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Control Relevant model of Synchronous Generator Attached to Turbine and Grid

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Summary:

A synchronous generator is one of the important units in the hydropower plant that plays a key role in the production of electricity. Much effort has been done for designing more accurate models of synchronous machines over the years. Here, a possible extension of the existing description of the synchronous generator model presented in Lie (2019) was carried out. First, a complete mathematical model of a synchronous generator is presented, and thereafter the model is connected to the waterway system including turbine and grid. Different behavior of the models was observed through simulation by using the open-source program OpenModelica.

The dq0 and phase space model of a synchronous generator was compared and the dq0 model was found to be stable than phase space. When large mechanical power was applied, synchronism was lost, and it was discovered that field voltage could be increased to regain synchronism. The frictional loss was also introduced to the model, and the results were found to be as expected.

Preface

This report, as a Master’s Thesis report, was undertaken as a part of the Electrical Power Engineering course FMH606 at the University of South-Eastern Norway (USN), under the supervision of Professor Bernt Lie, and co-supervisor Madhusudhan Pandey. This thesis contains the research work on the development of a synchronous generator model attached to turbine and grid. Due to time constraints, the saturation effect and the controller model were not included in this study.

Without Prof. Bernt Lie's constant supervision, my thesis work would not have been successful. I thank him from the bottom of my heart for his support. I would also like to express my gratitude to Madhusudhan Pandey, my co-supervisor. I would also like to thank Professor Thomas Øyvang and Dietmar Winkler for clarifying some of my concerns. My gratitude goes out to my family and friends for their encouragement and support throughout my studies during this difficult period of the COVID-19 situation.

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List of symbols

θ_e	Electric angle [rad]
θ_m	Mechanical angle [rad]
δ	Load angle [rad]
ω_e	Electric angular velocity [rad/s]
ω_m	Mechanical angular velocity [rad/s]
ψ_a, ψ_b, ψ_c	Stator flux linkage [V.s]
ψ_f	Field circuit flux linkage [V.s]
ψ_d, ψ_q, ψ_0	dq-axis flux linkage [V.s]
i_d, i_q	dq-axis stator currents [A]
J_a	Aggregate moment of inertia [kg.m^2]
$L_{aa} L_{bb} L_{cc} L_{ff}$	Stator self-inductances [H]
$L_{ab} L_{bc} L_{ac}$	Mutual inductances between stator windings [H]
L_{af}, L_{bf}, L_{cf}	Mutual inductances between stator and rotor windings [H]
L_{sl}	Leakage Inductance [H]

1 Introduction

Modeling synchronous machines has always been difficult, and much effort has gone into developing more accurate models of synchronous machines over the years. The bulk of the world's electricity is produced by synchronous generators, which are preferred by large power plants. The synchronous generator can both generate and consume reactive power, while the induction generator can only consume reactive power[1].

1.1 Background

Michael Faraday, who invented the principle of operation in 1831-1832, built the first electromagnetic generator[2]. The Faraday Disk generator generated a low voltage DC and was extremely inefficient. The first generator used in industry, the dynamo, was made in 1844. It used electromagnetic induction to generate direct current with commutator [2]. Later, the AC alternator came on the market, first with two phases, later with three phases. Today, the synchronous generator is by far the most widely used machine for generating electricity in the world.

Because of the inherent nonlinearities, rotating electrical machines, electric motors, and generators are a basic class of devices in electrical engineering that pose a challenge in control engineering applications. The simulation of synchronous generators for control purposes has gotten a lot of attention because of their specific characteristics and practical importance in power plants. There are numerous papers that explain modeling and use the built models for the design of different controllers, in addition to the basic textbooks that construct general dynamic models for synchronous generators [1].
.
The models of synchronous generators in [3], [4] are linear models that do not consider the mechanical equation of motion. This indicates that they do not describe the rotor position or the loading angle. However, the loading angle is an important system variable, since the synchronous generator can fall out of synchronism if the loading angle exceeds 90 degrees. The control of induction motors has been thoroughly covered in a number of excellent books. The modeling, stability, and control of the synchronous generator is a less researched area than the induction motor because of its limited use[1].

This is evident since a synchronous generator powered by a turbine is used in the final stage of power production in a hydroelectric power plant. This paper is about the modeling and control of synchronous generators found in hydroelectric power plants.

1.2 Scope and Objectives

Model libraries, such as those that exist in Modelica, are not directly useful for controller design. The goal of this work is to develop the models independently to provide opportunities for further studies on model simplification, estimation, controller design, etc.
The task has been outlined in the description as described in Appendix A, and are found below:

1. Give an overview of the elements/units in hydropower production and describe simple models of the waterway and the grid, e.g., an infinity bus.

2. Give a brief description of models of synchronous generators in the time domain, both in the abc and the dq0 coordinates. Extend existing description (Lie, 2019) with:
    a. A proper description of flux leakage,
    b. A description of how to find physical model parameters (resistances, inductances) from generator from the manufacturer,
    c. A model of the torque from the generator,
    d. Possibly with a description of magnetic saturation,
    e. Possibly with a description of the generator magnetization system.

3. Simulate the generator (i) with a fixed load angle and grid model, and (ii) as an integrated system with water way model including turbine (here, the load angle is given by the interaction between the turbine and the generator).

4. Discuss possibilities in OpenModelica with OMJulia/Julia to automate the finding of a linear model approximation that can be used for control. Design a controller (PI, MPC, etc.), and test the solution.

1.3 Software Requirement

For implementing mathematical models and many other tasks, open source program, OpenModelica is used in this thesis. Based on the Modelica modeling language, OpenModelica is a free and open source environment for modeling, simulating, refining, and analyzing complex dynamic systems [5].

1.4 Outline of Report

There are seven chapters in this thesis work. The first chapter contains the introduction and objective for the proper understanding about the topics.

In chapter 2, an overview of the units in hydropower production is presented.

In chapter 3, detail modeling of salient pole synchronous generator including the infinity bus model is presented.

In chapter 4, detail modeling of waterway system is presented with turbine included.

In chapter 5, simulation results and its discussion is presented. Conclusion and future work are described in Chapter 6 and 7 respectively. Appendix A contains the task description that should be done and Appendix B contains the code of the integrated system.

2 Overview of the units in Hydropower Production

Hydropower is a traditional renewable energy source. It is based on the natural circulating flow of water and its gradient from the higher to the lower land surface, which are the potential. To convert this potential into applicable electrical energy, the flow of water must be directed to and drive a hydraulic turbine, which converts the water energy into mechanical energy, which in turn drives an attached generator, which converts the mechanical energy into electrical energy.

This chapter contains an overview of the various components used in the hydropower generation.

2.1 Main Elements of Hydroelectric Power Plant

A hydroelectric power plant consists of a reservoir to store water, a diversion dam, an intake structure to control and regulate the flow of water, a piping system to carry water from the intake to the water wheel, turbines coupled to generators, the intake pipe to carry water from the water wheel to the tailrace, the tailrace, and a powerhouse, which contains the turbines, generators, accessories, and other miscellaneous items[6].

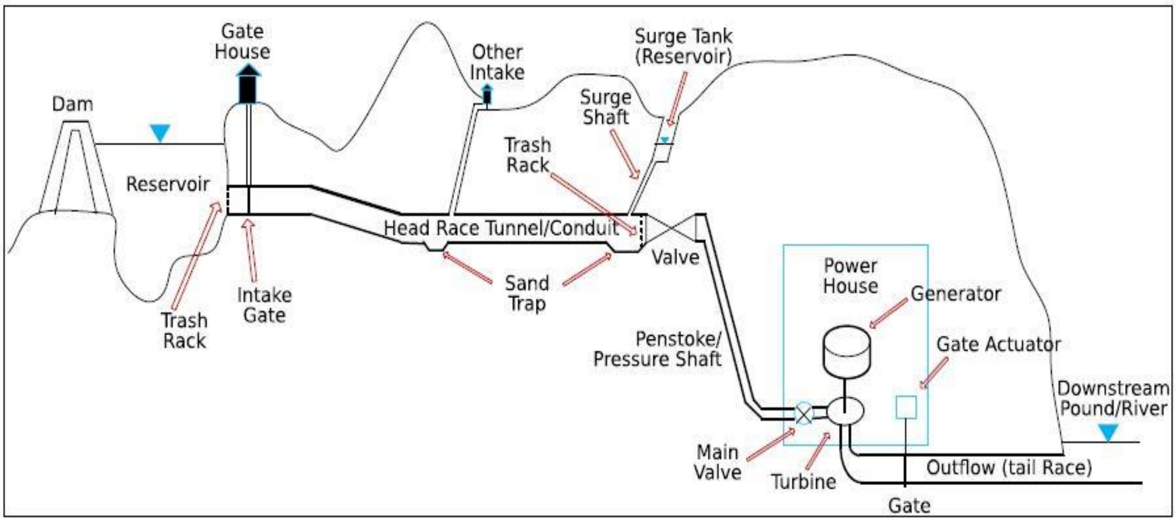

Figure 2.1 Typical high-head hydropower system[7]

2.1.1 Storage Reservoir

A hydroelectric power station's most important prerequisite is a reservoir. Its function is to store water during periods of high flow (i.e. rainy season) and to supply it during periods of low flow (i.e. dry season) (i.e. dry season). As a result, it assists in providing water to the turbines in accordance with the power plant's load. [6] .

A reservoir can be both natural and man-made. A natural reservoir is a mountain lake, while an artificial reservoir is formed by constructing a dam over a river.

2.1.2 Dam

The function of the dam is not only to raise the water surface of the stream to create an artificial head, but also to create a pond, a reservoir, or the possibility of diversion into channels. Dams may be made of concrete or stone masonry, as well as soil or rock fill. The style and structure are determined by the site's topography. [6].

2.1.3 Forebay

Forebay is a small reservoir of water at the end of the water passage from the reservoir and before the water is discharged into the penstock. It is a temporary regulating reservoir. When the load is light, it collects water and delivers the same water at high periods.[6].

There is no need for a forebay if the hydroelectric plants are situated directly at the dam's base since the reservoir serves as the forebay. If the plants are situated away from the reservoir, a forebay is provided. [6].

2.1.4 Spillway

A spillway is constructed as a safety valve. It diverts the overflow water to the downstream side when the reservoir is full, a condition that occurs primarily during periods of high water. These are usually constructed of concrete and have a water drainage opening that is shut off by metal control gates[6].

2.1.5 Surge Tank

The open pipes that carry water to the turbine do not need to be protected. When closed pipes are used, however, safety is required to prevent abnormal pressure in the pipe. For this reason, closed pipes are always equipped with a surge tank. A decrease in load demand causes an increase in the water level in the surge tank. This creates a retarding head and reduces the flow velocity of the water in the penstock. The reduction of the flow velocity to the desired level causes the water in the reservoir to fall and rise until it is dampened by friction [6].

The governor opens the turbine gates as the load on the system rises, allowing more water to circulate into the penstock to meet the additional demand, and a vacuum or negative pressure develops in the penstock. This negative pressure in the discharge line provides the necessary acceleration force and is undesirable in very long lines because of the difficulty of turbine control[6].

Also, under such conditions the additional water flows out of the surge tank As a result, the surge tank's water level drops, an acceleration head is formed, and the water flow in the penstock increases. Thus the surge tank aids in the stabilization of velocity and pressure in the penstock, as well as the reduction of water hammer and negative pressure or vacuum [8].

2.1.6 Penstock

Penstock is a closed conduit that links the forebay or surge tank with the turbine’s scroll case. Penstocks are made of reinforced concrete or steel. Long penstocks must be treated with extreme caution to prevent water hammer. The thickness must be sufficient to withstand both the usual hydrostatic pressure and the sudden surges caused by load fluctuations and by emergency condition both above and below normal [6].

2.1.7 Valves and Gates

In low-head plants, gates at the entrance to the turbine casing are needed to shut off the flow and allow for inspection and repair of the turbine during unwatering. Individual hoist-operated gates are provided in situations where periodic shutdowns are required and the time available for inspection is restricted [6].

2.1.8 Trash Racks

Trash Racks are made of long, flat bars that are placed vertically or nearly vertically in the turbine and spaced according to the minimum width of water passage. On very wide installations, the clear space between the bars will range from 25 mm to 40 mm to 150 or 200 mm. This are to avoid floating debris and other material from entering the turbine. As large diameter turbines are used, the racks are often removed, but skimmer walls or booms are usually used to prevent ice and other materials from entering the unit [6].

2.1.9 Tailrace

After having carried out its useful work in the turbine, the water is discharged to the tail race, which may lead to the same or another source. The tailrace configuration and size should be such that water has a free escape and the water jet has unimpeded passage after it exits the turbine.

2.1.10 Draft Tube

The draft tube is an airtight conduit of adequate diameter that connects to the runner outlet and transports water from the wheel to the tailrace, where it is discharged under the water surface. It safely releases water from the turbine runner to the tailrace. To decrease the kinetic loss at the outlet, it lowers the velocity of discharged water. This refers to the turbine that can be mounted above the tailrace without the water head being substantially lost. It is important to take proper account of the configuration of the draft tube. Otherwise, the draft tube may get damaged by cavitation[6].

2.1.11 Water Turbines

Water turbines are used as prime movers in hydroelectric power plants and their purpose is to transform water's kinetic energy into mechanical energy, which is further used to drive electrical energy producing alternators. Turbines can be classified as follow:

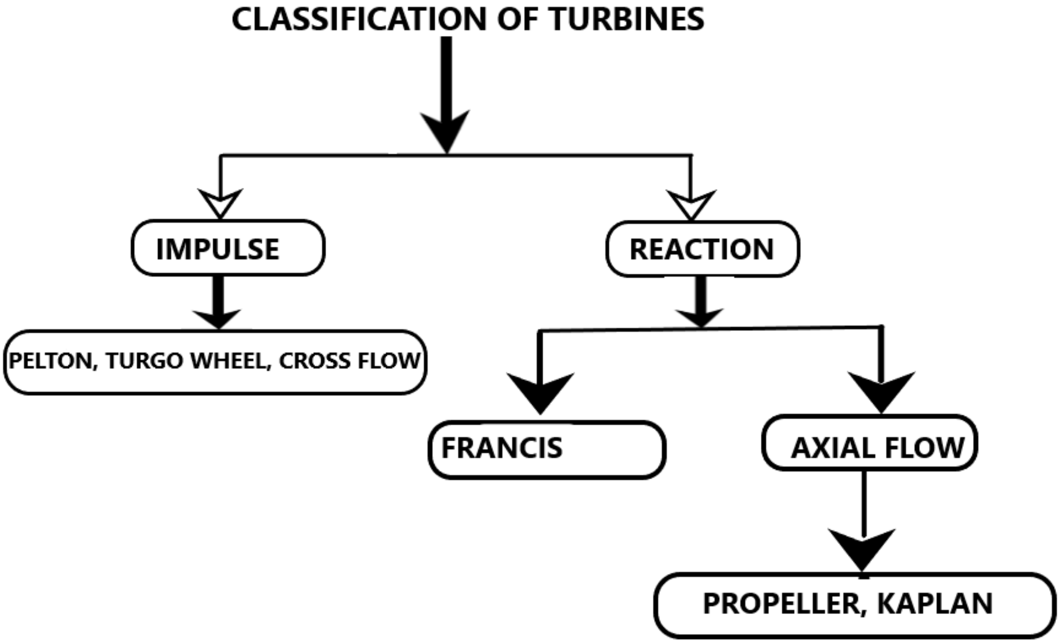

Figure 2.2 Classification of turbines

2.1.11.1 Impulse Turbines

The entire water pressure is converted into kinetic energy in a nozzle in an impulse turbine, and the speed of the jet pushes the wheel. This style of turbine is characterized by the pelton wheel which is shown in Figure 2.3 . It consists of an elliptical wheel with elliptical buckets along its periphery. The turbine is driven by the impact of the water jet entering the buckets on the wheel. A needle or spear fixed at the tip of the nozzle is used by the governor to monitor the needle's motion. If the load on the turbine reduces, the governor pushes the needle into the nozzle, reducing the amount of water striking the buckets. If the turbine's load rises, reverse action occurs [8].

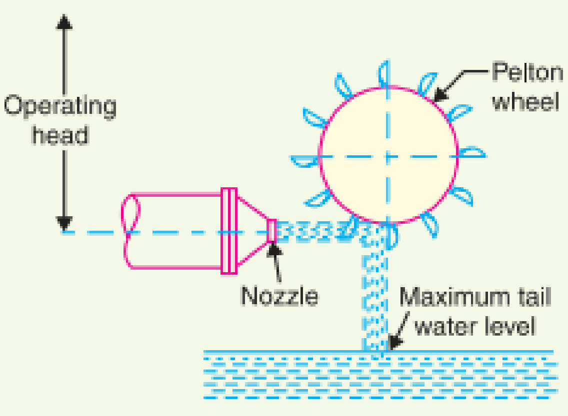

Figure 2.3 Pelton wheel [8]

2.1.11.2 Reaction Turbines

For low and medium heads, reaction turbines are used. Water meets the runner in a reaction turbine using a combination of pressure energy and velocity head. The significant types of reaction turbines are a) Francis turbines.

b) Kaplan turbines

For low to medium heads, a Francis turbine is used. It consists of an outer ring of stationary guide blades connected to the turbine casing and an inner ring of rotating blades forming the runner. The water flow to the turbine is controlled by the guide blades. When moving through the runner, water flows radially inward and shifts in a downward direction. As the water flows over the runner's "rotating blades," both water pressure and velocity are decreased, creating a reaction force that drives the turbine [8].

The Kaplan turbine is a water turbine of the propeller type, with adjustable blades and wicket gates. For low heads and large amounts of water, a Kaplan turbine is used. It is like the Francis turbine, except that water is collected axially by the Kaplan turbine runner [8].

2.1.12 Power Generation

The best way to generate electricity from a hydropower plant is to flow water from a specific head and rotate a hydraulic turbine coupled with a synchronous generator. Water's potential energy is trapped in a water pool built around a dam in a hydropower project, where gravity flow transforms it into kinetic energy. The water's kinetic energy is then transferred to a water turbine connected to an electromechanical generator, which produces electricity. The generated electricity will then be distributed to locals through a transmission line[9].

The hydraulic power (P_{hyd}), which is proportional to the flow rate of the water (Q) and the effective hydraulic head (H), determines the mechanical power (P_m) available from a hydro turbine. However, in practice, to account for turbine power losses, this is decreased by an efficiency factor (η_{turb}).

Mathematically[10]:

$$P_{\mathrm{m}} = \eta_{\mathrm{turb}} \rho g Q H = \eta_{\mathrm{turb}} P_{hyd} \tag{2.1}$$

where, P_{m} : Mechanical power [W]

ρ : Density of water [kg/m^3]

g : Acceleration due to gravity [m/s^2]

Q : Water flow rate [m^3/s]

H : Effective Head [m]

η_{turb}: Turbine efficiency

The generator is driven by the mechanical power from the turbine. The electrical power (P_{e}) generated form the generator is expressed mathematically in terms of mechanical power as follows:

$$P_e = \eta_{\mathrm{gen}} P_m \tag{2.2}$$

where, P_e : Electrical power [W]

P_m : Mechanical power [W]

η_{gen} : Generator efficiency

This is the final output power in the form of electricity. This power is then exported to the utility grid. Where η_{gen} denotes the generator efficiency, which is used to account for power losses during conversion.

2.2 Physical Description of a Synchronous Machine

Synchronous machine consists of two sets of windings[11]:

- Three phase armature winding on the stator distributed with centers 120^0 apart in space
- Field winding on the rotor supplied by DC (Direct Current)

Synchronous machine can be broadly divided into two groups as shown in Figure 2.4 and Figure 2.5. One is high speed machines with cylindrical also called non salient pole rotors and another is low speed machines with salient pole rotors.

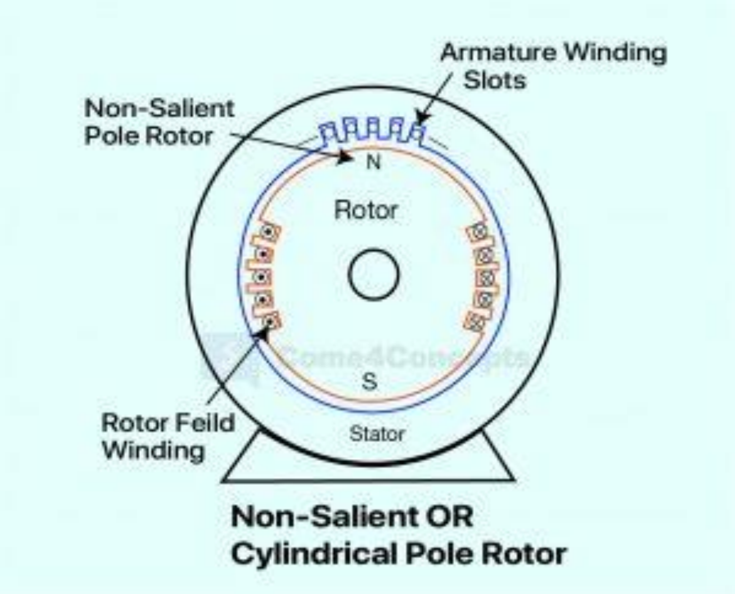

Figure 2.4 Non-Salient Pole Rotor[12]

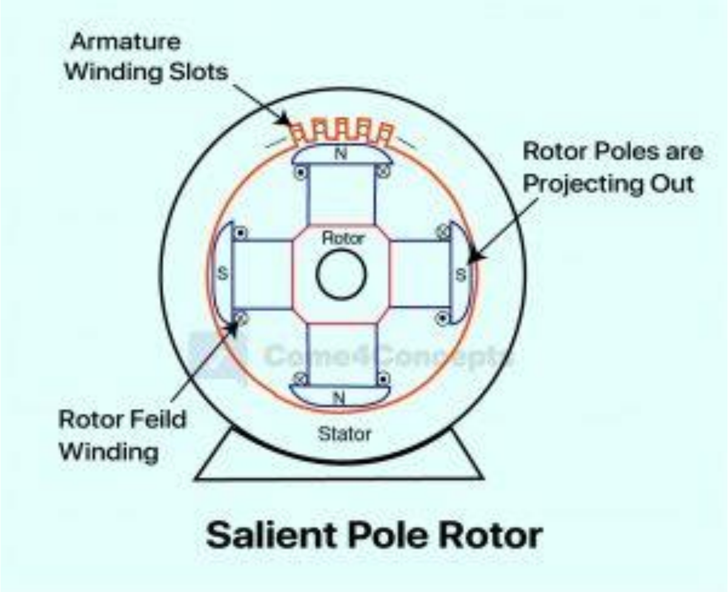

Figure 2.5 Salient Pole Rotor[12]

The mechanical and electrical parts of a generator can be explained as:

- Rotor: It is a rotating component of a generator.
- Stator: It is the stationary part that produces the electrical power.
- Field winding: Coil of wire which produces magnetic flux by an excitation system.
- Armature winding: When exposed to a varying magnetic field from the rotor field winding, this part generates an electric current.
- Excitation system: Based on the load and terminal voltage, this system provides and regulates current to the rotor windings.

By translating the potential energy of the water head into rotational energy or torque, the hydraulic turbine harvests mechanical energy. This mechanical energy is provided to generators in hydropower plants.This rotational energy is then converted to electrical energy by applying Faraday’s induction law. As a result this induces an electromagnetic voltage in the stator. This way an alternating current and electrical power are produced at the stator terminal.

3 The synchronous generator model

This chapter describes about the complete mathematical modeling of a salient pole synchronous generator.

3.1 Mathematical description of the synchronous generator

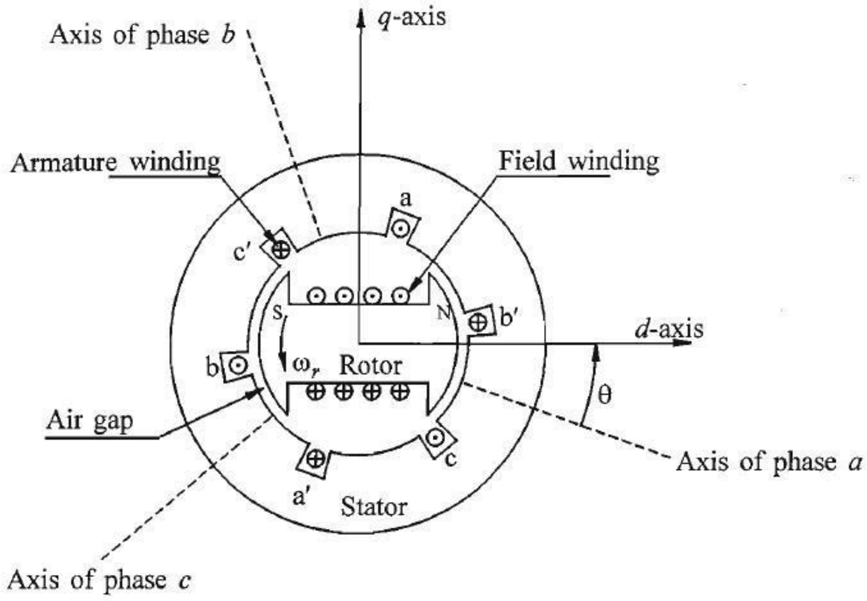

Figure 3.1 Configuration of three phase synchronous machine [7]

The schematic diagram of salient two-pole synchronous generator is shown in above figure. This has three stator windings that are distributed spatially. The currents in winding are represented by i_a, i_b and i_c. The current in the field winding of the rotor is denoted by i_f. The field winding is aligned with the axis called direct (d) axis. Also we define a quadrature axis (q) that leads the d - axis by 90^o. The angle between the d-axis and a - phase of the stator winding is denoted by θ.

The angle θ_e of the electric system and θ_m of the mechanical system are the same for a two-pole generator. The electrical angle θ_e of an N_p pole generator, on the other hand, is related to the mechanical angle θ_m of the turbine-generator as follows[13]:

$$\theta_e = \frac{N_p}{2}\theta_m$$

Similarly, the electrical angular velocity is given by,

$$\omega_e = \frac{N_p}{2}\omega_m$$

The diagram of the electric circuit that depicts the generator given in the Figure 3.2 is known as 1.0 model.

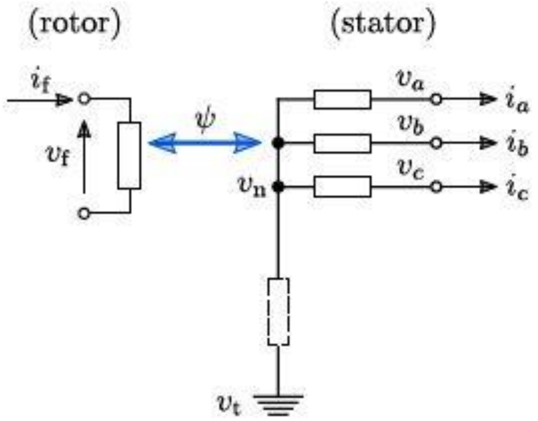

Figure 3.2 Circuit diagram; v_n is neutral voltage and v_t is ground voltage[13]

3.1.1 Model Objective

The model objective is illustrated by the functional diagram in Figure 3.3 .

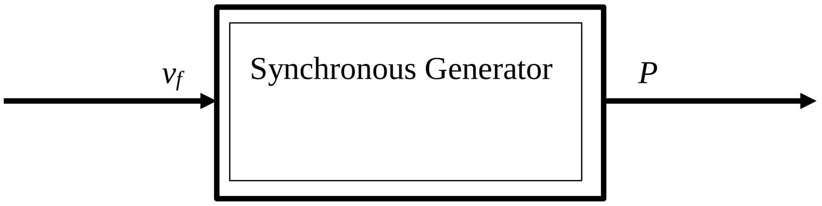

Figure 3.3 Functional diagram for model 1.0 synchronous generator

Based on the project task description and the functional diagram in above figure, a DAE model of the inputs u and the outputs y can be developed for the synchronous generator. The model input is:

$$u = (v_f) \tag{3.1}$$

where v_f is the field voltage [V].

and the model output is:

$$y = (P) \tag{3.2}$$

where P is the active electrical power [W].

3.1.2 Model development

The main assumptions that are made during the construction of the synchronous generator model are:

1. the machine consist of a symmetric tri-phase stator winding system,

2. the machine has one field coil,

3. there are no amortisseur or damper windings in the machine,

3. all the windings are magnetically coupled and

4. saturation of the machine is not included.

The model development is divided into three steps [13]:

- Step 1: Introduce the relevant balance laws.
- Step 2: Relate the quantities in the balance laws to the inputs and outputs.
- Step 3: Convert the model to a suitable format (DAE or ODE) for the computer language that will be used to describe or solve it.

Step 1: Introduce the relevant laws.

Here, we have applied the Kirchhoff's voltage laws in the stator and the rotor circuit.

The voltage across the field winding is expressed as:

$$v_f = R_f i_f + \frac{d\psi_f}{dt} \tag{3.3}$$

The armature phases are expressed as:

$$v_a - v_n = -R_a i_a - \frac{d\psi_a}{dt} \tag{3.4}$$

$$v_b - v_n = -R_b i_b - \frac{d\psi_b}{dt} \tag{3.5}$$

$$v_c - v_n = -R_c i_c - \frac{d\psi_c}{dt} \tag{3.6}$$

where $R_a = R_b = R_c = R$ is a symmetric armature. The stator currents are assumed to have a positive direction flowing out of the machine terminals. Here, we will neglect any voltage drop between the neutral point and the ground, i.e., $v_n = v_t = 0$.

Step 2: Relate the quantities in the balance laws to the inputs and outputs.

Since all magnetic fields interact, all flux linkages are reliant on all currents. The flux linkages are functions of self- and mutual inductances given by:

$$\begin{pmatrix} \psi_a \\ \psi_b \\ \psi_c \\ \psi_f \end{pmatrix} = \begin{pmatrix} L_{aa} & L_{ab} & L_{ac} & L_{af} \\ L_{ba} & L_{bb} & L_{bc} & L_{bf} \\ L_{ca} & L_{cb} & L_{cc} & L_{cf} \\ L_{fa} & L_{fb} & L_{fc} & L_{ff} \end{pmatrix} \begin{pmatrix} i_a \\ i_b \\ i_c \\ i_f \end{pmatrix} \tag{3.7}$$

Step 3: Manipulate the model into a suitable form

The terminal voltages are given by the electric angle θ_e of the grid, the phase/terminal voltages are:

$$v_a(t) = V_\phi \sin(\theta_e) = V_\phi \sin(\theta_a - \delta) \tag{3.8}$$

$$v_b(t) = V_\phi \sin(\theta_b - \delta) \tag{3.9}$$

$$v_c(t) = V_\phi \sin(\theta_c - \delta) \tag{3.10}$$

Introducing the angles for phases a, b, c of the rotor:

$$\theta_e = \omega_e t \tag{3.11}$$

$$\theta_a = \theta_e + \delta \tag{3.12}$$

$$\theta_b = \theta_a - \frac{2\pi}{3} \tag{3.13}$$

$$\theta_c = \theta_b - \frac{2\pi}{3} \tag{3.14}$$

In this model, the generator is assumed to rotate with a constant electric angular velocity ω_e, given by the nominal grid frequency, which is usually $\omega_0 = 2\pi \cdot 50$ rad/s (Europe, etc.). Similarly, phase voltages are considered to have a constant amplitude, and phase b lags 120^0 behind phase a, and phase c lags 120^0 behind phase b. So,

$$\omega_e \equiv \omega_0 \tag{3.15}$$

The voltage equation of rotor is:

$$v_f = R_f i_f + \frac{d\psi_f}{dt} \tag{3.16}$$

$$\psi_f = L_{fa} i_a + L_{fb} i_b + L_{fc} i_c + L_{ff} i_f \tag{3.17}$$

The voltage equation of stator:

$$v_a - v_n = -R_a i_a - \frac{d\psi_a}{dt} \tag{3.18}$$

$$\psi_a = L_{aa} i_a + L_{ab} i_b + L_{ac} i_c + L_{af} i_f \tag{3.19}$$

$$v_b - v_n = -R_b i_b - \frac{d\psi_b}{dt} \tag{3.20}$$

$$\psi_b = L_{ba} i_a + L_{bb} i_b + L_{bc} i_c + L_{bf} i_f \tag{3.21}$$

$$v_c - v_n = -R_c i_c - \frac{d\psi_c}{dt} \tag{3.22}$$

$$\psi_c = L_{ca} i_a + L_{cb} i_b + L_{cc} i_c + L_{cf} i_f \tag{3.23}$$

It can be seen that the self and mutual inductances for the stator circuit differ with period $2\theta e$, while the self-inductance of the field is constant and the mutual inductances between field and stator circuits vary with time θe.

The self-inductances of the stator circuit are:

$$L_{aa} = L_s + L_m \cos(2\,\theta_a) \tag{3.24}$$

$$L_{bb} = L_s + L_m \cos(2\,\theta_b) \tag{3.25}$$

$$L_{cc} = L_s + L_m \cos(2\,\theta_c) \tag{3.26}$$

The mutual stator inductances are negative as shown below:

$$L_{ab} = L_{ba} = -M_s - L_m \cos\left(2\,\theta_a + \frac{\pi}{3}\right) \tag{3.27}$$

$$L_{bc} = L_{cb} = -M_s - L_m \cos\left(2\,\theta_b + \frac{\pi}{3}\right) \tag{3.28}$$

$$L_{ac} = L_{ca} = -M_s - L_m \cos\left(2\,\theta_c + \frac{\pi}{3}\right) \tag{3.29}$$

Here, $L_m \equiv 0$ for cylindrical rotors, and $L_m > 0$ salient pole rotors.

The self-inductance for the field is constant:

$$L_{ff} = L_f \tag{3.30}$$

The mutual inductances between the rotor and the stator circuits are:

$$L_{af} = L_{fa} = M_f \cos\theta_a \tag{3.31}$$

$$L_{bf} = L_{fb} = M_f \cos\theta_b \tag{3.32}$$

$$L_{cf} = L_{fc} = M_f \cos\theta_c \tag{3.33}$$

The instantaneous power P running in the generator phases:

$$P = v_a\, i_a + v_b\, i_b + v_c\, i_c \tag{3.34}$$

The resistive power loss in the rotor is:

$$P_r = R_f i_f^2 \tag{3.35}$$

The resistive power loss in the stator is:

$$P_s = R\ (i^2_a + i^2_b + i^2_c) \tag{3.36}$$

Table 3.1 Parameters for simple 1.0 generator model: data of 555 MVA, 3-phase synchronous generator with power factor $\eta_P = 0.9$, taken from Sallam and Malik (2015).

Quantity	Value	Comment
R_f	0.0715 Ω	Field resistance
L_f	576.92 mH	Field inductance
M_f	32.653 mH	Field-phase mutual inductance
R_a, R_b, R_c	0.0031 Ω	Phase resistances
L_s	3.2759 mH	Phase self inductance constant
L_m	0.0458 mH	Phase inductance magnitude
M_s	1.6379 mH	Phase mutual inductance constant
ω_0	2π · 60 Hz	Nominal grid electric angular velocity
L_{sl}	0.4129 mH	Armature Leakage

Table 3.2 Operating conditions for simple 1.0 generator: data of 555 MVA, 3-phase synchronous generator with power factor $\eta_P = 0.9$, taken from Sallam and Malik (2015).

Quantity	Value	Comment
$i_f(0)$	1 kA	Field current initial value
$i_a(0)$	10 kA	Initial phase current, phase a
$i_b(0)$	$i_a(0) . \sin(\frac{-2\pi}{3})$	Initial phase current, phase b
$i_c(0)$	$i_a(0) . \sin(\frac{-4\pi}{3})$	Initial phase current, phase c
$v_f(t)$	400 V	Field voltage
ω_e	ω_0	Electric angulat velocity
δ	$\frac{\pi}{3}$	Load angle
V_ℓ	24 kV	RMS line to line voltage
V_ϕ	$\sqrt{\frac{2}{3}}\, V_l$	Phase voltage amplitude
$v_a(t)$	$V_\phi \sin(\omega_e t)$	Phase voltage, phase a
$v_b(t)$	$V_\phi \sin(\omega_e t - \frac{2\pi}{3})$	Phase voltage, phase a
$v_c(t)$	$V_\phi \sin(\omega_e t - \frac{4\pi}{3})$	Phase voltage, phase a

Model summary

The model can be expressed in the standard differential algebraic equation (DAE) form, which is a suitable form for the modelica computer language [13] .

$$\frac{dx}{dt} = f\ (x, z, u; \theta)$$

$$0 = g\ (x, z, u; \theta)$$

$$y = h\ (x, z, u; \theta)$$

Here,

State variables x = (i_a, i_b, i_c, i_f)

Algebraic $z = (v_a, v_b, v_c, \omega_e, \theta_e, \theta_a, \theta_b, \theta_c, L_{aa}, L_{ab}, L_{ac}, L_{af}, L_{ba}, L_{bb}, L_{bc}, L_{bf}, L_{ca}, L_{cb}, L_{cc}, L_{cf}, L_{fa}, L_{fb}, L_{fc}, L_{ff}, P, P_r, P_s, \psi_a, \psi_b, \psi_c, \psi_f,)$

Input $u = (v_f)$

Parameters $\theta = (R_f, L_f, M_f, R_a, R_b, R_c, L_s, L_m, M_s, \omega_0, \delta, V_{\phi}.)$

Output $y = (P)$

From equations (3.8) to (3.36), we have 35 equations. It follows that we have same number of unknowns and equations, and we can compute all variables.

3.2 Leakage flux model

In a magnetic circuit, leakage flux is the magnetic flux that does not obey the expected direction. This leakage flux does not add to the machine's usable flux. Magnetic flux leakage in rotating machines is the flux that connects only the stator or rotor windings. It indicates that they do not pass through the air gap. The armature winding is inductive in nature due to the leakage flux. The reactance caused by this leakage flux is called leakage reactance[14].

The concept of leakage flux can be observed in the below Figure 3.4.

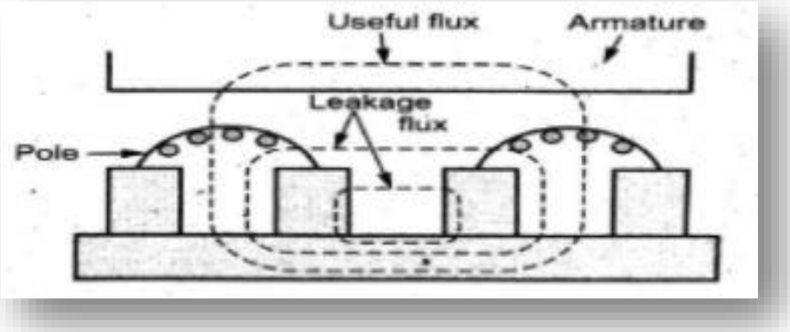

Figure 3.4 Synchronous Machine[15]

So for the complete definition of the self-inductance of phase a, the phase leakage inductance[16] L_{sl} has to be added (same for all three phases) in the model developed in 3.1.2:

$$L_{aa} = L_s + L_{sl} + L_m \cos(2\theta_a) \tag{3.37}$$

$$L_{bb} = L_s + L_{sl} + L_m \cos(2\theta_b) \tag{3.38}$$

$$L_{cc} = L_s + L_{sl} + L_m \cos(\theta_c) \tag{3.39}$$

Where, L_{sl} is the leakage inductane of the stator winding due to the armature leakage flux.

When simulating the above machine phase model which is the existing description of [13], following graphs are obtained.

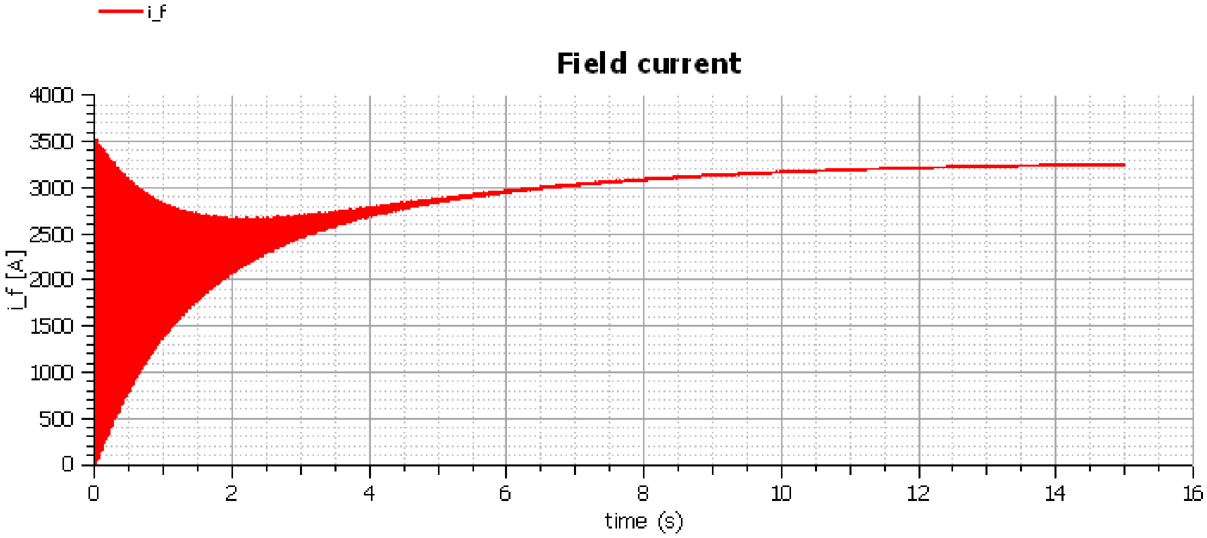

Figure 3.5 Field current

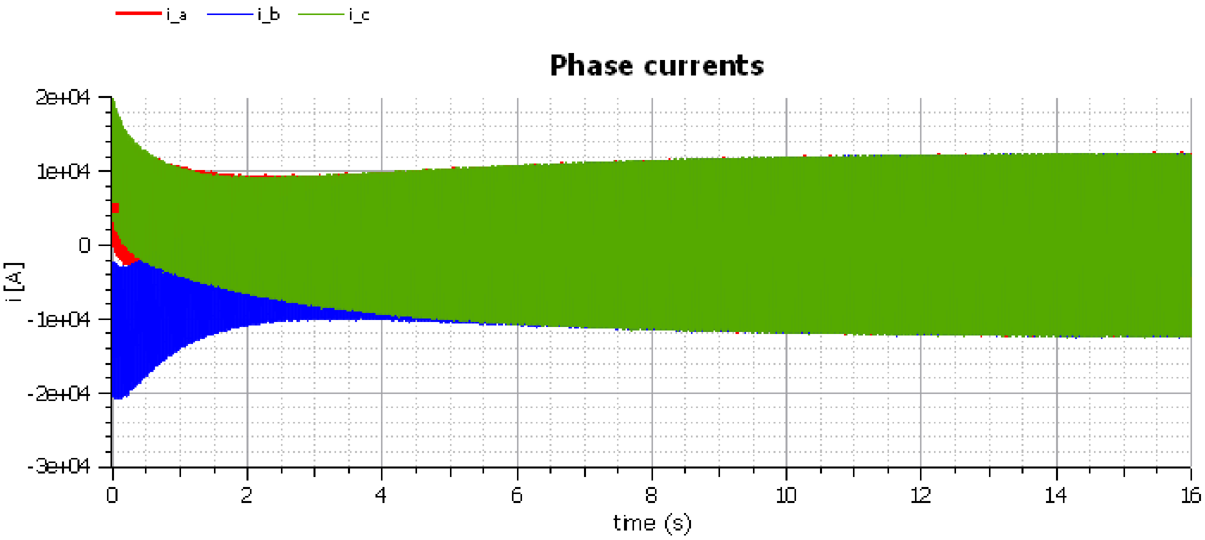

Figure 3.6 Phase currents

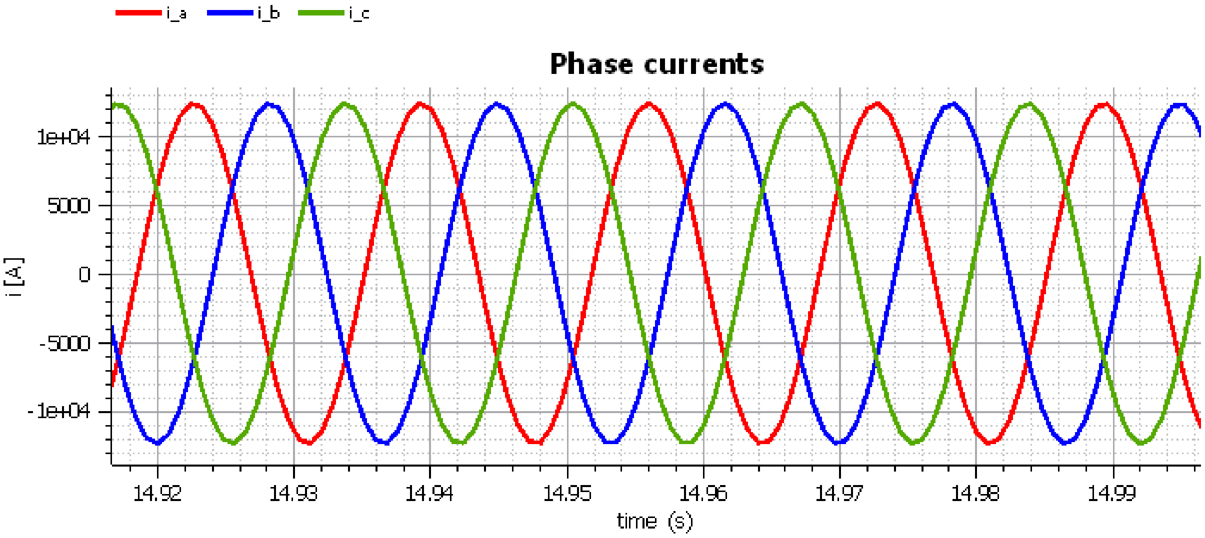

Figure 3.7 Phase currents (short term result)

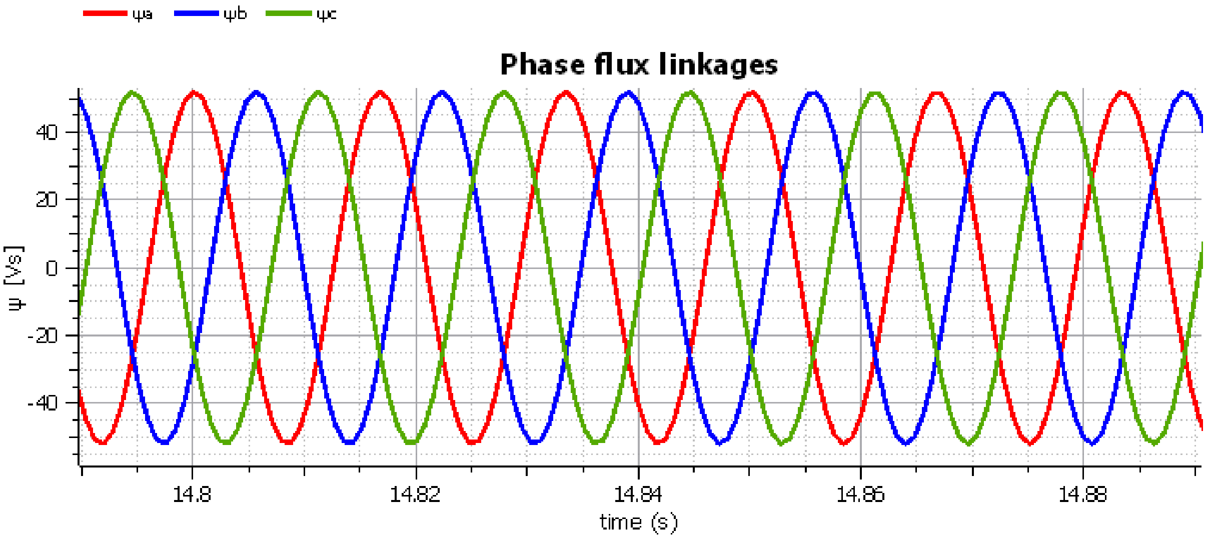

Figure 3.8 Phase flux linkages

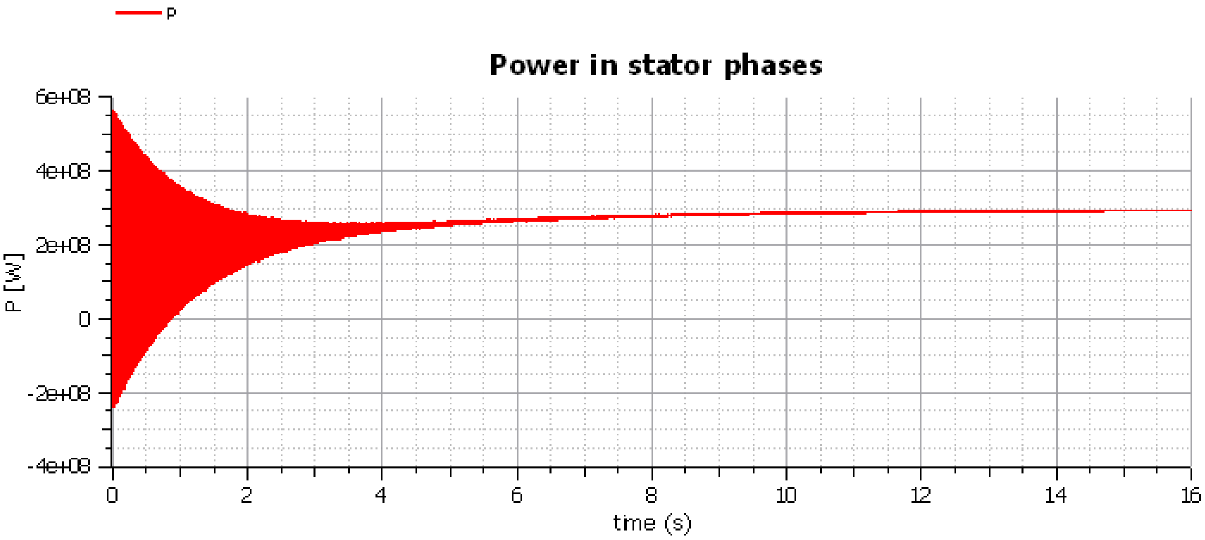

Figure 3.9 Active Power in stator phases

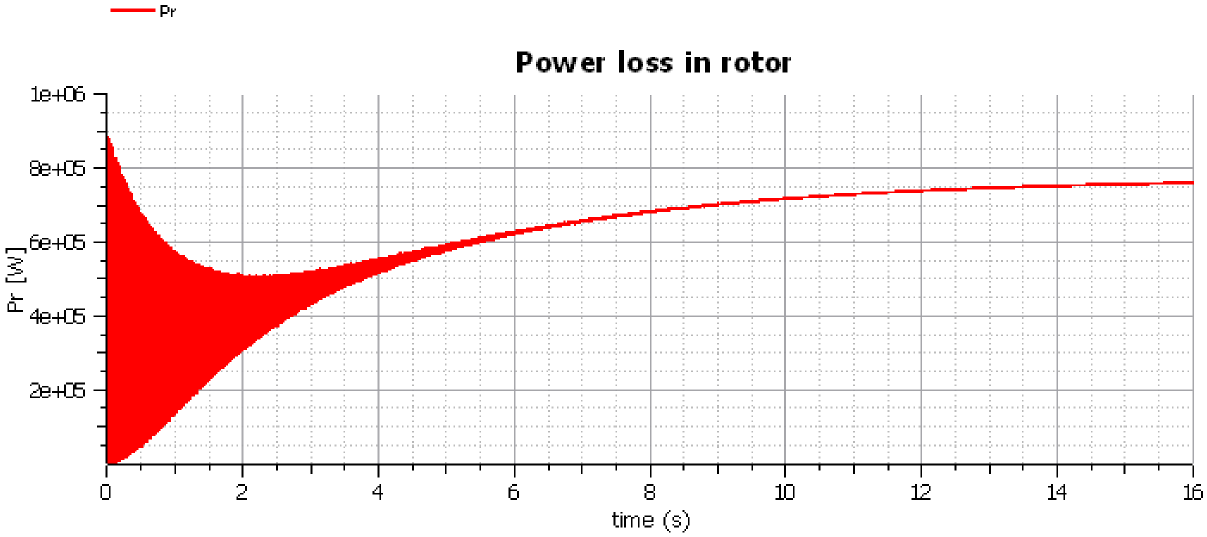

Figure 3.10 Power loss in rotor

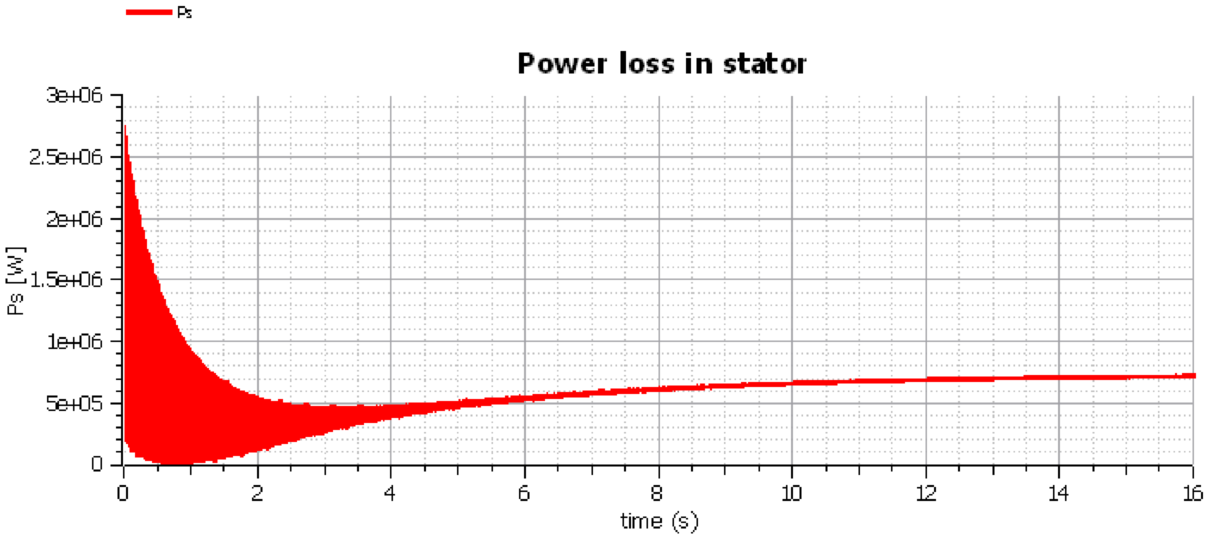

Figure 3.11 Power loss in stator

3.3 Park transformation and the dq-system

The model developed in section 3.1 is the model of the generator. However, the fundamental issues that occur are caused by variations in inductances as a function of rotor angular position. Park transformation is needed to address this issue. We can remove all rotor position dependent inductances from the calculations and simplify electric machine analysis by using this transformation. The Park transform is a mathematical transformation that is used to express rotor and stator quantities in a rotating reference frame that is aligned to the rotor's d- and q-axes. In addition to the direct and quadrature axes, a third coordinate axis, commonly referred to as the zero axis, is needed when there are three phases.

Let's call the signals in the three phases (s_a, s_b, s_c) the variables that need to be transformed to new variables (s_d, s_q, s_0).

$$\begin{pmatrix} s_a \\ s_b \\ s_c \end{pmatrix} = \sqrt{\frac{2}{3}} \begin{pmatrix} cos\theta_a & sin\theta_a & \frac{1}{\sqrt{2}} \\ cos\theta_b & sin\theta_b & \frac{1}{\sqrt{2}} \\ cos\theta_c & sin\theta_c & \frac{1}{\sqrt{2}} \end{pmatrix} . \begin{pmatrix} s_d \\ s_q \\ s_0 \end{pmatrix} \qquad (3.40)$$

In matrix notation,

$$s_{abc} = P_k . s_{dq0}$$

where

$$P_k = k \begin{pmatrix} cos\theta_a & sin\theta_a & \frac{1}{\sqrt{2}} \\ cos\theta_b & sin\theta_b & \frac{1}{\sqrt{2}} \\ cos\theta_c & sin\theta_c & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$s_{abc} = \begin{pmatrix} s_a \\ s_b \\ s_c \end{pmatrix} \text{ and } s_{dq0} = \begin{pmatrix} s_d \\ s_q \\ s_0 \end{pmatrix}$$

Coefficient $\sqrt{\frac{2}{3}}$ is chosen such that $P_k^{-1} = P_k^T$ which means Park's transformation is orthogonal.

We can compute the electric power either in phase space, or in dq0 space:

$$P = v_a\, i_a + v_b\, i_b + v_c\, i_c = \begin{pmatrix} v_a \\ v_b \\ v_c \end{pmatrix}^T \begin{pmatrix} i_a \\ i_b \\ i_c \end{pmatrix}$$

$$P = \left(P_k \begin{pmatrix} v_d \\ v_q \\ v_0 \end{pmatrix} \right)^T \left(P_k \begin{pmatrix} i_d \\ i_q \\ i_0 \end{pmatrix} \right)$$

$$= \begin{pmatrix} v_d \\ v_q \\ v_0 \end{pmatrix}^T P_k^T P_k \begin{pmatrix} i_d \\ i_q \\ i_0 \end{pmatrix}$$

$$= \begin{pmatrix} v_d \\ v_q \\ v_0 \end{pmatrix}^T \begin{pmatrix} i_d \\ i_q \\ i_0 \end{pmatrix}$$

$$P = v_d\, i_d + v_q\, i_q + v_0\, i_0 \qquad (3.41)$$

Thus power can be computed as $P = v_d\, i_d + v_q\, i_q + v_0\, i_0$ in dq0 space.

Similarly by using the above Park's transform, the stator voltage Eqs. 3.4-3.6 for the machine can be written as follows:

$$v_d = -Ri_d - \frac{d\psi_d}{dt} - \omega_e \psi_q \tag{3.42}$$

$$v_q = -Ri_q - \frac{d\psi_q}{dt} - \omega_e \psi_d \tag{3.43}$$

$$v_0 = -Ri_0 - \frac{d\psi_0}{dt} \tag{3.44}$$

The voltages in (d,q,0) coordinates are:

$$v_d = -\overline{V_l}\,\sin\delta \tag{3.45}$$

$$v_q = \overline{V_l}\cos\delta \tag{3.46}$$

$$v_0 = 0 \tag{3.47}$$

The rotor voltage equation remains the same as in Eqn. 3.16.

$$v_f = R_f i_f + \frac{d\psi f}{dt} \tag{3.48}$$

The flux linkages in dq-components are expressed as:

$$\begin{pmatrix}\psi_d\\ \psi_q\\ \psi_0\\ \psi_f\end{pmatrix} = \begin{pmatrix} L_d & 0 & 0 & \sqrt{\frac{3}{2}}\,M_f \\ 0 & L_q & 0 & 0 \\ 0 & 0 & L_0 & 0 \\ \sqrt{\frac{3}{2}}\,M_f & 0 & 0 & L_f \end{pmatrix} \cdot \begin{pmatrix} i_d\\ i_q\\ i_0\end{pmatrix} \tag{3.49}$$

where

$$L_d = L_s + M_s + \frac{3}{2} L_m$$

$$L_q = L_s + M_s - \frac{3}{2} L_m$$

$$L_0 = L_s - 2M_s$$

For the phase currents,

$$\begin{pmatrix} i_a\\ i_b\\ i_c\end{pmatrix} = P_k \,.\, \begin{pmatrix} i_d\\ i_q\\ i_0\end{pmatrix} \tag{3.50}$$

The resistive power losses in the rotor (P_r) is given as in Eq. 3.35.

The resistive power loss in the stator (Ps, dq0) can be expressed as follows using our chosen Park transform P_k:

$$P_s = R\,(i^2{}_d + i^2{}_q + i^2{}_0) \tag{3.51}$$

The above derived complete relationships are expressed in note [13].

When simulating the above dq0 model, following graphs are obtained.

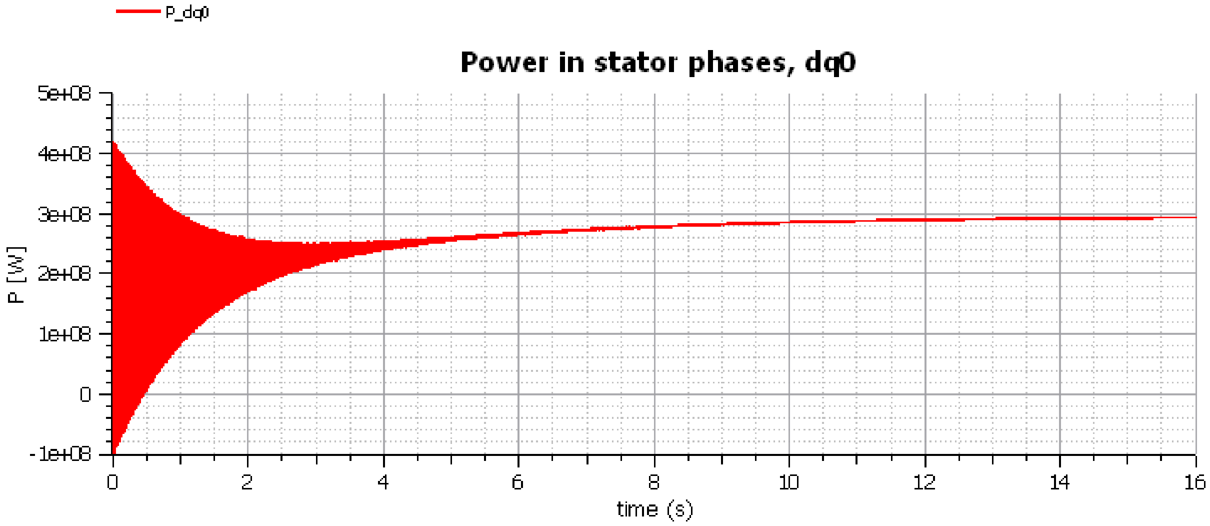

Figure 3.12 Active Electrical Power in dq0

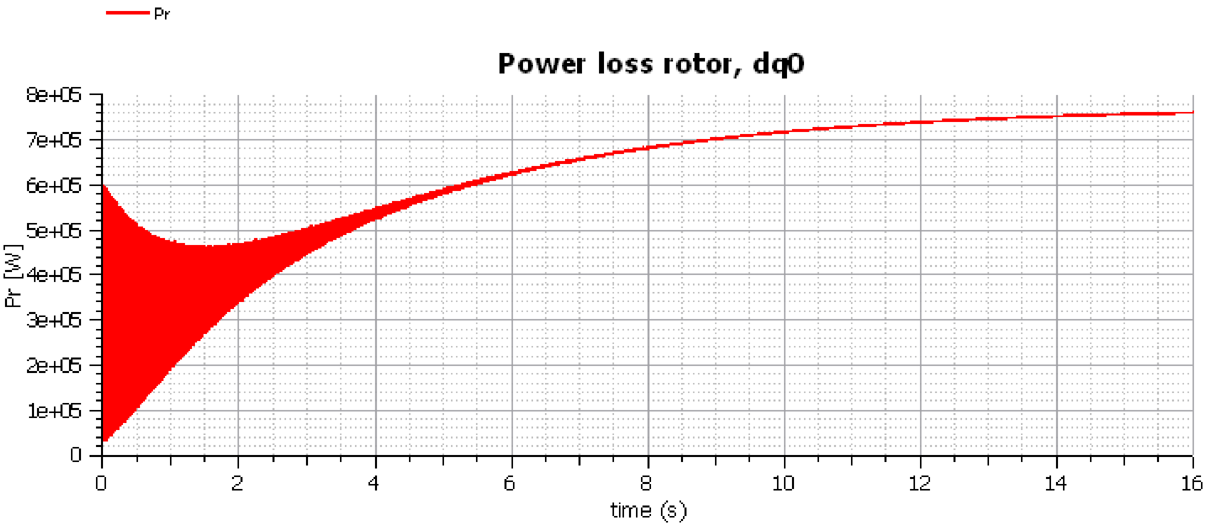

Figure 3.13 Power loss in rotor

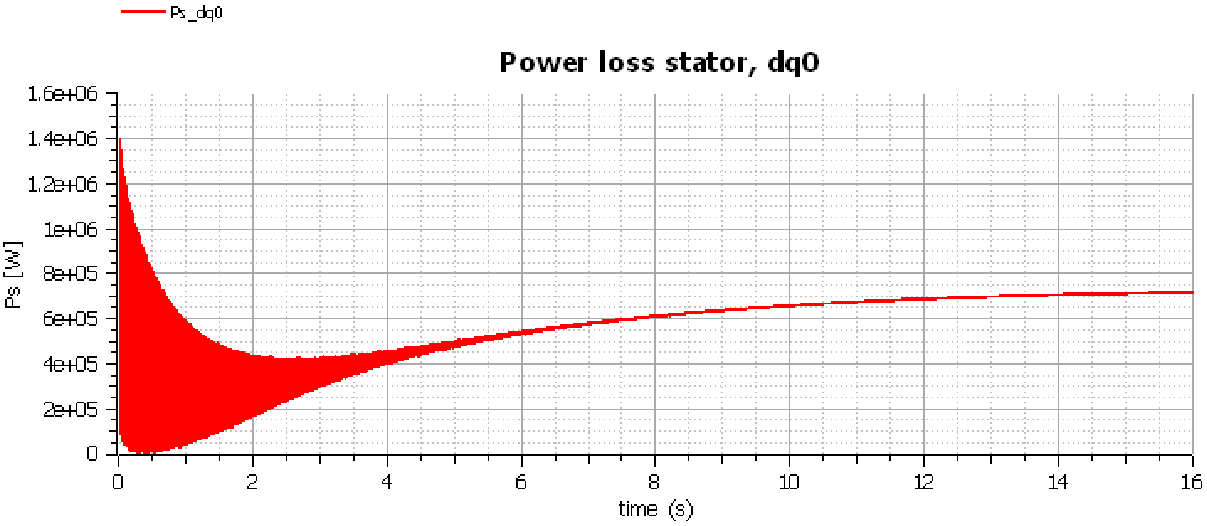

Figure 3.14 Power loss in stator

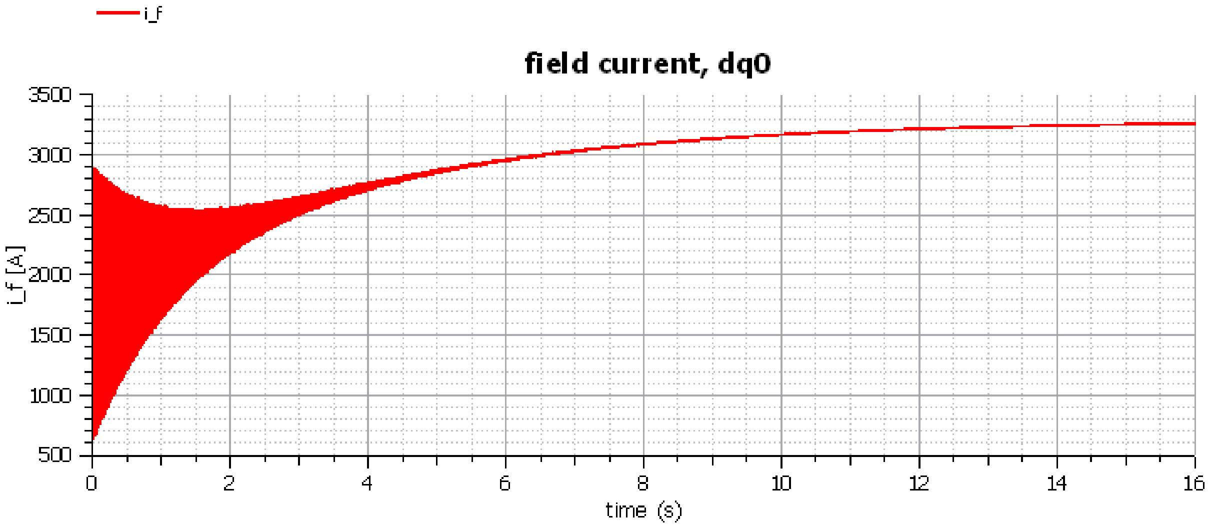

Figure 3.15 Field current

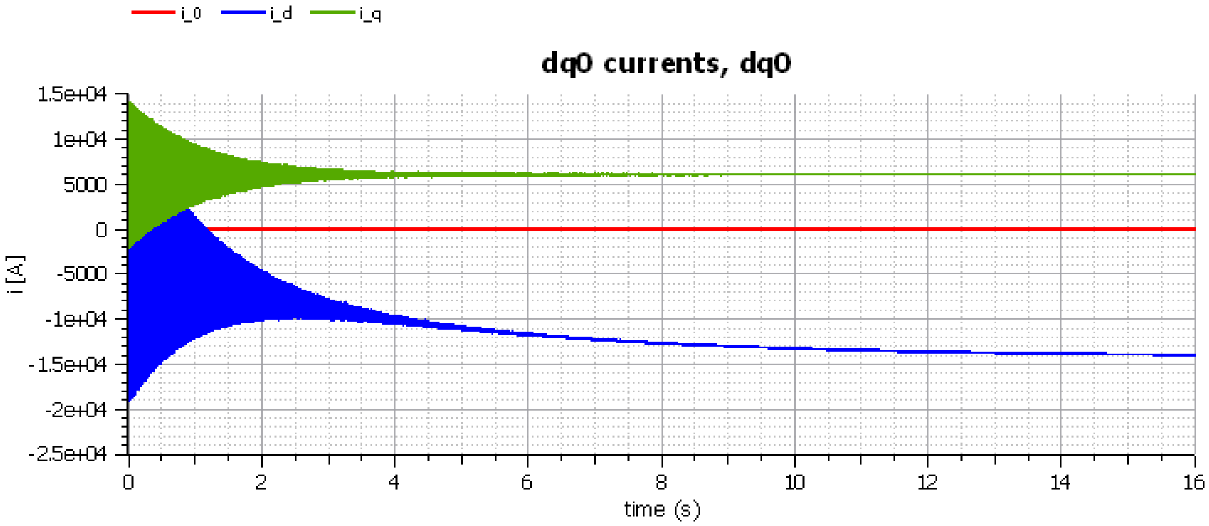

Figure 3.16 dq0 currents

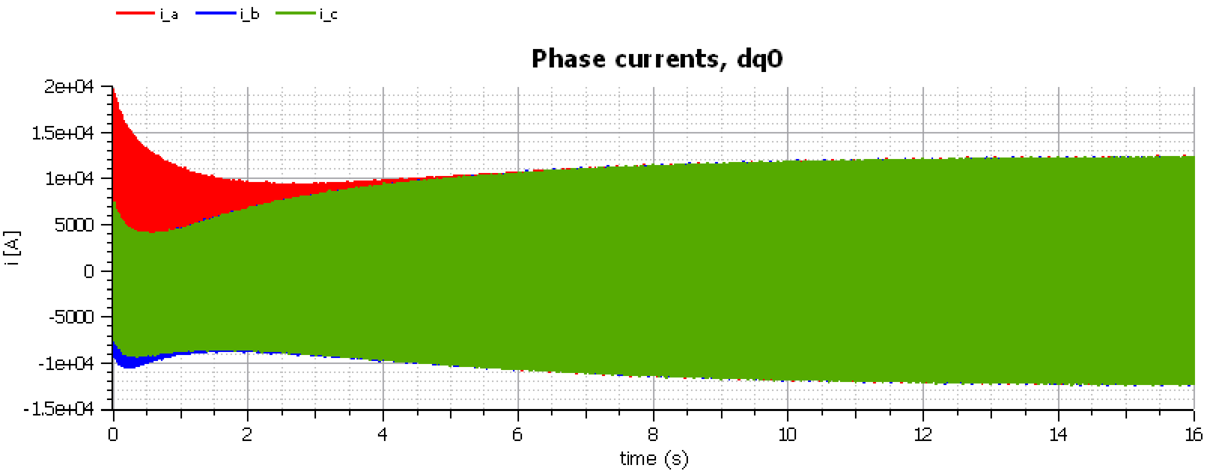

Figure 3.17 Phase currents

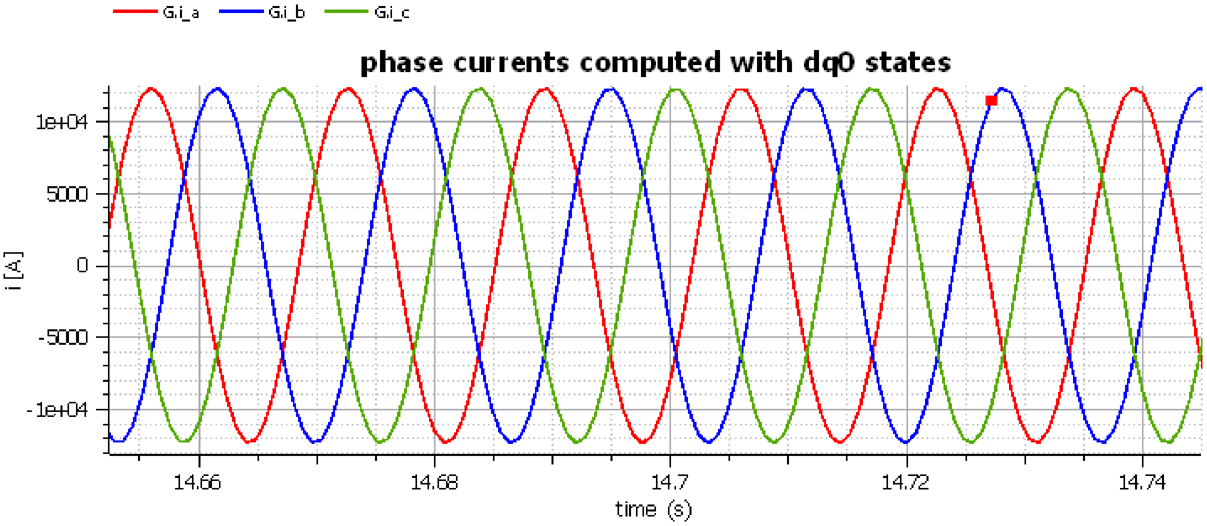

Figure 3.18 Phase currents (Short term result)

3.4 A model of the torque from the generator

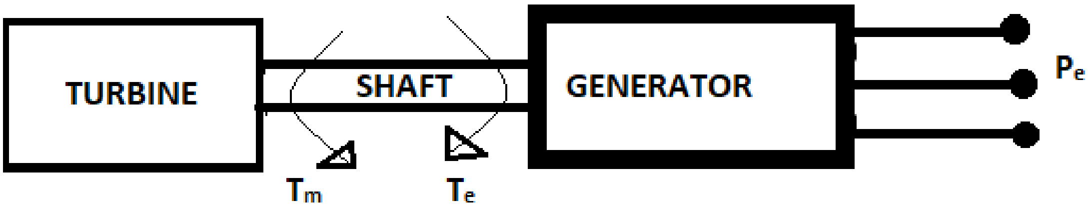

Figure 3.19 Mechanical and Electrical Torque

Let us consider that the rotational body is a shaft connecting a turbine to a generator, illustrated in Figure 3.19.

The torques on the shaft are:

From turbine: The shaft rotates due to a torque exerted by the turbine in one direction. This is a mechanical torque. This torque is referred to as T_m.

From generator: The generator generates torque in the opposite direction of the mechanical torque which retards the motion caused by the mechanical torque. This is an electromagnetic torque. This torque is referred to as T_e.

The instantaneous three-phase power output of the stator is:

$$P = v_a i_a + v_b i_b + v_c i_c$$

In terms of dq0 components, we have

$$P = v_d i_d + v_q i_q + v_0 i_0$$

In balanced condition:

$$P = v_d i_d + v_q i_q$$

Putting the equations of v_d and v_q :

$$P = (-Ri_d - \frac{d\psi_d}{dt} - \omega_e \psi_q) i_d + (-Ri_q - \frac{d\psi_q}{dt} - \omega_e \psi_d) i_q$$

$$P = (-i_d \frac{d\psi_d}{dt} - i_q \frac{d\psi_q}{dt}) + \omega_e (\psi_d i_q - \psi_q i_d) - R(i^2_d - i^2_q) \tag{3.52}$$

= (Rate of change of armature magnetic energy) + (power transferred across the air gap)-(armature resistance loss)

The air gap torque T_e is obtained by dividing the power transferred across the air gap (i.e., power corresponding to the speed voltages) by the rotor speed in mechanical radians per second[17].

$$T_e = (\psi_\mathrm{d}\, i_q - \psi_\mathrm{q}\, i_d)\,\frac{\omega_\mathrm{e}}{\omega_\mathrm{mech}}$$

$$T_e = (\psi_d\, i_q - \psi_q\, i_d)\,\frac{p_f}{2} \tag{3.53}$$

Here p_f is the field pole.

3.5 Rotor Swing Equation

When the load on the generator changes, the mechanical and electrical power become unbalanced. Where there is an imbalance between mechanical and electrical power in a synchronous generator, the generator compensates by either accelerating or decelerating the rotor speed. The following swing equation principle is used to analyze this generator process further[18].

Consider a synchronous generator with an electromagnetic torque T_e and a synchronous speed ω_s. Under steady state-operation with losses neglected we have,

$$T_m = T_e \tag{3.54}$$

Where T_m is the driving mechanical torque.

An accelerating ($T_m > T_e$) or decelerating ($T_m > T_e$) torque T_a on the rotor results from a deviation from steady state due to a disturbance.

$$T_a = T_m - T_e \tag{3.55}$$

The equation of motion of the machine rotor (neglecting frictional and damping torques) is given by:

$$J\frac{d^2\theta_m}{dt^2} = T_a = T_m - T_e \tag{3.56}$$

Here,

J : combined moment of inertia of the prime mover and generator in kgm^2

θ_m : angular position of the rotor with respect to a stationary reference axis in radian

T_m : torque supplied by the prime mover in N-m.

T_e : electrical torque output of the generator in N-m.

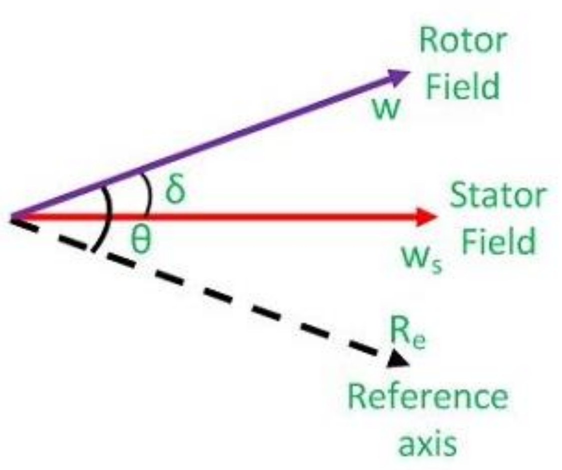

Figure 3.20 Angular position of rotor[19]

From Figure 3.20, the angular position $\theta = \theta_m$ of the rotor in a synchronous generator can be represented with respect to the synchronously rotating frame as given below:

$$\theta_m = \omega_s t + \delta_m \tag{3.57}$$

Where, ω_s is a constant representing synchronous rotational speed of the rotor in rad/s and $\delta_m$is the angular position in rad with respect to the synchronously reference frame.

By doing derivative of Eq. 3.5, the rotor angular velocity is obtained.

$$\omega_m = \frac{d\,\theta_m}{dt} = \omega_s + \frac{d\delta_m}{dt} \tag{3.58}$$

And the rotor acceleration is:

$$\frac{d^2\theta_m}{dt^2} = \frac{d^2\delta_m}{dt^2} \tag{3.59}$$

Substituting equation Eq. 3.59 into Eq. 3.56, we have

$$J\frac{d^2\delta_m}{dt^2} = T_m - T_e \tag{3.60}$$

By multiplying both sides of Eq. 3.60 with rotor angular velocity $\omega_m$gives equation of the rotor motion with respect to net mechanical power P_m and electrical power P_e in MW:

$$J\omega_m\frac{d^2\delta_m}{dt^2} = \omega_m T_m - \omega_m T_e$$

$$J\omega_m\frac{d^2\delta_m}{dt^2} = P_m - P_e \tag{3.61}$$

Figure 3.21 depicts the torque angle's relationship to mechanical and electrical power output, as well as generator acceleration and deceleration.

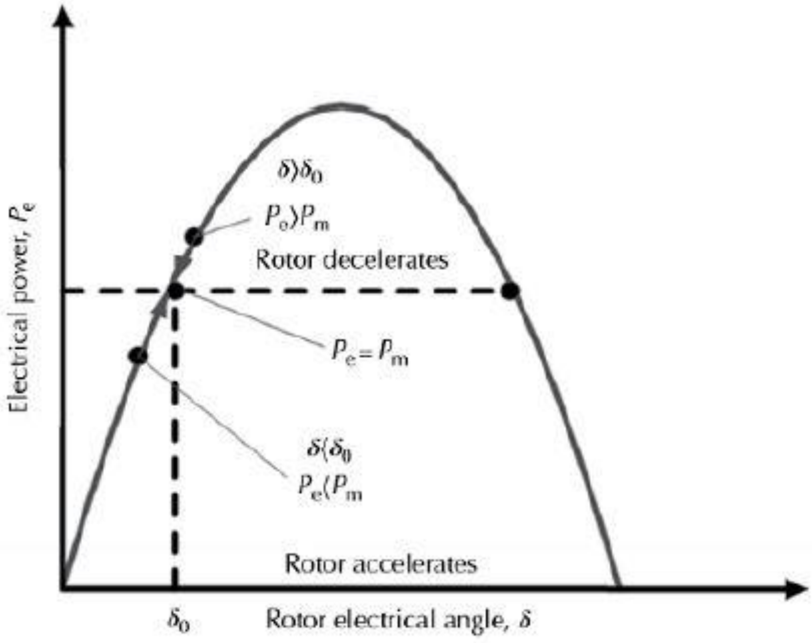

Figure 3.21 Power angle curve [13]

3.6 Grid connection: the infinity bus model

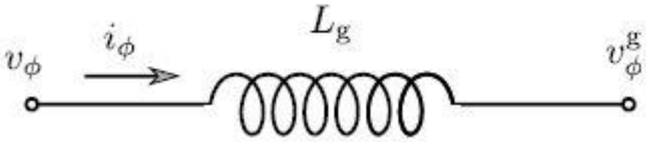

Figure 3.22 Infinity bus[13]

From Figure 3.22, the infinity bus is can be defined as the connection between a generator and the electric grid. Here, v_ϕ represents the synchronous generator phase voltages and v_ϕ^g represent the grid phase voltages where $\phi = \{a, b, c\}$. As current i_ϕ flows from the generator to the grid, it is defined as positive.

In a true conductor, there would be resistive losses and capacitances between the conductor and the ground. In the infinity bus model, we disregard resistance and capacitance and merely describe the relationship with a concentrated inductor.

$$v_\phi = \frac{d\psi_\phi^g}{dt} + v_\phi^g \tag{3.62}$$

Where $\psi_\phi^g = L_g i_\phi$

Compairing Eq. 3.62 to the model of generator Eqs. 3.4-3.6, we find

$$v_{\phi}^{g} = -R_{\phi} i_{\phi} - \frac{d\psi_{\phi}}{dt} - \frac{d\psi_{\phi}^{g}}{dt}$$

When we compare the expressions for ψ_{ϕ} and ψ_{ϕ}^{g}, we can see that the infinity bus effect is included in the generator model by replacing L_S with $\widetilde{L_s}$:

$$\tilde{L}_s = L_s + L_g$$

For a bus that is 100 kilometers long [13], $L_g \approx 0.1$ H.

4 Waterway Model

This chapter presents the waterway model from *Lie, Bernt (2016) course FM1015 Modelling of Dynamic Systems*.

4.1 Hydro-Turbine System

The model of hydropower system is based on mass and momentum balance as mentioned below.

$$\frac{dm}{dt} = \dot{m} \tag{4.1}$$

$$\frac{dM}{dt} = \dot{M} + F, \tag{4.2}$$

Here, m represents the mass of water, $\dot{m} = \dot{m}_i - \dot{m}_e$ is the difference between influent and effluent mass flow rates.

M is the momentum and $\dot{M} = \dot{M}_\iota - \dot{M}_e$ denotes the difference between influent and effluent momentum flow rates. The cumulative force acting on the system is denoted by F. Here, in-compressible water and inelastic pipes are considered.

Balance laws mentioned above can be further developed with series of algebraic equations considering a general pipe unit with influent and effluent pressures as p_i and p_e, cross sectional area A, Length L and height difference H as:

$$\dot{m} = 0$$

$$M = mv = \rho AL\frac{\dot{V}}{A} = \rho L\dot{V}\ , \dot{M} = \dot{m}v = 0$$

Force F consist of pressure forces, gravity and friction force.

$$F = F_p - F_f - F_g$$

Where, $F_p = (p_i - p_e)A, F_f = \rho L\frac{\pi D\dot{V}|\dot{V}|}{8A^2}f_D\ and\ F_g = mg\frac{H}{L}$

v is the velocity of water inside the pipe and ρ is the density of water.

The dynamic equation for volumetric flow rate $\dot{V}$ can be written as,

$$\frac{d\dot{V}}{dt} = \frac{A}{\rho L}(p_i - p_e) - gA\frac{H}{L} - \frac{\pi D\dot{V}|\dot{V}|}{8A^2}f_D\ ,$$

Here, based on Reynolds number $N_{Re} = \frac{\rho D|v|}{\mu} = \frac{4\rho|\dot{V}|}{\mu\pi D}$, Darcy friction factor f_D is calculated as,

$$f_D = \begin{cases} \dfrac{64}{N_{Re}} \\ \dfrac{1}{\left(2\log_{10}\left(\dfrac{\varepsilon}{3.7D} + \dfrac{5.7}{N_{Re}^{0.9}}\right)\right)^2} \end{cases}$$

Here, μ is the kinematic viscosity of water and ϵ is the pipe roughness height. f_D is calculated for the region with $2100 \leq N_{Re} \leq 2300$.

The dynamic equations of hydropower with intake, surge tank and penstock are mentioned below:

$$\frac{dh}{dt} = \frac{\dot{V}_s}{A_s} \tag{4.3}$$

$$\frac{d\dot{V}_s}{dt} = \frac{A_s}{\rho h}(p_n - p_a) - \frac{\pi D_s \dot{V}_s |\dot{V}_s|}{8A_s^2} f_{D,s} - gA_s \tag{4.4}$$

$$\frac{d\dot{V}_p}{dt} = \frac{A_p}{\rho L_p}(p_n - p_t) - \frac{\pi D_p \dot{V}_p |\dot{V}_p|}{8A_p^2} f_{D,p} + gA_p \frac{H_p}{L_p} \tag{4.5}$$

Also, Algebraic equations are:

$$p_t = p_a\left(1 + \left(\frac{\dot{V}_p}{C_v u_v}\right)^2\right)$$

$$\frac{d\dot{V}_i}{dt} = \frac{A_i}{\rho L_i}(p_a + \rho g H_r - p_n) - \frac{\pi D_i \dot{V}_i |\dot{V}_i|}{8A_i^2} f_{D,i} + gA_i \frac{H_i}{L_i}$$

$$\dot{V}_i = \dot{V}_s + \dot{V}_p$$

$$P_m = \eta_h (p_t - p_a)\dot{V}_p$$

Where h is the height of the oscillating water mass inside the surge tank,

p_t is the turbine pressure,η_h is the hydraulic efficiency of turbine,

$u_v$is the valve signal controlling water flow through the turbine,

p_n is the intake-penstock manifold pressure represented by bottom pressure of the surge tank and $p_a$is the atmospheric pressure.

Here, the turbine unit is modeled as a simple mechanical valve, as shown below.

$$P_m = \eta_h (p_t - p_a)\dot{V}_p$$

$$\dot{V}_p = C_v u_v \sqrt{\frac{p_t - p_a}{p_a}}$$

Where $C_v$is the turbine valve capacity.

4.2 Turbine-Synchronous Machine Aggregate System

The aggregate is the combined turbine rotor and electric generator rotor. The swing equation for the aggregate system dynamics is given by,

$$J_a \frac{d\omega_a}{dt} = P_m - P_e - P_{f,a} \tag{4.6}$$

where $\omega_a$is the angular speed of the aggregate and J_a is the total angular momentum.

$P_{f,a}$ is the frictional power loss in the aggregate. If we consider the bearing term dominating then,

$$P_{f,a} = \frac{1}{2} k_{f,b} \omega^2{}_a$$

where, $k_{f,b}$ is the bearing friction factor.

Following are the model parameters and operating conditions that are taken from *Lie, Bernt (2016) course FM1015 Modelling of Dynamic Systems* [20].

Table 4.1 Model parameters for hydropower system

Quantity	Value	Description
H_i	25 m	Vertical drop of the intake race
H_s	120 m	Reference vertical drop of surge tank
H_p	420 m	Vertical drop of the penstock
H_d	5 m	Vertical drop of the discharge race
L_i	6600 m	Length of the intake race
L_s	140 m	Reference length of the surge tank
L_p	600 m	Length of the penstock
L_d	600 m	Length of the discharge race
D_i	5.8 m	Diameter of the intake race
D_s	3.4 m	Diameter of the surge tank
D_p	3.3 m	Diameter of the penstock
D_d	5.8 m	Diameter of the discharge race
ε	1.5e^{-5} m	Pipe roughness height
C_v	6 m^3/s	Turbine valve capacity
η_h	0.90	Turbine hydraulic efficiency
η_e	0.99	Electric generator efficiency
J_a	3e^6 kg m^2	Aggregate moment of inertia
$k_{f,b}$	10^3 Ws^2/rad^2	Friction factor in aggregate bearing
ρ	997 kg/m^3	Density of water at ambient temperature
μ	8.9e^{-4} Pa s	Dynamic viscosity of water at ambient temperature
p_a	1.013 e^5 Pa	Atmospheric pressure
g	9.81 m^3/s	Acceleration due to gravity

Table 4.2 Operating conditions for hydropower system

Quantity	Value	Description
H_r	50 m	Height of water in the reservoir
H_t	10 m	Height of water in th tail
$\dot{V}^0$	20 m^3/s	Nominal volumetric flow rate through the turbine
u_v	0.5	
$\dot{V}_p(t=0)$	$\dot{V}_{rate}$	Initial water flow rate in th penstock
$\dot{V}_i(t=0)$	$\dot{V}_{rate}$	Initial water flow rate in the intake race
$h_s(t=0)$	$H_r + H_i$	Initial water level in the surge tank

5 Results and Discussion

This chapter properly describes and presents the important simulation results that have been implemented using OpenModelica.

5.1 Phase space model and dq0 space model

When the stator coordinate structure is converted to rotate with the rotor, it is much easier to analyze the model. And this implementation of the model in dq0 space appeared to be more stable than that in phase space. The generated active power in the phases, resistive power loss in the rotor, and resistive power loss in the stator with dq0 space formulation are displayed in Figure 3.12, Figure 3.13, and Figure 3.14 respectively. The field current in dq0 state is shown in Figure 3.15.

The dq0 currents are found to respond as in Figure 3.16. From Figure 3.16, we can also see that $i_0$reaches steady state extremely fast. It is so because, $L_0 = L_s - 2\ M_S$ is very small or close to zero, so that the time constant tends to be zero. Therefore,

$$v_0 = -Ri_0 - L_0 \frac{di_0}{dt}$$

$$i_0 = -\frac{1}{R} v_0$$

With symmetric terminal voltages,$v_0 = 0$ leading to $i_0 = 0$ in steady state.

More than this, Figure 5.1 shown below compares the power in phase space with the power in dq0 space. It is clearly seen that the behavior of the active power in the phase space is oscillating whereas dq0 power is converging to steady solutions.

Similarly, Figure 5.2 compared the resistive power loss to stator in phase space and dq0. It is observed that the power loss in phase space is oscillating whereas computing power loss in dq0 space is more accurate.

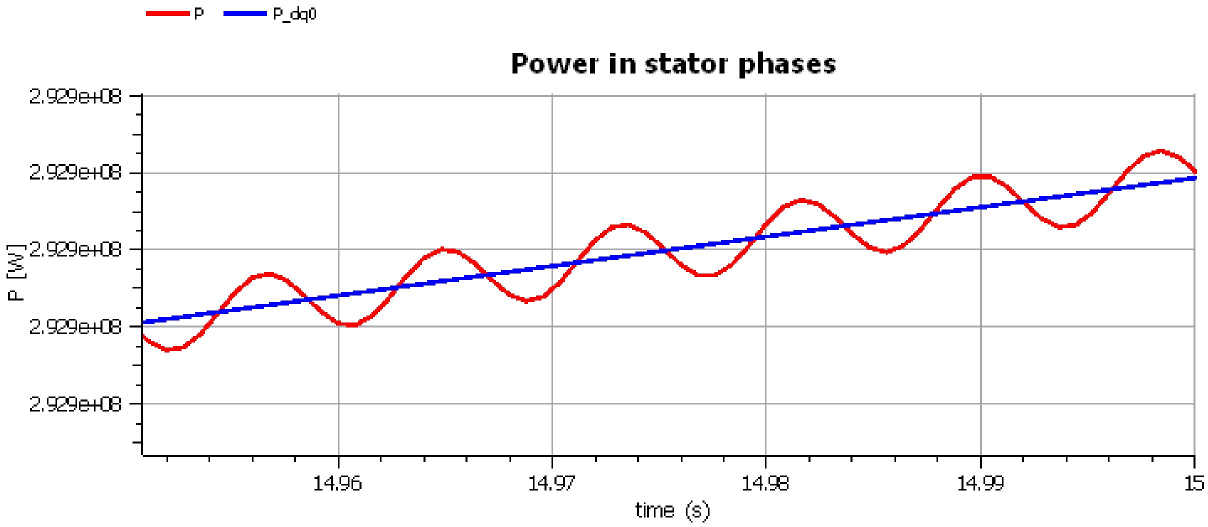

Figure 5.1 Active power in phase space and dq0

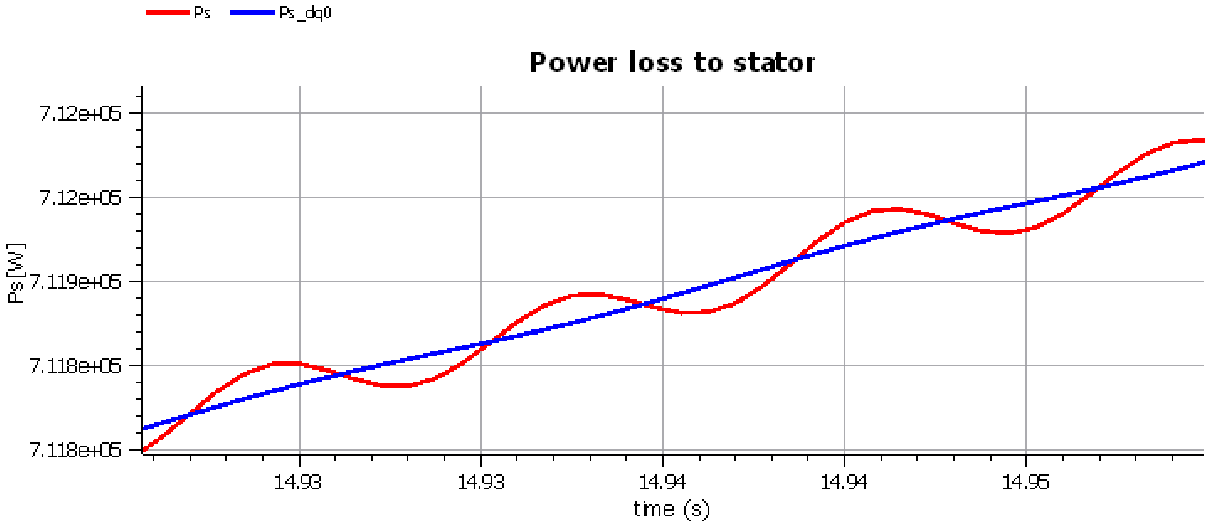

Figure 5.2 Stator power loss in phase space and dq0

5.2 Model connected to grid

The synchronous generator model is then connected to the infinity bus model. Here, the generator is simulated with a fixed load angle. In a typical operation, the load angle is in the range of 30^0 or $\frac{\pi}{6}$. When simulating the system over 200 s, the graphs are found to respond in the way mentioned below:

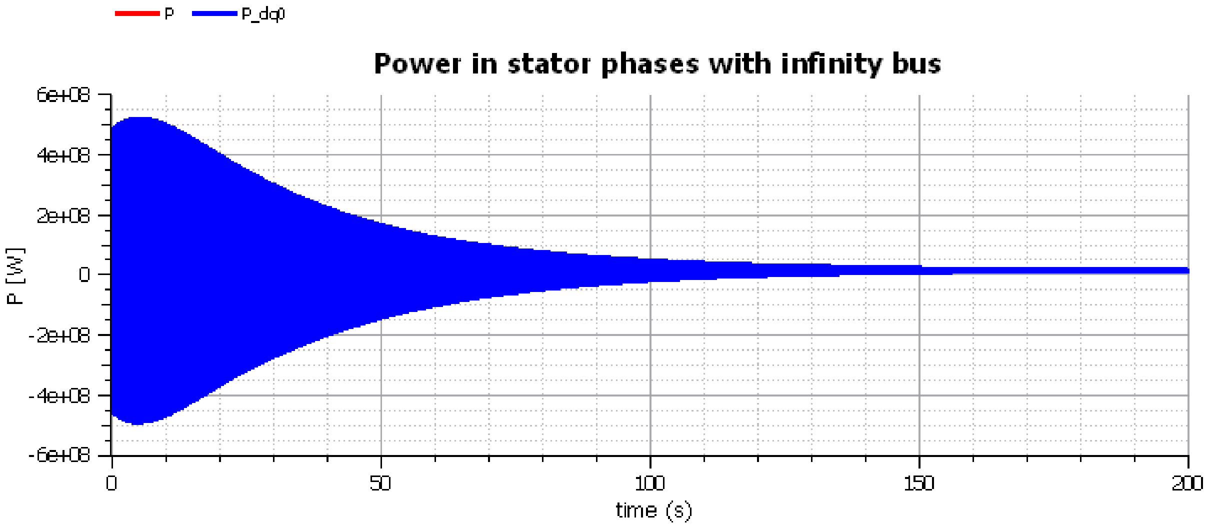

Figure 5.3 Active electrical power with infinity bus

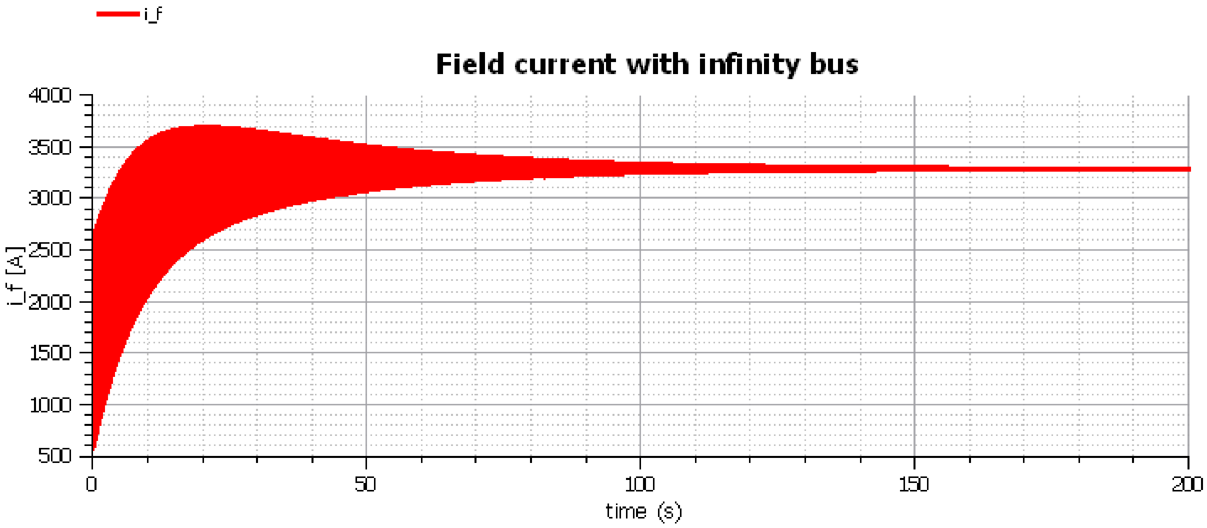

Figure 5.4 Field current with infinity bus

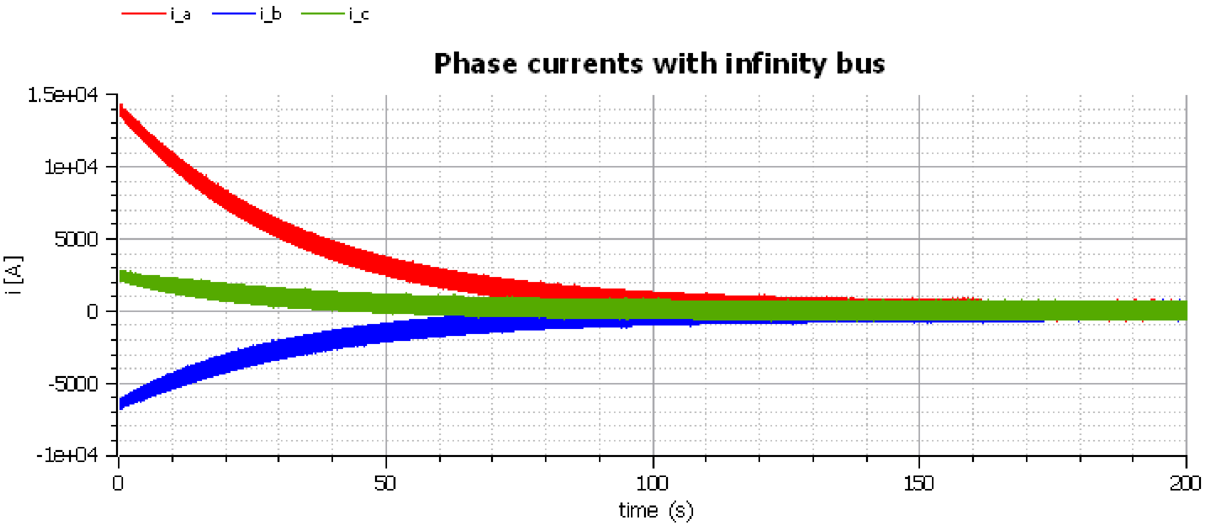

Figure 5.5 Phase currents with infinity bus

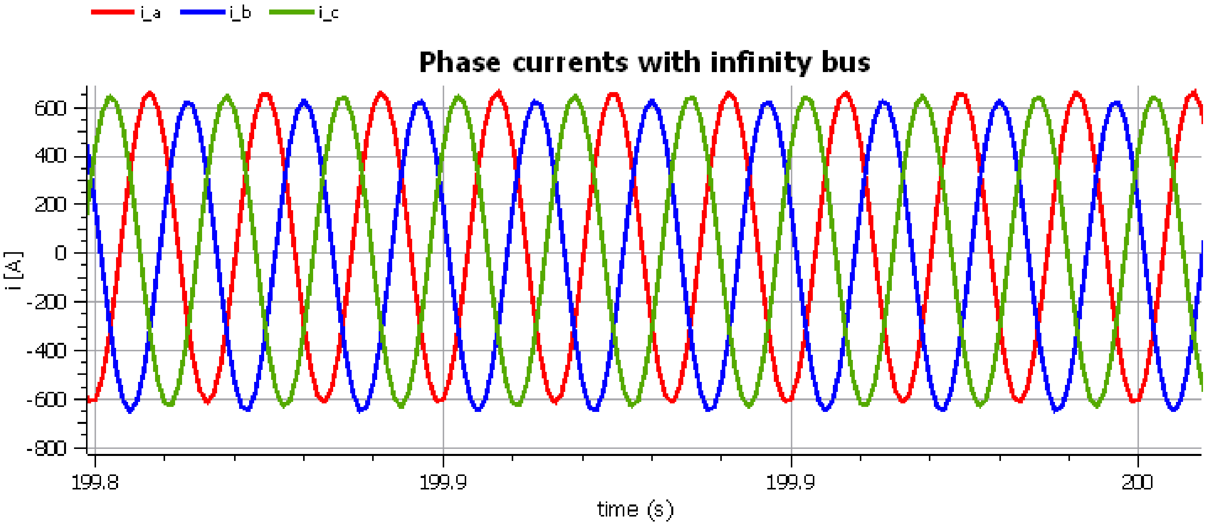

Figure 5.6 Short term phase currents

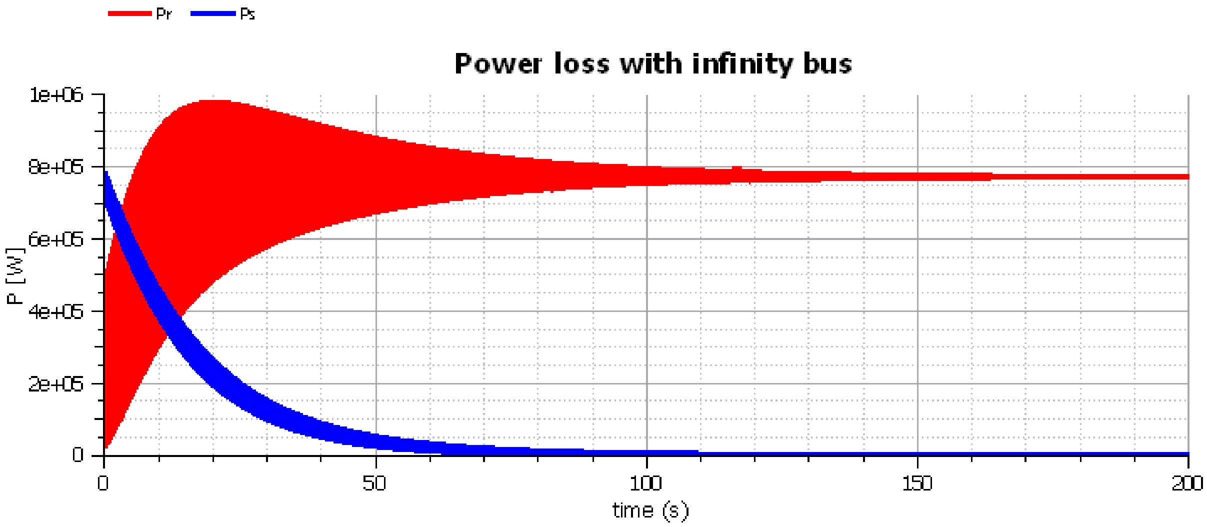

Figure 5.7 Rotor and Stator power loss with infinity bus

The generated active electrical power (approximately 15 MW) is shown in Figure 5.3. The field current and the phase currents (i_a, i_b, i_c) are shown in Figure 5.4 and Figure 5.5. A short term simulation for phase currents computed by Park transformation of the dq0 currents is shown in Figure 5.6. Carefully observing Figure 5.6, we can see that the stationary solution is still not reached perfectly even after 200 s. The resistive power loss in the rotor and stator is displayed in Figure 5.7.

5.3 Model connected to grid and turbine

It is necessary to observe the different behavior of the synchronous generator when it is connected to the turbine. Here, 3 phase synchronous generator is connected to the grid and turbine. Now, the load angle is given by the interaction between the turbine and generator, and it is now no more fixed.

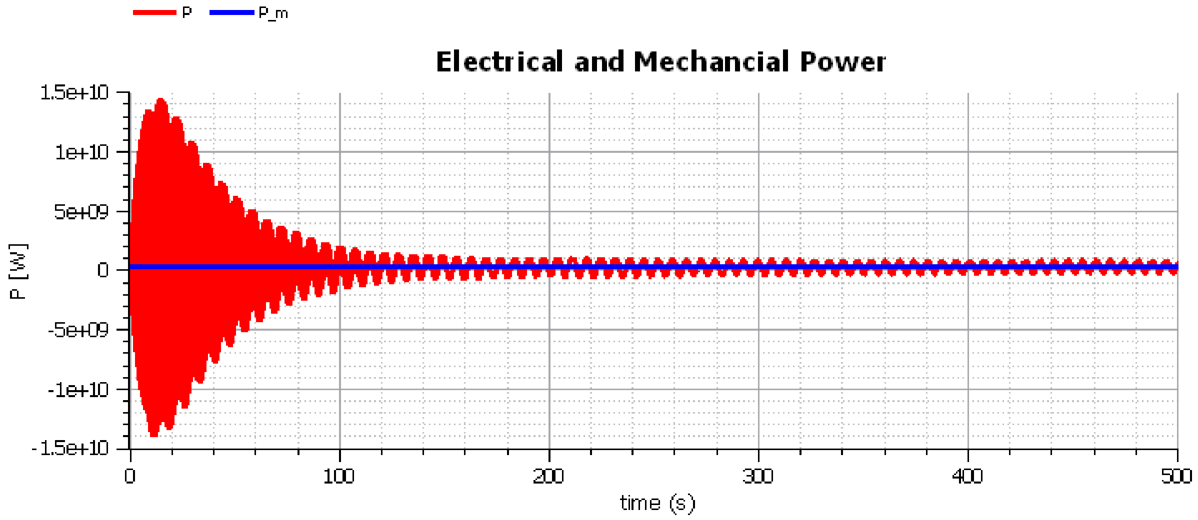

Figure 5.8 Active electrical and mechanical power

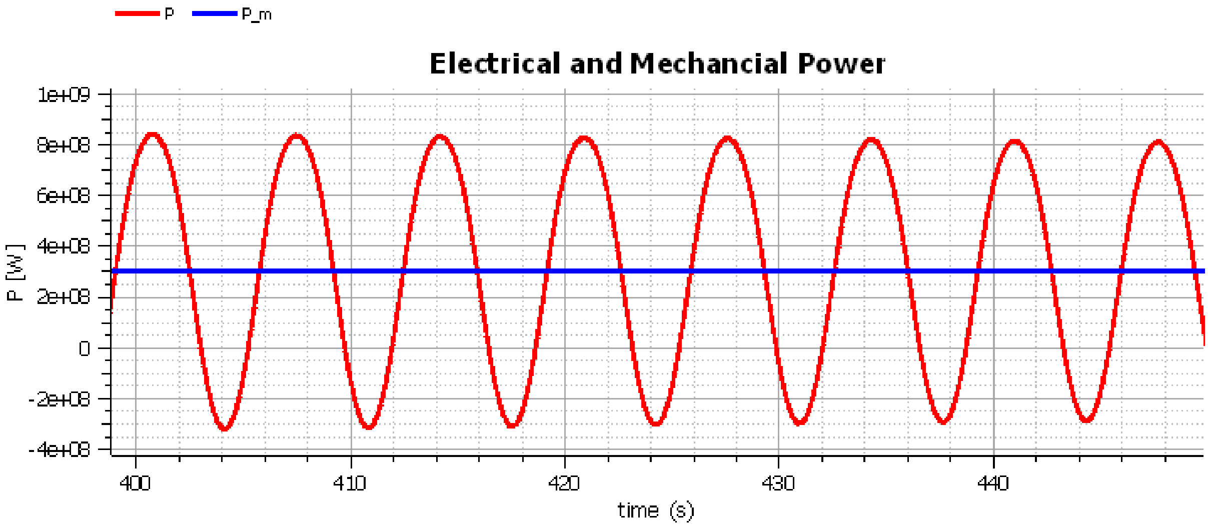

Figure 5.9 short term result of Electrical and Mechanical power

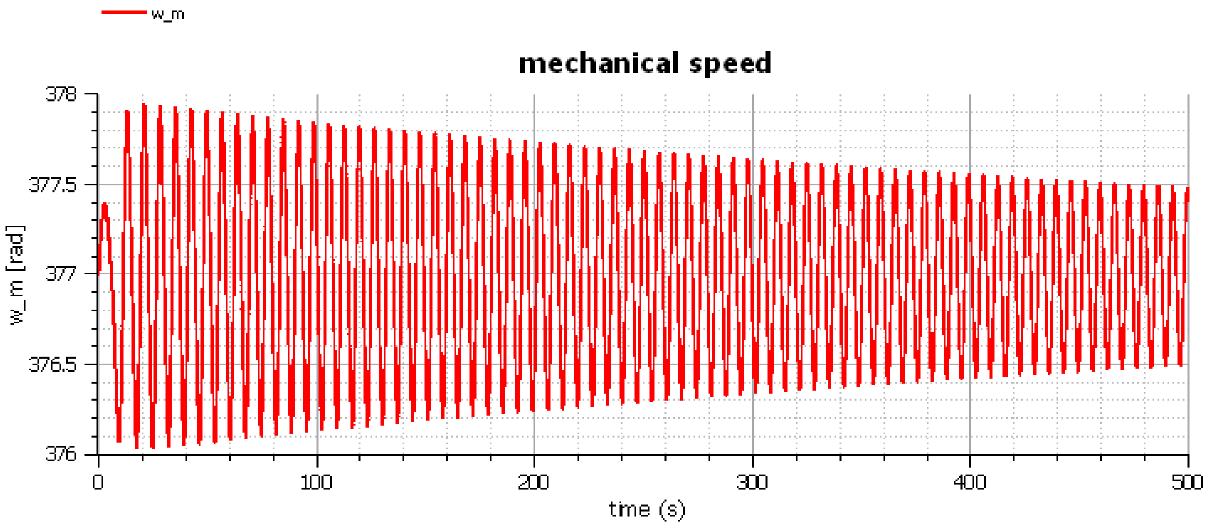

Figure 5.10 Mechanical speed of a rotor

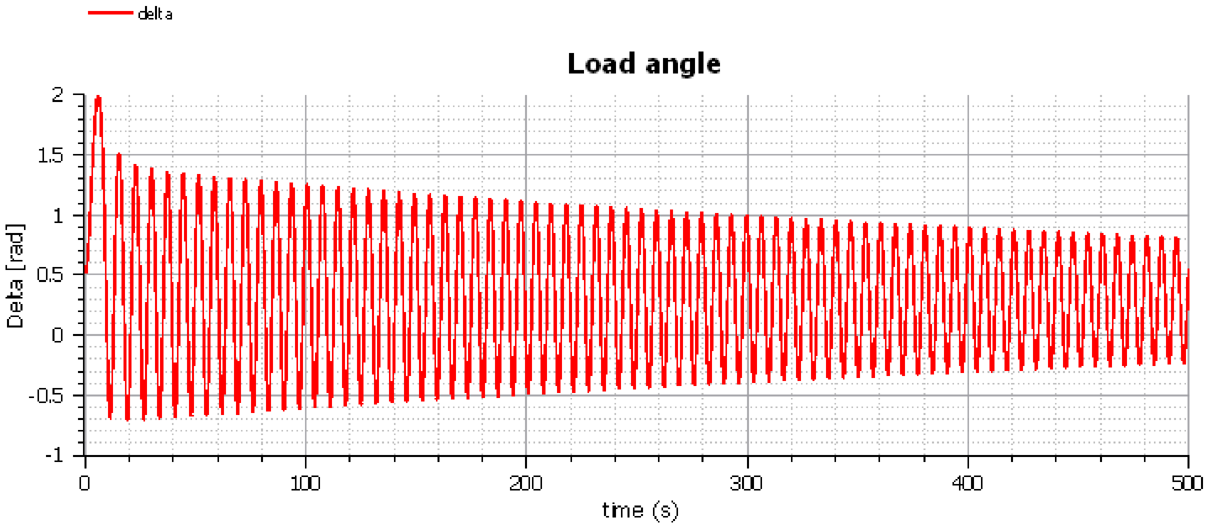

Figure 5.11 Load angle

It is known that the generator is driven by the mechanical power from the hydro turbine. This mechanical power is obtained from the water energy of the hydropower. But here a constant mechanical power is given for the simulation without the connection of the waterway. The main purpose of the waterway is to provide mechanical power.

In Figure 5.9, the short term result of the active electrical and mechanical power can be observed. Here, mechanical power supplied is: $P_m = \omega_m T_m = 2\pi * 60 * 800000 \approx 300\ MW$ In this simulation result, we can see that mechanical power $P_m$and active electrical power P on average are similar as long as they stay in synchronism. Also, the friction losses of a generator are not included, so all mechanical power is converted to electrical power.

The load angle and mechanical speed of a rotor are shown in Figure 5.10 and Figure 5.11. The load angle and mechanical speed of a rotor displayed seems to be oscillating.

The important thing to be discussed is about loss of synchronism which is shown below:

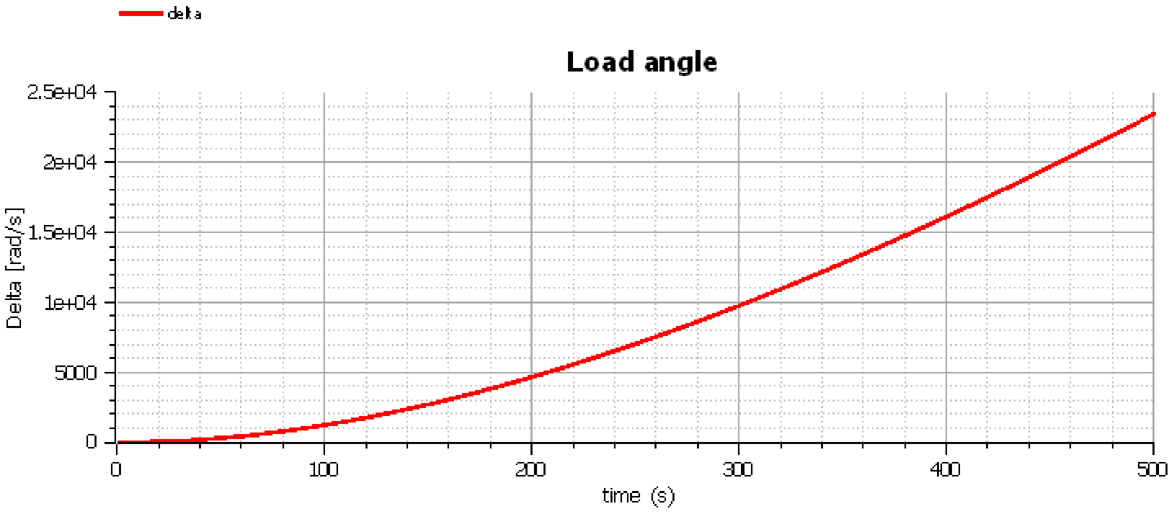

Figure 5.12 Load angle increasing linearly

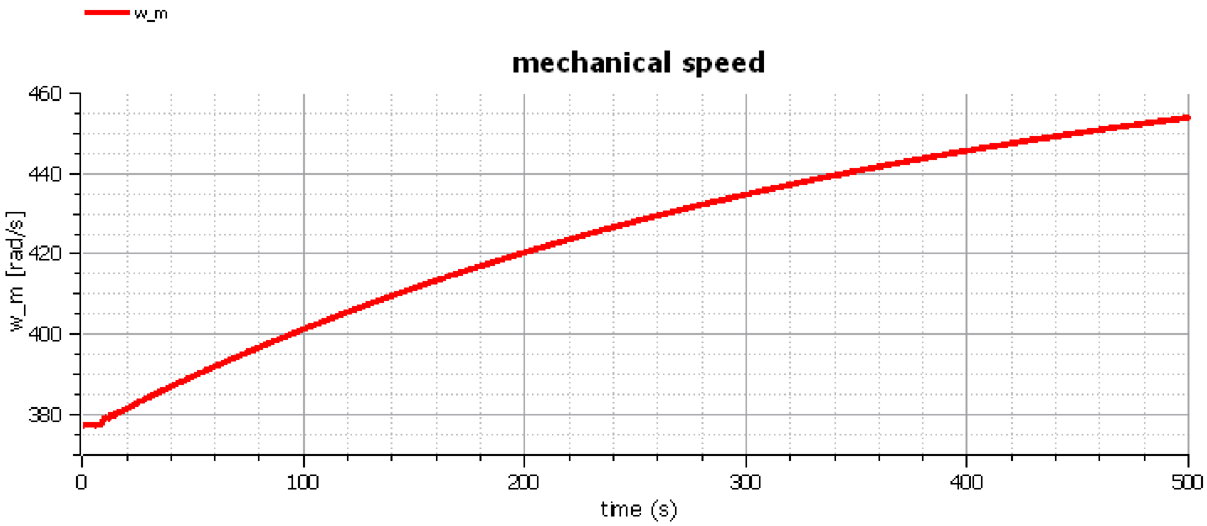

Figure 5.13 mechanical speed losing synchronism

In the above Figure 5.12 and Figure 5.13, we can see that the load angle and the mechanical speed of a rotor are increasing linearly. This is the case when a large mechanical power is applied to the generator due to which it loses its synchronism.

The synchronous machine will stay at synchronous speed if it is loaded up to the limit of electrical power for a generator. But the machine loses synchronism if large torque is applied to the generator. On the other hand, the value of the torque can be increased by increasing the value of field voltage v_f [21].

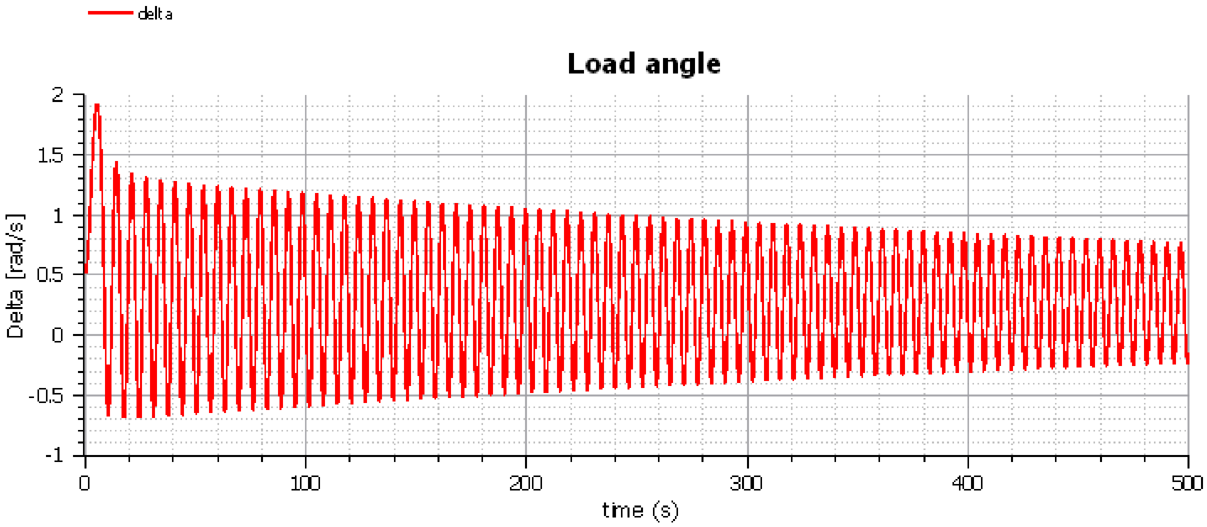

Figure 5.14 Load angle after gaining synchronism

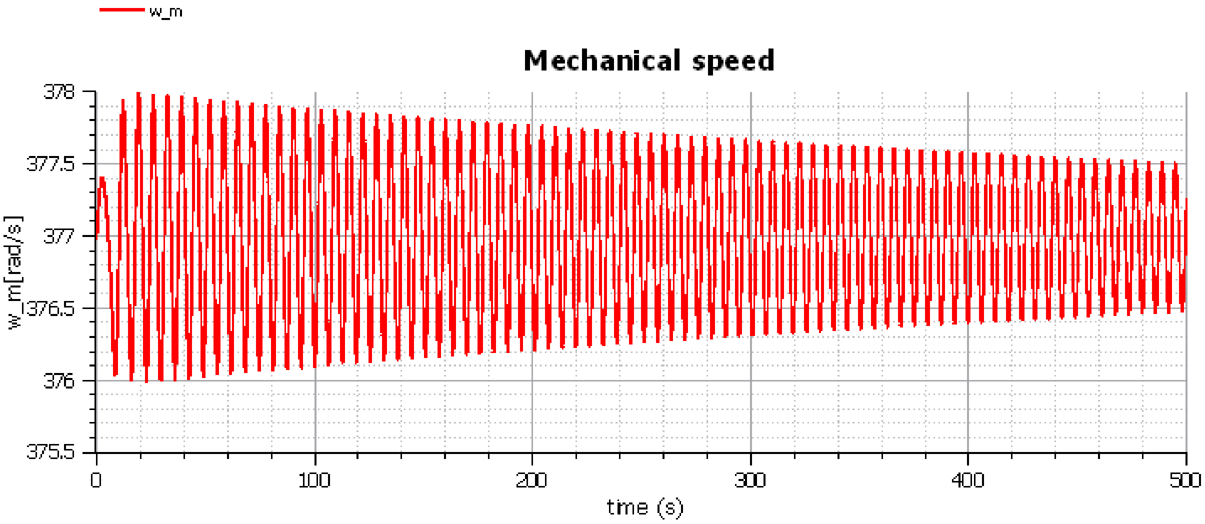

Figure 5.15 Mechanical speed

Figure 5.12 and Figure 5.13 are the cases of synchronism loss when the mechanical power is increased keeping field voltage 8500 V. Now in Figure 5.14 and Figure 5.15, the field voltage is increased to 10000 V due to which synchronism is gained. Thus, by increasing the field voltage, a synchronous generator that has lost synchronism due to the prime mover rotating the unit at speeds faster than the synchronous speed may be brought back into synchronism. Increasing the field voltage raises the counter-torque and slows the system to synchronous speed[21]. Hence, a good controller is required for this.

5.4 Model connected to water way including turbine

Here, the grid connected generator model is linked with the complete model of the waterway including the turbine. Now, the waterway provides mechanical power to the generator for the production of electricity. When a full model is set up, the model's different behavior is observed as shown below.

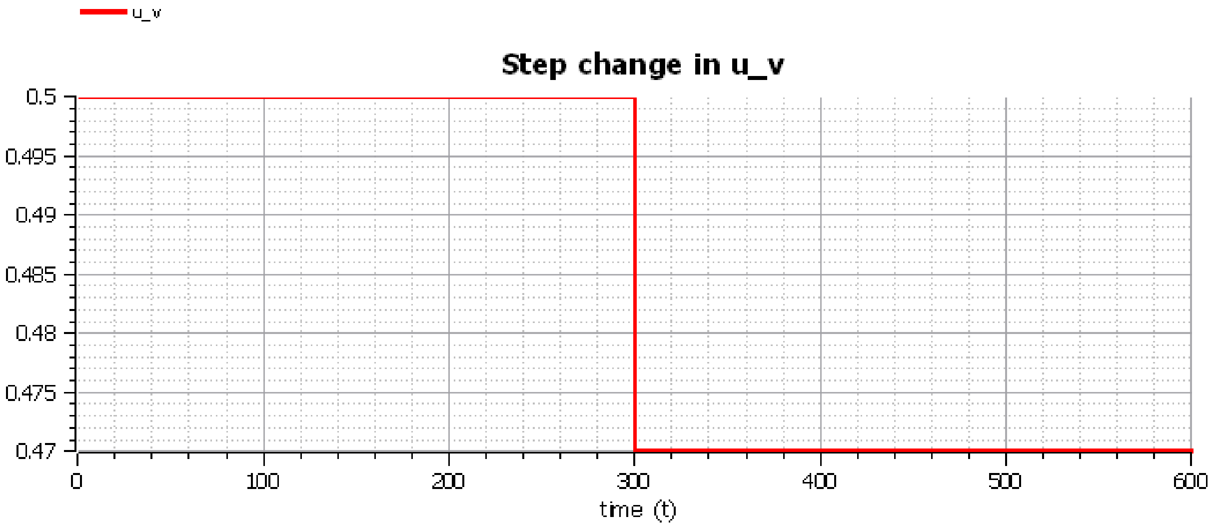

Figure 5.16 Step change in turbine valve

Figure 5.16 shows the step change in the opening of turbine valve. The valve opening initially is 0.5 and after 300 seconds, it is step down to 0.47.

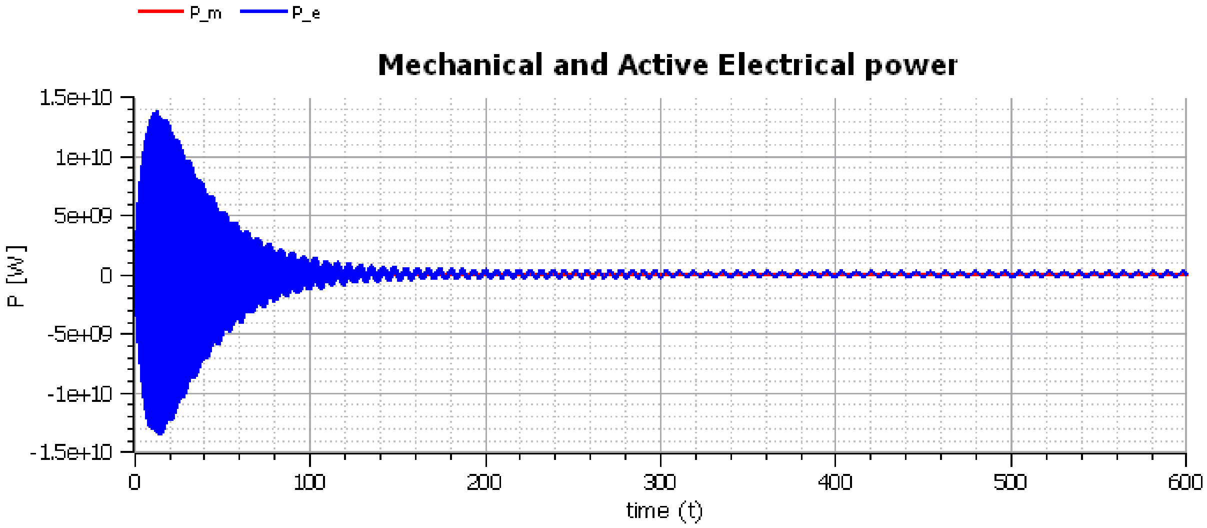

Figure 5.17 Mechanical and Active Power

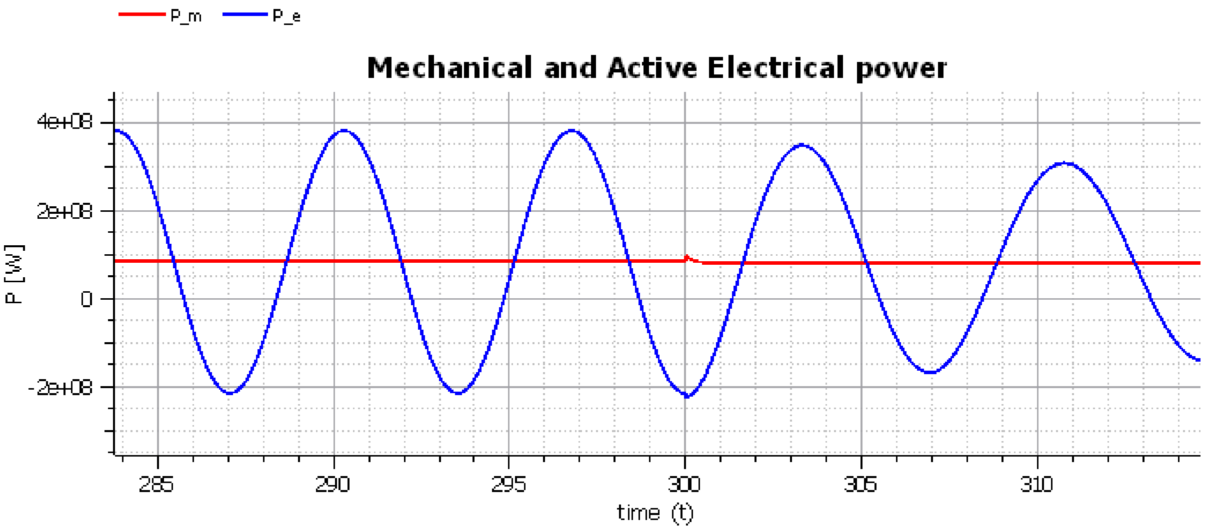

Figure 5.18 Short term view of Power

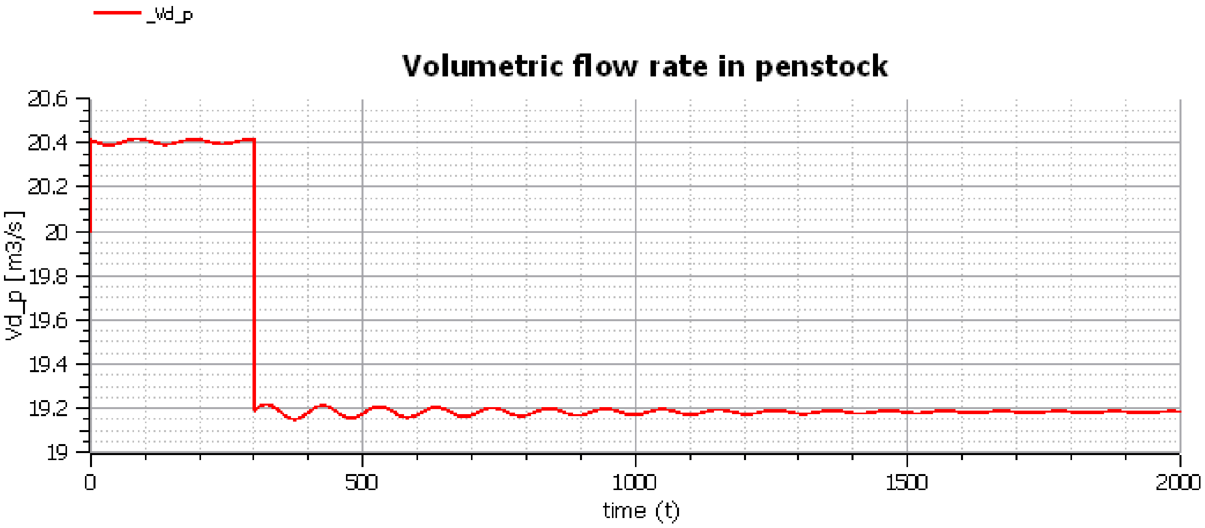

Figure 5.19 Flow rate in penstock

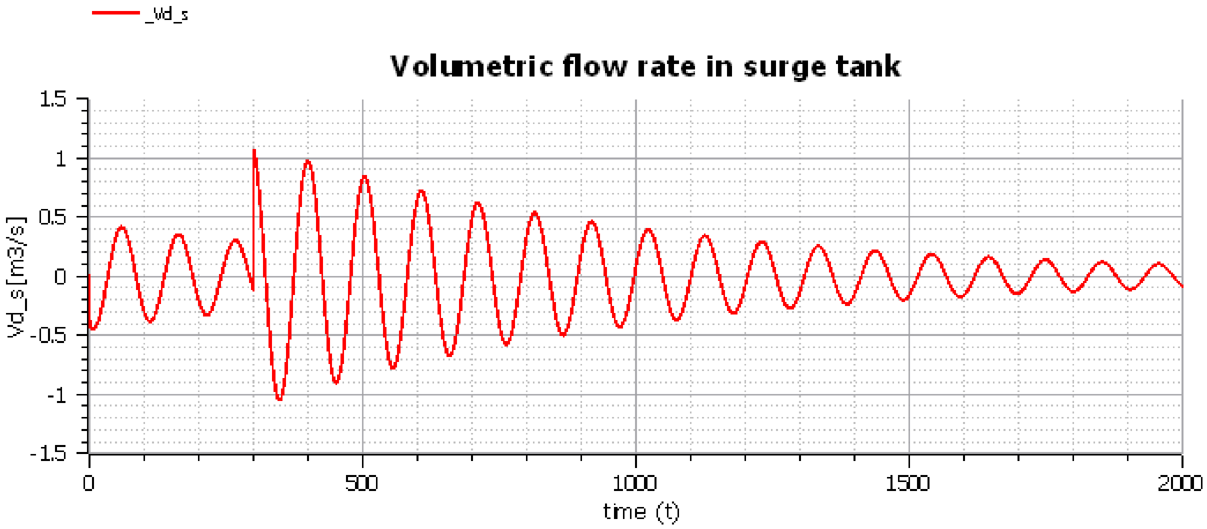

Figure 5.20 Flow rate in surge tank

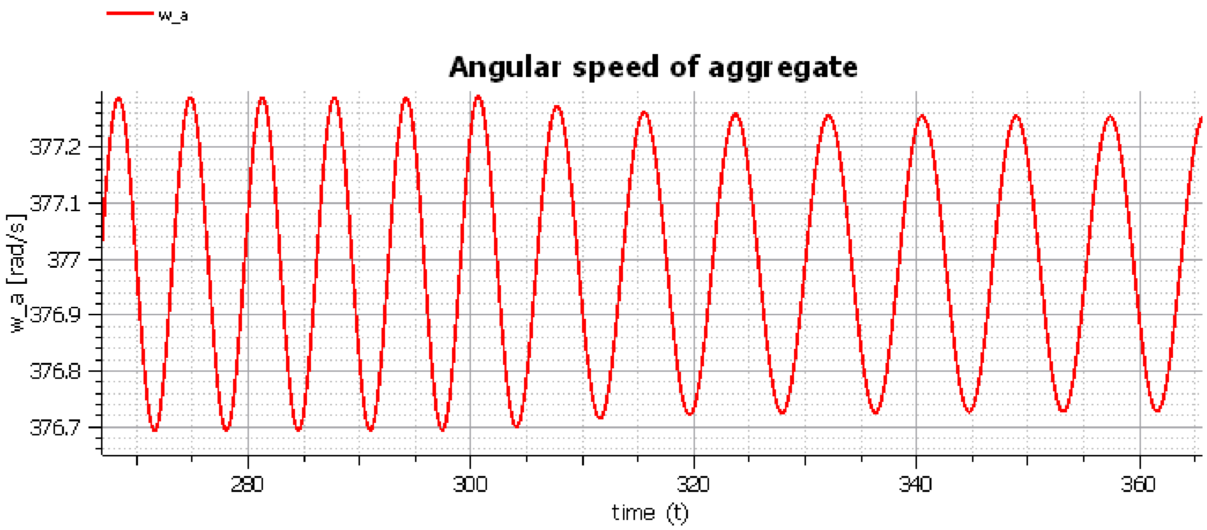

Figure 5.21 Aggregate speed

In Figure 5.18, a short term view of the mechanical and electrical power is shown. It can be seen that the mechanical and electrical power on average is similar. It is so because no friction loss is included. Before 300 seconds when the valve opening is 0.5, the mechanical power obtained from the waterway is 86 MW, and thereafter it is 80 MW due to u_v being 0.47. Also, we can see in Figure 5.18 that as mechanical power decreases, average electrical power decreases as well.

Figure 5.19 shows the volumetric flow rate of water in the penstock. Here, the volumetric rate of water decreases in penstock due to the decrement of valve opening. Figure 5.20 shows the volumetric flow rate of water in the surge tank. Aggregate angular speed is displayed in Figure 5.21.

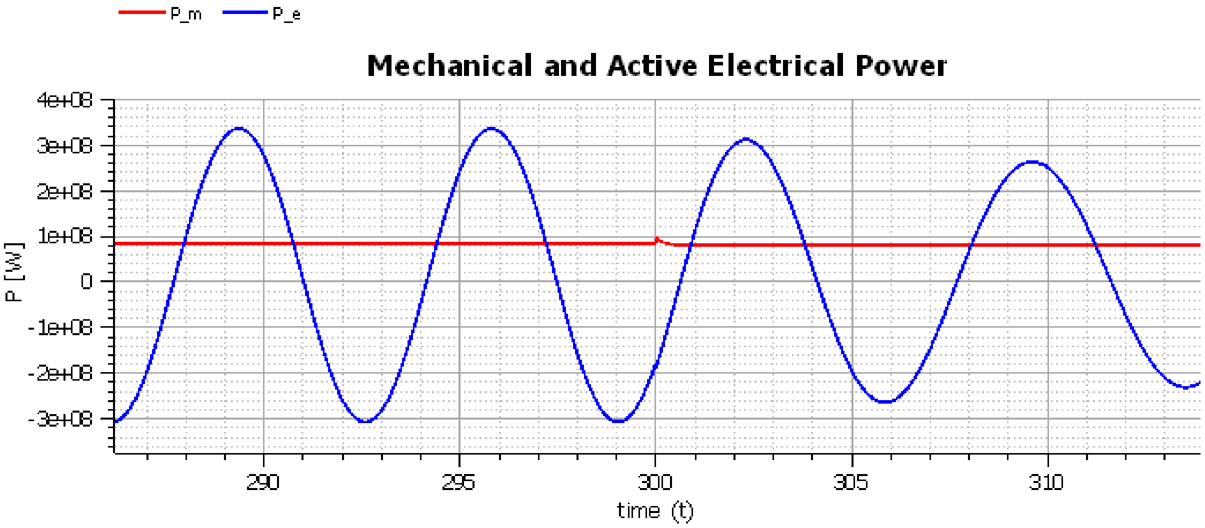

Figure 5.22 Power with friction included

The mechanical and active electrical power is displayed in Figure 5.22. But in this case, the friction loss is included according to Table 4.1. Comparing Figure 5.22 with Figure 5.18, it can be easily seen that electrical power and mechanical on average are not similar. This is because all mechanical power generated from the waterway is not converted to electrical power due to the presence of friction.

6 Conclusion

In this thesis, an overview of hydropower production units was given, as well as a possible extension of the existing description of the synchronous generator model presented in Lie (2019). Initially, the dq0 and phase space model of the generator were compared, and it was seen that the dq0 model was more stable than phase space. After that, a model of a grid-connected synchronous generator was simulated with a fixed load angle of 30°. The generator was also connected to a real waterway system, which included a turbine, and the load angle was calculated using the interaction between the turbine and generator. During the model's simulation, it was noticed that when the mechanical power is much greater than the electrical power, the generator's synchronism is lost. By increasing the field voltage, a synchronous generator that has lost synchronism due to the prime mover rotating the generator at speeds faster than the synchronous speed was brought back into synchronism. Furthermore, when the generator was connected to the waterway, the mechanical and active electrical power of the synchronous generator was on average identical when friction loss was not taken into account. When the frictional loss was introduced, however, all of the mechanical energy was not converted to electrical energy. This proved that the model performed as predicted.

7 Future Work

The research presented in this thesis has opened the way for the improvement of the models. Following further works can be done to enhance the model:

- Magnetic saturation effects can be introduced to the synchronous generator model to make the model more practical.
- An advanced controller like Proportional-Integral (PI), model predictive controller (MPC), etc. can be added.
- It is often interesting to implement the models in Julia and compare the plotting results.

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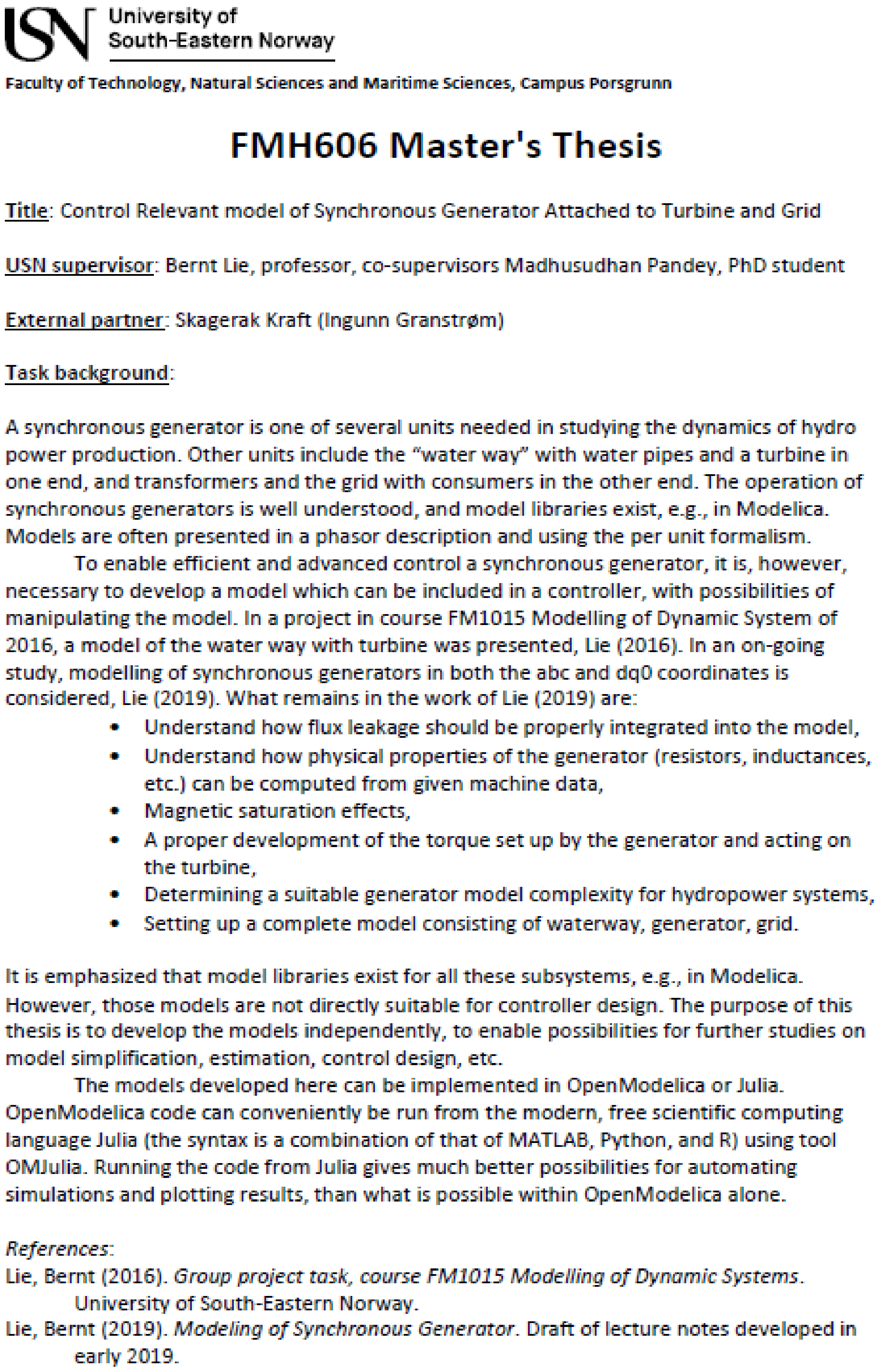
Appendices

Appendix A : Task Descriptions



University of
South-Eastern Norway

Faculty of Technology, Natural Sciences and Maritime Sciences, Campus Porsgrunn

FMH606 Master's Thesis

Title: Control Relevant model of Synchronous Generator Attached to Turbine and Grid

USN supervisor: Bernt Lie, professor, co-supervisors Madhusudhan Pandey, PhD student

External partner: Skagerak Kraft (Ingunn Granstrøm)

Task background:

A synchronous generator is one of several units needed in studying the dynamics of hydro power production. Other units include the "water way" with water pipes and a turbine in one end, and transformers and the grid with consumers in the other end. The operation of synchronous generators is well understood, and model libraries exist, e.g., in Modelica. Models are often presented in a phasor description and using the per unit formalism.

To enable efficient and advanced control a synchronous generator, it is, however, necessary to develop a model which can be included in a controller, with possibilities of manipulating the model. In a project in course FM1015 Modelling of Dynamic System of 2016, a model of the water way with turbine was presented, Lie (2016). In an on-going study, modelling of synchronous generators in both the abc and dq0 coordinates is considered, Lie (2019). What remains in the work of Lie (2019) are:

- Understand how flux leakage should be properly integrated into the model,
- Understand how physical properties of the generator (resistors, inductances, etc.) can be computed from given machine data,
- Magnetic saturation effects,
- A proper development of the torque set up by the generator and acting on the turbine,
- Determining a suitable generator model complexity for hydropower systems,
- Setting up a complete model consisting of waterway, generator, grid.

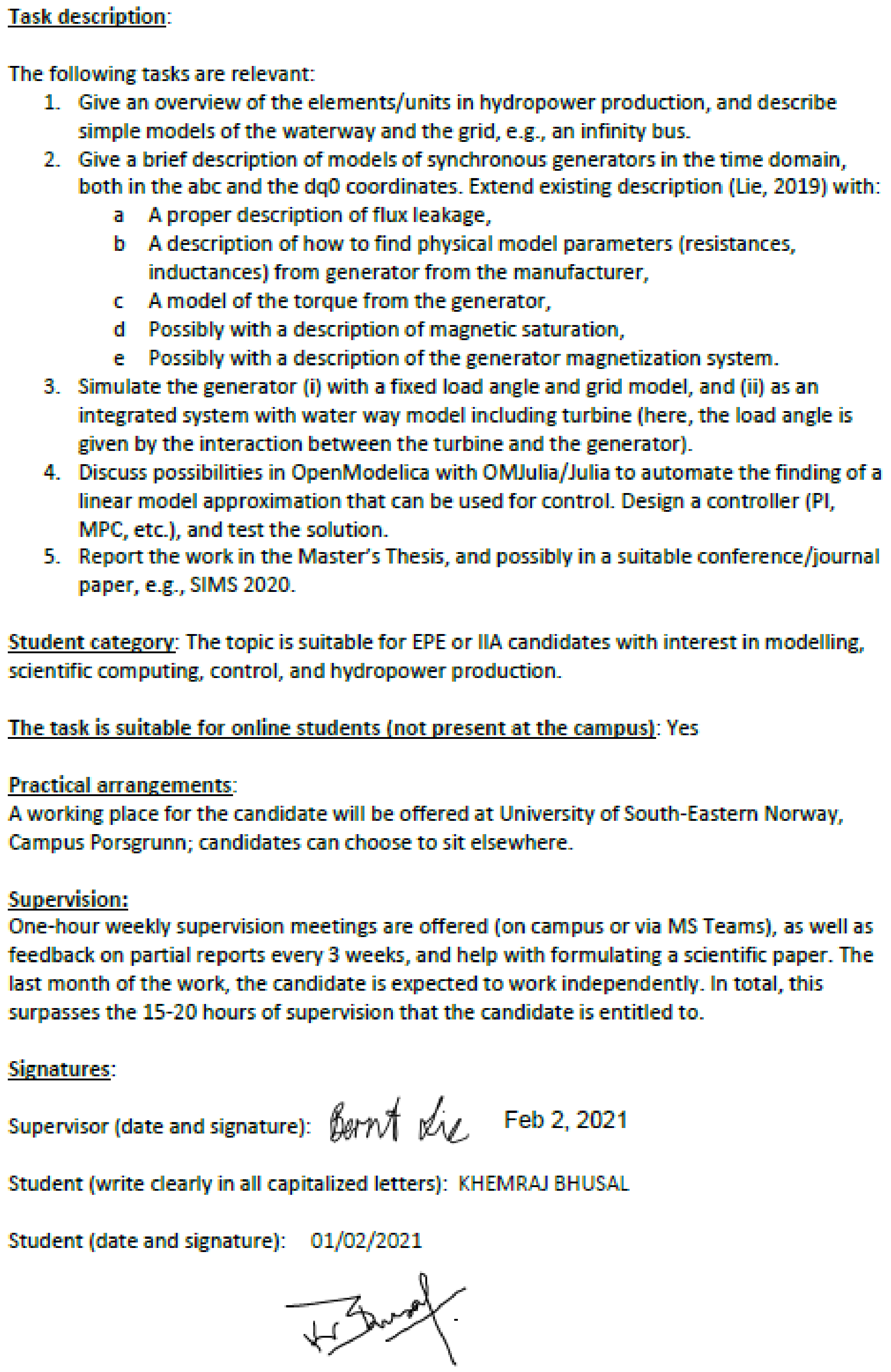
It is emphasized that model libraries exist for all these subsystems, e.g., in Modelica. However, those models are not directly suitable for controller design. The purpose of this thesis is to develop the models independently, to enable possibilities for further studies on model simplification, estimation, control design, etc.

The models developed here can be implemented in OpenModelica or Julia. OpenModelica code can conveniently be run from the modern, free scientific computing language Julia (the syntax is a combination of that of MATLAB, Python, and R) using tool OMJulia. Running the code from Julia gives much better possibilities for automating simulations and plotting results, than what is possible within OpenModelica alone.

References:

Lie, Bernt (2016). *Group project task, course FM1015 Modelling of Dynamic Systems*. University of South-Eastern Norway.

Lie, Bernt (2019). *Modeling of Synchronous Generator*. Draft of lecture notes developed in early 2019.



Task description:

The following tasks are relevant:

1. Give an overview of the elements/units in hydropower production, and describe simple models of the waterway and the grid, e.g., an infinity bus.
2. Give a brief description of models of synchronous generators in the time domain, both in the abc and the dq0 coordinates. Extend existing description (Lie, 2019) with:
    - a A proper description of flux leakage,
    - b A description of how to find physical model parameters (resistances, inductances) from generator from the manufacturer,
    - c A model of the torque from the generator,
    - d Possibly with a description of magnetic saturation,
    - e Possibly with a description of the generator magnetization system.
3. Simulate the generator (i) with a fixed load angle and grid model, and (ii) as an integrated system with water way model including turbine (here, the load angle is given by the interaction between the turbine and the generator).
4. Discuss possibilities in OpenModelica with OMJulia/Julia to automate the finding of a linear model approximation that can be used for control. Design a controller (PI, MPC, etc.), and test the solution.
5. Report the work in the Master's Thesis, and possibly in a suitable conference/journal paper, e.g., SIMS 2020.

Student category: The topic is suitable for EPE or IIA candidates with interest in modelling, scientific computing, control, and hydropower production.

The task is suitable for online students (not present at the campus): Yes

Practical arrangements:
A working place for the candidate will be offered at University of South-Eastern Norway, Campus Porsgrunn; candidates can choose to sit elsewhere.

Supervision:
One-hour weekly supervision meetings are offered (on campus or via MS Teams), as well as feedback on partial reports every 3 weeks, and help with formulating a scientific paper. The last month of the work, the candidate is expected to work independently. In total, this surpasses the 15-20 hours of supervision that the candidate is entitled to.

Signatures:

Supervisor (date and signature): Bernt Lie Feb 2, 2021

Student (write clearly in all capitalized letters): KHEMRAJ BHUSAL

Student (date and signature): 01/02/2021

Appendix B: Listing Code

The complete model of generator with waterway and grid can be found in:
https://github.com/Khemraj44/Thesis_2021_code_ModHydroPower