

# Stochastic MPC For Optimal Operation of Hydropower Station Under Uncertainty

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**Abstract:** The operational condition at the Dalsfoss power station is complicated due to many requirements related to environmental regulations and safety constraints such as the seasonally varying water level requirement at the reservoir. However, the operation becomes more difficult due to uncertainties in the system. In this paper, at first a certainty equivalent MPC is applied to the uncertain hydro power system and it has been shown that its robustness property is poor. Secondly, to prevent the constraint violations due to the uncertainties in the system, two measures are taken. One measure is to introduce a safety margin for the constraints and further design a certainty equivalent MPC. The other measure is to implement a multi-stage MPC for robust constraint satisfaction. Two types of multi-stage MPC are considered in this paper. The first employs all of the possible 50 scenarios of the uncertainty of an input disturbance variable, and the latter generates and uses three synthetic scenarios to approximate all of the possible 50 scenarios. All of the simulation results are compared for their robust performances and computational time.

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**Keywords:** Model predictive control, Multi stage model predictive control, Stochastic analysis, uncertainty, flood management

## 1. INTRODUCTION

The Kragerø watercourse is located in Telemark, Norway. The catchment area of the watercourse is over 1200 square kilometers. There are five hydropower plants along the watercourse. The locations of the five plants are shown in Figure 1. The uppermost hydro power plant is the Dalsfoss power plant. The plant is operated by Skagerak Kraft. The plant has two flood gates and one intake to a turbine (SkagerakKraft, 2021a,b).



Fig. 1. Overview of the Kragerø watercourse. (SkagerakKraft, 2021b).

For proper operation of the power plant concession requirements are imposed by the Norwegian Water Resource and Energy Administration (NVE). Some of the requirements are related to safety while others are related to the ecosystem around the watercourse (NVE, 2021). However, it is arduous to satisfy all of the requirements due to the presence of uncertainty in the system. The prediction of how much water will flow into the reservoir (lake Tokke) from the surrounding terrain is quite uncertain. The power production is decided by Skagerak Kraft from considering aspects such as the power demand and the electricity price change in the future. The prediction of water inflow is given as 50 possible scenarios for the next 13 days. The prediction is constructed based on the weather forecast and complex hydrological models. The water inflow prediction is updated every 24 hours.

Model predictive control (MPC) is an attractive multivariable constrained optimal control approach to deal with dynamic system with multiple inputs, outputs and constraints (Morari and H. Lee, 1999; Mayne et al., 2000). In a previous work, a reference region tracking deterministic MPC was suggested for the operation of the Dalsfoss power station. This MPC was designed to let the water level at the reservoir remain within a specified region (Lie, 2014). As a further improvement, a stochastic MPC based on multi-objective optimization (MOO) was proposed (Menchacatorre et al., 2019) to consider the uncertainty on the water inflow. For the MOO based stochastic MPC, the optimal control problem (OCP) formulation was similar to the reference region tracking MPC. A new OCP, based on the maximization of the water level at the reservoir is

formulated, simulated and compared to the OCP of the reference region tracking MPC. Although a deterministic MPC based on the new OCP maintains a higher level of water at the dam, it shows that robust constraint satisfaction cannot be guaranteed for all 50 possible inflow scenarios (Jeong et al., 2021).

In this paper, further works on Dalsfoss hydropower plant have been discussed with the aim to prevent the potential violation of constraints due to the uncertainties in the system, and to systematically handle large computational time associated with stochastic MPC. For this, a certainty equivalent MPC with a safety margin on the water level constraint, and a multi-stage MPC are proposed and discussed in detail in this paper.

The paper is organized as follows: In section 2, a brief introduction of the multi-stage MPC is presented. The system model, constraints and the optimal control problem are described in section 3. In sections 4 and 5, it discusses the simulation condition and the results of the simulation for certainty equivalent MPC and multi-stage MPC. Finally, the conclusion is written in section 6.

## 2. MULTI-STAGE MODEL PREDICTIVE CONTROL

A standard formulation of a deterministic MPC does not deal with uncertainty in the system. It is because the optimization technique does not consider uncertainty (Birge, 1997; Shapiro et al., 2009).

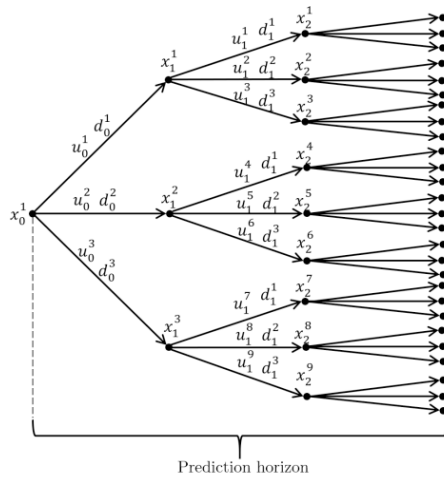


Fig. 2. Scenario tree representation of uncertainty evolution for multi-stage MPC (Lucia et al., 2013)

Multi-stage MPC includes the uncertainty of the system, as a form of scenario tree, into the optimization problem. A typical structure of a scenario tree is displayed in Fig. 2. The scenario tree describes the possible evolution of the uncertainty at each time step. It branches out at each node. Every node has expected states by branched uncertain events and control input from the events. For the Dalsfoss hydropower, the scenario tree will have 50 branches with each branch corresponding to each possible water inflow prediction. OCP for multi-stage MPC is described as follow:

$$\min_x \sum_{j=1}^S \omega_j \sum_{i=1}^N J(x_{i,j}, u_{i,j}, p_{i,j}) \quad (1a)$$

$$\text{subject to } g(x_{i,j}, u_{i,j}, p_{i,j}) \geq 0, \quad (1b)$$

$$h(x_{i,j}, u_{i,j}, p_{i,j}) = 0. \quad (1c)$$

$$\sum_{j=1}^S \bar{E}_j u_j = 0 \quad j \in 1, \dots, S \quad (1d)$$

where  $\omega$  denotes the probability or weight for each scenario, (1a) is the cost function, (1b) and (1c) are inequality and equality constraints. The last constraint, (1d), is the non-anticipativity constraint which guarantees to have the same control input for all branches arising from the same parent node (Lucia et al., 2013).

Since the future uncertainty is included in the optimization problem, the size of the OCP increases exponentially as the number of branches or scenarios increases. Therefore, when designing a scenario tree, it is critical to include the uncertainty in the future but also to have a tractable size of the OCP for feasible computational time. A good trade-off between robustness and computational cost can be achieved by constructing a simplified scenario tree that uses only the minimum, maximum and mean values of the scenarios, rather than using all possible scenarios. The simplified scenario tree will then have less number of branches but can still describe uncertainty of the system well. (Thangavel et al., 2018).

## 3. SYSTEM DESCRIPTION

### 3.1 System model

Lake Toke is located next to the Dalsfoss power station and it works as a water reservoir for the hydro power plant. The simplified layout of the lake is displayed in Fig. 3. The lake is divided into two compartments described by two different water levels ( $h_1$  and  $h_2$ ) in Fig. 3. The left side is the upper part of the lake called Merkebekk and the right side is Dalsfoss which is the lower part of the lake near to the dam and the plant. The dynamic model of the lake Toke is described in Lie (2014)

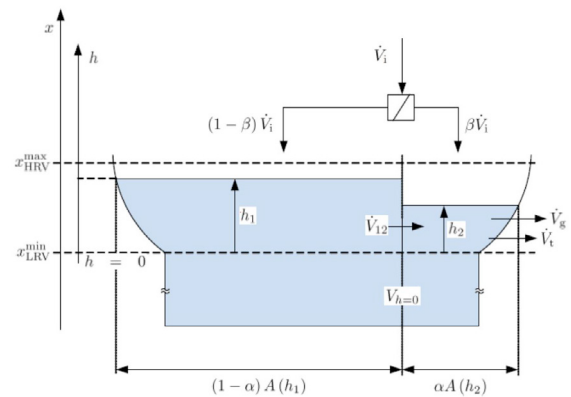


Fig. 3. Schematic of lake Toke (Lie, 2014)

The states of the systems are water levels at Merkebekk and Dalsfoss which are denoted as  $h_1$  and  $h_2$  respectively. The water levels above the sea level at Merkebekk and Dalsfoss,  $x_M$  and  $x_D$ , can be expressed as:

$$x_M = h_1 + x_{LRV}^{\min} \quad (2)$$

$$x_D = h_2 + x_{LRV}^{\min} \quad (3)$$

where  $x_{LRV}^{\min}$  means the low regulated value of the water level.

The area of the lake surface is calculated based on the water level as:

$$A(h_i) = \max(28 \times 10^6 \cdot 1.1 \cdot h_i^{\frac{1}{10}}, 10^3) \quad (4)$$

The inter compartment flow,  $\dot{V}_{12}$ , is defined based on the height difference of the water levels as follow:

$$\dot{V}_{12} = K_{12} \cdot (h_1 - h_2) \sqrt{|h_1 - h_2|} \quad (5)$$

where  $K_{12}$  is the inter compartment flow coefficient.

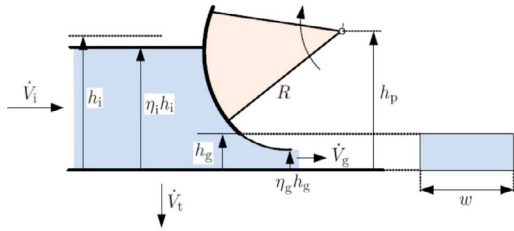


Fig. 4. Structure of floodgate (Lie, 2014)

The Dalsfoss dam has two flood gates. The height of the gate openings are denoted as  $h_{g1}$  and  $h_{g2}$ . The layout of floodgate structure is displayed in Fig. 4. The water flowrate through floodgate is expressed as:

$$\dot{V}_g = C_d w \cdot \min(h_g, h_2) \sqrt{2g \cdot \max(h_2, 0)} \quad (6)$$

where  $C_d$  means the discharge coefficient and  $g$  represents the acceleration of gravity. The width of the floodgate is denoted as  $w$ . The maximum opening height of floodgates is up to 5.6m.

The water inflow to the lake and the power production are disturbances in the system. The water inflow is denoted as  $\dot{V}_i$ . The water inflow prediction is updated everyday based on weather forecast data. The power produced is written as  $\dot{W}_e$ . The water flowrate through a turbine,  $\dot{V}_t$ , is given by:

$$\dot{V}_t = a \frac{\dot{W}_e}{x_D - x_q} + b \quad (7)$$

In (7),  $a$  and  $b$  are coefficients obtained from the data fitting method and  $x_q$  means the water level at quoy. The water level at quoy is calculated by solving the following cubic equation.

$$\begin{aligned} 0 &= c_1 x_q^3 + (c_2 - c_1 x_D) x_q^2 \\ &+ (c_3 - c_2 x_D + c_4 \dot{V}_g) x_q \\ &+ \dot{W}_e - c_3 x_D - c_4 \dot{V}_g x_D - c_5 \end{aligned} \quad (8)$$

where  $c_1, c_2, c_3, c_4$ , and  $c_5$  are obtained by coefficient from polynomial data fitting.

The total water outflow from the Dalsfoss power station,  $\dot{V}_o$ , is a summation of water flowrates through the floodgates and the turbines as follow:

$$\dot{V}_o = \dot{V}_t + \sum \dot{V}_g \quad (9)$$

The dynamic model of states,  $h_1$  and  $h_2$ , are described by the following equations:

$$\frac{dh_1}{dt} = \frac{1}{(1-\alpha)A(h_1)} ((1-\beta)\dot{V}_i - \dot{V}_{12}) \quad (10)$$

$$\frac{dh_2}{dt} = \frac{1}{\alpha A(h_2)} (\beta\dot{V}_i + \dot{V}_{12} - \dot{V}_o) \quad (11)$$

Parameters for the model are specified in Table 1.

### 3.2 Operational constraints

The operational requirements are decided by NVE. It is mandatory to satisfy those requirements for (i) operational safety, (ii) securing ecological diversity, and (iii) avoiding property damage. The constraints are:

- (1) The flowrate of the total water outflow,  $\dot{V}_o$ , should not change abruptly. This constraint ensures that people and animals downstream do not experience sudden large changes in the outflow.
- (2) The flowrate of the total water outflow,  $\dot{V}_o$ , must be kept bigger than  $4m^3/s$ . The ecosystem is not disturbed and fishes in downstream can move freely by satisfying this constraint.
- (3) The water level at Merkebekk,  $x_M$ , must be maintained within a specified water level range:

$$x_M \in [x_{LRV}, x_{HRV}]$$

where  $x_{LRV}$  and  $x_{HRV}$  mean the low and the high regulated value for the water level respectively. The seasonal level requirement changes are shown in Fig. 2.

- (4) When severe flooding occurs,  $x_{HRV}$  can be extended to  $x_{HRV}^{\max}$ . However, after the culmination of flooding ends,  $x_M$  must reach  $x_{HRV}$  as soon as possible.
- (5) The flow rate through the turbine,  $\dot{V}_t$ , is limited up to  $36m^3/s$ .

### 3.3 Optimal control problem

When handling the flood gates, care should be taken that the water from the dam is not thrown out through flood gates unnecessarily. This would result in loss of water which otherwise could be used to produce electricity. In this sense, saving as much water as possible (i.e. having as high water level as possible) in the dam while still satisfying the concession requirements becomes necessary. The objective function in the OCP is designed to maximize the water level at Merkebekk and to minimize the control action and its rate of change as: (Jeong et al., 2021)

$$\min_u \sum_{i=1}^N \omega_R R_{\text{new}}^2(x_{t+i}) + \omega_{\Delta u} \Delta u_{c,t+i-1}^2 + \omega_u u_{c,t+i-1}^2 + p^2 \omega_p \quad (12)$$

Table 1. Parameters for Lake Toke model

Parameter	Value	Unit	Comment
$\alpha$	0.05	-	Fraction of surface area in compartment 2
$\beta$	0.02	-	Fraction of inflow to compartment 2
$K_{12}$	800	$\text{m}^{\frac{3}{2}}/\text{s}$	Inter compartment flow coefficient
$C_d$	0.7	-	Discharge coefficient, Dalsfoss gate
$w_1$	11.6	m	Width of Dalsfoss gate 1
$w_2$	11.0	m	Width of Dalsfoss gate 2
$x_{\text{LRV}}^{\text{min}}$	55.75	m	Minimal low regulated level value
$x_{\text{HRV}}^{\text{max}}$	60.35	m	Maximal high regulated level value
$g$	9.81	$\text{m}/\text{s}^2$	Acceleration of gravity
$a$	124.69	$\text{Pa}^{-1}$	Coefficient in equation 7
$b$	3.161	m	Coefficient in equation 7
$c_1$	0.13152	$\text{W}/\text{m}^{-3}$	Polynomial coefficient in equation 8
$c_2$	-9.5241	$\text{W}/\text{m}^2$	Polynomial coefficient in equation 8
$c_3$	$1.7234 \cdot 10^2$	$\text{W}/\text{m}$	Polynomial coefficient in equation 8
$c_4$	$-7.7045 \cdot 10^{-3}$	$\text{Pa}/\text{m}$	Polynomial coefficient in equation 8
$c_5$	$-8.7359 \cdot 10^{-1}$	W	Polynomial coefficient in equation 8

Table 2. Seasonal level requirement

Date	$x_{\text{LRV}}$ [m]	$x_{\text{HRV}}$ [m]
Jan. 1 - Apr. 30	55.75	60.35
May. 1 - Aug. 30	58.85	59.85
Sept. 1 - Sept. 14	55.75	59.35
Oct. 28 - Nov. 11	55.75	59.85
Nov. 12 - Dec. 31	55.75	60.35

The first term in (12) is to maximize the water level at Merkebekk and expressed as:

$$R_{\text{new}}(x_{t+1}) = x_{\text{M},t+1} - x_{\text{HRV}} \quad (13)$$

The last term,  $p^2\omega_p$ , is the penalty for violation of level constraints. The variable  $p$  is the slack variable which is automatically decided by the optimizer. It allows the violation on  $x_{\text{LRV}}$  to satisfy the minimum flowrate constraints.

#### 4. SIMULATION

##### 4.1 Simulation setup

To implement the simulations, the two disturbances, the power production and the water inflow to the lake Toke, must be defined.

The synthetic data of the power production plan used for the simulation is described on Fig. 5. In this paper it is assumed that the power production is perfectly known.

For the water inflow, the real data of the water inflow prediction stored by the plant operator as historical data is utilized. The water inflow prediction data is updated every 24 hours. The prediction consists of an averaged water inflow on each day for the next 13 days and there are 50 such prediction scenarios. Since the prediction does not reflect hourly changes in water inflow, the water inflow is assumed to be constant during each day. The real historical data is multiplied by 3 to simulate the flooding situation.

The period of the simulation is set from April 15 to May 15. This period involves the drastic change of the water level constraints at Merkebekk. The step size is set as 1 hour. The length of the prediction horizon is 3 days i.e. 72 hours. The weighting parameters for OCP are determined from trial and error and are stated in Table 3. For the solution of

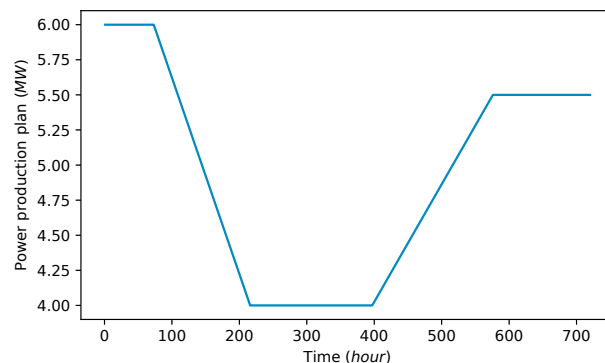


Fig. 5. Synthetic plan for power production during the simulation period

Table 3. Parameters for the simulations

Parameter	Value	Unit
$\omega_R$	10	-
$\omega_{\Delta u}$	1	-
$\omega_u$	1	-
$\omega_p$	100	-

multi-stage MPC, multiple shooting method and IPOPT solver in CasADi are used (Andersson et al., 2019).

##### 4.2 Robustness analysis

Robustness analysis is a tool that shows how the controller performs due to the influence of uncertainty. It is performed to show the number of potential violation of the constraints when the realized disturbance is different from the one that was used for prediction. The procedure of the robustness analysis is displayed in Fig. 6. When the optimal control input is calculated by the optimizer, it is applied not only to the nominal system but also to imaginary systems each having different realization of the input disturbance. However, the state is not updated through the imaginary systems but only stored for analysis.

##### 4.3 Control strategies

In this subsection, MPC formulations which can handle the effect of future uncertainty is described. In this paper, 3 types of formulations have been studied as follow:

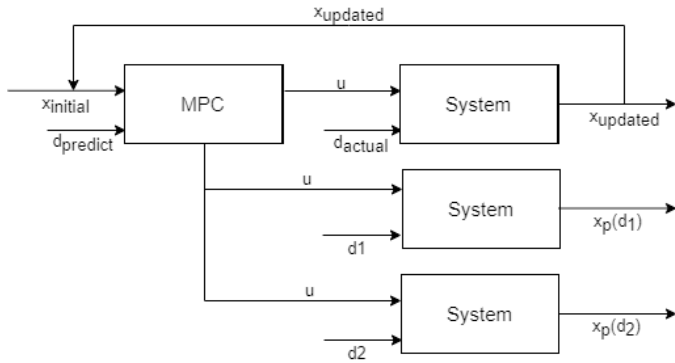


Fig. 6. The procedure of robustness analysis

- **Certainty-equivalent MPC with safety margin:**

With this method, a virtual upper bound for the water level is created by subtracting a small value (safety margin) from the actual upper bound. This virtual upper bound of the water level is then used to formulate a certainty-equivalent MPC. The safety margin decreases the maximum allowed water level at the reservoir by a small value. The idea is that any fluctuations of the water level due to uncertainty will remain within the safety margin and will never go above the actual upper bound. In this study, the value of the safety margin is set as 5cm.

- **Multi-stage MPC using all the 50 scenarios:**

The weight on each scenario is set as 1 for the implementation of the multi-stage MPC because every scenario has an equal probability of occurrence. However, using all 50 prediction scenarios increases the size of the OCP by 50 times, which increases the complexity of the problem and will therefore also require significantly larger computational time.

- **Multi-stage MPC using 3 synthetic scenarios:**

To have the balance between the robustness and the computational demand, only three synthetic scenarios are generated from the all of the 50 possible scenarios by extracting maximum, minimum and mean values along the prediction horizon. Therefore, these 3 synthetic scenarios cover the whole range of the 50 possible scenarios.

## 5. RESULT

Fig. 7 shows the results of the robustness analysis when the certainty-equivalent MPC is used. There are 1287 cases of total possible violations counted by robustness analysis throughout the whole simulation period. Although certainty equivalent MPC maintains the water to the maximum possible level, it is shown that there is a danger that the constraints may be violated in the future. However, there were zero constraint violation counted by robustness analysis when the two multi-stage MPC techniques, and certainty equivalent MPC with safety margin are employed.

The level changes and the control input throughout the simulation period are displayed in Fig. 8 and Fig. 9. CE-MPC means the certainty equivalent MPC and CE-MPC(S) stands for the certainty equivalent MPC with a safety margin. MS-MPC shows the water level changes when multi-stage MPC with all of 50 possible scenarios

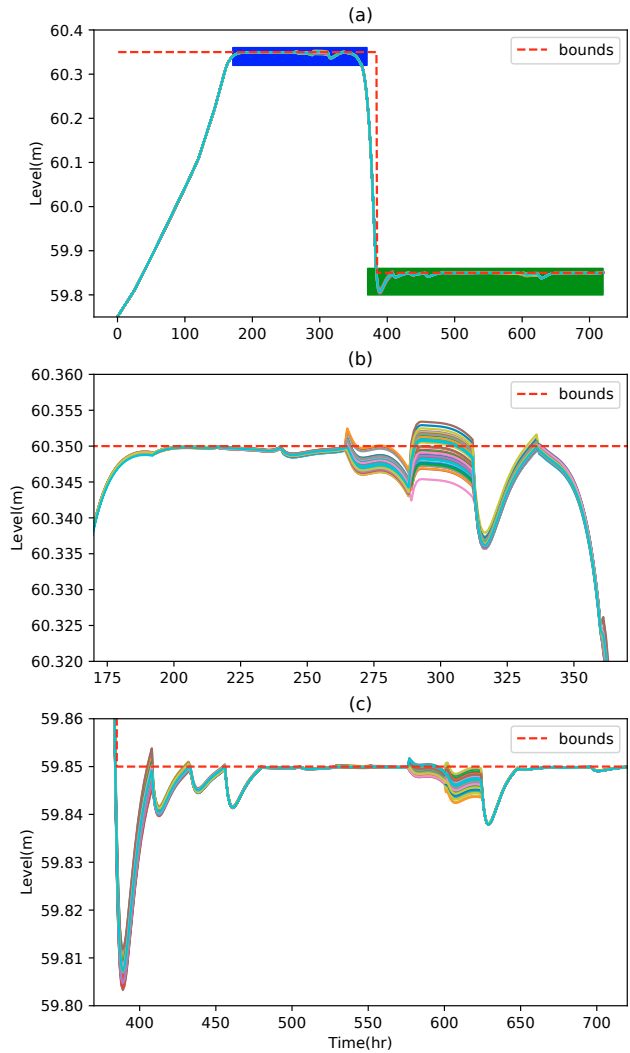


Fig. 7. The robustness analysis of certainty equivalent MPC: (a) robustness analysis through the whole simulation period, (b) detailed image of robustness analysis (blue color area in (a)), (c) detailed image of robustness analysis (green color area in (a))

are used. MS-MPC(R) shows the simulation result, using multi-stage MPC with only the three synthetic scenarios. Certainty equivalent MPC(CE-MPC) pursues the most maximized water level, but the robustness is proven to be vulnerable as shown in Fig. 6. The performance of the certainty equivalent MPC with the safety margin(CE-MPC(S)) is the worst compared to the other approaches, i.e. it maintains the lowest water level at the dam. Multi-stage MPC with all of 50 possible scenarios(MS-MPC) shows more conservative performance compared to the certainty equivalent MPC but better performance than the certainty equivalent MPC with a safety margin. Multi-stage MPC with the three synthetic scenarios(MS-MPC(R)) has similar (only negligible performance loss) compared with the MS-MPC. MS-MPC and MS-MPC(R) shows less drastic changes on control input through simulation period.

Table 4 is the summary of the computational time required to complete simulations. When CE-MPC is used for the simulation, it takes only 163s(avg. iteration time: 227ms).

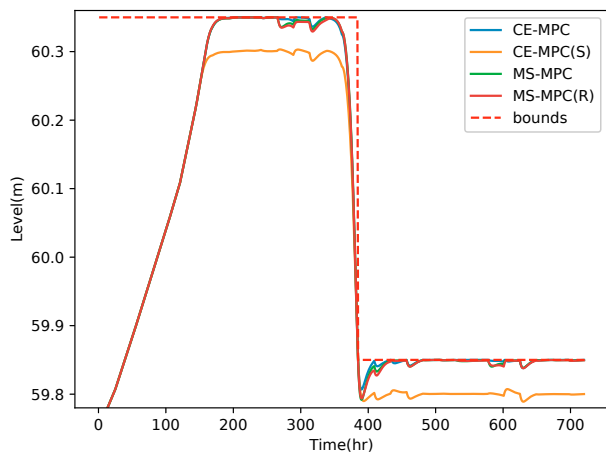


Fig. 8. The level changes throughout the simulation period

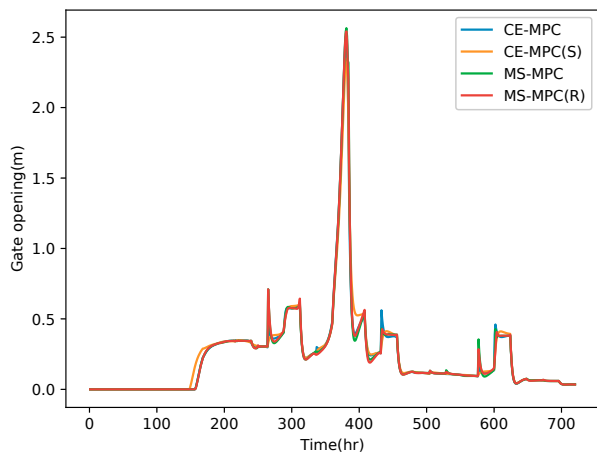


Fig. 9. The control input throughout the simulation period

Table 4. Computation time for simulations[s]

CE-MPC	MS-MPC	MS-MPC(R)
163	2977	257

However, it takes 18.2 times more, 2977s (avg. iteration time: 4135ms), when the MS-MPC is applied. The computational time decreases significantly to 257s (avg. iteration time: 358ms) when MS-MPC(R) is used.

## 6. CONCLUSION

Although with the certainty-equivalent MPC, constraint violations can occur, this paper presents three approaches to eliminate the influence of future uncertainty. One is to add a conservative safety margin on the certainty equivalent MPC. The other two is to employ multi-stage stochastic MPC. From the simulation results, all of approaches perform well. However, multi-stage MPC shows better performance compared to certainty equivalent MPC with safety margin. Therefore, the implementation of multi-stage MPC on the Dalsfoss hydropower station seems beneficial because it gives strong robustness while it keeps

the water level as maximum as possible. Also, The computation time for implementing multi-stage MPC can be significantly reduced without performance loss by selecting only the three synthetic scenarios instead of using all of the 50 scenarios. The performance is not significantly degraded when the three synthetic scenarios are used with the multi-stage MPC.

Although multi-stage MPC shows advantages compared to the other approaches, there is still more uncertainty to consider. Firstly, there may be a case that the actual water inflow might be out of the considered uncertainty range. Secondly, the water inflow may change every hour or every minute unlike how it is assumed in this paper. It may be necessary to investigate how to implement the multi-stage MPC under these conditions.

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