Is Engineering Mathematics Really an Engineering Tool?

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Abstract: *A set of final exams at Telemark State University has been studied to check if there is a symbiotic relationship between the mathematical tool and some technical courses offered at the Electrical Engineering Department. The study reveals some major differences with respect to culture, scope, form and content between the mathematical and technical world; maybe explained by the traditional role of mathematics as a 'sorting mechanism' of entering students. However, recent internal development has shown that a bridging process is on its way. Thanks to this development, the answer to the question raised in the heading must be a cautious 'yes' but further development work is necessary to ensure good linking between the Department of Mathematics and the engineering staff.*

Keywords: *engineering mathematics, engineering culture, taxonomy, didactics, student sorting, academic level*

1 Background

During the last decades the emphasis on engineering and engineering education has shifted to a more broadscoped viewpoint. At the same time engineering teachers worry that too many of the entering freshmen are stopped because they don't pass the entry-level exam in engineering mathematics.

The question has been raised: Is there a consistency between the introductory engineering mathematical course and the mathematical level actually used in the senior class? Or, are the contents, eventually the weighting of the individual components of it, fairly irrelevant to the technical courses?

To try to get a partial answer to these questions, the content of the introductory mathematics final exams and the use of mathematics as a tool in the introductory Electrical Engineering course and some senior semester engineering courses final exams at Telemark State University is checked, compared and evaluated.

2 Scope

In order to get a fairly complete impression of relevant connections between mathematics and engineering, the paper will first examine the *form* of the exams as well as the single problems, and then compare the *content*. This approach is chosen to check if there are more basic, maybe even *cultural* differences between mathematics and engineering than can be found from a mere inspection of content alone.

Thus, some aspects of the examination form will be described and shortly discussed in Part 3. In Part 4 it will be tried to identify eventual variations in the form of which the problems are given, while it in Part 5 will be taken a closer look at the factual content. The overall summary will then be given in Parts 6 and 7.

3 Sampling

The survey is based on material from 18 final exams in the Electrical Engineering Department and 6 in mathematics at Telemark State University during the period 1995-98.

The period is chosen because the task of making the final exams in mathematics up to 1996 was considered a *national* responsibility. From 1996 this responsibility was delegated to the engineering schools. Thus, this survey is intended to detect eventual changes in style and content as a result of this shift in responsibility.

Further, as it is assumed that the electrical engineering students are subject to more extensive use of mathematics while pursuing their studies compared to fellow students at other engineering departments, the engineering reference final exams were chosen from this department. The selection covers the electrical engineering entry course, some main technical courses in electronics and power engineering plus advanced courses in control engineering - all at the senior level. An overview is given in Table 1.

The Electrical Engineering Department may be considered divided into three parts, control, electronics and power engineering. In practice, control and electronics overlap and are representred with 11 final exams compared to power's 7.

In reading the table, it should be kept in mind that the sum and numbers in the rows and columns do not always add to expected results. The reason for this is that some numbers may be used more than one time, f.inst. the use of calculator, which may be represented in more than one column. This is done for not to overload the table, and is believed to have negligible importance for the main findings of the present work.

Table 1: Sampling data

	Number of exams	Number[1] of questions	All[2] kinds of support allowed?	Modern[3] calculator allowed?	Basic[4] calculator allowed?	Pen & paper only	Sheet of 'formulas' attached
Control/ signals	5	22	0	0	0	5	5
Electronics	6[5]	18	0	2	3	1	1
Power	7[5]	21	6	3	2	0	1
Mathematics	6	14	3	3[6]	3	0	3

Table comments:

[1] These have been summed to check if there is a significant difference between engineering and mathematics problem formulation with respect to 'problem fragmentation'
[2] 'All kinds' includes the use of a 'modern' calculator, handouts and textbooks - in one case even notes
[3] 'Modern' calculator is technical, alphanumerical, graphical and programmable
[4] 'Basic' calculator is arithmetic and non-programmable
[5] Electronics and power share the responsibility of the introductory courses, and have been credited with two exams each
[6] Modern calculator was allowed after the era of centrally given introductory exams in mathematics ended in 1996

What, then, can be read from Table 1?

1. Splitting problems into several subquestions at different taxonomic levels to calm down the nervousness of the student, is a widely used technique. This splitting appears more specificly used in engineering, as the number of subquestions are about 36% higher than in mathematics. Together with other signals, this *could* indicate that some cultural differences exist

2. There is apparently a difference in final exam culture within the Electrical Engineering Department, as the signal/control finals are all set up for no use of supporting tools like calculator, textbook, notes etc. This could indicate a more clear-cut theoretically than practically oriented level in these courses

3. It may also be read that the form of the power final exams appear to be at the leading edge with respect to setting up 'real-life-situation' type of problems, as more supporting material and the use of modern calculator are allowed

4. The row for mathematics shows extreme variations, spanning from 'all kinds' of support material to the acceptance of only a basic calculator. The most liberal variation is dated

1998; the most restricted final exams are given by the Ministry's Council for Engineering Education in 1995 and 1996. Thereafter, modern calculators have been an obviousness

Finally, it can be mentioned that all electrical engineering faculty members (14) plus two parttime, adjunct professors have produced the technical exams. In formulating the exams in mathematics, the entire mathematics staff of four have contributed, though not with the same course at the same time.

4 Problem formulation

By inspecting the formulation of 390 problems, it is possible to group these into several categories, like:

1. 'Show that', 'prove that' etc. - a formulation used in mathematics only
2. 'Formulate', 'model' - including circuit transformation, small signal models, control systems modeling, etc.
3. 'Define' - used only once
4. 'Describe', 'explain', 'know' - 27% of a total of 100 questions in this group requires sketching or graphical explanations
5. 'Calculate'
6. 'Discuss'

However, it appears to be no strict or well-defined borders between these categories. If these numbers are collected into only three groups - each representing the description of an assumed intellectual level, the result could be:

1. 'Know', 'calculate'
2. 'Define', 'discuss'
3. 'Show that', 'prove that', 'model'

ranged with escalating demands on intellectual challenges.

It is assumed here, that modeling represents the engineering equivalent to the mathematician's search for proofs. It is also believed that the mathematical problems of defining limits and integrals represent the intermediate level equivalent to engineering problem formulation and discussion of solutions. On the other side, it should be mentioned that the 'calculate' term often include a significant ability to formulate and describe simple technical problems, which is much more than just 'plugging numbers into formulas'.

The other aspect is grouping of courses. In this case, 'technical courses' include control, electronics and power. In Part 3 it felt natural to share the responsibility for the entry course in electrical engineering between these sections. However, this may not be the case here, as the introductory course is taught concurrently with the courses in mathematics. Thus, it has been chosen to tabulate the introductory course and mathematics in separate columns, while the data of the joint control, electronics and power exams have been accumulated in the third column labeled Senior year, see Table 2.

Table 2: Problem classification

Definition	Entering level	Senior year	Mathematics
Know, calculate	51 (77%)	166 (68%)	21 (26%)
Define, discuss	12 (18%)	55 (23%)	45 (56%)
Show/prove/model	3 (5%)	23 (9%)	14 (18%)
Sum	66 (100%)	244 (100%)	80 (100%)

In Norway the grading of each course is normally based on the final exam exclusively. It is interesting, then, to note that it apparently seems to be a difference of 'exam culture/ pedagogics' between the Electrical Engineering and the Mathematics departments.

An inspection of the Entry level and senior year Engineering columns reveals that a solid majority of the problems ask for the lowest intellectual level solutions. The intellectual demands seem to increase slightly as the students proceed on to the senior year.

The Mathematics column shows that quite a few problems are of the 'know/calculate'- type. A weighty ¾ of all problems require significant independent way of thinking and intellectual maturity to obtain a good grade, or maybe even passing the exam.

This difference may be explained by reminding that mathematics, in addition to being an engineering tool, in many educators's opinion even serves as a tool for mind expanding, ability to abstract thinking and problem formulation - even as *the* definition of the 'academic level' of the engineering profession. On the other hand, it is claimed by others that such levels exist in *all professions*, which include engineering.

This difference also indicates that the final exam in mathematics may be perceived as a *sorting* mechanism based on some, maybe arbitrarily chosen criteria. On the other hand, the engineering exams, in addition to their evaluational purpose, even indicate an instrument for *personal growth*. If this is the case, a difference in culture and scope for the daily work with students may be said to exist.

5 Mathematical content

Table 3 aims to list the factual mathematical tools necessary to solve the problems given in the entering mathematical as well as the engineering courses.

The entry-level mathematical courses are 'Mathematics 1'/'Mathematical methods 1' and 'Discrete mathematics and linear algebra'. The slash indicates that the survey include two sets of finals given centrally and two sets given locally after 1996 respectively, see Part 3 Sampling. Inspection of these finals reveals that there are some important differences with respect to adaption to engineering culture between the centrally and locally given exams. These will be referred to under the overall discussion but not tabulated for not to overload Table 3.

There are, of course, a number of details which remain invisible in this tabulation. For example: 'Guldin's Rule' applied for finding areas under arbitrarily rotating curves is abundantly represented in Mathematics 1 and Mathematical methods 1. This method is here listed as 'integration'. Another noteworthy detail is the apparent absence of technical relevance found in the linear algebra problems. As this course is taught concurrently with circuit analysis, it should be expected that traces of explicit solving of simultaneous equations, even in complex form could have been found.

With respect to high-level mathematics, the control and signal courses are, at least 'on the paper', the most ambitious courses in electrical engineering at Telemark State University. Even if the exams seldom ask for explicit solutions of the many mathematical expressions, a good mathematical understanding is an absolute requirement to obtain even a passing grade.

Table 3: Mathematical content (beyond basic elements listed above)

Element	Introductory Electrical Engineering	Main courses, senior year level	Mathematics 1/ Mathematical methods 1	Discrete mathematics and linear algebra
technically oriented?	all	all	5 Math 1 5 Math meth 1	1
complex	yes	yes	yes, with 'non-engr.' numbers	yes, applicable
series	none	control & signal only	3 Math 1 5 Math meth 1	none
lim	none	none	4 Math 1 5 Math meth 1	none
laplace transf.	none	control & signal	none	none

fourier series	none	electric machinery & signal (understand, not calculate)	none	none
matrix	none[1]	control engineering	none	if technical, presumedly at PhD-level
derivation	sinewave, const. & di/dt- underst.	understand importance of rates	some technical applications	none
integration	sinewave, circle	constant, sinewave, basic understanding	hard to relate to technical EE-applications	1, not particularly technical
1^{st} order differential equation	none	control & signal; 'understand'	technically oriented but 'odd'	nontechnical
2^{nd}/higher order differential equation	none	control only, 'know' level or verbal	Math 1: None Math methods 1: techn. approach	none
parameter equation	none	none	1	none

Table comment:

[1] An unexpected result!

From inspecting Table 3, it may be seen that the entry level course Discrete mathematics and linear algebra contains very little ready-to-use material for the technical studies. The emphasis on abstract matrix manipulations may, maybe, match some needs of advanced control courses at the Ph.D.-level taught at the college.

On the other hand, the treatment of complex numbers, so important to electrical engineering, tends to be more close to the engineering way compared to the material found in the finals of Mathematics 1 and Mathematical methods 1.

Another interesting information is, that the Introductory electrical engineering course is mostly based on basic mathematics. The only exceptions are the derivation and integration of very fundamental functions, mainly sine, cosine, and constants. This, again, may indicate a pedagogical approach to improve student retention.

Finally, Table 3 reveals that most problems even in the engineering senior exams require *basic mathematics* for their solution. The Control and Signal courses lean heavily to higher level mathematics but the applications may be labeled as 'verbal' or 'conceptual' more than 'actively applied'.

6 Discussion

This survey shows that mathematics is an important tool for the description, solving and discussing of technical problems at the Electrical Engineering Department at Telemark State University. In addition, it is clear that solving most final exam problems in engineering call for 'basic mathematics' as the tool. Sketching and the use of graphics are also important tools and ways of expressing engineering ideas and solutions.

Most important, maybe, is the engineering culture of caretaking, as expressed by a larger number of subquestions and the fact that the heavy bulk of problems are defined at the cognitive level and slightly above. This *may* be interpreted as a genuine interest in student growth as well as a mean to assure high retention.

On the other hand, it has been registrated a significant 'technification' of the introductory course in mathematics, showing that the shift from Mathematics 1 to Mathematical Methods 1 has had a deeper meaning that just a change of name. The acceptance of modern, fully programmable calculators and new problem formulations are examples of this. Looking at the content, it appears reasonable, though wrapped in a form often unfamiliar to engineers.

Finally, it has been found, that Discrete mathematics and linear algebra exams in the present form appears fairly irrelevant with respect to the ordinary engineering 3-year's study. Some adjustments in content together with an even more clearly pronounced engineering-point-of-view didactics could be the solution to this problem.

7 The answer

Before giving the answer, it should be remembered that this paper is written on informations extracted from 24 final exams. These do not necessarily reflect *all aspects* of the learning processes proceeding this final test. This may be most heavily pronounced in the mathematical exams, which have intededly been given a content and form to serve the needs of *all* engineering departments. In other words, textbook examples and problems may be more directly linked to specific engineering applications than reflected in the final exam. To find a reliable and generally valid answer should then require a much larger base of information than used for this survey.

Thus, if there is an answer to the question 'Is engineering mathematics really an engineering tool' based on the findings in this survey, this should be a cautiously 'engineered' *yes*.