University of South-Eastern Norway
Faculty of humanities, sport, and educational sciences

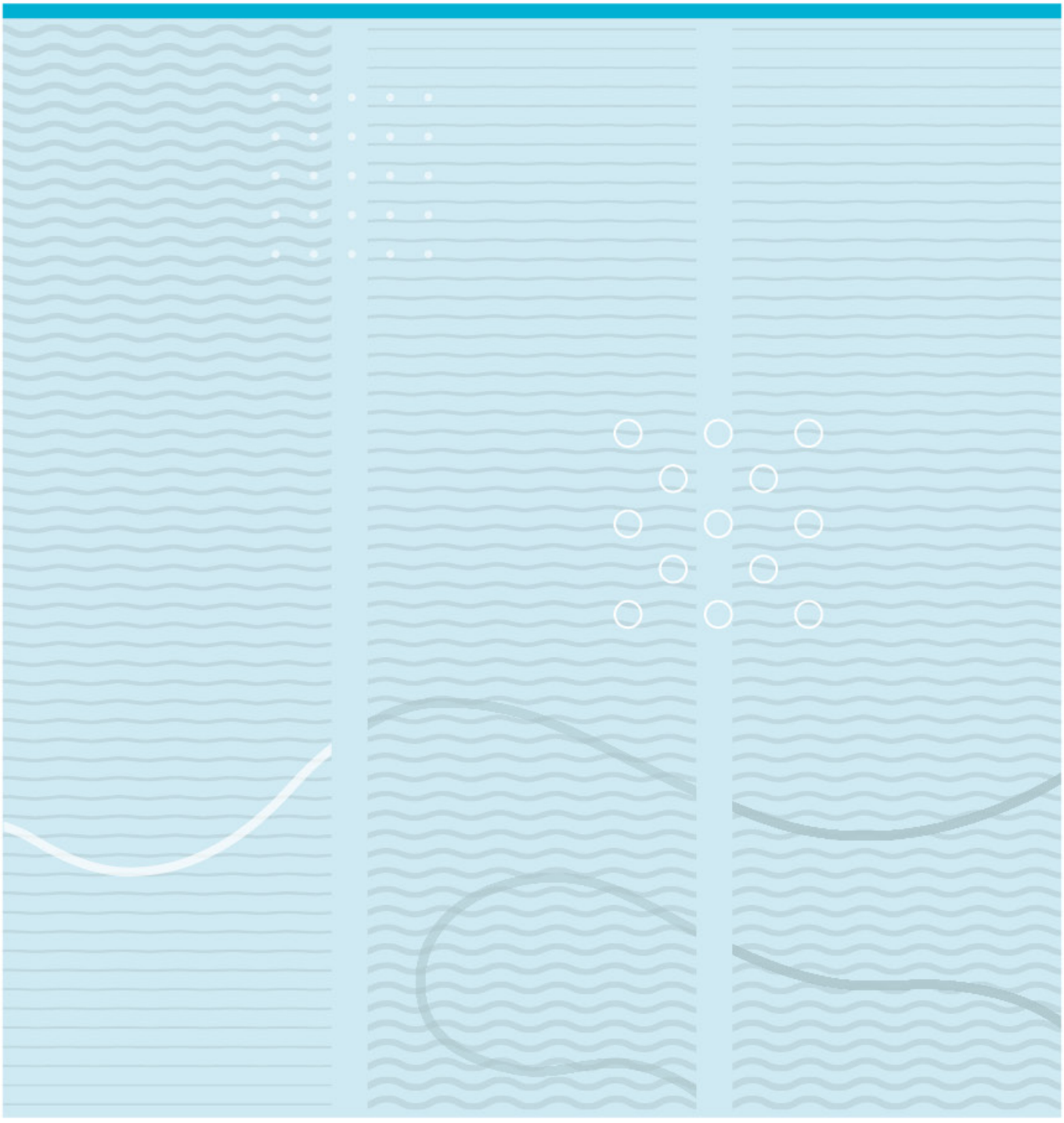
–

Master's Thesis MG1MA7-1 22H
Study programme: GLU 1-7
Spring 2023

Heidi Dansholm Stokmo

Resonative mathematical practices

Subtitle



University of South-Eastern Norway
Faculty of humanities, sport and educational sciences
Institute of mathematics and science
PO Box 235
NO-3603 Kongsberg, Norway

http://www.usn.no

This thesis is worth 45 study points

Sammendrag

Dette dokumentet presenterer forskningsfunnene til Heidi D. Stokmo i masteroppgave om matematikkundervisning. Oppgaven utforsker hvordan migrasjon i matematikk oppleves gjennom teorien om "resonans", som refererer til det dynamiske samspillet mellom individet og det kollektive under matematiske praksiser.

Oppgaven starter med en introduksjon til begrepet resonans og dets relevans for studiet av migrasjon i matematikkundervisningen. Forfatteren argumenterer for at resonans kan gi innsikt i de affektive og sanselig dimensjonene ved matematiske praksiser, dimensjoner beskrevet av Gutiérrez & Rogoff (2003), og kan hjelpe til bedre å forstå ulike dynamisk kompenter av en individets kulturhistoriske repertoarer.

Oppgaven er delt inn i kapitler seks, som hver utforsker et annet aspekt ved resonans i matematiske praksiser. Kapittel 1 og 2 gir en oversikt over bakgrunnen og det teoretiske rammeverket for denne forskningen. Kapittel 3 beskrev forskningsmetodikken, mens kapittel 4 og 5 presenterer en analyse av hvordan resonans manifesterer seg i elevenes engasjement med matematiske oppgaver og setter søkelys på funnene i denne studien som er mønstre av resonans og dissonans som man møter i matematikklasserommet.

Den første delen av kapittel 6 setter søkelys på lærerens rolle i å tilrettelegge for resonans under matematiske praksiser, mens den andre delen utforsker implikasjonene av resonans for matematikk forskning i større omfang. Oppgaven avsluttes med en diskusjon av resonansteoriens mulige bidrag til matematikkundervisningsforskning og -praksis, spesielt dens evne til å legge merke til resonans i både kjente og ukjente matematiske praksiser.

Samlet argumenterer oppgaven for at resonans gir et verdifullt rammeverk for å forstå de dynamiske dimensjonene til kulturhistoriske repertoarer i matematikklæring og -undervisning, særlig i tilknytning til opplevelsen av migrasjon i matematikk. Forfatteren foreslår at fremtidig forskning innen matematikkundervisning bør fortsette å utforske begrepet resonans og dets mulige implikasjoner for å forstå og fremme mer engasjerende og effektive matematiske praksiser i mangfoldige klasserom.

Abstract

This document presents the research findings of Heidi D. Stokmo´s master's thesis in mathematics education. The thesis explores how migration in mathematics is experienced through the lens of "resonance," which refers to the dynamic interplay between the individual and the collective during mathematical practices.

The thesis begins with an introduction to the concept of resonance and its relevance to the study of migration in mathematics education. The author argues that resonance can provide insight into the affective and embodied dimensions of mathematical practices, dimensions described by Gutiérrez & Rogoff (2003), and can help in better understanding the dynamic components of an individual's cultural-historical repertoire.

The thesis is divided into chapters six, each of which explores a different aspect of resonance in mathematical practices. Chapter 1 and 2 provides an overview of the background and theoretical framework of this research. Chapter 3 described the research methodology, while Chapter 4 and 5 presents an analysis of how resonance manifests in students' engagement with mathematical tasks and focuses on the findings of this study which are patterns of resonance and dissonance encountered in the mathematics classroom as well as the dynamic interplay of the various components.

The first section of chapter 6 focuses on the role of the teacher in facilitating resonance during mathematical practices, while the second section explores the implications of resonance for mathematics education more broadly. The thesis concludes with a discussion of the potential contributions of resonance theory to mathematics education research and practice, specifically its ability to notice resonance in both familiar and unfamiliar mathematical practices.

Overall, the thesis argues that resonance provides a valuable framework for understanding the dynamic dimensions of cultural-historical repertories in mathematics learning and teaching, specifically when investigating the experience of migration in mathematics. The author suggests that future research in mathematics education should continue to explore the concept of resonance and its potential implications for understanding and promoting more engaging and effective mathematical practices in diverse classrooms.

Foreword

The title of this work contains the conjugated term resonative. What I attempt to convey through the term resonative is the active process that occurs in encountering and performing mathematical practices. The term “resonate” is often associated with the term “reverberation”, which is associated with a sound, a pattern of repeated sounds, or harmony. This connection points to resonance as an event, perhaps familiar or meaningful, that contains a quality abundance. Resonative experience, regardless of their cause or origin, represent practices that are meaningful to the individual. The conjugation of the term resonance is purposeful as it not only indicates but emphasizes the activation states demonstrated in this research.

Senja, 10.03.2023
Heidi Dansholm Stokmo

Table of Contents

1 Introduction

Amid the studies on mathematics education, the topic of diversity and migration in mathematics seems to be receiving attention both in the West and in developing countries, and how to supports student´s prior knowledge and improve achievement. Alongside this research, there has been a shift in curriculum emphasis which requires us as educators to reinvigorate our mathematical practices as educators and allowing students to select and rationalize a wide range of mathematical approaches in addition to completing tasks and solving problems. This line of inquiry is a response to structural practices and discourses that disregards the knowledge skills, and thus, opportunities of multicultural students. One way to respond is by examining the mathematical repertories of students with multi-cultural educational background. The students´ accounts of repertoires may not be definitive in the limited empirical sources, such as transcripts, but it does counter the oversimplification that transcends the construction of mathematical universalism and presents noticing resonative practices as an approach to alternative discourse. It seeks to emphasize the rationalization of mathematics education, starting from the source, i.e. the individual student, and reflected in experienced mathematics practices.

1.1 Background

My supervisor, Yasmine Abtahi, introduced me to a research project as part of my master thesis journey. I joined the "Migration in Mathematics Classrooms" (MMC) research project, which seeks to look closer at how mathematics learning impacts student learning by collecting insight from the perspectives of students and teachers. This is because, when it comes to interacting with mathematical concepts, students from diverse cultural, linguistic, and educational backgrounds may take different approaches. They may also have a greater or different repertoire of mathematical strategies and approaches. The researchers of the MMC project view these repertoires with potential and employ research skills with the goal of enhancing instructional strategies for mathematics.

The intent of the MMC project is to promote intercultural dialogue in school mathematics. By intercultural dialogue I mean an open and respectful exchange of mathematical reasoning, approaches and tools, an approach that is relevant given both the ambition of mathematics education the objective of inclusive educational practices. The reason for this focus has been highlighted in research which shows how many migrant students struggle with math and perform below average. This research project hypothesizes that the variety of mathematical skills and approaches practiced and encountered by students of migration, may create blind spots that limit mathematics teachers' interaction, educational support and mathematical enrichment. Students from other cultures may find the teaching and study of mathematics to be rather foreign, and or discover new mathematical ideas and methods. The experience of mathematics in a variety of educational settings may cause some students to feel apprehensive and unsettled. These feelings may also be experienced by educators who teach mathematics in classes with diverse cultures. The project aims to comprehend how students and teachers learn and teach mathematics in the context of migration, as well as encourage discourse between students and teachers about these experiences; and examine the impact of this dialogue on teachers' practice. The MMC-project is situated in Canada where both the research team and data collection are situated.

Given that the intent of this research is to promote intercultural dialogue in school mathematics it became essential to me to highlight student feedback which entailed noticing and reflecting on students' experiences with migration in mathematics in a rich and productive way. The theories on learning and pedagogy that have changed from individual cognitive processes to collaborative, social, and dialectical learning have been influenced by this type of empirical research. These pedagogical

changes, along with sociocultural changes, create a vacuum in our approach to teaching mathematics. An approach to this is narrative inquiry, an analysis is based on the work of many scholars such as Bruner (1986), specifically his use of narratives, which in contrast to paradigmatic ones, the aim is to create accounts of “rich and multilayered meanings of historical and personal events” as well as Gillian (1982) where the approach is applicable when seeking to investigate “voices of experience as expressed in a narrative” (In Wertz 2011, p. 63 & 65). In order to be able to connect concepts such as students cultural-historical repertoire in mathematical practices, we as researchers must apply several tools of inquiry which enable us to notice varied experiences and expressions.

When considering how to address didactical practices based on student feedback, we looked to previous research projects for inspiration and insight regarding appropriate methodology. The following projects employed digital resources as a design technique which provided a variety of affordances. A recent use of “voices” as empirical basis can be found in Reid´s et al (2015)'s SSHRC-funded study of mathematics classroom pedagogies, which entailed the development of multi-layered dialogue among teachers from various regions of Canada, was informed by Tobin's strategy. Tobin, Wu, and Davidsons technique or design of multivocal ethnography facilitated an ongoing dialogue between insiders and outsiders, practitioners and researchers. The intention being examining the idea of pedagogy by examining the "implicit cultural practices" of instructors. This is referred to as “embedded cultural views” about how children learn and how teachers should instruct, by Bruner (1996) (Citet in Reid et al 2015, p. 369). What I understand from this work is that the focus is not on the actions that teachers take in each culture, but rather on how they perceive what they are doing. That is why I view this research as both theoretical understanding as well as an approach to posing queries about an “external world” (Reid et al 2015, p.370). The use of multi-layered narratives allows us to focus on first-hand accounts of students who have experienced migration in mathematics; the participants accounts´ of mathematical practices. The MMC-project has drawn inspiration from these research initiatives, as evidenced by the several investigation phases that involve multiple participant groups, allowing for multi-layered narratives and numerous voices.

My involvement in this project has included participating in research meetings, interacting with student transcripts, planning, and structuring the research design, presenting a conference paper outlining ethical dilemmas encountered during recruitment, helping to develop a conference paper based on preliminary findings, and assisting the research team through reflection and knowledge mobilization. Knowledge mobilization seeks to make information useable and accessible and the academic community benefits from collaborative research because it enables multiple academics to

share information and findings, which can result in high-quality research through the exchange of ideas.

1.2 Research question

The research question of my thesis is as follows:

How could engaging with migrant students' accounts (*) provide a basis for recognizing cultural-historical repertoires in unfamiliar mathematical practices?

(*) *migrant students accounts of their experiences in the mathematics classroom*

The MMC-project examines the theoretical viewpoint known as cultural-historical repertoires (CHR) in its mathematics education research. I recognize that my research question is a limited query into the nature of student's mathematical practices, and I could not attempt to define only one specific practice as a student's cultural-historical repertoire. However, these ideas are largely connected to the research questions of the MMC project where the focus of my participation, phase one, has been migrant students' experiences in Canada as well as previous mathematical experiences. The MMC project defines migrant students as students with educational experiences from other countries prior to their education in Canada.

While becoming acquainted with the MMC project ambitions, I was drawn to the second phase of the project where the focus is on discourse between students and teachers about these experiences, and the impact of this dialogue on teachers' practice. I was curious to know if we as researchers, as well as teachers as participants in the second phase of the project, were able to notice students' repertoires in these transcripts. These are the reflections that lead me to my research question, and include questions such as.

> Am I able to notice different aspects and properties of cultural-historical repertoires? Do accounts from student interviews provide context for reflection on mathematical repertoires? Do these accounts illustrate aspects and themes of participants history, cultural and the make-up of their personal repertoires? Can we engage with unfamiliar mathematical practices through the discipline of noticing?

I move forward with the understanding that mathematics has frequently been regarded as objective, universal, and neutral, resulting in diminishing of the situated and cultural heritage of mathematical

practices tied to a community´s history and historical story. I proceed with the knowledge that the contextual and cultural heritage of mathematical practices connected to a community's history and historical story has been diminished and as a result mathematics has generally been viewed as objective, universal, and neutral.

1.3 Objective

The objective of this thesis is to contribute to the MMC-project and the development of my own research practices through participation and creation of a methodological framework that is beneficial to both researchers and educators when relating to students´ varied and situated mathematical experiences. This thesis also consolidates the completion of my teacher training and mathematics education in Norway, GLU 1-7, USN. This process includes learning how to investigate aspects of mathematics education through the process of scientific research. My objective is to take advantage of the strong theoretical background of the MMC project, by applying my theoretical framework to empirical data in order to build on my understanding both as a researcher and an educator.

1.4 My positionality

My interest in connecting the perspectives of students, teachers and researchers is due to my educational background. The educational requirements for primary school teachers in Norway is a master's degree, which focus on conducting and implementing research as part of professional development, this is a change from previous educational requirements of educators. The prioritization of research as professional growth within the teaching profession and the changes to educational policies may potentially improve communication between the academic and educational communities and result in more effective didactical practices. One could argue that the implementation of these educational requirements impacts the use of cross-cultural communication between academics and educators. Given the intellectual common ground afforded by requirements of higher education, it could be considered "intracultural communication" when one is both researcher and educator (D'Ambrosio, 2001, p. 45). This statement demonstrates my broad definition of culture which notes how collective groups such as researchers and educators with common norms and practice can be defined as a cultural group. This view has been investigated and defined by others such as Cohen (2009) who describes how those having similar beliefs and meanings can be referred to be cultural groups or cultural centres (Cohen 2009, p. 200). Curricular and didactic changes have often been fronted by policy makers, whereas these qualification changes imply that the ambition is

that improvements will occur as a result of teachers incorporating research into their professional practice, and through working closely with researchers. I am passionate about the cross-culture communication and finding common ground both in regard to the diversly cultural backgrounds of students, as well as habituating intracultural communication between academics and educators. I have enjoyed exploring the link between research data and teaching practice and am curious about how to navigate the benefit of both roles in a powerful and meaningful way.

The second aspect of my positionality that I will be drawing on throughout this research is connected to my prior academic studies, in social anthropology, as well as my work experience with educational and self-sustainability projects in Tanzania, and in recent years, the management of the municipality´s re-settlement department in my local town. These experiences have allowed me to observer both the disparities and virtues surrounding migration as well as strengthen my desire for reflexivity. Given my academic and lived background, the MMC project which investigates mathematical practices in connection to cultural-historical repertoires allows me to utilize my positionality. However, given the discourse of intersectionality and my own experiences with it, I would not assume to categorize a person's practice as their "culture". Rather, it is my intent by participating in this project and highlighting student voices, that my practice as a researcher-educator will be strengthen.

1.5 Brief Methodology

Given that the term cultural-historical repertoire (CHR) has not been fully explored in terms of mathematical practices, it requires that I approach this thesis abductively. By this I mean allowing the empirical material to impact my theoretical framework and guide my analysis. This does not mean that I will not be building on a strong theoretical background presented in my literature review, gained through my academic studies and MMC participation. It is simply meant to indicate the abductive process of my research: a process which has involved the adaptation and creation of a methodological framework which builds on Mason´s phenomenological approach. The justification for this strategy has two parts: The first of which is that this research falls within the umbrella of both socio-cultural disciplines and the field of mathematics education; Secondly, first-person narratives of students' mathematics experiences are the empirical evidence of this investigation. These factors require a responsive approach both towards the two scientific disciplines as well as the participants accounts. I view responsiveness as both a general state of being as well as a research approach; it is intended to be versatile and adaptive, enabling researchers to react to developing problems and conditions. Moreover, culturally responsive research can take into account various ways of knowing,

understanding and doing that are based on situated context and cultural practice. This strategy encourages inter- and transdisciplinary research that meets the demands of a world that is changing quickly.

1.6 Chapter summary and thesis overview

The main body of the thesis consists of six chapters with the structure: introduction, review of the literature, theoretical framework, and methodology, findings, discussion and implications. The literature review discusses previous studies related to the topic as well as relevant theories relevant for this research. I begin by presenting a literature review as the background for this study as well as the basis of my theoretical framework. The literature review examines significant theories of this research, both from the MMC-projects theoretical framework as well as other relevant theories and earlier investigations on the subject. The data collection and analysis procedures are described in the methodology chapter. The notes on significant assumptions made in the study are also included in this part, along with an explanation of how these assumptions might affect the study's outcomes. The analysis' results are presented in the findings section, and the discussion chapter discusses how those findings relate to earlier theories and studies. The thesis' work is summarized through a concluding discussion on its implications, how it relates to research and classroom practice, and reflections are offered for additional study.

2 Literature Review

This section is structured by first presenting a theoretical description of mathematical activity, before addressing the historical value of mathematics as cultural capital. The following literature depicts how representation functions and well as indicates how these perspectives might impact mathematic practices, specifically classroom regularities and variations. By representation I mean to make present and represents the variety of ways in which mathematical activity is demonstrated. The literature discusses what is commonly regarded as mathematic activity in connection to cultural-historical repertories and students' interactions through the lens of "multivocal ethnography", which is an important qualitative part of the projects' ethos. In the final section of this chapter, we examine a theory of noticing which is the basis of our theoretical framework.

2.1 What is mathematic activity.

How we define mathematical learning matters. The mastery of a more sophisticated and formal mathematical discourse has been described as the goal of mathematical learning (e.g., Sfard, 2008; Chapman, 1997; Barwell, 2016; Roth & Radford, 2011). Mastery is connected to the historically established, culturally specific, and tool-mediated modes of communication employed by proficient users of mathematics. The term "mastery" indicates a discourse regarding formal and informal mathematics that impacts both teachers and students. Maturanas (1988) describes science as a domain of explanation, a realm in which characteristic implicit criteria for creating and accepting explanations apply; he terms this idea "emotional orientation" (Reid et al. 2015, p. 369). This could apply to the investigation of mathematical practices because it describes our preferences for particular practices as an emotional orientation. This is because many of our practices or reasoning are implicit, we understand something, and it is logical to use because we have been exposed to it before and it is an accepted practice. Within these two theories, we find a definition of mathematics as a set practice to be mastered, as well as mathematics as a form of reasoning, which implies that this type of knowledge is subject to tendency. By tendency I mean preference, inclinations or dispositions prescribed by internal and external inference.

One way to view and describe mathematical activity is through the lens of "cultural-historical repertoires" (Gutiérrez & Rogoff, 2003). This is because mathematics is a system of thinking and doing that uses mathematical principles to accomplish specific objectives. It is both a discourse and a practical activity. In linking culture to mathematical practices, the term "cultural-historical repertoires" may be relevant (Gutiérrez & Rogoff, 2003). In contrast to other theoretical frameworks, this one does not aim to categorically characterize distinct cultural qualities. Instead, it highlights distinctions and makes us aware that they may be rooted in contrasting practices. Distinction is the unique ability to recognize significant elements and patterns. Distinctions frequently exist within wider social and/ or educational discourses that may define advantageous or disadvantageous practices, for this reason caution is advised. This can be shown, for instance, in the perception that formulaic expression is superior to ideograms for mathematical representation. I use this tool as a means of identifying and note differences through an in-depth inquiry, with the aim of recognizing and highlighting the distinctive aspects of an experience, despite the challenges involved in doing so which include the various discourses surrounding them. The goal being to transition away from the categorization of mathematical practices, as advantageous or disadvantageous, toward a distinction involving the value that individuals place on them by taking into account students' preferences and approaches to mathematical practices as well as their cultural and historical significance.

2.1.1 Sociocultural aspects of mathematical practice

The term “Ethnomathematics” was introduced by Ubiratan D'Ambrosio. It studies the relationship between mathematics and culture with the goal of understanding and appreciating the connection between the two. He describes how the knowledge acquisition process, knowing and doing, is a “dialectic relation” accomplished across several dimensions, these dimensions are individual, social, environmental, and military (D'Ambrosio, 2001, p. 41). The aim is to acknowledge and respect different mathematical procedures, and practices developed by members of varied cultural groups through recognition and respect. His work is relevant as it reminds us of the intricate dimensions of both math and culture and how they are connected. Therefore, teaching mathematics in a heterogenous society is not unproblematic. That is why the dialectic action of gathering and listening to empirical data, such as the experiences of minority students in Canada, provides a glimpse into the way tension between culture and mathematics is experienced in our society today. The institution of education often dictates and sets boundaries, regarding what is seen and identified as valued representation and repertories of higher mathematical reasoning. I believe these predefined parameters of cultural capital make societal changes, such as migration, and the change from homogenous to heterogenous societies, more challenging. What is applicable in one context, such as a popular repertoire which is typical fixed in a situated context, may not be applicable in another thus affecting the accessibility and applicability of previous repertoires. This is why D'Ambrosios prerequisite of acknowledgement is valid as these dimensions impact the applicational value of mathematical repertories.

Historically, society has sought to define, categorize, and streamline artefacts and procedures of value. Bourdieu, in his book “Distinction” (1979), describes how cultural capital is a hegemony determined by the dominate class instilling in us individuals the need to acquire social representations and repertoires of value. In other words, cultural capital has been used as a mechanism for sustaining conventionality. I recognize a consequence of this in regard to mathematics, in the fact that efficient and abstract formulas, typically obtained through schooling, are often viewed as superior to the mathematical practices of everyday life. As such, different practices and methods are prescribed as have different value in different settings. While Bourdieu’s (1979) description of social constructs prescribing value is only one perspective and not an absolute representation of cultural capital, this ontologically fixed standpoint is one that remains and dominates the consciousness of our social constructs. A contradicting ontological perspective that navigates and negotiates cultural value can

be found in Bakhtin´s (1981) work where he describes social conflict through the tendencies of discourse in language (Bakhtin 1981, p. 40). This contrasting standpoint recognizes and highlights tensions between cultural values and is relevant to my research as Bakhtin encourages investigation of these negotiations through identifying terms, as a tool to demonstrate elements of tension. This relativistic approach encourages me to notice the ongoing negotiation of mathematical practices. I recognize that these two theorists are ontologically contradictory with one presenting a fixed standpoint and the other a fluid dialectic negotiation, however I chose to include these contrasting perspectives as they provide a broad range of world-view description to my investigation, as well as illustrate the contrasting ontologies present in our societies. Choosing to include both Bourdieu's and Bakhtin's perspectives is partly because of the ethos of the MMC-project with features aspects of Tobin (2015) design of multivocal ethnography which is an ongoing dialogue between insiders and outsiders, facilitated by practitioners and researchers (Reid et al 2015, p. 368). It is also due to my previous academic background in social anthropology that I am aware that when supporting several "voices," specifically those who have experienced a variety of cultural and academic environments, one must be willing to consider the potential possibility that they might have internalized or had to make accommodations for opposing ontological positions. That is why awareness of both fixed and fluid ontological perspectives is necessary in order to be able to recognize the ways in which these may impact personal inference, practice, and experience.

The sociocultural components of mathematics could be characterized as the ways in which students are influenced in the classroom. Research looks at these influences and attempts to categorize and contextually distinguish them from mathematical conceptions. One such term used to describe mathematics, as a practice of one's community, is "funds of knowledge" (González, Moll, & Amati, 2005). This is the knowledge that learners carry with them from their homes and communities to school. This could be described as situated math providing the context or image one visualizes when doing geometry, working with cardinality or patterns. Students from an agricultural background might visualize a field when working with area, or a quantity of fish when working with measurement and volume. It might also be visualization of a knitting pattern that shows the development and structure of a pattern. Studies on *funds of knowledge* place a strong emphasis on family knowledge and experience, and they also call for curricular reforms and a greater desire to understand students and their families (Appelbaum & Stathopoulou, 2015, p.763).

Educational strategies also influence how pupils' structure and solve problems. One term that describes addressing this aspect of mathematics education is the *landscape of learning*. It

characterizes educational practices as “complex network of social practice" which is made up of various “interrelated dimensions” (Alrø, H., Skovsmose, O., & Duenas, P. X. V. 2007, p. 1567). The complexity of the learning environment is reflected in the pedagogy and didactics utilized by the teacher, which have an impact on how interactions are structured and how subject lessons are organized. One example of this is how students are asked to “display their work”. This obviously depends on the sort of assignment, whether it's a math problem, a word puzzle, or a task that calls for a number of steps, often described as “mathematic modelling” (Blum 2015). Mathematic didactics may be a component of the educational strategy outlined in "display your work”, although students who are adept at mental calculation might not find this aspect of mathematics education to be very engaging or meaningful. Another example that illustrates the complexity of the learning environment is the current emphases on mathematical discussions (Kazim, 2015). This kind of mathematical thinking may be more engaging for students from communities where they were bought up taking part in discussions involving problem-solving. However, students from cultures where engagement in participation is mostly through observation and modelling may find these behaviours foreign and stressful.

Cultural and educational practices are dynamic and change over time. They influence people's behaviours and patterns of involvement and are why individuals are described as "carriers of culture" (Gutiérrez & Rogoff, 2003, p.21). Because of this, the focus of my thesis is empirical and involves analysing the experiences of students who have taken part in various educational and cultural practices. The task of noticing and reflecting on mathematical experiences, of student’s who have experienced these differences through the process of migration, might shed light on how this aspect of ethnomathematics is experienced. When considering these theoretical foundations, the term cultural-historical repertoires may be viewed as an overarching idea which describes how various mathematical practices are a part of an individual’s skillset. These mathematical conceptualizations and ideas include the description of mathematical practices as an individual’s reserve of abilities and resources gained through participation in education, community, and daily life. These and other theories attempt to provide descriptions of what mathematics activity is.

2.1.2 Cultural-Historical Repertories

There have been many frameworks describing styles of learning, whether it be types of learning styles or cultural traits. Unfortunately, these categorizations can often be less effective as they lack the relational aspect of mathematics as a practice of the community. In addressing the link between

culture and mathematics, the theory of cultural-historical repertoires (CHR) may be relevant (Gutiérrez & Rogoff, 2003). This framework is situated within socio-cultural research and, in contrast to previous learning frameworks, does not aim to categorically characterize distinct cultural qualities. Given my academic background in social anthropology, I am aware that early anthropological investigations have defined culture poorly through descriptions of cultural traits and predispositions. These definitions were often based on preconceptions and bias. That is why I appreciate Gutiérrez & Rogoff´s (2003) framework that calls for understanding the relationship between the cultural practices and the historical situations. Rather than aiming to characterise, it emphasizes the distinctions and makes us aware that they may be rooted in contrasting practices.

The term "cultural-historical repertoires" (CHR) is described in Gutiérrez & Rogoff (2003) and the term indicates the historical and cultural aspect of mathematical practices. It explains how factors influencing our problem-solving strategies are connected to the environment from which they arise, or how we behave in a situated setting. These practices are historical because they are situated in a time and place, in the past and future. They are cultural because meaning arises through contact with others, through observation and participation in cultural practices (Gutiérrez & Rogoff, 2003, p. 368). Rogoff uses these three terms when discussing repertoires as cultural ways of learning: regularities, activities, practices (Gutiérrez & Rogoff, 2003). These refer to the dialogical aspects of relationships where the process takes place within an ongoing activity. The word dialogue indicates that the relation is not monolithic with one entity supplying all the information without exploration and clarification of meaning through interaction and discussion. Rather it is a collaborative effort that explores meaning and interpretation. Due to the nature of teaching and learning, understanding the history and cherished customs of many cultural groups is both beneficial and significant when working in math classrooms. I build on the social-cultural description of contrasting practices when I talk about contrasting mathematical practices. By this, I refer to the many approaches that students can take to learn the subject, which requires employing the right tools strategically and recognizing their benefits and limitations.

The relationship between educational and cultural practices is referred to by Tobin et al. (2009), as implicit cultural logic. This work resides within a larger body of research and describes how pedagogical methods, also those not being explicitly taught, my contain a cultural logic (Tobin in Reid et al. 2015, p. 369). An example of this would be how scholastic standards for accepting an explanation in science are often based on fundamental criteria, leading to explicit requirements and expectations, however an individual's ideas, knowledge, norms and habits that direct practice are usually unspoken and in this way are implicit (Reid et al. 2015, p. 368). This affects mathematical

practices, as standards by which teachers decide on their practice are often based on explicit criteria, but in the moment are also implicit in some way. Even though the focus of my study is on students' mathematical repertoires rather than teachers' pedagogical practices, this body of work demonstrates the many factors that influence practice. Regardless of how dogmatic or non-dogmatic the collective's approach may be, it affects and influences the criteria for scientific explanation, which in turn affects teacher and student practice. Given the that the nature of scientific explanation, how standards are both explicit and implicit, students must navigate and evaluate the strategic benefits of various mathematical practices. In this way Tobin´s (2015) description of the relationships between educational and cultural practices, their implicit logic, is relevant to my research as students' narration often includes inference on the logic behind the mathematical practice.

One body of work researching the repertories of multi-lingual learners in the mathematics classroom is by Uribe & Prediger (2021) which builds from Barwell´s (2018) characterization of repertories as connection processes (Uribe & Prediger 2021, p. 3). This body of work both describes the dynamic conceptualization of multilingual mathematics classrooms and contains findings which challenges classical assumptions in instructional approaches. Although this article focuses on linguistic connection processes it contributes to theorizing the repertories students use in mathematics. It reminds us that the repertories students use are varied and dynamic, even the repertories of students from similar cultural background (Uribe & Prediger 2021, p. 21-22). These dynamic interactions may be more apparent in multi-cultural classrooms as this is the context in which repertories from a variety of cultures are encountered and negotiated. As this article questions assumptions and requires that the intricacy and numerous underlying reasons and elements impacting the repertoires pupils utilize be articulated, it is pertinent to my thesis. Given the fact that a student's repertoires cannot and should not be overgeneralized, I continue the research process seeking to utilize a method that facilitates for the complexity of this supply of skills. This supply of skills consists of the repertoires that teachers and students select, apply, and engage with on a regular basis based on their own preferences, backgrounds, and current situations.

2.2 Theoretical Framework

Mason´s framework of noticing is relevant to both the theoretical framework of my investigation as well as the methodology and research design. It creates a platform for noticing as well as recognizing differences prevalent in culturally diverse mathematics classrooms. I was directed to Mason's work on the "discipline of noticing" since the basis of my inquiry is empirical and I needed a framework to enable me to observe and reflect on the experiences being shared (Mason 2002 & 2021). His work is

relevant as it requires me to structure my observation in a way that encourages working with accounts in a way that both creates an awareness and allows for noticing phenomenon. Wertz (2011) has described how the term “phenomenon” has been used more broadly to characterize any work in research, theory, or practice that emphasizes first-person experience (Wertz 2011, p.52). This form of scientific approach describes a theory of knowledge that is qualitative and seeks to suspend the researchers' previous notions about the phenomenon in order to investigate the daily experiences of people in an effort to comprehend and express the essence of a phenomenon. Although a phenomenology can be understood as pragmatic approach where one seeks to investigate the essence of a matter, it is important to take into account that knowledge structures are different. Bruner (1986) makes a distinction between paradigmatic and narrative knowledge frameworks. Whereas narrative information is structured around stories or a series of events, paradigmatic knowledge is based on logical categories and relationships (Bruner 1986, p. 11). Recognising the difference creates awareness of the fact and narrative knowledge enables us to make sense of the world in a more holistic and context-dependent way and is fundamental to human understanding. This is important when working with participants experiences as they may contain both pragmatic and narrative knowledge structures. This understanding is reflected in many other theoretical ideas and methodological approaches such as the narrative theory of storyline from Herbel-Eisenmann which investigates the way the stories we tell ourselves impact our sense of self (Herbel-Eisenmann 2002, p. 42). These approaches characterize my theoretical understanding and research practice, specifically in regards to the way one attends to the voices and narratives of first-hand accounts. In my thesis these understanding effects my research method of noticing student experiences with familiar and unfamiliar mathematical practices. The hope is that listening to these stories, which contain both pragmatic narration and meaning-making inference, will lead us to a better understanding of cultural-historical repertoires of mathematics.

In order to implement this approach, I was inspired by Tjora's (2017) stepwise-deductive inductive (SDI) model, which captures the essence of abduction without overgeneralizing the deductive and inductive procedures used in scientific inquiry (Tjora, 2017, p. 18-20). He offers a realistic, in-depth, and practical approach to qualitative practice while also challenging many of the traditional conventions and presumptions of modern qualitative research practice. The SDI model is systematic and insightful as it describes both the different steps of qualitative research as well as the theoretical processes taking place. The SDI model describes the process of induction that takes place after the empirical data has been collected. The steps he categorises as inductive are treatment of the raw data, preliminary coding tests and a test of the categories created from coding, grounded in the empirical

data. The test of categorization he labels as abduction along with the final steps of conceptual development models and concepts base on theory. The process of looking back to the previous step to evaluate the quality he describes as a deductive process as it involves testing each step and checking their connections. This model is valuable to me as it is systematic and takes advantage of the different approaches to scientific knowledge and although one might describe this approach as abduction it encourages awareness of the multiple approaches to knowledge. It also requires that the findings be thoroughly grounded to avoid having personal interpretations and preconceptions affect the research. I will go into more detail about how I have used this model in my work in the methodology chapter.

2.2.1 What is noticing

Mason´s work is essential when working in the mathematics classroom; he defines noticing as a form of reflection implying the ambition of "reflective practices" (Mason 2002, p. 5). Reflection is sometimes used to describe the results of "attention flowing outwards from an incident to the broader socio-cultural-historical – political forces" which are acting to constitute and constrain the individuals and the group to act, speak, and work in particular ways (Mason 2002, p. 15). As an educator, I am able to notice this pattern because there are numerous times during the course of a school day when we, as teachers, pause to consider the effects of our pedagogical, didactical and ethical practices on our students by gauging their responses, reactions, and subject-matter progress. Similar to the limitations of research, there is insufficient opportunity in the classroom to fully explore the factors that led to student reactions; therefore, one must rely on various hypotheses or cause-and-effect scenarios. I will never be able to fully understand the more general socio-cultural, historical, or political forces that have an impact on the individuals, both as a researcher and an educator. I can, however, make an effort to distinguish between instances that offer opportunities for meaning making and learning from those that don't. That is why key theoretical concepts I have chosen to utilize, from Mason´s work, are the concepts of resonance and dissonance (Mason, 2002 & 2021). Resonance is evidence of validation, when something experienced as meaningful, helpful or makes sense to them (Mason 2002, p. 191-192). The opposite of this would be dissonance, when an experience doesn´t make sense or is unfamiliar (Mason 2021, p. 232). These key concepts are relevant when working with first-hand accounts, specifically the way in which students describe and understand their experience with migration in mathematics.

2.2.2 Why this theoretical framework

Alongside my work with the MMC project, I have been slowly reading and reflecting on John Mason´s book, “the discipline of noticing”, in order to see how this theoretical framework could be relevant to my work with cultural-historical repertoires. What I understand to be the intent of Mason´s work is a platform for noticing. His work describes an aspect of professionalism that is relevant to both researchers and educator; the desire being to understand and learn from encountered experiences. This requires being systematic about noticing and putting in the work to separate between the incident and the interpretation. Mason refers to this as the precision principle and states how the more precisely we describe someone´s behaviour, the more is revealed about our reflexivity, or sensitivity as an observer, in our role in describing what we have observed. “The more precisely someone’s behaviour is described, the more that is revealed about the sensitivity of the observer to observe what is described” (Mason 2021, p.234-235). The way we as researchers structure our empirical data as well as our analysis impacts investigation and reflection.

The vast variety and variation in our classrooms, as well as the generalizations provided by literature that describe a generalizable technique, make precise descriptions difficult. These variables influence teaching strategies, which in turn influence empirical findings. However, students’ mathematical repertoires are dynamic and cannot be categorized, therefore I perceive Mason´s (2002) application of noticing as being relevant to this research. This is because we can no longer "prescribe" instruction and rely on assumptions about specific groups of learners (Gutiérrez & Rogoff, 2003, p.20). The difference between reflection and noticing is its intention and systemization. This theoretical framework is important because it asks us to be intentional both in creating and probing accounts. This intentionality and these systematic methods enable us to respond professionally to disturbance by looking for underlying assumptions, familiar scripts and issues of importance (Mason 2002, p. 139). By doing so we focus our sensitivities and search for possibilities. Mason's first enquiry asks if one can take a normative teaching approach and articulate both its merits and disadvantages. The purpose of these reflective activities isn't necessarily to effect change, but rather to "problematize the norm" and to re-examine details that could otherwise go unnoticed. This is, in my opinion, a test of how well we comprehend our own reflective practice, which is a goal of this thesis, the product of my education, and potentially a practice of my participation in the teaching profession. Instead of attempting to problematize the norm, the objective is to take into account the experiences of participants by systematically identifying and differentiating their elements.

The goal of using this framework is to interrogate instances of difference by attempting to differentiate between the narration, or description, and the inference or interpretation. The goal is not to assume that what I am noticing is aspects of cultural-historical repertoires but to assist in interrogating the ideas of what these experiences and preferred mathematical practices are, what they look like in order to be able to notice, and hopefully in the future be able to recognize them, by looking at how they are experienced and understood by the individual as well as the educator.

2.2.3 Preparing to notice

The structural elements of Mason´s intentional noticing makes me conscious of the characteristics of intentionality, moving beyond casual observation to seek out identifiers actively. Coding is crucial in order to be able to identify tension, differences, and things that surprise us or are not as expected; the goal being to set oneself out to look for things, and carry through, not just observe passingly. The three forms of intentional noticing addressed are noticing, marking, and recording (Mason 2002, p.33-34). I understand marking to be the act of highlighting something for further discussion with others. When the purpose is analysis and reflection, the intention of recording is the desire to externalize in order to access the event. All three aspects of intentional noticing have been relevant to my theoretical framework.

In recording encounters, Mason differentiates between “accounting of” and “accounting for” and links these to impartiality (Mason, 2002, p. 40). I view this as an ethical connection. This is so because it is the duty of scientific research to offer accurate explanations of what happened in a form that permits clarity of both the methodology and interpretation. In order to achieve this, it is essential to present in such a way that readers can discern between facts and interpretations. This is what I understand to be the difference between accounting “of” and accounting “for” (Mason 2002, p. 40). One is interpretive in its presentation whereas the other is not. The reason Mason urges that straight forward description be given as much attention as possible is due to the fact that the events are often described with biased or prejudiced values. That is why he highlights the benefits of keeping description and judgement distinct.

In order to accomplish this task, Mason (2002) describes a method of creating accounts that provide a “brief-but-vivid” description; accounts that provide vivid images in order to enable for “re-entry” (Mason 2002, p.51). Creating descriptive accounts that provide vivid imagery and enable other participates to enter or re-enter the moment is both a guiding principle of this thesis as well its practice.

In order to enhance communication, we are encouraged to use reflective accounts of reader-recognizable examples. What I understand as the intention behind both forms of accounting is to enable one to act “non-habitually” in the future (Mason, 2002, p. 41). The task of accounting for events should take place after the events have been objectively described. The intention behind examining the specifics of a collection of narratives is to notice in detail the good, bad and intriguing aspects of mathematics practices, both similarities and differences. Mason refers to this act as "wisening" (Mason 2002, p.41).

Mason´s (2021) article gives an example of structuring analysis by dividing it into three sections: the incident or example, the account of enabling readers to recognize aspects of the experiences through providing the context or some details surrounding the event. The final section is accounting for the event, which would be the interpretation. In my theoretical framework, I will be using the terms “instance”, “narration” and “inference” as analytical differentiation. The term “instance” is used to indicate a moment or experience of difference, the incident or example. The “narration” is the student’s description of the event, an account of the experience. The word “inference” is used to mark the students reasoning, interpretation, or definition of an event, as well as their understanding and explanation oof how they perceive and comprehend it.

2.2.4 Recognize and validating noticing

One part of systematize noticing which I would like to highlight, alongside systematic reflection, is validating with others, as this is a key aspect of my research. This is because systematic observation and contemplation by any individual could be extremely idiosyncratic; what I observe might or might not match what you perceive. For this reason, the transparency and understanding that comes with Mason´s structuring is valid. Validation of noticing is based on whether the other can perceive what is being described or suggested and whether they discover that their own sensitivities to notice are improved. This contrasts other forms of validation through logical argument, quantitative practices, or traditional theories of deduction. The theoretical concepts I will be using as stated earlier, are resonance and dissonance as they support my process of validation. These concepts are used as identifying tools and reference the work of Bakhtin (1981) who uses terms as tools for identifying tensions within a dialog. Given the dialogical nature of mathematical practices, the identifying terms of resonance and dissonance support my investigation into the students’ inference, how they perceive a practice and whether they feel it supports their learning, if it is experienced as preferred a practice and one that can be supported by their implicit logic. These inferences are dialogical, reflecting an

internal debate negotiating the benefits and drawbacks of a course of action or practice within an experience. This project contains aspects of what Mason (2002) refers to as collective validation which means interweaving strands of one´s own experience with those of others (Mason 2002, p. 90). In reflecting together, both researchers and educators, we reveal our own noticing to others with the goal of accenting resonance, identifying issues of similarities and difference, and cultivating understanding.

3 Methodology

I begin my methodology chapter by presenting the specifics of the project organization before presenting my research strategy along with the theoretical concepts and critical viewpoints that were pertinent to the creation and operationalization of my theoretical framework. By this, I mean the way I interacted with the empirical data as well as some of the affordances and constraints. I conclude this chapter with additional core values that have guided me through the various ethical considerations as well as reflections on my experience with collaborative research and my own positionality.

3.1 Project organisation

This study is situated in Canada, where students of migration, that is students with educational experiences from more one country, were invited to participate in an interview regarding their experiences with mathematics education. The design of the initial research phase was centred on middle school students who have studied mathematics in two or more different countries. Recruitment was done via MMC-project initiatives both prior to and throughout my participation. The empirical material I had been working on as part of my contribution to the MMC project was primarily six of the 10 interview transcripts. These six transcripts were selected as they demonstrated key themes described in the collected material and are representative of the data collected by the MMC-project this far. The demographic information collected from students included, their age, the cities, or countries they have gone to school in, and the number of years they have been in Canada. These six participants' experiences are situated within the countries of China, Saudi-Arabia, Turkey, the United Kingdom and Canada. At the time of our interaction, they were all middle-school students, apart from one who primarily addressed her experiences from secondary school.

The six interview transcripts that make up the population of my research thesis each feature a number of accounts relating both familiar and unfamiliar mathematical practices. I chose one excerpt from each of the six student transcripts as they provide details of an experience the student had while

participating in a mathematical lesson. My rational for the selection and structure of these accounts was in connection to a larger MMC initiative pertaining to phase two of the research project which involves sharing student accounts with teachers. Thus, a great deal of collaborative effort was generated with the purpose of structuring the empirical data in order to produce participant content for the professional development phase of the project. The reason for this initiative is the "multi-vocal" aim of the MMC project which involves highlighting the voices of both students, teachers and communities who experiences migration in mathematics (Tobin 2015).

In qualitative research, it can be beneficial to prioritize and provide space to the voices of the participants by reducing the amount of data and concentrating on fewer transcripts. The reason for this is that studying a smaller dataset enables researchers to focus more intently on the particular experiences and viewpoints of each participant, which can aid in highlighting the distinctive voices and viewpoints of each individual (Wertz 2011, p.125). Researchers can commit more time and effort to doing a more in-depth and nuanced analysis of the data by concentrating on fewer transcripts. As a result, patterns and themes that might not have been obvious if the research had been done on a bigger sample can be identified. Researchers can make sure that participants' views are heard and that their contributions are valued in the study process by giving priority to their experiences and viewpoints. Ultimately, giving participants' voices priority in qualitative research is crucial to ensuring that the study is ethically and respectfully conducted and that the experiences and viewpoints of the participants being researched are accurately represented. One strategy to facilitate this prioritization and give participants the room they merit in the research process is to condense the data and concentrate on fewer transcripts. That is why I have restricted the amount of empirical data to just six student excerpts. This is my endeavour to act in accordance with the practice of highlighting students' voices and respecting these voices by giving them room and prominence in this work.

3.2 Research Method

Simply described, a research method is the process used to respond to a research topic, the focus of my research being a qualitative investigation of students' first-hand accounts. I view the research method as qualitative as it involves the investigation of students´ experiences with migration in mathematics. The research is also situated within a larger project where HCR, as a phenomenon in mathematical practices, is being investigating. Since the research requires us as researchers to notice and reflect on first-hand accounts, the steps taken to that end could best be described as a phenomenological approach. In this way, I connect my research strategy to a phenomenological

approach, as reflections on the students´ stories, are the starting point of reflection. In identifying what the students are saying, I do not primarily attempt to interpret the accounts, what Mason terms as "account for"; rather I attempt to showcase essential elements of HCR, which may be identified through the various accounts and their illustrations (Mason 2002, p. 52). As phenomenology contains a strong essentialist ontological description that fundamentally leads to a definite account of the essence or core matter, I proceed cautiously, aware of the limitations of my research in defining the essence of mathematical repertories within the framework of migration, culture, and history. As show in my theoretical framework, historical understandings on the nature of human existence consists of contrasting views which produces personal uncertainties as to how much of one's experience can be defined within a world of fluctuation. However, dialog demonstrating patterns of resonance could produce insight into valued mathematical practices of students. The way I understand the term phenomenon, as stated in my theoretical framework, is in its broad sense when it "characterizes any work in research that emphasizes the first-person experience" (Wertz 2011, p. 52). The way I use the term in my investigation is through attempting to recognize what individuals' experience as resonative in their mathematical practices. The characteristics of these dynamic experiences, whether it be familiarity with a practice, instructional motivation, or other essential elements indicate the composition of the phenomenon, the phenomenon being the experience of mathematical education from one or more country. In this way, I recognize models of narrative and phenomenological approaches as being helpful as they provide me with a guide as to how to interact with first-hand accounts. The benefits I recognize within this approach are ones I connect to ethnographic practices where authentic personal experience is the basis of reflectivity and responsiveness. This approach is valid when it enables us as researcher to recognize and reflect on significant components and identifiers of a phenomenon (Wertz 2011, p.52). While phenomenology is both a theoretical and methodological framework, I will primarily be applying it to my research as a methodological approach, mirroring Masons understanding and application of the method.

3.2.1 Data collection

In order to investigate student experiences of migration in mathematics, I was given access to the MMC projects pre-existing data. This access was gained through my participation as an assistant researcher in the project; thus, I did not personally participate in recruitment or conduct student interviews. I recognize this as one of the benefits of working with and contributing to existing research. This pre-existing data, in the form of student interview transcripts, was the basis of my inquiry. The interview guide, designed by the MMC-researchers, contained open ended questions on

students´ mathematic experiences. The interview guide included questions pertaining to similarities and differences in mathematics, such as mathematical activities and tools, language, homework as well as experiences with other mathematical practices. The transcripts contain a wealth of narration and inference, each describing a variety of unique experiences.

Since my thesis is part of a larger research project, the process of data selection and reduction was essential. The empirical material I had been collaborating on, as part of my research assistant role, was the student survey responses and interview transcripts. Each student transcript contained several accounts relating different aspects of mathematics, such as teacher practices, mathematical content, and other indelible experiences. Due diligence and ethical thought were therefore required when thinking about my data selection and reduction. Initially, I had planned for the basis of my inquiry to be focus on the profession development phase of the project; teachers experience and interaction with student material. The reason for this focus was, 1) my positionality as an educator and 2) the added learning benefit of participation in data collection. However, due to the project´s time constraints and the fact that my involvement during the first six months had mainly consisted of working with student transcripts, I made the decision to base my investigation on pre-existing data. Given the thesis limitations, I have chosen to only analysis one account from each of the six-student transcript, as each account demonstrates how the student experienced and/or navigated the mathematical practice.

3.2.2 Preliminary holistic reading of the data

The first stage of my research consisted of reading though the literature review and theoretical framework of the MMC project a process which took place from spring 2021 until fall 2021. These comprehensive sources provided me with a strong theoretical foundation for moving forward, but that does not mean that my approach has been solely deductive. What I have learned from reflecting on the literature was the need for an inductive approach. I understand the term inductive to refer to a more recursive process in which observation, in this case reflection over the empirical data, is often the first step (Nyeng, 2020 p. 59). In contrast, a deductive approach is when research is approached from a specified theoretical perspective. Having a strong theoretical foundation is beneficial, especially given our role as researchers which entails ongoing inquiry based on established discovery. Ongoing research builds on established discoveries, and the learning process involves research, inquiry, questioning, and establishing connection with the real world in order to transform information and facts into actionable knowledge. This is an approach in which a question, problem or scenario is first presented, and the established theoretical foundations are compared with current

knowledge and findings. However, working from a deductive standpoint impacts inquiry as each theoretical perspective carries its own assumptions which may in turn may predetermine or limit the analysis. Given that this research project resides within the academic disciplines of mathematics education and social-cultural research, I felt a solely deductive approach would not be adequate. This is because I, as a researcher, may be misguided in my assumptions when seeking to base my finding on established theories, consequently limiting the research's potential. That is why I felt it important to be respectful and responsive to the theoretical ideas the students´ experiences communicate when working with student accounts, and as such my work required the process of abduction. The combination of both strategies is defined as abduction, which starts with empirical data but acknowledges the value of theories and viewpoints at the beginning or during the research process and has been acknowledge as an approach that attempts to reconcile the two theoretical approaches of induction and deduction through a recursive process.

As part of my contribution to the MMC project, I assisted in condensing the transcripts by creating several accounts or stories from the student interviews. The goal behind the creation of several accounts was to create authentic bite size stories for teachers to reflect on in phase two of the MMC project. Each account contains an element that stands out as an experienced incident and contains students' narration of different elements surrounding the event. Working with student transcripts over a longer period of time, from August 2021 until March 2022, has helped me to stop and notice each account thoroughly, through a micro perspective. This approach impedes pre-interpretation in the form of a summary of the interview, one which focuses on the overall picture rather than the specific details, a macro perspective.

3.2.3 Creating grounded meaning units

My research process involved utilizing the discipline of noticing when analysing students´ experiences on migration in mathematics education. I can appreciate the efforts of individuals, such as Tjora (2017), who have provided us with analyzation models that enable us to better systematize the research processes. Given that qualitative research is frequently rich and complex, the models provide support, both when analysing empirical data and when conceptualizing the methodological framework. Analyzation models, such as the one from Tjora (2017), have assisted me in the creation of a methodological framework, allowing for an abductive balance that is both structured and inductive. Given that I as a researcher must be careful not to make assumptions about previous cultural repertoires or jump to conclusions about how certain experiences relate to other cultural or

mathematical ideas, the process of abduction has been important to me. In light of these potential research pitfalls, I have given Tjora's stepwise-deductive inductive (SDI) model some thought, as well as its application and ramifications for my research process (Tjora 2017, p. 18–21).

I have taken inspiration from this model for scientific research, specifically when it comes to coding. The process he describes involves creating codes that provide detailed insight while still reducing the amount of data. I chose to take inspiration from this model for several reasons, but primarily in the creation of codes using text excerpts from the data as they can help prevent over-interpretation. I found this to be true when working with the student interview transcripts. A thematic approach would be creating thematical subheadings in order to categorize or group the findings. After reflection I recognized this as a form of pre-analyzation or interpretation of the data, an act that could possibly affect the transparency and validity of the research. Thus, I decided to draw on this model and use text excerpts from the transcripts as subheadings. The goal was to prevent my own interpretation of the topic affecting the reader. The desire was to present open accounts of mathematics in migration so that we as collective researchers could reflect, without over-interpretation or researcher categorization interfering. I also recognize the positive ethical aspect of this approach, how it allows for openness, in other words transparency and reliability, where readers can identify the subheading in the text and evaluate whether it is a valid descriptor of the subject matter. Another reason I took inspiration from Tjora´s (2017) SDI coding method is because of its correspondence to my theoretical framework, specifically what Mason (2012) describes as “brief-but-vivid” accounts (Mason 2012, p. 46). After recognizing the connection between the two approaches, I could draw on their value as a foundation in this process, knowing that the structure of my work was in line with scientific methods of coding as well as based on my theoretical framework. The pre-existing data of student transcripts provided access to multiple accounts of migration in mathematics upon which to reflect and apply my methodological framework to. Structuring these accounts has been with the intent of making it easier for me, as well as us as the research team, to notice similarities, regularities, and differences within their experiences. When noticing phenomena, one is looking at the subjective experience, in this case how the various mathematical activities and experiences are described by the students. The systematic approach has assisted me in noticing the meaning making process behind these accounts; by this I mean the students inference regarding the experiences and why they described the mathematical experience as “fun” or “hard”. That is why the discipline of noticing, by Mason (2002), became my chosen research strategy, as it contains a structured practical approach to working with student accounts.

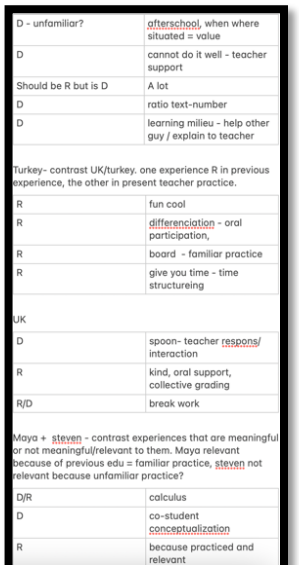
3.2.4 Organization and transformation

The inductive process of data collection and selection constituted the initial processing and structuring of empirical data. The next stage included building on the initial codes to create identifiers that demonstrate the empiricism and recontextualization. This process required abduction as I recognized some overlap between my theoretical and methodological framework. This is because the discipline of noticing is both theoretical, with concepts such as resonance and dissonance, as well as a methodological tool for structuring analysis.

The coding is based on my understand of Mason´s (2002 and 2020) process of creating and probing accounts. The first step was to present an account of what was said by the participant. I do this by attempting to differentiate between the narration, or account of the instance, and the student's inference, their account of the event. The next step is for me as a researcher to give an account of what I noticed and finally an account for my analysis. I struggled with this process, and although I had been reflecting on Mason´s description of resonance for some months, I had not been able to see how to utilize it in my investigation. That was not until I began my analysis of the empirical material which led to the development of the terms resonance and dissonance, as the key concepts or identifiers of my analysis. I feel these terms reflect the dialogical tension described in the literature review while still maintaining their originality as a method. It was through this process, working with the MMC-project, creating accounts from transcripts, and reflecting on theoretical concepts that led me to this theoretical framework that assists me in answering my research question. The framework for noticing assists me in exploring accounts of familiar and unfamiliar mathematical practices and their implications.

3.2.5 Synthesis of data

Synthesizing the data was challenging due to the fact that many of the influencing factors of the experience overlap and interweave. The empirical data is limited and does not offer the foundation for evidentiary discovery. An example of this would be a mathematical activity that included tools, props, and teacher support. In this context, the researcher would be unable to identify which aspect had the greatest impact on the experience. These qualities made me cautious when deconstructing, developing, and proposing the categorizations of these findings.



D - unfamiliar?	afterschool, when where situated = value
D	cannot do it well - teacher support
Should be R but is D	A lot
D	ratio text-number
D	learning milieu - help other guy / explain to teacher
Turkey- contrast UK/turkey. one experience R in previous experience, the other in present teacher practice.	
R	fun cool
R	differenciation - oral participation,
R	board - familiar practice
R	give you time - time structureing
UK	
D	spoon- teacher respons/ interaction
R	kind, oral support, collective grading
R/D	break work
Maya + steven - contrast experiences that are meaningful or not meaningful/relevant to them. Maya relevant because of previous edu = familiar practice, steven not relevant because unfamiliar practice?	
D/R	calculus
D	co-student conceptualization
R	because practiced and relevant

Figure 1- mapping of R&D

Differentiating between the different types of components that students' narration and inference referenced required careful consideration. Reading Mason (2002 & 2021) had made me aware of the significance of resonance, thus the patterns of resonance and dissonance had caught my attention. For this reason, I made an effort to map these perspectives in each account, as seen in Figure 1. In this image the letter D stands for dissonance and the letter R stands for resonance and the process involved going through each account and attempting to identify these sentiments.

The next image, Figure 2, provides insight into the process of data synthesis. The process involved firstly noting down the variety of communication on post-its before creating three rough categories. The next step was creating columns for resonance and dissonance under each category to see if this process would reveal any patterns. Influencing factors are frequently intertwined and make me unable to present distinct factors quantitatively as some instances such as mathematical activity could be regarded as both pertaining to content and teacher interaction. For this reason I chose not to present findings statistically but rather describe how approximately one third of the students´ communication pertain to aspects of mathematical content, a little less than one third describe teacher interaction and support and over one third give a description of what mathematics is. This process and these distinctions served as the foundation for my data synthesis.

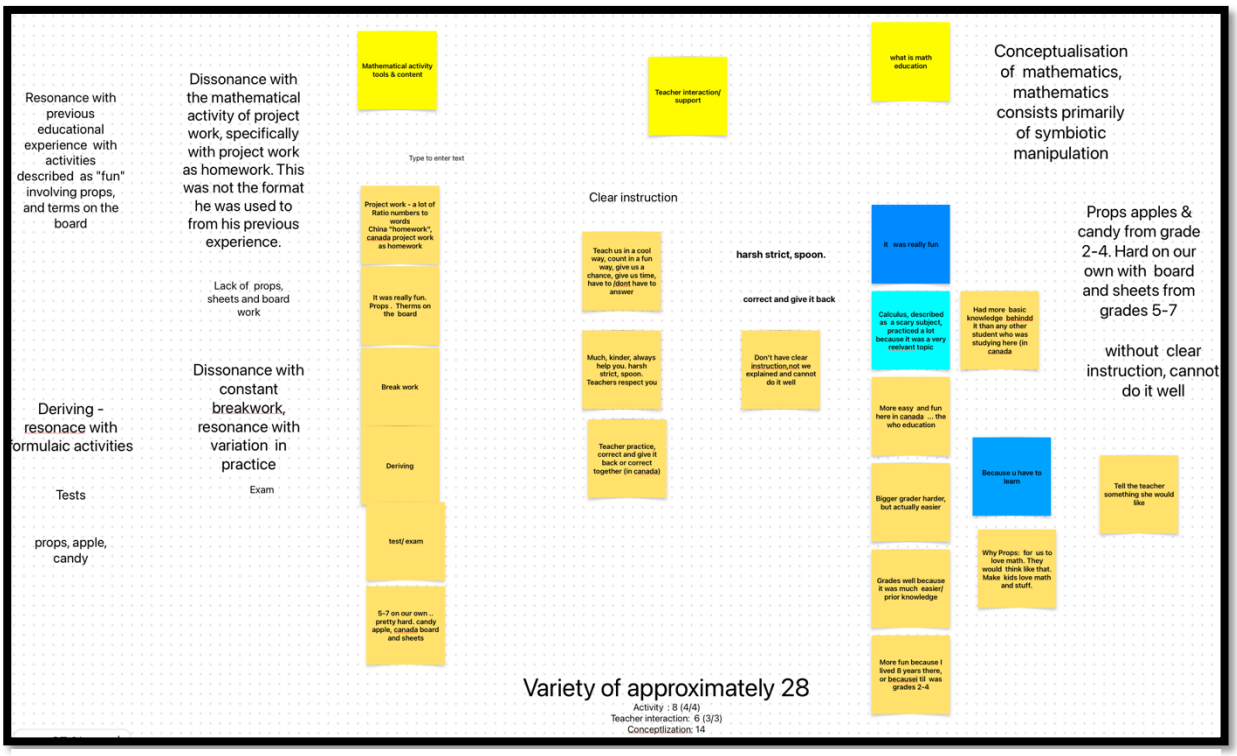


Figure 2 - Mapping content of communication

The final process of analyzation was to synthesize the analysis through a discussion of the findings and concepts that are demonstrated in the empirical data and the ways in which they are reflected in the theoretical framework. This step required a deductive analysis and involved going back to the theoretical foundation of the research in order to identify whether or not my literature and methodology were relevant, given the findings. This step also enabled me to assess the affordances and constrains of the research method as well as their implications both for classroom practices as well as research.

3.2.6 Affordances and constraints

Given the parameters of my research, I cannot assume the following empirical data provides proof of CHR, nor do I have the in-depth material needed to provide insight into the tensions arising from these experiences. Given the parameters, these tendencies cannot be sufficiently linked between the data and the framework. I can, however, use noticing to create awareness and increase sensitivity. Throughout my work, I attempt to use and share a research design and collection of strategies for noticing and reflecting on experiences of students who have experienced migration in mathematics. Working with this framework has produced a report of findings as well as an evaluation of the responsiveness of both the theoretical and methodological framework, and reflections on how

beneficial the identifiers of the methodological framework were in analysing instances of resonance and dissonance.

The affordances of my theoretical framework are as follows.

1. It promotes reflective practices by concentrating on accounts within an experience.
2. It has a structure for reflection that separates the narration from the inference and offers a clear framework to employ while reflecting. Moreover, it features a system of support requiring the separation of instance from interpretation.
3. It also makes use of key terminology to identify tension, both fixed and fluid tension.
4. With unambiguous resonance that can be followed directly from the empirical data to the conclusion, it supports the development of concepts. This encourages external evaluation and provides a way for the researcher's reflections to be externally assessed and validated.

Some of the constraints of the theoretical framework are the following.

1. Subjectivity is restricted as the framework is based on students' individual experiences and inferences from cases that may not be generalizable.
2. The framework is time-consuming; this method of analysis necessitates a number of processes, including differentiating between the student's narration and inference and the researcher's summary of inference and interpretation.
3. The framework's limited reach prevents it from revealing more general contextual elements that can affect these experiences. Instead, it frequently depends on the researchers' systematic work leading to an interpretation of students' narratives.

In conclusion, my theoretical framework includes a number of affordances for structured, transparent research that create an environment for internal and external attention and reflection. It does, however, also have limitations due to subjectivity, time, and scope. Despite these limitations my hope is that these efforts, as well as the efforts of the MMC project, will support an “alteration in the structure of attention”, allowing us to recognize moments of difference and resonance (Mason 2002, p. 165).

A primary concern regarding the affordances and constraints of my research has been my responsiveness; this includes responsiveness towards, literature, methodology, collaboration and participate. When defining issues and presenting research, there is always a component of power. Because of this, it is critical to understand and be transparent about the research's goals. That is why

I have been clear about my positionality and the reflections behind my chosen research method. It has also been vital for me to consider the presumptions and discourses around migration in mathematics throughout the research process. My collaboration with the MMC-project has aided me in navigating the field of migration in mathematics, specifically reflecting on the need for responsiveness when recruiting and interacting with communities, participants, and parents of participants.

3.3 Ethical norms and guidelines

In this section, I will discuss ethical considerations and address factors pertaining to the quality of the methodology employed and conclude by sharing some experiences from the collaborative research. I begin by discussing what scientific theory describes as internal and external research norms and guidelines (NESH). One example of this would be validity which pertains to both the internal trustworthiness of the research as well as the external validity, the extent to which the results can be generalized. What I understand as the intent of ethics is ensuring I have not harmed or betrayed anyone's confidence in the research process, maintaining my word and carrying out what I promised, without misrepresenting the participants, the empirical data, or my conclusions. The three pillars of the humanities are respect, protection, and justice, which are based on human rights concepts (forskningsetikk.no). Research ethics, such as these, are standards for conducting research.

In educational research and educational practice, the ongoing interaction between theory and practice is has become quite evident to me. In light of changing research and educational standards, teachers, such as myself, are reconciling their roles as practitioners and researchers. That is why I have attempted to remain conscious of my positionality throughout the research process and the underlying traditions upon which I am building. My academic studies and participation have provided an initial foundation, both theoretically and practically, for the task of scientific research into mathematics education. However, the task of navigating the field of scientific research as a master student within a research project provided many more opportunities to learn and apply the rules and guidelines pertaining to ethics. What I have learned is that each aspect of research requires ethical consideration, and even after careful thought and evaluation, one will never be able to predict every possible result or effect of a research procedure.

3.3.1 Ethical considerations

Many of the ethical considerations taken while planning and conducting this research design have already been mentioned. However, I will attempt to consolidate, and I synthesize some of these reflections by evaluating my research process using the terms, reliability, validity and generalizability or transferability, described by Tjora (2017), as criteria to evaluate the quality and responsiveness of my research (Tjora, 2017, p. 231). The terms, reliability, and validity are quantitative constructs which are not directly transferable to qualitative research. Attempting to make any kind of claim regarding the validity or reliability would be in vain as they link to questions of transferability and replicability, that is, whether one can replicate the process and achieve the same results. It has been highlighted in research that you cannot replicate the research due to the fact that the context will be different. However, given that this work is situated in within a larger body of research, my research colleagues will be able to test my methodology and see if it is reliable, if they see similar results and gain similar insights from using this method. The validity of the research is also assessed through my discussions with my colleagues when validity is explored through assessing the focus as well as the previous research and concepts I refer to in my work.

The component of reliability relates to the organization and systematization of the research process, as well as transparency ethics. The internal context during the research process is affected by this matter. Through working with the MMC research team, I abided by the guidelines of the Social Sciences and Humanities Research Council of Canada. The guidelines pertain to aspects such as access, organizing and storage of recordings and transcripts. Collaboration with and abiding by the guidelines of the research council assisted the research logic and coherence, an effort that increases reliability and internal transparency of the research.

Since transparency is crucial when analysing data, I based my analysis on the phenomenological investigations' theoretical underpinnings. Phenomenological methods put more emphasis on the experiences and the process of producing meaning than they do on justification or interpretation. It attempts to view the phenomenon from the perspective of the individual, and as a result, it is a helpful tool in terms of reliability (Nyeng 2020, p. 36). When addressing the method of coding, I link this element to Tjora's dependability considerations where it is critical to keep the codes authentic and near to the empirical data (Tjora, 2017, p.19). This helps avoid misunderstandings and keeps the analytical process rooted in the empirical evidence. These factors are quite important given my role as a teacher. I must be aware of my own involvement in the study and how it can skew the findings if I want to do reliable research.

I also relate the external value-related characteristics of reliability to sensitivity responsiveness. The societal duty that comes with research is one component of reliability that I attach to the overall ethics of research. Relevance, political and practical ramifications, and research material are some of these. The subject of migration in mathematics is pertinent because inclusive educational practices are required as our societies diversify. That is why it is crucial for researchers to be conscious of the tension these elements cause as well as the discourses surrounding the issue, some of which have a deficiency perspective. Some discourses see migration in mathematics as a deficiency and position various contextual mathematical methods on different value scales. It is crucial for researchers to be aware of these tensions and to consider how they might affect the study process, the participants, the findings, and their ramifications. One of my most significant ethical obligations as a researcher is to treat participants with respect before, during, and after the research process. The implications of their agreement, the context in which they will participate, and how the research will be utilized, including their right to decline involvement and their anonymity in publications, must all be made clear to the participants. Respecting the voices and experiences of the participants while also being aware of the discourses around the topic are also essential to me as shown in my methodological considerations and choices.

Validity refers to the claims I make about my results and how sure one can be if the results show what I claim they do. This links to what kind of steps I take to protect the research and make sure my methods produce valid results. By reflecting upon them, they allow me to consider the validity of scientific methods applied with the goal of answering my research question. The question of how the research question fairs to the internal validity depends on how far these questions are appropriate and whether this form of enquiry is suitable to the purpose of the research. Whether or not the questions and research method actually answer the question, or if there are aspects which are misunderstood or misinterpret which could possibly lead to answers not related to the topic or which do not answer the research question. The collaborative research process has required a period of orientation to the projects design as well as its theoretical foundations. The preliminary orientation has been necessary prior to interaction with the empirical data and application of my methodological framework, based on my theoretical framework, the discipline of noticing. Taking the time to recognize the theoretical concepts of the research as well as how it is conceptuality situated within the academic community, provided me with a basis to evaluate whether my investigation and methodological application was beneficial and facilitated my answering the research question.

The internal guidelines refer also to the generalizability, transferability, and replicability of the research. -- The question being if there is any ability to generalize from my findings, or if it was purely an exploration that could merit trying out further in other cases. The person generalizing, in this case, me, the researcher, is primarily accountable for transferability. By offering in-depth descriptions of the experience under investigation and creating clear connections to cultural and social contexts, the qualitative researcher can increase transferability or show that similar results have been found in other types of research. Towards this purpose, a thorough and in-depth overview of experiences from data collection is provided via a thick description. However, when it comes to external validity and generalizability, I examine the relationship and my responsibility toward participants, researchers and the projects scientific value.

3.3.2 Collaborative research

Collaborative research is a form of knowledge mobilization, where the reflections and discussions are part of the collective's knowledge. A major part of my research process has involved finding my role within the research team, being both respectful of the collective knowledge, theological and methodological preferences as well as actively participating in reflection and design strategy. Part of the research process has been navigating my own role as an independent researcher as well as an assistant researcher to the MMC-project. Working with the research team has had multiple benefits, such as collective reflection as well a support team who provide tips and guidance throughout the process. One of my contributions to the MMC-project was presenting a conference paper regarding the ethical dilemma's experienced during recruitment (MIM2022). Although I had not participated in the recruitment process or collection of data, my participation exposed me to the process as well as critical feedback from conference attendees, which were then addressed by the researchers. In this way, my ethical reflections and experiences have not been limited to my thesis but broadened due to the scope of my participation.

I have, however, been mindful of the need for originality and independent thinking in my master´s thesis. This is because collective discussions and reflections are the intellectual property of the research team as these contain concepts that were created in collaboration. Given that this work is situated within a larger research project, it was important for me to be able to develop my own independent theoretical framework based on my understanding of relevant literature. Within the collaboration, the concentration of my participation has been primarily student transcripts which has resulted in a conference paper (MIM2023). It was necessary to organize the accounts by category in

a way that was responsive to the integrity of the student transcript when working with the findings and preparing to present these preliminary findings in the form of a conference paper. This collaboration has served as a support system for my participation in the field of scientific research. Nonetheless, the partnership has challenged me to balance the ethical dimensions of my obligations to the research team and to my thesis while also considering their distinctions. This challenge was most present at the time of my decision to change my focus of inquiry, from teacher interviews to student interviews. This transition was based on the time constrains of my thesis as well as based on the consideration of the content of my contribution thus far in the research process; this decision was not taken lightly. The transition required a great deal of ethical reflection, given that the research team was already in the process of writing a conference paper based on the same empirical data. Due to this fact, I felt there was an ethical dilemma in differentiating between the analyzation for the conference paper and my thesis. Given the empirical data of the conference paper is the same as my master's thesis, I decided it would be prudent to develop and apply a methodological approach that would be distinguishable from that of the conference paper.

My perspective of cooperative scientific research has greatly benefited from my introduction to a research community that regards knowledge in this way, as shared property, as initially I was concerned about the overlap in findings. However, in order to collaborate effectively, it was crucial for me to understand and reflect on the process of knowledge construction; in this case shared intellectual property through an exchange of reflective dialogue. An awareness of this process strengthens my understanding of collaboration as a significant research instrument and a platform for knowledge mobilization. These reflections and the MMC collaboration have assisted in the development of both this research thesis and me as a researcher.

3.3.3 Attending to positionality in education research.

This thesis is a reflection of my participation in collaborative research projects as well as my personal exploration and growth as a researcher and educator. The organized, methodical effort completed over a longer time span, from May 2022 to May 2023, illustrates my academic work as a researcher. My literary research studies, specifically Mason´s work (2002) and other ethnographic approaches, impacted my structuring choices when developing content for phase two of the MMC-project, the creation of shorter "stories" or accounts from the student transcripts. The practice of storytelling is one that I link to both dialogic and narrative research as well as didactic and pedagogical practices where one seeks to create context and engage participants. I was motivated to engage with the practice

of responsiveness included in the ethical facets of scientific research because I acknowledge that describing another person's culture is an act of "othering." Choosing to use directs excerpts as titles for each story was a result of these ethical reflections as well as my studies on the scientific method. These methodological actions were based on qualitative research processes and models.

It is because I had work with the empirical data for a longer amount of time, from August 2022 until May 2023, that the discussion of influencing factors and inference is so extensive. As a result of the collaboration and timeframe, I had a number of reflections from both the perspective of a researcher and an educator. When structuring and introducing noticed elements of the account I took a pragmatic approach of describing what the account contained. In contrast, my inference of the account is narrative and demonstrates the process of meaning-making. My interpretation of student accounts is reflective of the patterns of a teacher's professional development, where one is continually looking for ways to comprehend and accommodate the needs of the learner.

This synopsis shows how my thoughts, choices, and actions are related to one another, how they were influenced by my positionality as well as the environment in which I worked and acted. As a researcher, I have worked toward reflexivity by being aware of my positionality. This entailed challenging my assumptions and finding strategies to address them. I believe researchers can conduct ethical and inclusive research by addressing their own positionality as well as the perspectives and experiences of their research participants. The same is true in the role of an educator where understanding students' perspectives and experiences can help us to engage in more ethical and inclusive mathematical practices.

4 Empirical findings

In this section, I will show the analysis leading to the findings regarding the familiar and unfamiliar practices experience by students who have migrated to Canada, to answer the research question,

How could engaging with students' accounts of migration in mathematics provide a basis to recognize cultural-historical repertoires in unfamiliar mathematical practices?

In general, the analysis of data showed the following three findings:

1. Familiar or unfamiliar mathematical practice does not predetermine the student's understanding and explanation of how they perceive and comprehend these practices.
2. Experiences of resonance and dissonance are dynamic, rather than a fixed state.

3. During the process of orientation, from one educational context to another, the significant presence of teachers is heightened. It is the combination, of teacher presence and students´ perceptions of what mathematics is, that creates the make-up of a students´ CHR. These dynamic relations portray the way a student´s CHR is shaped and made visible.

The following structure presents six student transcript excerpts, and each excerpt contains an experience from the mathematics classroom, both abroad and from within Canada. I investigate these accounts by applying my methodological framework and differentiating between the students´ narration and inference of an experience. Narration is the students account of an event, and inference is the students´ description and perception of an event. Inferences are often identified by descriptive nature and use of adjectives. In doing so, I attempt to notice mathematical experiences that contain familiar and unfamiliar practices, referring to my identifying terms of resonance and dissonance. The preliminary analysis is structured to clearly distinguish between the summary of the account and the inference of the account. In the rest of this chapter, I present these findings supported by detailed examples of inference and narration as described earlier.

4.1 Instance: project work - “Financial literacy”.

Inference	here we do a lot of project work
Inference	sometimes the math project is not well explained,
Narration and Inference	we don’t have clear instruction to follow and its, well, very hard to say, is, it’s just. Uh you don’t have clear instruction to follow to you cannot do it very well.
Inference	I think [the project] is more English than math.
Narration	Because we write like five pages,
Narration and inference	and you rarely see numbers.
Narration and inference	Uh yeah. Explain to teacher how you help the other guy how to do their tax[es].
Narration	Well::, one of the differences is, in China and Montreal, we always do some calculations and something like that for homework, but now here, we mostly do project, after school.

What was noticed in the student's description of the account: This student's narration of his experience with project work contains descriptions such as regularity, "a lot", quality of instruction, not "clear", scope of work "five pages", text format as opposed to "numbers", as well as the task description; how to do taxes, and where the mathematical activity was situated, "after school". I also noticed the statement "explain to teacher how to help the other guy" and was curious as to what it may imply. What initially stood out to me in the reading of the transcript, was the statement "do the other guys taxes". A statement that caused me to reflect on how meaningful or motivating mathematical tasks are experienced by student. However, after applying the identifying terms to the story, what strikes me is the statement "cannot do it well", implying outside factors as the cause of success or failure. One thing I noticed in the interview transcript was the fact that this example came after the statement "here we do many projects", yet the student ends the topic of project work by relating it to homework and stating, "here we mostly do project, after school". These somewhat conflicting statements made me curious as to how project work fits with his conceptualization of mathematics education. By conceptualization I mean the methods and activities the student recognizes as "classroom" or "homework". This query relates to resonance and whether students are able to affiliate the task of project-work as relevant to mathematical competency.

Reflection on students' inference of the experience - - one way to account for students' narration and inference:

Within the description of where the work is done, I recognized an aspect of inference. This is because the account describes not only what the math project was, but also how the work is placed. The term "after" school may contain an inference describing project work as extra-curricular. This infers that perhaps project work is not a dominate part of "the learning of mathematics" in this classroom despite the conflicting statement "here we do many projects". This may a be description of the way project work is situated in relation to other classroom activities. It may also be an inference on the value of project work, and as it is situated outside of the classroom and this student may question whether project work is central to the learning of mathematics. In other words, how the student perceived the mathematical value of the project, and whether project-work, as a mathematical task, was experienced as helpful to the learning of mathematics.

The student's description of project instruction is closely followed by a statement or inference on how well they achieved. The statement "cannot do it very well" links teacher support to student´s

academic success. I find this statement to be revelatory as it places the responsibility of success or failure on the task structuring, which is the responsibility of the educator. This would indicate that a lack of teacher support is a source of dissonance for the student.

The student uses the term “a lot” to describe the regularity of the activity but does not explicitly compare or contrast it to other forms of mathematical activity. Although the frequency of this format, “a lot”, implies project work as being a format they are acquainted with, the statement “mostly after school” may indicate dissonance. Could this dissonance be caused by lack of instruction or is project-work a format that students from China would be less accustomed to? Or could it possibly be the placement of the task, “after school”, that prescribes its value as that of homework or extra-curricular, therefore creating dissonance between the type of tasks that resonate with the student, the mathematical practices that are valued. By resonate I mean tasks that fit the student’s conceptualization of helpful mathematical activity.

The student reflects on the ratio between text and numbers, in the statement, “you rarely see numbers”, which infers an assumption about math. The statement may indicate a conceptualization of how mathematics is about numbers and therefore class activities containing more text than digits should be categorized as language class. As an educator, I recognize the changing proportion of text and numbers in mathematical tasks as source of dissonance. I connect this dissonance to the conceptualization and purpose of school mathematics as being that of utilizing numbers to produce efficient and absolute solutions. This student’s inference is one that has been identified in research through low scores on text problems and limited text responses and explanations to show students mathematical reasoning.

Within the narration of the task description, “help the other guy”, I recognize an inference in regard to the mathematical value of the project. Knowing how to do taxes is recognized as an important competency involving mathematical procedures. That is why I was surprised by the inference that implied the task did not resonate with the student. One way to account for this statement is that the task implied helping some hypothetical person with their taxes, or perhaps the dissonance could be linked to the introducing statement “explain to the teacher”. Whether or not the statement was explicit or implicit in the task description we do not know, but it may be an insight into how the varied mathematical tasks are conceptualized by students. The fact that the task required “explaining to the teacher” could imply a dissonance between school “problems” and real-world problems. This brings

up the question of what type of learning milieu this student has most experience with, as it may impact his conceptualization of relevant mathematical tasks.

4.2 Instance: How the teachers teach in Turkey

Inference	In Turkey, it was really fun.
Narration	They give us a lot of props to use as a grade three
Inference	They teach us in a fun way … like they count the numbers in a fun way. They teach us in a really cool way. And, the best part is, …
Narration	they give us like a chance to say something, if you don't want to answer the question you are allowed to
Inference	Because you´re in grade three, but if it's grade four, like you have to answer because you're in grade four and you need to learn to, like, know what to say to like grade five or six or seven. …math, it´s like really easy, because they teach us in a good way,
Narration and inference	and they also give us terms on the board, and they give you time, and yeah, it´s just like that.

What was noticed in the student's description of the account:

Within this excerpt I find descriptions on counting numbers, voluntary and non-voluntary student participation, learning tools, "prop" and "board", as well as how the work is paced, with "time". One part of this account that stands out to me is when the "chance to say something" is described as "the best part", although it is not clear if it was the act of responding or the option of answering that the student experienced as the "best". This account contains an element of student autonomy in the classroom given that they had a choice to answer or not. The student goes on to explain that in the future, they do not have the option of responding or not, and this is because you "have to learn". I was curious about this differentiation and what it might indicate regarding the conceptualization of mathematics. This student summarizes with a definite statement, "just like that", which indicates the conclusion of her narration and inference on the teaching of mathematics.

What I noticed in this student's narration of her educational mathematics experience in Turkey, was the regularity of the pronoun "they". In this transcript excerpt, it is used in nearly every sentence. The

second thing I noticed were the descriptors, “really, a lot, cool, best”; this made me curious as to both the characters and concepts these terms refer to.

Reflection on students’ inference of the experience - - one way to account for students’ narration and inference:

Given these descriptions one could say that, overall, the mathematical practices she accounted for from Turkey resonated with her. While description of how they counted numbers, “in a fun way”, does not proved further insight, as an educator one could visualize this activity through a variety of pedagogical practices such as songs, and tool such as posters and math aids. The terms “fun” and “cool” could imply that the activity was experiences as meaningful or one that resonated with her.

The same goes for oral participation, both the participation and differentiation. By differentiation, I mean the option to answer or not. Within this account one could identify resonance within the statements “best part”, “give us a chance” and “don’t have to answer”. Following comes the description of a change in practice where the option to answer or not is removed. This change in practice seems to be something this student has noticed, perhaps indicating dissonance. However, her inference as to why the practice changed comes in the form of an explanation, due to advancement in grade, “you have to answer … you need to learn”. How that explanation evolved is unclear as this student began her Canadian education in the middle of grade three and since she did not attend grade four in Canada, we might assume that she did not experience the differentiation first-hand. Regardless of how she came to know the differentiation of optional oral participation and non-optional participation, whether it be internal or external factors, it clearly was an experience she has reflected on as seen in her description, “best part” and explanation, “you have to learn”. The account of autonomy in participation resonated with me because it indicates a practice for student autonomy within the classroom. This account reminded me of the principles of motivation and self-determination which describe the impact of student autonomy on resonance.

The account of visualization of “terms” on the board, as well as the expedient timeframe for performance can be described as teacher practices that resonated with her as they supported her learning. The use of visualization in classrooms is not an unfamiliar practice to many of us educators. However, what piqued my interest was the statement “they give you time”, as it contains an element of inference describing resonance with the practice that provides ample time and dissonance when working within a limited timeframe. One would assume that learning takes time and teachers plan

lessons and activities to fill the time allotment. However, as an educator I am keenly aware of the time constraints within the classroom, specifically regarding the individualization of student needs. Perhaps this account implies that this student or other students in her class where able to accomplish their tasks within the given timeframe. Perhaps the element of time was noted by the student due to contrasting experiences of having too little time wherein to accomplish the task. Regardless, this inference demonstrates “time” as source of resonance for her mathematical learning practices.

Given the frequent use of the pronoun "they," which suggests a dominate third party, it is possible that the teacher's teaching methods served as the major protagonist or main source of her resonance and dissonance. This assumption is based on the accounts repeated instances where this indefinite party dictates the instructional and structural elements of the mathematical practice. This leads to the inference that teacher instructional practices play a leading role in students´ educational experiences. This account uses the pronoun “they” to describe an indefinite subject and within this account there are many indefinite descriptions of both persons, activities, and tools. The fact that this story is a reconstruction of earlier educational experiences with mathematics may be the cause of these ambiguous subjects, however the students´ inferences clearly indicate pre-exiting resonative experiences.

4.3 Instance: Teacher support

Inference	Yeah. Over here [in Canada], the teachers are much kinder, they always like help you.
Inference, narration, inference	And the UK, they were kind of harsh and strict. So, like, if you asked the question, they would not really, like be happy?
Narration	Like they, usually I remember, they used to tell me, “Do I have to take a spoon and feed you?” That's what they used to tell me.
Inference	Yeah. So, that's why I find that over here the teachers are much kinder, and they are really helpful.
Narration + Inference	So, [in the UK], when we were given work in class, we wouldn't do it together. We were like, we had to do it on our own. And, and then once you finish, she takes our papers, and then she'd correct it and give it back to us, whereas here we all correct it together. Yeah.

Narration	In the UK, we were given a lot of homework, like, we would stay up late just to finish our homework. And also, in like the winter break, and those like short breaks that we had, we were given like homework still. In Canada, we do get homework, but not as much as in the UK. And also, sometimes the teacher like, is being kind to us and doesn't give us, like in the break, worksheets to do. So, yeah.
Inference	… The teachers [in Canada], are more, like, nice to-, they, like respect you in a way.

What was noticed in the student's description of the account:

This student's narration includes the following accounts of a statement made by a teacher involving a "spoon", the ways work was corrected, "on our own" in the UK and "together" in Canada, an indication of the amount of homework both on weekdays as well as school breaks and includes descriptions of how the task involved staying up "late" in the UK. In contrast, the student experienced that some school breaks in Canada do not involve worksheets. Within the students´ inference she uses the following terms to describe teachers: kinder, always help you, harsh, strict, not happy, helpful, nice, and respect. This made me curious as to how the identifying terms of this account demonstrated mathematical practices of resonance and dissonance. This is because the contrast of her experiences, from school in the UK and school in Canada, is demonstrated through descriptions of teacher interaction as well as instructional practices.

Reflection on students' inference of the experience - - one way to account for students' narration and inference:

Teacher interaction and instructional practices that seem to resonate with her are kindness and helpfulness. Within the narration of asking a "question", the manner in which the teacher responded impacted the experience. Although this account does not state who was asking the question, herself or one of her co-students, one can assume that this account is based on a first- hand experience. The description of "not happy" implies that the teacher practice that resonates with her is when the teacher responds "happily" to student inquiry. The student elaborates on this experience by narrating an account of a teacher´s response. Within the reply is the conceptualization of instructional not involving "spoon feeding" students the answer to questions. This statement may be connected to a foundational principle for the instructional mathematical practice, perhaps the principles of self-

study, and repetition. However, the part of this account creating dissonance for me is, firstly the statement, and secondly the fact that this student was only in the UK until grade three. The statement itself could have been the result of a trying day and not an example of standard instruction, however, the fact that this incident occurred sometime between grade one and three, at a time she was new to the mathematics education system may provide insight into the expectations placed on primary school students. Educative expectations often impact instructional practices therefore affecting the experiences of students in the classroom.

In order to make any statements in regards to the difference in educational standards between the UK and Canada one would need large amounts of both qualitative and quantitative data. We can however notice how this account was experienced by the student, through the contrasting words used to describe instructional practices. The terms harsh and strict could be primarily linked to the one instance of teacher interaction or they could be a cumulation of experiences with instructional practices. The confines of this account do not enable us to determine whether practices concerning individual and collective grading practices as well as break work are structural, pertaining to the practice of the school, or instructional pertaining to the individual teacher´s instructional practices. However, the descriptors on "our own" and "together" describing grading, as well as "a lot", "stay up late", contrasting "kinder", and describing worksheets during school breaks, exemplify the instructional practices that resonated, and did not resonate with her. The instructional practices that were a source of dissonance were the accounts that described mathematical activity as something she was expected to master on her own, through her independent reasoning and studying. The instructional practices that resonate with her were oral support, collective grading, and a balanced amount of independent study. Combined, these contrasting practices indicates her conceptualization of mathematical learning, one which involves support, collaboration, and balance.

4.4 Instance: calculus

Narration + inference	We started deriving just a bit and I was like, wait, I took this before...
Narration	I always hear like, oh, calculus is such a scary subject
Inference	and I was like, is it? Like, what is it about? Like, why is everybody so scared?
Narration	… And then it was like, oh, deriving

Inference	and I was like, oh, it's deriving like, “what's scary about it?” So, because I've had like, knowledge about it, it was really easy
Narration	and I got like an 88 in that class, … But we went over it more in Saudi in grade 8 and in my 11th grade.
Inference	We had a lot of integration and deriving problems.
Narration	And I practiced it a lot
Inference	because it was a very, like, relevant topic.
Inference	And so, I had more of a basic knowledge behind it than any other student would who was studying here [in Canada].

What was noticed in the student’s description of the account:

The student’s narration contains several accounts, one describing the task of deriving as a familiar one, a mathematical activity she had previously done in another place. The next statement describes an encounter with an unfamiliar mathematical term, described by her co-students as “scary”. Following comes her experience and inference with calculus along with descriptions of previous work with the subject, the outcome measured in test result scores as well as the value of the subject as being “relevant”. She concludes by comparing her knowledge of the subject to that of her co-students. This account contains an observation of her co-student’s perception of the mathematical subject which I was curious about as it describes how the conceptualization of the community, her collective students, is one the affects personal experience and impression. In this account, I also recognize a transition from an unfamiliar mathematical term to a familiar mathematical practice; the transition occurred when the connection between deriving, and calculus was made.

Reflection on students’ inference of the experience - - one way to account for students’ narration and inference:

This account contains three occurrences of both resonance and dissonance. The accounts I reflect on here in the account assists me in noticing a pattern that moves from the one to the other, the process of inference. The first occurrence is found in the statement, “wait, I took this before”. One could assume that this student was studying calculus in class and eventually made the connection between the phrase and the mathematical skill of deriving. If we suppose that the student had no prior established notion of what the mathematical topic of calculus involved or contained, that would mean the term "calculus" was unfamiliar to her. The case that the phrase itself was new to her, despite the

fact that she had previously used calculus in another context, illustrates how contexts have an impact on how things are conceptualized.

Along with the term's unfamiliarity, her initial experiences with the term are affected by her co-student's conceptualization of calculus labelled as "scary", that she "always hears" in Canada. This shows a patterned transitioning from dissonance to resonance. She moves from a description of "scary", afforded to her by her co-students, to the mathematical topic being "not scary". The adjective "scary" is used in relation to the topic, a description influencing its conceptualization. Her pervious mathematical knowledge and experience led to the statement of dissonance "why is everybody so scared". Her previous experiences constructed an understanding that contrasted with her co-student description of this mathematical practice. Her familiarity with the subject matters contrasted the collective sentiment of her co-students.

The mathematical knowledge of deriving involves the process of creating a formula. Given her previous experiences with deriving, she had a conceptualization of it being a relevant topic as she "practiced a lot". This statement describes the different scholastic emphasis often found when comparing structural and instructional mathematical practices. What I mean by structural and instructional mathematical practice is the structuring of the curriculum as well as the form and focus of instruction often dictated by assessment standards. The account gives the indication that her previous educational context contained a strong emphasis on teaching and learning calculus and that is why "they went over it in Saudi". Differential calculus, which determines instantaneous rates of change, and integral calculus, which adds an unlimited number of small factors to determine some whole, both have practical uses but are often initially taught as the structuring of formulaic expressions, otherwise known as rote learning. This may indicate a difference in instructional practices, whether the instructional process begins with practical application and ends with formulaic expression or the opposite. This student´s previous educational experience included frameworks, tools, and formulas that gave her "more knowledge" than "any other student". The process of migration, experiencing mathematics education in one context and then another, illustrates why she experienced increased resonance with formulaic expressions, it was more "relevant" in the previous educational context. This account shows how educational standards, constructs and explanations impact students experiences and conceptualizations of mathematics.

4.5 Instance: Math as hard and easy

Inference	It's just that it's more fun here [in Canada] and like, easy and fun…
Narration + inference	Uh it, like here [in Canada] you get like, math activities in games, but, in London, you just do the work and you have no fun. It's just work… And there [in the UK] it's just strict and just "do your work"…
Inference	The difference like, uh, is that in London it's more strict. And then, here [in Canada] it's like, fun and easy…In the whole like, like, the whole, what's it called, uh, education.
Narration	And like, like, at the end, you have exams [in the UK], here you have a test every single time [after every topic]. You don't have the end of year exam. [In Canada]
Inference	[In Canada] it's easier, and it's like, every, uh yeah, every topic, we do a topic and then we do a test after it and then we learn a different topic until the end of the year. We just tests.
Narration	I like when stuff was hard for you. Like, you have to like, like, you get harder work…
Inference	When I came to Canada, I thought like going on a bigger grade would be harder, but it was actually easier.
Narration	Like the teacher, even like in grade four I remember when I would tell the teacher something she would like, I would like I remember I would always get like my grades really well, although when I came here [Canada],
Inference	because it was much easier.

What was noticed in the student's description of the account- - one way to account for students' narration and inference:

This account contains the description of mathematics as fun, drawing on the mathematical practices of activities and games as examples of "fun". In contrast, her experience with mathematics in the UK were described as "work", depicting the practice as "strict" and not fun. This student narration illustrates contrasting experiences through statements that describe the mathematical practices that resonate and did not resonate with her. These narrations include the educational practice of exams and tests, and where she describes "tests" as the practice that was "easier". There is also a narration

describing the students' enjoyment "when stuff was hard", linking mathematical advancement to "harder work". The account also contains the student´s own conception of how "going on a bigger grade would be harder", but goes on to state that her grades went "really well". This made me curious as to the experiences that constructed the description of mathematics as "easy" and "hard".

Reflection on students' inference of the experience - - one way to account for students' narration and inference:

This account describes "the whole education" in Canada as fun and easy; the testimony of her statement lies in the example of activities and games as preferred instructional practice. The fact that she did not mention experiencing these forms of learning in the UK is surprising as she left the UK in grade two. Although she is now in fifth grade, and her recollection of the experience may be limited, what we can deduce is that this narrative reflects a mathematic educational experience that did not resonate with her provided by the descriptions of "strict" and "work". Perhaps it was the contrasting experiences that brought her to a conceptualization of "the whole education". While the term "work" could describe any number of instructional practices, the term "strict" is more closely linked to a personal trait of either the teacher or institution. Given the details of her account, the phrase could refer to both the teacher and the school as an institution.

The format of the exam illustrates the approach of summative assessment, a practice that is related to the institutions' procedures for measurement; in contrast, topic tests illustrate the focus of formative assessment. This account demonstrates the practice which resonated with the student, but it is unclear whether topics tests were a practice she was familiar with from previous experiences. Both forms of assessment link to achievement but the implications differ; a test can either represent the sum of your knowledge, a depiction of your skills as a student, or the scope of your knowledge on a certain topic, a depiction of your proficiency in a given theme. The first depiction is a summary of one's knowledge as a mathematics student, and the other is a more condensed view of specific skills sets. Whether or not the use of summative and formative assessment was an institutional practice we do not know. What is demonstrated, however, is that intuitional practices impact teachers' instructional practices and as such impact students' conceptualization of mathematics education, which is a cumulation of their experiences.

The statement regarding her enjoyment of "harder work" maybe be surprising, but within this statement I can recognize the discourse that links mathematical knowledge to status. It is related to

ideas that portray students as smarter or better mathematicians if they can solve complex issues and produce complex answers. The concept of mathematics as a fundamental aspect of success links prospects to value, although values differ between countries and contexts and consequently educational emphasis. The necessity of success is often dictated by the cultural environment such as opportunity for economic stability and / or advancement. Whether or not this concept was implicitly or explicitly imparted to her, it describes how her prospects for success may be predicted together with the effectiveness of her answers. In this account, success is related to educational advancement and she refers to test results and teacher feedback as instruments for evaluating her success.

When a teacher is described as "strict", it could be in conjunction to behaviour or mathematical performance, the assumption being that that strict teachers have higher expectations for their students. Regardless, the actions and responses of the teacher are valued by this student as seen in the statement "I would tell the teacher something she would like", which links positive feedback to success. Her conflicting mathematical experiences, which both have an impact on her conceptualization of success, are described in these two accounts of teacher interaction. Although we do not have access to a description of her academic accomplishment from the UK, she does provide us with evidence of success, the conceptualization of her abilities as a mathematics student, confirmed in the grades received in Canada. There is, however, an element of dissonance in the inference of "bigger grades" means "harder work". Her experience testifies to the opposite, though it does not explicitly say what was easier or why she experienced it as easier other than the "resonative" practice of formative assessment and positive feedback.

4.6 Instance: Math props

Narration + Inference	It's like the same questions. Like they explain, they help you, they, you know, give you a chance to do your own work so they can understand if you like, understand or not.
inference	It's kind of the similar, it's like the similar questions, you know, but it's just that like, I would prefer Turkey more. I don't know. I just feel like it's closer to like...Like, I feel like it's more fun. I don't know because I live like eight years there. Or something.
Inference	School was more fun because like maybe it was because it's grade four, and like two, and three.

Narration	Like, they gave us like props to use with maths, and like they gave us like apples to use or candy to use
Inference	So it was more fun for kids, but here now it's grade seven, six, five so like we have to like do it on our own, and like if we have and all that you know like-, pretty hard...
Narration	like in grade two they used to like give us like apple you know, candy to like count like one plus one, like they gave us apple plus apple like actual apple....
Inference	We would like, you know, it would be more fun to us like to love it more. They would like think that way"
Narration + Inference	In grade, like, in grade four [in Canada], they used to don't do that... Yeah, they used to not do that. They used to not like get actual apple and candy to make the kids love math and stuff. They just used the board. Like give us sheets.

What was noticed in the student's description of the account:

This account begins with the students' inference on the similarities between mathematics education in Turkey and Canada, specifically regarding the use of mathematical props. She begins by presenting how the teachers are helpful as well as her understanding as to why they do "work". The students´ inference links mathematical tasks to mathematical understanding. The account connects her preference to the duration or length of her stay as well as grade level. The account presents situated experiences with "props to use with maths" like the use of "apples and candy" along with the description of "fun". The account also presents her current contrasting experience, in grades 5-7, where they must "do it on our own" with instructional tools such as "board" and "sheets", which she describes as "pretty hard". This made me curious as to how students' preference is linked to understanding of the purpose and intent behind student activity.

Reflection on students' inference of the experience - - one way to account for students' narration and inference:

The student begins this account with an inference on the similarities between the two educational experiences and seems to recognize familiar aspects, as shown in the statement "it's the same/similar questions". The teacher practice that resonates with the student is displayed within the statement "they help you, … they give you a chance". This inference is link to the student's conceptualization of

mathematics education on how students are obliged to provide insight, in the form of mathematical reasoning and solutions, to the teachers “so they can understand if you understand or not”. There is a clear inference on the purpose and intent behind the mathematical content, its value being the demonstration of mathematical reasoning. In this way the intent behind teacher interaction is presented as a constant, the constant influence being educational support and mathematical assessment. The previous educational setting is described as preferential and the student proceeds to explain her preference through the following fragments “closer to, more fun, and eight years”. The students´ inference leads to two conclusions that influence her preference; the first is the duration of her stay, and the second is the mathematical tools and resources she was familiar with in lower grades.

The inference on the change of mathematical practice is linked to the increased grade level. In contrast to the intent behind teacher interaction as a constant through the use of props such as apples and candy are inconstant. The use of props, such as apples and candy, might be defined as both conceptual tools as well as incentives, linking the practice to behaviourism. However, this assumption is negated by the student’s concurrence on the interaction being linked to mathematical achievement. What strikes me is the student’s support of the inference, regarding teacher interaction, through the statement “they would think that way”. This statement depicts a clear implication about the motivation and intent of mathematical tools. Although it is unclear whether this conclusion was deduced from her educational experiences or explicitly stated, it stands out as one of the few unambiguous statements about educators´ objective. The objective being to make students “love [math] more”. It is unclear whether it is the change of location or grade that generates the dissonance found in the narration “they just use the board” and “give us sheets”. What is noteworthy is the pattern of transition in the students’ resonance as well as her inference on the change in practice and how she experiences it. The student´s account reflects an inference on the purpose and intent of props, connected to the purpose and intent of teacher support, both of which are connected to learning of mathematics.

5 Discussion of findings

The following discussion chapter interrogates the accounts through the key concepts of my theoretical framework. It also systematises my findings as to identify the similarities, regularities, and differences within these experiences, curiosity, interrogation of distinction and systematisation. These distinctions enable us to notice the variety of mathematical practices being addressed. The task of noticing regularities addresses the collective practices of my findings and interrogates the concept of cultural-historical repertories in mathematics education. These findings are linked to the terms Rogoff (2003) uses to discuss repertoires as cultural ways of learning: regularities, activities, practices

(Gutiérrez & Rogoff, 2003). I address the mathematical practices through differentiation between instances of resonance and dissonance, displayed in the findings. This is because each instance contains a collection of mathematical experiences, some familiar and meaningful, while others unfamiliar containing less resonance and or are undesired.

The reason for this structure is to enable me to address the similar or collective practices behind the individual experience and answer the research question,

How could engaging with students' accounts of migration in mathematics provide a basis to recognize cultural-historical repertoires in unfamiliar mathematical practices?

The analysis of data has led to the following three findings.

1. Familiar or unfamiliar mathematical practice does not predetermine the student's understanding and explanation of how they perceive and comprehend these practices.
2. Experiences of resonance and dissonance are dynamic, rather than a fixed state.
3. During the process of orientation, from one educational context to another, the significant presence of teachers is heightened. It is the combination, of teacher presence and students´ perceptions of what mathematics is, that creates the make-up of a students´ CHR. These dynamic relations portray the way a student´s CHR is shaped and made visible.

I do this as one approach to engaging with unfamiliar mathematical practices through the discipline of noticing, building off theoretical foundations which both theorizes mathematics and collective practices.

Before attempting to notice familiar and unfamiliar mathematical practices, I must first refer to the theoretical concepts previously addressed which describes mastery as the goal of both mathematical learning and mathematical practices. This implies formal aspects found in accounts of calculus, formulaic expression, and informal aspects described by Maturanas (1988) as emotional orientation (cited in Reid et al 2015, p. 369). I have used the terms of resonance and dissonance to signify the pattern of emotional orientation through these accounts. This is because mathematics is both a systemic, deductive, emotional, and inductive, form of reasoning. Mathematical knowledge or reasoning is subject to preference, tendencies which we can identify in these accounts as being applied by external and internal forces. External forces being contextual or situational factors affecting the

institutional practices which then impact the daily practices, mathematical actions, and inferences of these students. This description supports the theorization of the system of mathematics as being both a discourse and a practical activity. Within these accounts we can recognize mathematics described as a situated conceptualization when the students describe mathematics as a system that is “hard” or “easy”. These accounts also describe the system of mathematics though the conceptualization of mathematics as a practical activity, described by terms such as “fun” or “work”.

I will begin by interrogating the accounts and descriptions of mathematics by first summarizing aspects pertaining to classroom activities and teacher interaction before addressing the student’s conceptualization of mathematics as a discourse.

5.1 Mathematic activity

Roughly one third of the accounts describe a specific mathematical practice, such as project-work, sheets, tests, exams, apples, candy, and other forms of mathematical content. Approximately half of the instances indicate dissonance with the activity while the other half indicate resonance with the mathematical activity. The indicators reveal when a learner perceives mathematical activities as positive or negative through descriptive terms such as fun or not fun.

Examples that demonstrate resonance with previous practices are activities involving props, visualization with terms on the board, as well as the process of deriving as it represents familiar formulaic activity. These mathematical activities were familiar practices. New or unfamiliar mathematical activity that students indicated resonance with were the practices of formative assessment in the form of tests as well as flexible break work practices.

Examples of familiar practices as a source of dissonance were standard break work, and summative exams. Unfamiliar practices that were experienced with dissonance were project-work as the format of homework, and sheet and board work in contrast to props. These examples show that it is not solely familiarity that prescribes resonance, as unfamiliar activities were also described as positive and "better" than previous familiar practices.

Since these accounts are situated within the mathematics classroom, one can recognize the practices of teachers through the mathematical content they provide for their students. These accounts position the educator as the external force who selects and administers the mathematical content to the

students. Given the scope of the empirical data I am unable to connect the use of such mathematical content to collective pedagogical practices of the school, district, or country. I do, however, recognize that mathematical content, such as project-work, can be connected to pedagogical theories found in literature. Within the findings we can identify three mathematical formats that students react to with dissonance. These mathematical formats are project-work, calculus, and exams. Project-work and calculus represent two contrasting learning milieus. This distinction between "school" and "world" problems, is discussed by Skovmose (2005) in his description of the different types of learning milieus as they represent mathematics education as a complex network of social practice (Skovmose, 2005, p.1567). In this way the interpretation of mathematics education is dependent on situated and associated variables, such as ambition, as well as the implementation-related realities. The example of this framework demonstrates how mathematical activities are often situated in other learning milieus, are guided by theoretical principles and applied by educators as a community. Other mathematical activities we can link to pedagogical theories are homework, founded in the concept of repetition, board work and props, founded in the principle of visualization and conceptualization that assist in symbolization of sizes and amounts. Another example of mathematical practice is break-work; the accounts show that constant break-work was a practice that incited dissonance whereas occasional break-work was a practice that resonated with the student. This account could have been viewed as an instructional practice, as homework is a didactic practice to support learning based on the concept of repetition. However, such practices are often situated within implicit school practices, found within debates such as "homework free" or "homework" schools. As such, I link this collective practice to school practices, or school culture which impact and dictate after school learning tasks the students are given.

The practical activity can be connected to the practice of one's community where, through participation an individual gains access to the communities "funds of knowledge" (Appelbaum & Stathopoulou, 2015, p.763). Participation in classroom practices involves exposure to the educational strategies and principles of the educator community. One might define this as the collective practices of the students, dictated by the collective practice of the teacher and school. We are not able to deduce whether these instructional practices were a result of the teachers' professional pedagogical practice or the school practice. The principles behind the mathematical content cannot be evidentially demonstrated within these accounts and I am therefore unable to connect them to the collective teacher practice of the school, district, or country. However, one assumes an extent of autonomy in teachers' instructional practices and therefore I cannot directly link mathematical activities to the collective practices of the teachers.

What has been demonstrated in these accounts is that experiences of resonance and dissonance can be identified regardless of the student's familiarity or unfamiliarity with the practices. The instance of dissonance, with calculus being a "scary" subject, was resolved through the process of participation which enabled the student to recognize and connect the activity to familiar mathematical content thus providing resonance. Dissonance is not illustrated as a constant state, rather these accounts demonstrate transitions from dissonance to resonance or unresolved dissonance as in the account of break-work.

5.2 Teacher interaction

Participation involves interaction, and the main character of these interactions is presented as the teachers. Alongside the mathematical content presented in the accounts, the student remarks often refer to classroom and teacher interaction. Examples of this has been, tell the teacher, show your work, do it kindly, and oral participation. Previous teacher interaction that resonated with the students were clear instruction, teaching and counting in a "cool", "fun" way, giving the students time to answer as well as the option of not answering. Previous teacher interaction as a source of dissonance were harsh and strict interaction and statements, such as the one involving "spoon feeding", as well as teachers correcting and grading of student work.

Current teacher interaction that resonated with the students were kind, helpful and respectful, and student work being corrected collectively. Current teacher interaction as a source of dissonance were lack of clear instruction and explanation. The variety of interaction within these accounts cannot be differentiated through the terms of familiar or unfamiliar practices as teacher interaction appears to be subject to the constant ideal of respect and support. The instances of resonance and dissonance are linked to teacher support and describe interaction that supports or does not support their learning as well as respectful or disrespectful actions towards the students.

These interactional practices, as with mathematical activity, can be recognized through pedagogical theories such as principles of formative assessment behind collective grading, and principles of autonomy in time allocation and communication in oral participation. The principle of autonomy I link to Deci and Ryan's description of intrinsic motivation and view it as a key pedagogical principle (Ryan & Deci, 2000). The principle of communication has been addressed by D'Ambrosio (2001) who connects it to qualitative thinking through communication received through collective

participation, thus becoming shared knowledge (D'Ambrosio, 2001, p.21). The historical foundation for collaborative group behaviour is the accumulation of these interactions.

Within the description of cultural-historical repertories we find the description of collective practices influencing and impacting one's personal repertories. The collective practices addressed in student accounts are primarily educational practices pertaining to curricular standards as well as pedagogical practices, didactics. I distinguish between these forms of collective practices through the terms structural and instructional practices. The term structural practices refer to curricular standards such as marking, exams as well as format or construction of mathematical tasks. The interactional practices, demonstrating teachers support, can also be defined as interactional norms, the way a teacher speaks and interacts with the student. Interactional norm can be viewed as collective practices, as students accounts demonstrate clear perspectives on what teacher interaction should be. Given that all six students addressed aspects of teacher interaction, it would appear to be a substantial component of mathematics as a practice. In noticing the students´ accounts, I recognized the orientation and navigation process described in differences and similarities they experienced in the transition from one situated educational experience to another. Remarks on teacher interaction reminded me of how one is heavily reliant on support when engaging in an unfamiliar task or attempting participation in an unfamiliar environment.

The students rely on the teachers to navigate the mathematical practice and resolve a state of dissonance. The instances which portray a constant state of dissonance are the ones which remain unresolved, found in the statement "you cannot do it well", reflecting the importance of teacher support in resolving mathematical dissonance. Teacher as a valued agent can be recognized in the student statement on bigger grades, demonstrating task navigation through teacher interaction. This student would "tell the teacher something she would like", an action which resulted in good grades. This illustrates the instrumental aspect of mathematics which connects potential access to success to external confirmation of learning (Skovsmose, 2005, p.1571, 1574). Perhaps this is the reason teacher interaction is so markedly noted because it links success to positive teacher interaction and displays teachers as valued participants. This brings us to ideas of achievement and students' conceptualization of mathematics along with its value.

5.3 Students conceptualization of mathematics

Conceptualization is described as the process of forming an abstract idea or concept, and a large majority of the students´ inference is connected to mathematics as construct. Throughout these accounts we can see how student participation and interaction with mathematical activity impacts the description or conceptualizations of these practices as well as mathematics as a "whole". The lived experience of these students, provided as a memory, is often a combination of what they did and why it was important. This is relevant when interrogating the ideas of different mathematical practices specifically the instances that were experienced as familiar as it is often affected by the way in which the student defines mathematics as hard or easy. Some ideas adhere to the concept that the "bigger" the grade, the harder, and less fun, the tasks. This is seen in the account which reflects on the use of props in lower grades and the reduction of props in "bigger" grades. These accounts provide a description of mathematical content demonstrated in the accounts of project-work which was "less relevant" due to the conceptualization of mathematics as the process of digit manipulation, where "hard" subjects such as calculus are more "relevant". Statements such as "I like when math is hard" demonstrate pride in their mathematical knowledge and skill. Other statements describing emotions such as fear and lack of symbolic mathematical representation are signs of dissonance and indicate disruption of previous conceptions of what math looks like.

Students also demonstrated clear ideas about why certain tools, such as props, were used, "for us to love math", with statements implying the goal of mathematics educators being to "make kids love math". One account also addressed how the length of time in a situated context affected their conceptualization as well as why certain subjects were easier, pertaining to prior knowledge. This is seen in the statement it was "more fun because I lived 8 years there". These ideas, also found in the statement "because you have to learn", are connected to the relevance of mathematics. The inference on what mathematics education is varies between situated mathematical experiences and application, ranging from structural to instructional practices, and are the basis of student conceptualization.

Within these accounts I identify the discourse of mastery in connection to its symbolic value. The accounts that describe success or failure reflect the relevance of achievement within this system as being one of higher value. The societal value of mathematics as a system impacts the inference of its relevance and status. The norms of the community, what some might more broadly call cultural norms, prescribe value to different systems. Obtaining access to these systems and demonstrating mastery in their utilization produces what Bourdieu (1979) defines as social capital (Bourdieu, 1979).

6 Implications

To conclude this investigation, I will address some of the implications of these findings for both educators and researchers before concluding through offering some unanswered questions which provide the basis for further exploration.

6.1 Classroom practice

This research describes how the conceptualization of mathematics affects the student's resonance with the mathematical activity and is tied to what one describes as mathematical activity; both the mastery of mathematics as well as the process of emotional orientation that takes place with its use. The greater variety of mathematical content and exercises evoke inferences about preference and relevance, even when pupils notice many commonalities among the many mathematical practices. Through the discussion of students´ inference, I have identified elements that may represent implicit preferential logic, what was referred to in my theoretical framework as implicit cultural logic (Reid et al. 2015, p. 368). Given the limitations of my research, I choose to describe these tendencies as preferential rather than cultural and in reflecting over the pedagogical value of these findings I recognize the resource of the term preference. The dictionary defines preference as the "power or opportunity of choosing" and "the act, fact, or principle of giving advantages to some over others" (Merriam-Webster (n.d.). These definitions describe a position, one which we as educators facilitate through our pedagogical practices. This facilitation strikes me as critical to my practice when interacting with all students, but especially those in a process of transition or with a large repertoire of resources. The students´ accounts demonstrate their ability to navigate both practically and conceptually throughout the crucial processes of transition to a new educational setting. The discipline of noticing dictates my response as an educator when encountered with these experiences; it necessitates that I facilitate learning, by making at attempt to recognize students' implicit preferential logic and emotional orientation.

These finding, I recognize as being relevant to my classroom practice, as it affects the planning and teaching of mathematics which includes the structural elements of mathematical content. They highlight the importance of mathematical resonance, whether it be mathematical activities, or curricular emphasis as the present community's practices may be experienced as less or more relevant. These finding require us as educators to be responsive to our students, they also remind us of the critical stage of transition and the importance of teacher support during the time of transition

from one situational context to another. This process is supported through instructional elements such as verbal support and interaction. These findings are beneficial to me as an educator and serve as a useful reminder of the advantages and challenges of transition, as well as my responsibility to support students during this process. These responsibilities include a greater understanding of the capabilities, preferences, and inferences of pupils as they are critical to supporting resonative practice. Masons´ (2002) work is helpful in this task, reminding us of the importance of context when noticing and reflecting on mathematical practices and providing responsive teaching and learning experiences. The task of facilitating resonative practices in diverse classrooms reminds us as educators that mathematics is described and experienced in a variety of ways, and as such we, like our students, must broaden our range and be open to facilitating wider depiction of mathematical activity.

6.2 Research practice

How could engaging with students' accounts of migration in mathematics, provide a basis to recognize cultural-historical repertoires in unfamiliar mathematical practices?

Recognizing students' cultural-historical repertoires involved noticing not only the mathematical activities, tools and content; it also required recognizing the inference and the connections made to the conceptualization of mathematics as well as the educational support. The ways in which these aspects of mathematical experiences are interconnected is demonstrated throughout the student accounts. It is important to recognize that CHR has been primarily researched in social cultural studies, and as such there is no established link between CHR as a construct and educational practices involving migration in mathematic. Due to this, the underlying assumptions of this research are predominantly reflective. They call for the recognition of the dynamic relations that are a part of each person's experience and are shaped by external and internal variables. In acknowledging that individuals' experiences cannot be researched or understood in the same manner as other scientific studies where objective results are produced, this research requires reflectivity.

The research question links to a concept from socio-cultural studies which describes how individual repertories are often rooted in collective practices. When considering repertoires as being cultural-historical, I have noted the terms regularities, activities, and practices which Rogoff (2003) uses as descriptors (Gutiérrez & Rogoff, 2003). That is why I have attempted to investigate the instances that may represent collective practice as they may indicate regularities. What I mean by collective practices are the instructional and structural practices situated within the educational system. The

students´ accounts relate primarily to school mathematics connecting mathematics to educational practices which is not surprising as classroom driven practice is often teacher driven practice. However, the instances of dissonance and resonance cannot be predetermined as they are not confined to one cultural context or another. Instead, they are situational, dependent on previous conceptualizations and current experiences. These finding are similar to the description of ethnomathematics, where identity, the world, alterity, and the local terrain, meet (Vergani cited in D'Ambrosio, 2001, p. 34). Since this research resides within the intersected territory of anthropology and mathematics, the combination of my academic background has enabled me to reflect over the meeting between the domains of mathematics education and personal experience.

The terms familiar and unfamiliar mathematical practices have assisted my investigation but have also demonstrated their limitations as "familiar" is not always experienced as relevant. In thinking about cultural-historical repertoires, Masons´ terms of resonance and dissonance aided my inquiry, enabling me to interrogate structural and instructional practices as well as their conceptual value. This investigation has demonstrated fluctuation, presenting a broader perspective of the elements impacting migration in mathematics. This awareness can be found in D'Ambrosio discussion on the fragmentation of epistemology, such as knowledge paradigms, and how it is inopportune for analysing mathematical knowledge of secondary cultures (D'Ambrosio, 2001, p. 26). That is why I am unable to produce categorical distinctions; I am only able to note some transitional patterns as well as the values of the theoretical concepts of resonance and dissonance, that assisted me in identifying the variety of interactional and conceptual forms impacting the experiences. Doing so requires a broader parameter and understanding of what makes a practice relevant and resonate. A welcome finding, however, was the report showing fluctuating and temporal patterns of dissonance. This finding should encourage researchers to continue exploring resonating cultural and mathematical patterns. The work of this thesis demonstrates the benefits of noticing, as a methodological approach, in engaging with and investigating student accounts. The affordances of the methodological approach were its broad support of wider considerations that demonstrated the limitations and possibilities of identifying terms as instruments of the investigation. Specifically, the affordance of observing how these experiences is dynamically composed and how personal repertoires are developed through the dialogical interaction of structural, educational, and personal components.

The student accounts conveyed three interrelated and dynamic aspects of situational transition in mathematics: communication about mathematic activity, communication about interactions with teachers and communication conceptualising mathematics as a discourse. These three relations

demonstrate the "connection process" on the "micro-level", by connection process I refer to the ongoing interaction, and navigation of meaning and learning that took place in the mathematics classroom (Uribe & Prediger, 2021, p. 21-22). The dynamic aspect of three elements were identified through noticing the patterns of resonance and dissonance. These terms as identifiers were informative because they were broad enough to capture the interrelated aspects of mathematics conveyed by the students, demonstrating the mathematical and contextual awareness of the participants. The identifying terms of the framework are also responsive, respective of individuals experiences. They enable us to notice what the mathematical process feels like to the participants and how they illustrate or make sense of it. The culmination of these processes and experiences composes the make-up of their personal repertories. Further research on these resonance and dissonance patterns would be intriguing, particularly to observe the patterns that emerge over time and how they are formed. This is because these relations are dynamic, as the empirical evidence shows, and distinguishing patterns of resonance and dissonance aid in observing the way an individual's cultural-historical repertoires are shaped.

6.3 Continued exploration

My appreciation of these reports is way that they present contradictions and prevent me, as a researcher, from making definitive and categorical statements about mathematics and culture. What I have noticed in these accounts is how the description of the activities, found in the narration, as well as the interpretation within the accounts´ inferences often overlap, leading to appraisal and generalized statements. The student's inference, assessment of mathematical practices, are connected to their conceptualizations of what mathematics is as well as a description of what mathematics educations should be. There seems to be an element of cross-culture collective practices, as well as collective assumptions which prescribe to the concept of what is mathematical mastery, how mastery is dictated, and the necessity of mastery. I am lead to this question due to the absence of situated knowledge within the classroom which presents as a homogenized environment.

Perhaps an approach for further research would be noticing what is there an absence of in accounts such as these. I have noticed an absence of identifiable culture since there are elements of collective practice in all three conveyed forms. If collective practices are a form of cultural-historical repertoires, we could categorize mathematics education as its own collective form, which is often so far removed from the communities that one is unable to identify mathematical practices from the daily practices of the situated communities. Perhaps what this thesis indicates is that classroom

research limits our ability to see mathematics as an individual's cultural-historical repertoire, given that classroom practices do not clearly demonstrate "home" practices. Perhaps parents of students are also unable to identify local mathematical practices unrelated to school or curricular practices. Perhaps the only thing we can document in this investigation of CHR in mathematics are they tensions displayed in the mathematics classroom. Unless we define formulaic preference as a CHR, although I argue this preference may be a result of historical systemic violence from Western nations. If we define formulaic approaches as part of a student's CHR, should we as educators respect their approach and allow them to work in a way that suits their historical or cultural preference? This would perhaps entail not subjecting them to new curriculum or enhancing the transition process to include a more gradual change in content. Given that the findings demonstrate that cultural-historical repertoires are portrayed through the many interactions, impacting elements as well as personal perception, further investigation requires noticing dynamic relations in play. It might also be valuable to investigate other contexts of mathematical practices and mathematical experiences outside of the classroom in order to identify other way in which CHR are shaped and made visible.

7 References

Alrø, H., Skovsmose, O., & Valero, P. (2005). Culture, diversity and conflict in landscapes of mathematics learning. In M. Bosch (Ed.), Proceedings of the fourth Congress of the European Society for Research in Mathematics Education (pp. 1141- 1152). Sant Feliu de Guíxols, Spain: FUNDEMI IQS, Universitat Ramon Llull.

Alrø, H., Skovsmose, O., & Duenas, P. X. V. (2007). Landscapes of learning in a multicultural mathematics classroom. In *CERME 5 European Research in Mathematics Education: Proceedings of the Fifth Congress of the European Society forResearch in Mathematics Education* (pp. 1567-1576). < Forlag uden navn>.

Bakhtin, M. M. (1981). *The dialogic imagination: Four essays*. (Ed., M. Holquist; Trans, C. Emerson & M. Holquist). Austin, TX: University of Texas Press.

Bourdieu, P. (1987). *Distinction: A social critique of the judgement of taste*. Harvard university press.

Bruner, J. (1986). Actual minds, possible worlds. Harvard University Press.

Cohen, A. B. (2009). Many forms of culture. *American psychologist*, *64*(3), 194.

D'Ambrosio, U. (2001). *Ethnomathematics*. Rotterdam

González, N., Moll, L. C., & Amanti, C. (Eds.). (2006). *Funds of knowledge: Theorizing practices in households, communities, and classrooms*. Routledge.

Gutiérrez, K. D., & Rogoff, B. (2003). Cultural ways of learning: Individual traits or repertoires of practice. *Educational researcher*, *32*(5), 19-25.

Herbel-Eisenmann, B. A. (2002). Mathematics teachers' stories: Understanding and influencing identity development. Journal of Mathematics Teacher Education, 5(1), 37-59.

Kazemi. (2014). Intentional talk: how to structure and lead productive mathematical discussions (Hintz, Red.). Stenhouse Publishers. https://bibsys-almaprimo.hosted.exlibrisgroup.com/permalink/f/ucso2h/BIBSYS_ILS71549852650002201

Mason, J. (2002). *Researching your own practice: The discipline of noticing*. Routledge.

Mason, J. Learning about noticing, by, and through, noticing. *ZDM Mathematics Education* **53**, 231–243 (2021)

Merriam-Webster.com dictionary. Retrieved March 14, 2022, from https://www.merriam-webster.com/dictionary/preference

Nyeng. (2020). *Nøkkelbegreper i forskningsmetode og vitenskapsteori*. Fagbokforl.

Reid, D. A., Simmt, E., Savard, A., Suurtamm, C., Manuel, D., Lin, T. W. J., Quigley, B., & Knipping, C. (2015). Observing observers: Using video to prompt and record reflections on teachers' pedagogies in four regions of Canada. Research in Comparative and International Education, 10(3) 367-382.

Ryan, R. M., & Deci, E. L. (2000). Intrinsic and extrinsic motivations: Classic definitions and new directions. *Contemporary educational psychology*, *25*(1), 54-67.

Tjora. (2017). Kvalitative forskningsmetoder i praksis (3. utgave). Gyldendal.

Tobin, J., Hsueh, Y., & Karasawa, M. (2009). *Preschool in three cultures revisited: China, Japan, and the United States*. University of Chicago Press.

Uribe, Á., & Prediger, S. (2021). Students' multilingual repertoires-in-use for meaning-making: Contrasting case studies in three multilingual constellations. *The Journal of Mathematical Behavior*, *62*, 100820.

Wertz, F. J. (2011). *Five ways of doing qualitative analysis: Phenomenological psychology, grounded theory, discourse analysis, narrative research, and intuitive inquiry*. Guilford Press.