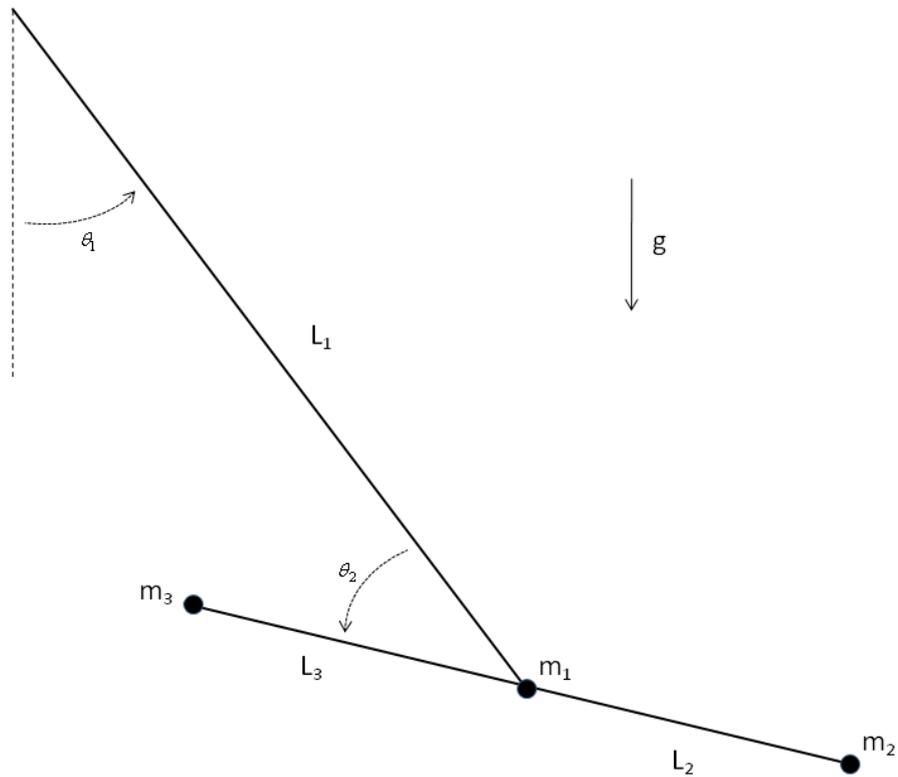
Figure 1 Playground swing with a stiff and mass-less rope, having length L_1 and an angular position θ_1 with the vertical. The swinger is idealized as a rigid dumbbell with length $L_2 + L_3$ and an angular position θ_2 relative to the ropes. All mass is located in point-masses m_1, m_2 and m_3. Angles are defined positive in the directions shown.

Angular position of ropes

E0: W

Angle with vertical (deg)

Time (s)

Angular position of ropes

E0: C1

Angle with vertical (deg)
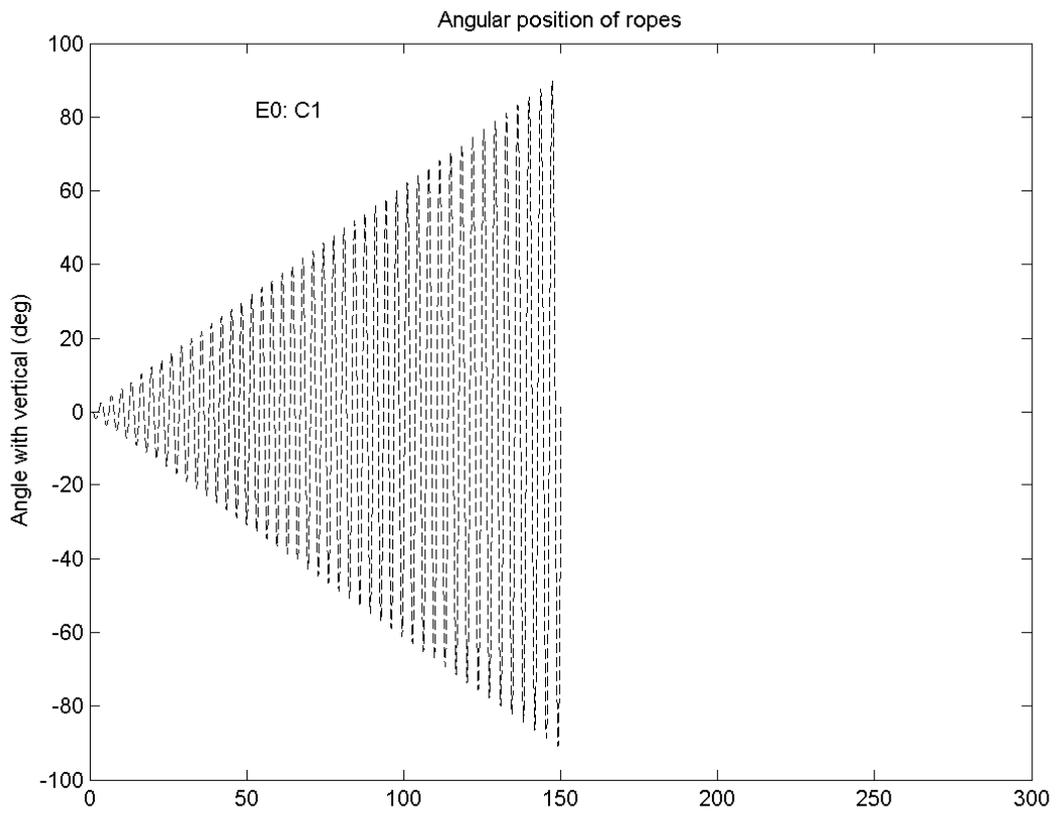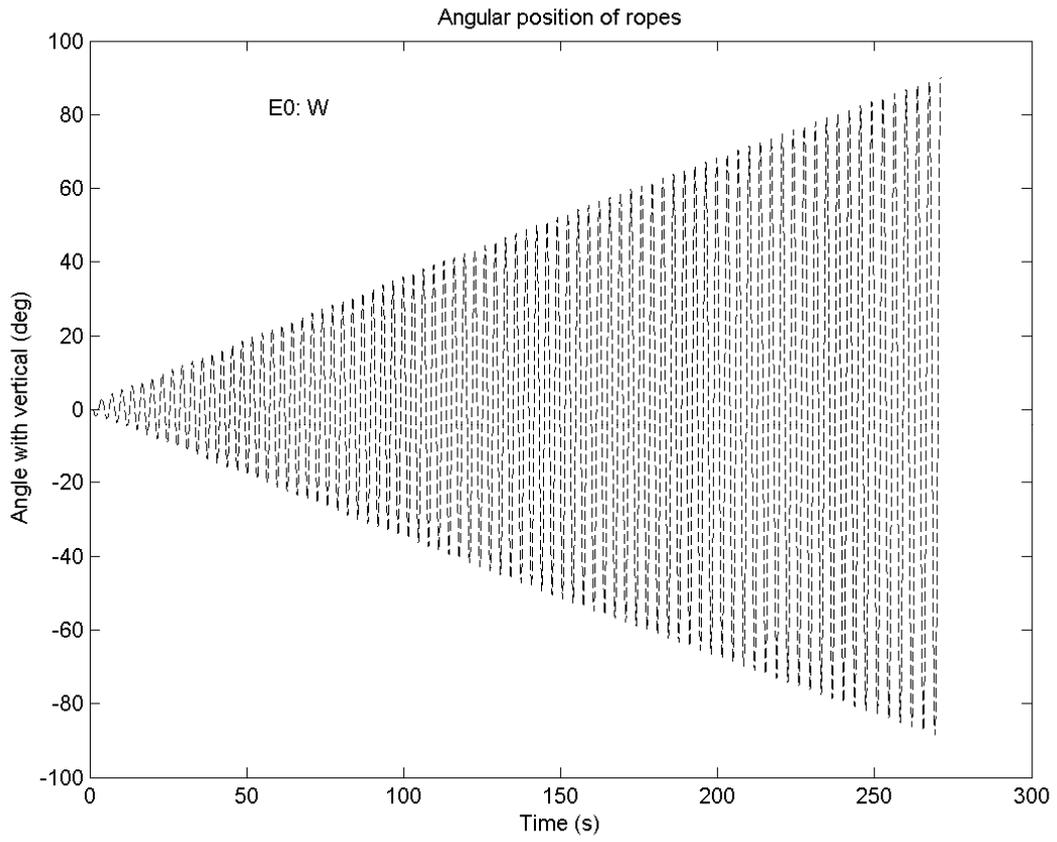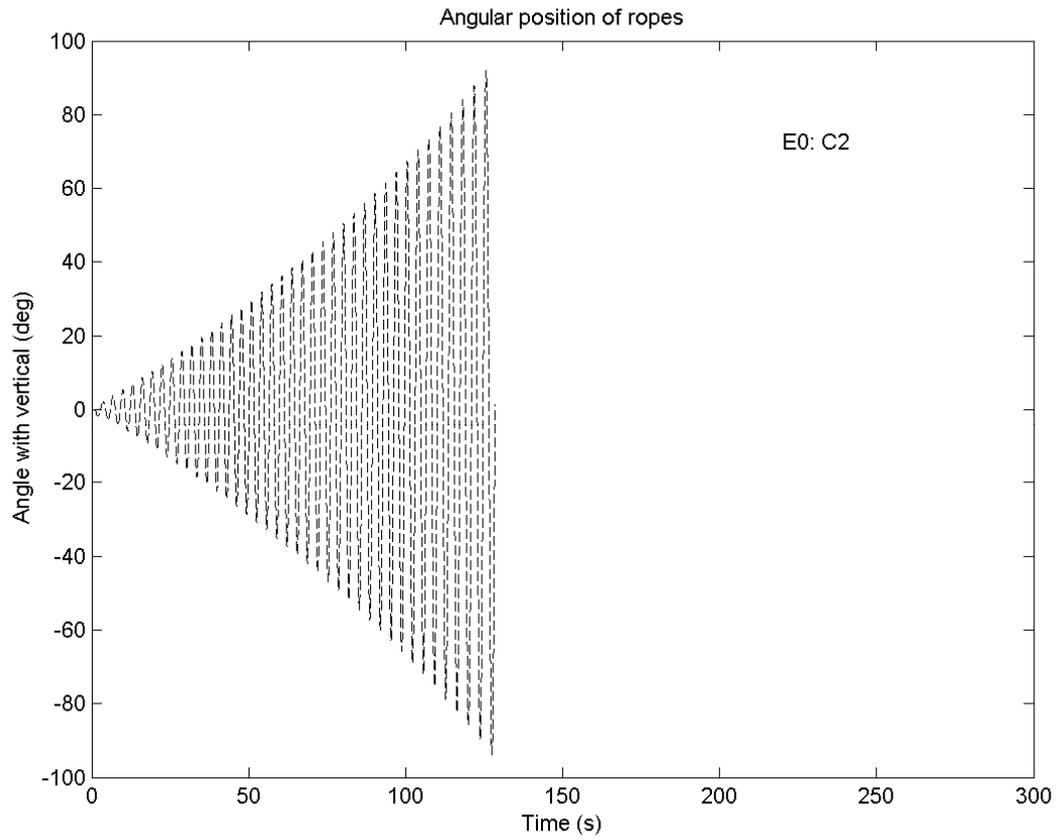
Figure 2 Amplitudes of the swing as they grow with W (top), C1 (middle) and C2 (bottom) during E0 ($k = 1$, $d = 0$, $N = N_a$). A linear growth is seen with W and C1 as opposed to the faster growth of C2. The corresponding torques are shown in Fig. 3.

Torque required by swinger

Torque required by swinger
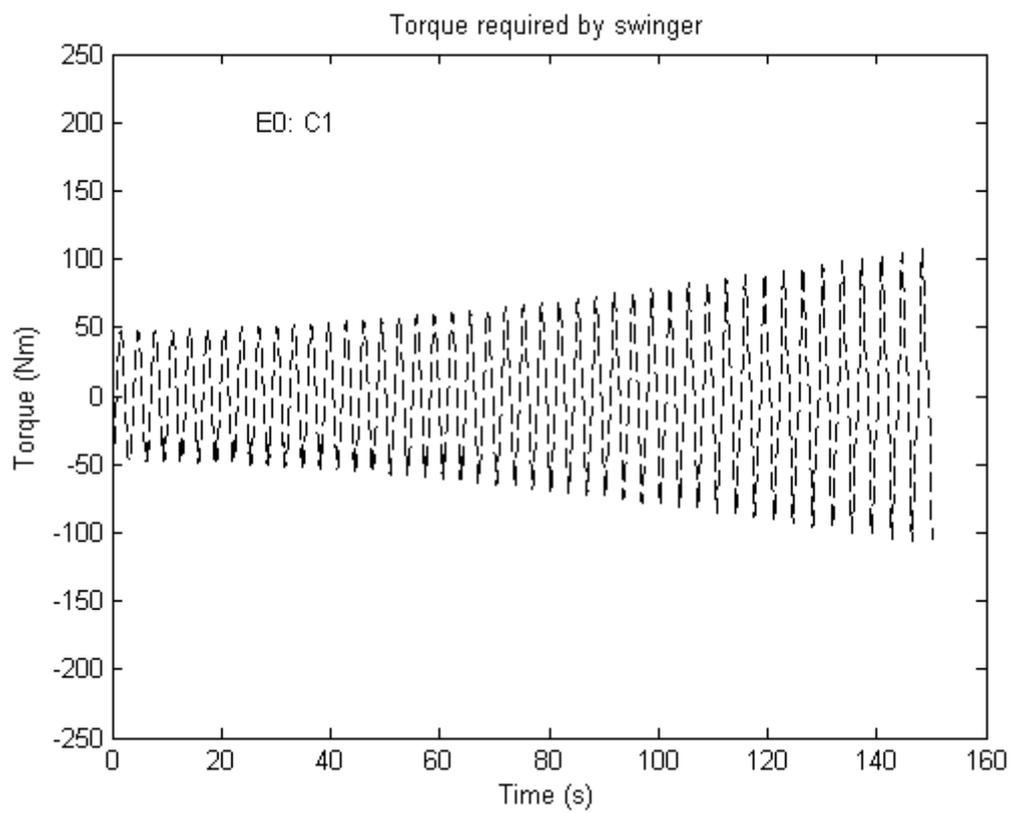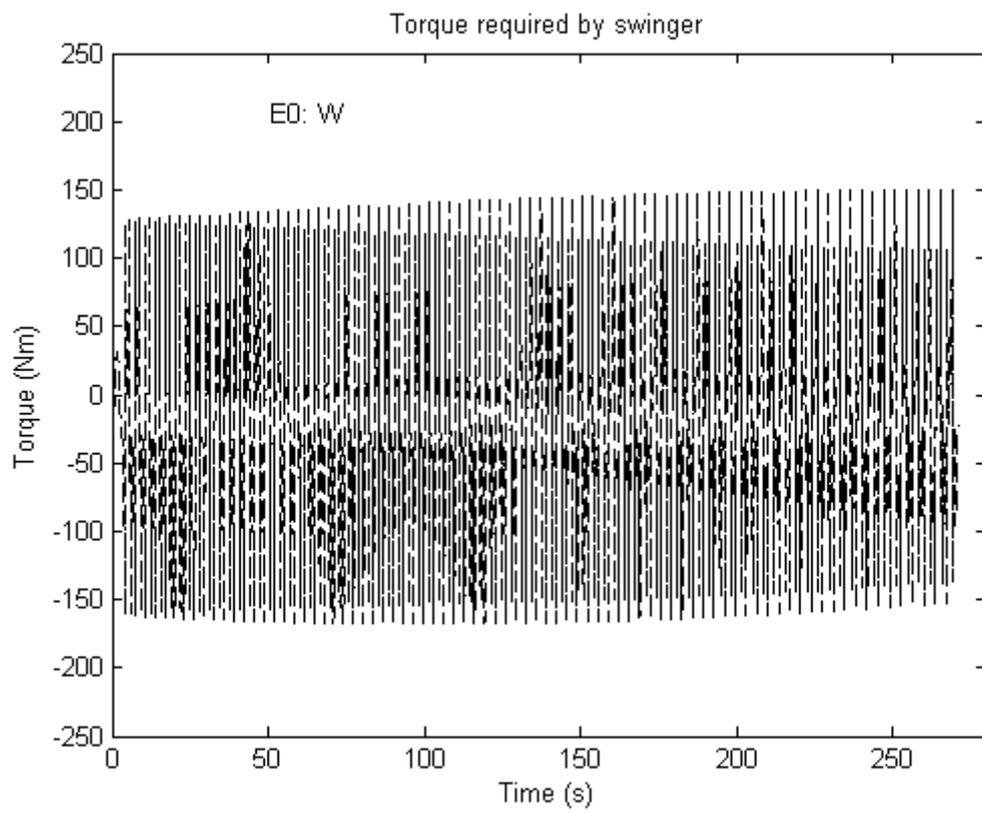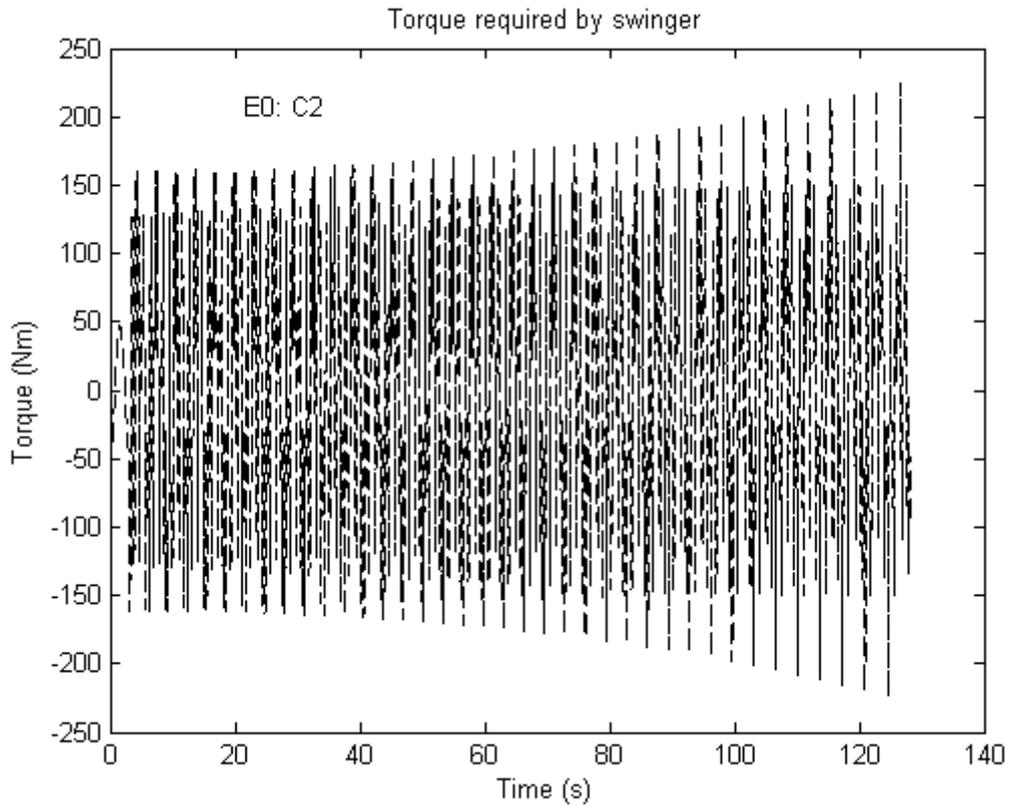
Figure 3 Torques required by the swinger with W (top), C1 (middle) and C2 (bottom) during E0 ($k = 1$, $d = 0$, $N = N_a$). The corresponding amplitudes are shown in Fig. 2. Note the different time scales.

Angular position of ropes

E3: C1
d = 1

Angle with vertical (deg)

Time (s)

Torque required by swinger

E3: C1
d = 1

Torque (Nm)

Time (s)
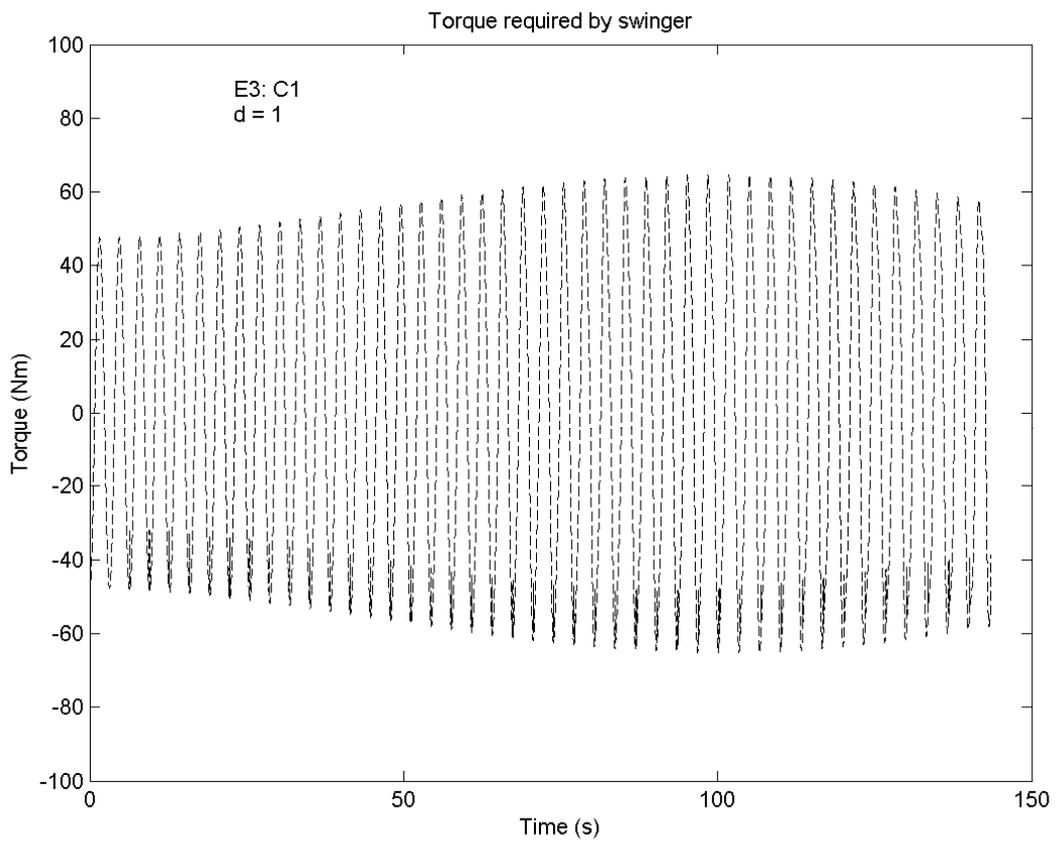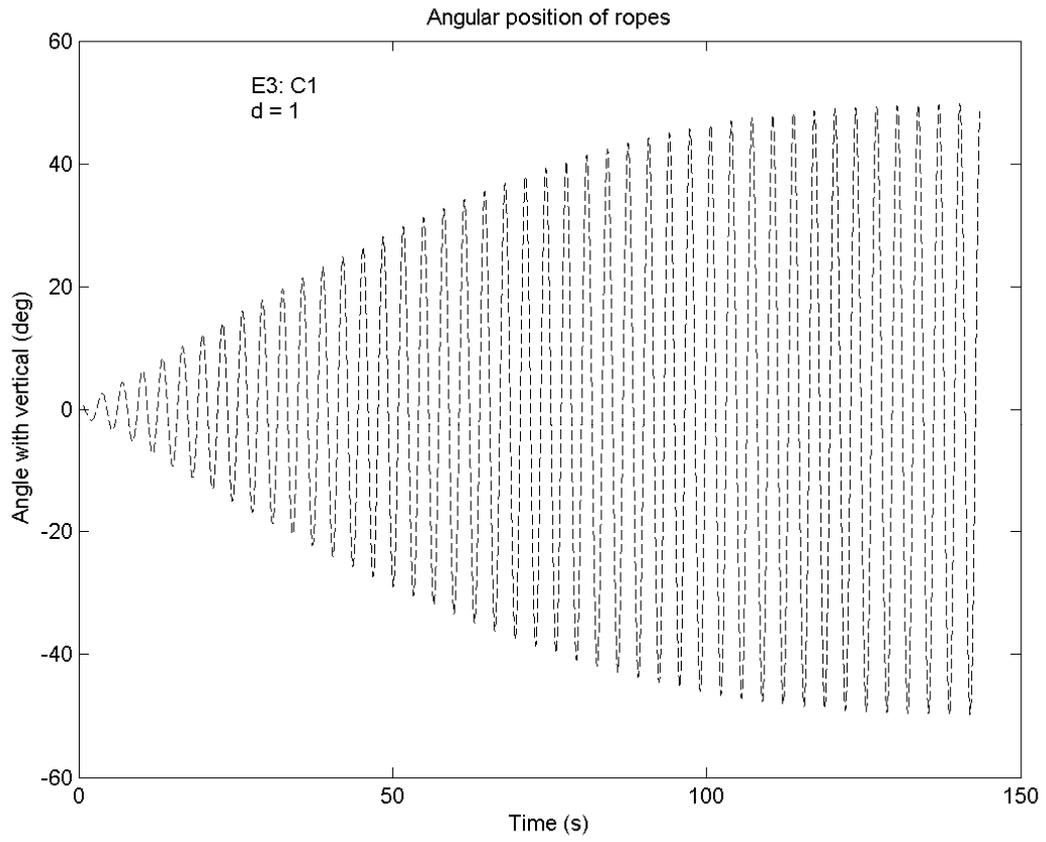
Figure 4 Amplitudes (top) and torques (bottom) required by the swinger with C1 during E3 ($k = 1$, $d = 1$, $N = N_a$). The amplitudes do not grow further than $50°$ when swinger body rotation is delayed by only 1% of T_0.
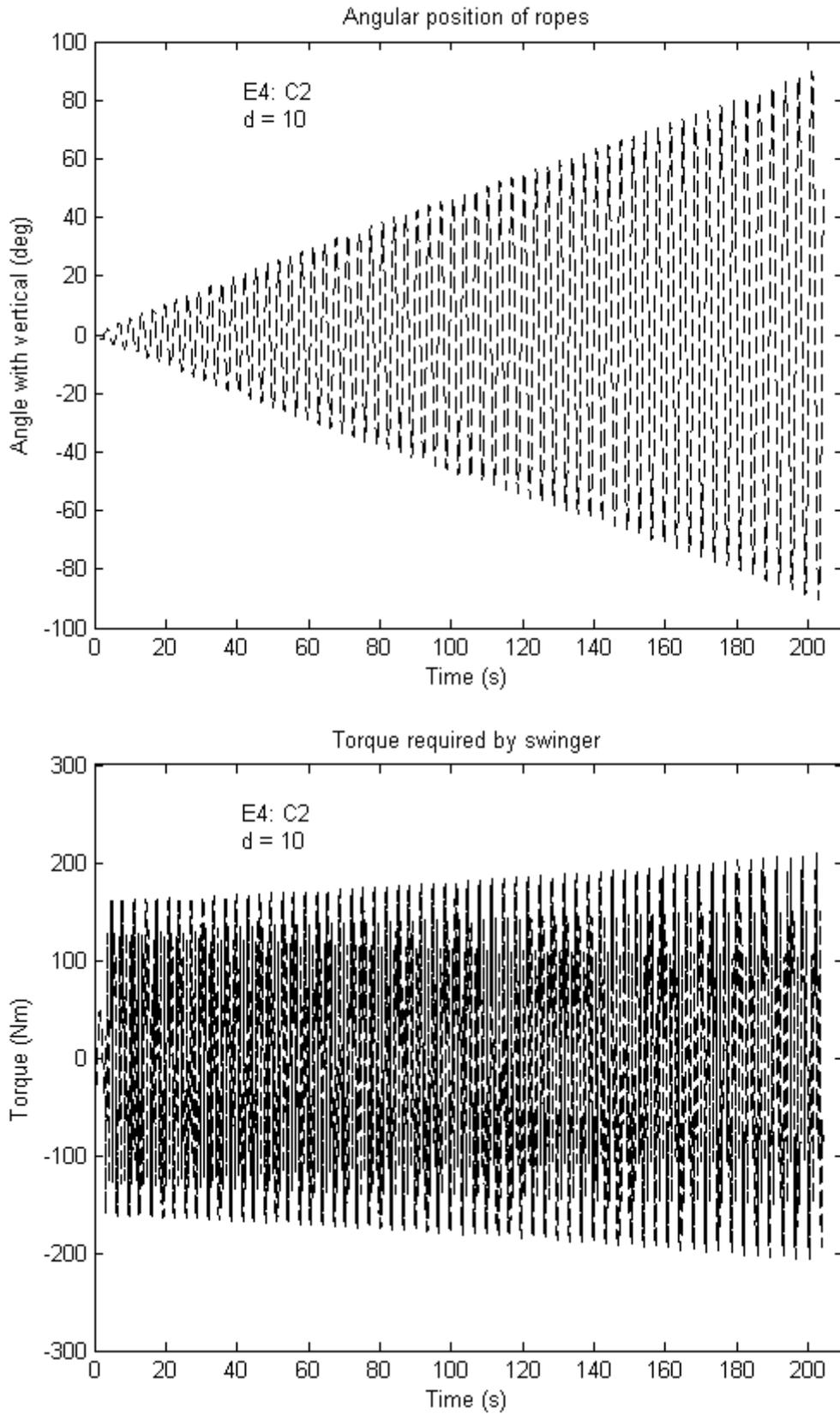
Figure 5 Amplitudes (top) and torques (bottom) required by the swinger with C2 during E4 ($k = 1$, $d = 10$, $N = N_a$). The amplitudes now grow linearly only and much longer time is needed to reach $90°$.