

Optimal Settings of Fast Active Power Controller: Nordic Case

¹Acosta Montalvo, Martha Noheми; ²Gonzalez-Longatt, Francisco; ³Denysiuk, Serhii; ³Strelkova, Halyna

¹School of Mechanical and Electrical Engineering - Universidad Autónoma de Nuevo León

²Department of Electrical Engineering, Information Technology and Cybernetics - University of South-Eastern Norway

³Power Supply Department - IEE Igor Sikorsky KPI

Acosta, M. N., Gonzalez-Longatt, F., Denysiuk, S., & Strelkova, H. (2020). *Optimal Settings of Fast Active Power Controller: Nordic Case*. 2020 IEEE 7th International Conference on Energy Smart Systems (ESS), 63–67. <https://doi.org/10.1109/ESS50319.2020.9160281>

Publisher's version: DOI: [10.1109/ESS50319.2020.9160281](https://doi.org/10.1109/ESS50319.2020.9160281)

© 2020 IEEE. Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collective works, for resale or redistribution to servers or lists, or reuse of any copyrighted component of this work in other works.

Optimal Settings of Fast Active Power Controller: Nordic Case

M. N. Acosta
*School of Mechanical and Electrical
Engineering*
Universidad Autónoma de Nuevo León
Nuevo León México
martha.acostamnt@uanl.edu.mx

F. Gonzalez-Longatt
*Department of Electrical Engineering,
Information Technology and Cybernetics*
University of South-Eastern Norway
Porsgrunn, Norway
flongatt@flongatt.org

S. Denysiuk, H. Strelkova
Power Supply Department
IEE Igor Sikorsky KPI
Kyiv, Ukraine
spdens@ukr.net

Abstract— The Nordic power system is continuously changing, and it has been experiencing a growing replacing of conventional power plants with renewable power plants, this together with other factors are causing reduction of the total inertia of the Nordic power system. The application of technologies that emulates the dynamic response of the synchronous generators has been a feasible solution. This paper focuses on finding the bests control adjustment of the fast-active power injection/absorption (FAPIA) model by using an optimization algorithm. The FAPIA model has two frequency sensible control actions: a proportional control ($K-f$) and a derivative control ($K-df/dt$). The optimization problem is defined using the gains of the proportional and derivative control together with the volume of FAPIA model contribution as decision variables. Two objective functions are determined based on two system frequency response indicators: minimum frequency, the and steady-state frequency. A simplified version of the Nordic power system is implemented for system frequency response studies.

Keywords— Fast active power injection/absorption, frequency control, frequency response indicators, low inertia, optimization.

I. INTRODUCTION

The constant evolving of the Nordic Power System (NPS) has raised several challenges related to frequency control. Particularly, one of the challenges that are facing the NPS is the reduction of the total system inertia [1], [2]. The total system inertia has been decreased due to the increase of the wind power plants installed in the NPS, closure of thermal power plants, closure of Swedish nuclear power plants and more than 50% increase of the capacity of the interconnector between the NPS and other power systems [3], [4]. It has been estimated that the total kinetic energy of the NPS will be below 120–145 GW·s around 1–19% of the time, this value will be depending on the climate year [5], [6]. Therefore, it is essential to develop a simple and robust methodology that allows facing a low inertia problem.

An alternative to deal with the low inertia in the NPS is to insert synthetic inertia (SI) to the system by using SI controllers [7]. The SI controller family uses a derivative function as its main controller; however, it has several disadvantages in wind turbine implementations and other forms of power converter interfaced technologies [8], [9].

Consequently, an efficient procedure to deal with the low inertia and improve the system frequency is the fast-active power injection/absorption (FAPIA) controller. It is a frequency

sensitive controller that imitate the inertia response of the synchronous generators by using a proportional ($K-f$) and derivative ($K-df/dt$) control actions [4]. The extremely fast response within 1 second and the very short time-delay related to measurement are two advantages that highlight the FAPIA controller performance.

This paper presents the concept of FAPIA controller and an intensive-search methodology to quantify the volume of FAPIA require to fulfil the future low inertia scenario of the Nordic Power System. However, finding the correct settings of FAPIA controllers and the total FAPIA volume represent a challenge. The principal objective of this research paper is finding the proper settings of FAPIA controllers in the low inertia scenario by using the interior-point optimization algorithm. Moreover, carried out an assessment of the frequency response indicator when the total system inertia decreases.

The paper is organized as follows: A full description of the FAPI controller is presented in Section II. Section III presented a methodology applied to optimize the FAPI controller parameters. Two objective functions are defined base of two frequency response indicators: steady-state frequency and minimum frequency. Section IV describes the simplified model of the NPS used to evaluate the proposed objective functions and assessed the frequency response indicators. Section V presents a discussion of the results obtained by assessing the frequency response indicators in low inertia scenario. Furthermore, the optimization solution for the FAPIA controller parameters is described and evaluated. Section VI presents the main conclusion of this research paper.

II. FAST ACTIVE POWER INJECTION/ABSORPTION

The modern power converters (MPCs) based on voltage source converters can modify the active power production in a limited spectrum defined by the availability of energy in the DC side of the MPC [3][10], this MPC power converter is distinguished by its fast response [1]. The fast-active power injection/absorption (FAPIA) is a frequency response model that is activated by the controller in the MPC [11], [5]. The main purpose of using the FAPIA controller is to fast compensate the frequency deviation by (i) injecting active power when the system has an under-frequency condition and (ii) absorbing active power when the system has an over-frequency condition. In general, the FAPIA controller uses the frequency deviation as an input and the output is the active power to be delivered. The FAPIA controller follows the classic linear control theory and

implement two control actions: (i) proportional control action and (ii) derivative control action.

A. Proportional control action (K - f control)

The proportional control action (K - f control) is a linear feedback control in which the active power injection is controlled by a correction factor (gain). The gain is proportional to the frequency deviation (Δf), i.e., the difference between the nominal frequency (f_0) and the current measured frequency (f). The classical power-frequency (P - f) characteristic, considering a dead band and saturation, of the K - f control used in FAPI controller is shown in Fig. 1.

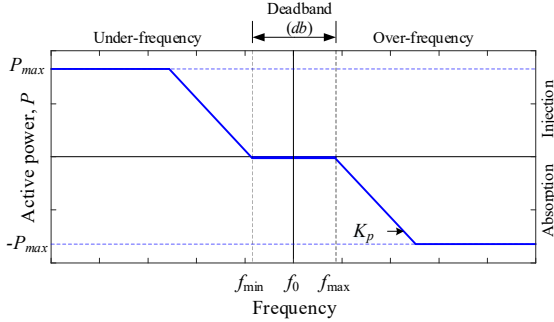


Fig. 1. Classical power-frequency (P - f) characteristic of the K - f control implanted un the FAPI controller.

The K - f control action considering the deadband, shown in Fig. 1, can be mathematically described as:

$$P = \begin{cases} 0 & \text{if } f_{\min} \leq f \leq f_{\max} \\ K_p f & \text{if } f > f_{\max} \\ -K_p f & \text{if } f < f_{\min} \end{cases} \quad (1)$$

The physical limitations of the MPCs are included by writing two physical restrictions as follows

$$P = \begin{cases} P_{\max} & \text{if } P \geq P_{\max} \\ -P_{\max} & \text{if } P \leq -P_{\max} \end{cases} \quad (2)$$

The isolated effect of varying the K_p values in a range $[K_{p,\min}, K_{p,\max}]$ is reflected on the P - f characteristic, in which the slope changes, this change is illustrated in Fig. 2. Moreover, Fig. 3 presents the power-time (P - t) characteristic considering K_p variations in the range $[K_{p,\min}, K_{p,\max}]$.

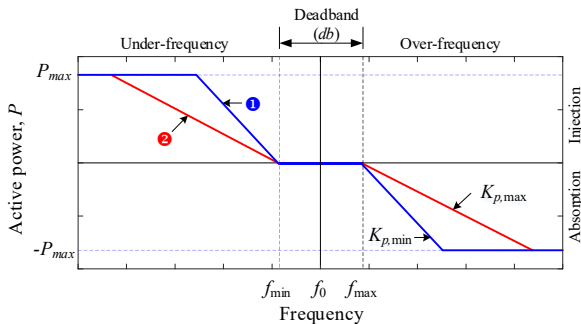


Fig. 2. Classical P - f characteristics of the K - f control considering K_p variations, i.e., $K_p \in [K_{p,\min}, K_{p,\max}]$.

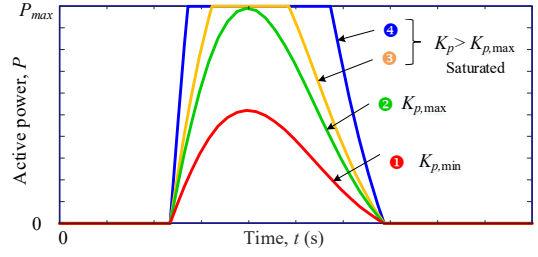


Fig. 3. Power-time (P - t) characteristics of the K - f control K_p variations, i.e., $K_p \in [K_{p,\min}, K_{p,\max}]$.

From Fig. 3, it can be observed that as the values of K_p increases the active power (P) delivered by the FAPIA controller also increases during frequency events outside the deadband. Moreover, high values of K_p tend to increase the speed at which P reaches its maximum limit, i.e., the FAPIA controller reaches its active power production limits faster than low values of K_p . Meanwhile, low values of K_p make the frequency response slower and lower.

B. Derivative control action (K - df/dt control)

The derivative control action (K - df/dt control) refers to the rate of change of the error concerning the time. The K - df/dt control is used to overcome the drawbacks presented in the K - f control. The K - df/dt control is mainly related to the rate of change of frequency ($ROCOF$), i.e., $ROCOF = df/dt$. Using this control action on the FAPIA controller allows that the FAPIA controller behavior emulates the dynamic of the synchronous generator.

The active power injected/absorbed by the FAPIA controller during the system frequency disturbance is calculated using a mathematical expression analogous to the swing equation of a synchronous generator:

$$\Delta P = K_d \frac{df(t)}{dt} \quad (3)$$

where K_d represents the gain of the derivative controller, sometimes called synthetic inertia (seconds) and f is locally measured frequency (p.u) and ΔP represents injected/absorbed active power by the FAPIA controller.

C. Combining control actions

A combination of the K - f control and K - df/dt control would take advantage of the dynamic performance of both functions; it is mathematically formulated as:

$$\Delta P = K_p \Delta f - K_d \frac{d\Delta f}{dt} \quad (4)$$

where term K_p represents the gain of the K - f control of the FAPIA controller without considering saturation and deadband, and the term K_d represents the inertial contribution of active power which is proportional to the $ROCOF$ of the system.

The frequency responsive term K_p has a strong influence on the steady-state frequency, and the $ROCOF$ mainly influences the K_d term after the disturbance. The effect of K_d term is equivalent to an increase in the system's kinetic energy, and therefore it reduces the initial $ROCOF$.

III. OPTIMIZATION OF THE FAPIA

The frequency response of a power system can be significantly modified by the appropriate volume of frequency response. However, defining the appropriate settings of FAPIA controllers and the total FAPIA contribution can be represented as an optimization problem. In this section, an optimization approach is used to define the optimal values of the FAPIA controllers: K_p and K_d and the total volume of FAPIA contribution α_{FAPI} . The next section presents the system frequency response indicators and then the optimization problem is presented.

A. System Frequency Response

The system frequency response of a power system is mainly evaluated by employing the dynamic response (time-domain plots) of the system frequency caused by a disturbance in the power system. The system frequency response is evaluated by three main indicators [12], [13]:

- (i) *Minimum frequency (f_{min})*: The minimum value that the frequency reaches in the dynamical response after a disturbance.
- (ii) *Steady-state frequency (f_{ss})*: is the value in which the frequency settles (final value) after the dynamical response.
- (iii) The rate of change of frequency (*ROCOF*) represents the speed at which the frequency changes concerning with the time $ROCOF = df/dt$ [Hz/s].

B. Optimization approach

The optimization problem in this paper is formulated as a continuous optimization problem because the decision variables are continuous. The decision vector \mathbf{x} has three main decision variables, the K_d the gain of the derivative control action, K_p represents the proportional control action of the FAPIA controller, and the volume of FAPIA controller contribution is α_{FAPIA} .

$$\mathbf{x} = [K_p \quad K_d \quad \alpha_{FAPIA}]^T \quad (5)$$

Acting on the three decision variables, the active power contribution of the FAPIA is defined, and the frequency response can be modified.

The decision variables (x_i) are restricted in order to fulfil two main conditions: (i) reduce the searching space and (ii) more important to keep the parameters inside realistic physical values. Therefore, the bound are defined as:

$$\mathbf{x}_{lower} \leq \mathbf{x} \leq \mathbf{x}_{upper} \quad (6)$$

where $x_{lower,i}$ and $x_{upper,i}$ define the upper and lower bounds of the i -th decision variable.

The previous sections defined the main indicators to consider in the system frequency response. Now, those indicators are used to define the objective function. In this paper, two objective functions are examined:

1) Minimizing steady-state frequency deviation

The steady-state frequency (f_{ss}) is a very important indicator as it is defined by the droop used in the governors of the synchronous generators and the size of the system frequency disturbance (ΔP). In this paper, the steady-state frequency

deviation (Δf_{ss}) is minimized using the difference between the nominal frequency and the steady-state frequency. The objective function is written as:

$$\min(\Delta f_{ss}) = \min[\|f_0 - f_{ss}\|] \quad (7)$$

However, using the equation above the steady-state frequency will ideally back to the rated frequency, f_0 , and that is an unsatisfactory solution because the FAPIA controller will be used for frequency control not to overlap the Automatic Generation Control (AGC). The AGC is responsible for modified the active power injections of the synchronous machines at post-contingency to recover the steady-state frequency to the nominal frequency, f_0 . Consequently, this paper considers the FAPIA contribution to recover the steady-state frequency into a pre-defined frequency f_{set} , and the objective function expressed in (7) now is written as:

$$\min(\Delta f_{ss}) = \min[\|f_{set} - f_{ss}\|] \quad (8)$$

2) Minimizing the deviation of the minimum frequency

The minimum frequency (f_{min}) is an important indicator of the frequency response. This indicator represents the minimum frequency that the power system reaches after a disturbance. In general, the synchronous generators maintain a continuous operation when the frequency is inside its operative limits, i.e., $f_L \leq f_0 \leq f_U$ where f_0 is the nominal frequency, f_L and f_U represent the lower and upper limit of frequency. If the frequency reaches values below f_L , the under-frequency protection of the synchronous generator can be activated.

Therefore, minimizing the deviation of the minimum frequency (difference between f_{min} and the f_L), Δf_{min} , ensures that f_{min} will not take values below f_L and therefore avoiding the activation of the under-frequency protection of the synchronous generators. The objective function to minimize the deviation of the minimum frequency is written as:

$$\min(\Delta f_{min}) = \min[\|f_L - f_{min}\|] \quad (9)$$

IV. SYSTEM FREQUENCY RESPONSE MODEL

The Nordic power system is composed of four countries: Norway, Sweden, Denmark and Finland. Typically, the NPS has a simplified model represented by three control areas (i) Area 1: Sweden, (ii) Area 2: Norway and (iii) Area 3: Finland. An illustrative diagram of the simplified NPS is presented in Fig. 4. Each control area is considered to have local generation and load, and interconnection transmission lines are included between Norway-Sweden and Sweden-Finland.



Fig. 4. Simplified model of the NPS. ΔP_{tie} is used to represent the incremental active power change during a system frequency event

The details of the block diagram of a single control area are presented in Fig. 5. $G_{gov}(s)$ is the transfer function of the governor, $G_{turb}(s)$ is the transfer function of the hydro turbine and the term $1/Ms+D$ (where M is the equivalent moment of inertia and D represents the power demand of the frequency-

dependent loads) represent the transfer function of the generator and load.

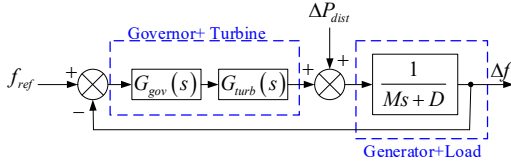


Fig. 5. Block diagram of a single control area: The main controllers involved in the frequency response are represented in the control area. ΔP_{dist} represents the system frequency disturbance.

In this paper, the equivalent model of Area 1, Area 2 and Area 3 has been implemented using MATLAB® R2019b for the proposes of assessing the frequency response indicators and evaluate the two objective functions proposed in Section III. The full details of the model and the parameters are described in [6].

V. SIMULATIONS AND RESULTS

The implemented equivalent model of the Nordic Power system is used to assess the sensitivity analysis of the frequency response indicator in the scenario of low inertia. Moreover, the optimal settings for the FAPIA controller are obtained by using the two objective function defined in Section III.B.

A. Sensitivity analysis of the frequency response indicators

The frequency response indicators are affected by the decreasing of the total system inertia in the NPS. Therefore, this section is dedicated to carrying on a sensitivity analysis of the frequency response indicators. For this study, a sudden step increases in the load demand (ΔP) is considered as a frequency disturbance.

The frequency response is assessed by observing its indicators: (i) minimum frequency (f_{min}), (ii) steady-state frequency (f_{ss}) and (iii) maximum *ROCOF*. The load demand increase is $\Delta P=0.0280$ p.u. and the nominal inertia is $H_0=4.84$ seconds. The nominal inertia is gradually decreased until the frequency response cannot be recovered and the NPS is unstable. Fig. 6 show the behavior of f_{min} when the system inertia of the NPS is progressively reduced. From this figure, it can be concluded that as long as the system inertia is smaller, the frequency deviation increases and therefore f_{min} reaches lower values for the same frequency disturbance.

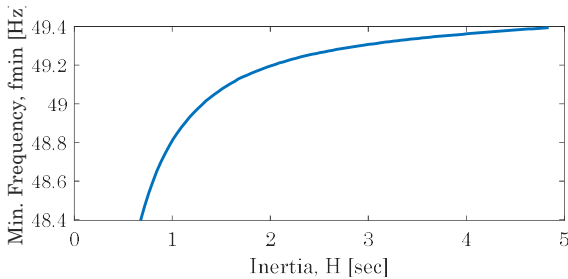


Fig. 6. Minimum frequency (f_{min}) behavior by gradually decreasing the system inertia of the NPS.

Meanwhile, if the system inertia is smaller, the maximum *ROCOF* increases for the same step increases in the load demand, it is shown in Fig. 7. Finally, the frequency response

of the NPS is gradually deteriorated when the system inertia is decreasing. The unstable frequency response means that the kinetic energy stored in the rotating masses of the NPS is not enough to recover the frequency as it is presented in Fig. 8.

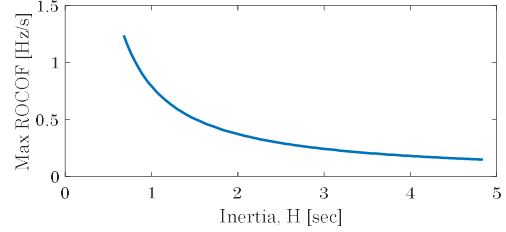


Fig. 7. Maximum *ROCOF* behavior by gradually decreasing the system inertia of the NPS.

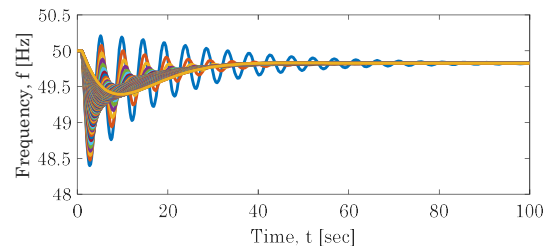


Fig. 8. Frequency response by gradually decreasing the system inertia of the NPS.

The sensitivity analysis highlights the downsides of having a low level of inertia in the power system and how it deteriorates the power system security in case of a disturbance. Moreover, for the test system, the minimum inertia that the NPS can tolerate is $H_{min}=0.6776$ seconds which represent 14% of the nominal inertia (H_0).

B. Optimization solution of the two objective functions proposed

This section is dedicated to evaluating the solution of the two objective functions proposed in Section III.B. The objective of evaluating the proposed objective functions is to obtain optimal setting of the FAPIA controller parameters (K_p , K_d and α_{FAPIA}) for a given disturbance. The interior-point optimization algorithm given in the MATLAB® R2019b is applied to solve the optimization problem. Details of the interior-point optimization algorithm are described in [14], [15]. The frequency disturbance is $\Delta P= \Delta P= 0.0280$ p.u., representing a sudden step, increases in the load demand. Moreover, the total system inertia is set 50% of the nominal inertia; therefore, $H =0.5H_0=2.42$ seconds.

1) Minimizing steady-state frequency deviation

The objective function defined in (8) to minimize the steady-state frequency deviation is evaluated. For illustrative purposes in this paper, the pre-defined steady-state frequency is $f_{ser}=49.9$ Hz.

The optimization solution provides a decision variable vector as $\mathbf{x} = [2.5172 \ 2.2628 \ 2.5173]$ which represent the optimal settings of the FAPIA controller. The first element of the vector is the gain of the K - f control, $K_p=2.5172$. The second element represents the gain of the K - df/dt control, i.e.,

$K_d=2.2628$. The third element of the vector is the volume of FAPIA controller, and it is $\alpha_{FAPIA}=2.5173$.

The FAPIA controller parameters are set using the results of the optimization, and a frequency disturbance is inserted in the NPS. The frequency response is shown in Fig. 9. The minimum frequency that the NPS reaches is $f_{min}=49.836$ Hz, the maximum *ROCOF* is -0.144 Hz/sec, and the steady-state frequency is $f_{ss}=49.9$ Hz demonstrating that the optimal setting of the FAPIA controller parameters are fulfilling the minimization equation describe in (8) and the steady-state deviation is $\Delta f_{ss}=0$ Hz.

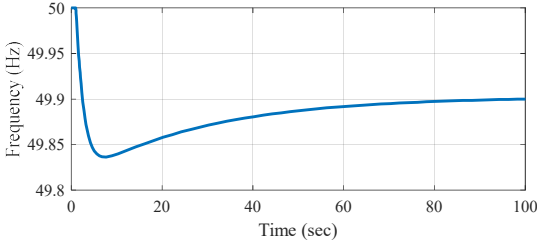


Fig. 9. Frequency response using the optimal settings for minimizing the steady-state frequency deviation (Δf_{ss}).

2) Minimizing the deviation of the minimum frequency

The minimization of the minimum frequency deviation of the NPS is performed using the equation defined in (9). For this study, the lower limit of frequency is selected as $f_L=49.5$ Hz; this value is chosen for illustrative proposes.

The decision variable vector for the objective function defined in (9) that contains the optimal settings of the FAPIA controller is $\mathbf{x} = [0.7166 \ 0.6927 \ 0.7133]$. The gain of the *K-f* control is $K_p=0.7166$, the gain of the *K-df/dt* control is $K_d=0.6927$, and the volume of FAPIA controller is $\alpha_{FAPIA}=0.7133$.

Fig. 10 present the frequency response for a frequency disturbance using the optimal settings for the FAPIA controller parameters. It is observed that the minimum frequency does not exceed the lower limit of frequency (f_L) demonstrating that the optimal setting of the FAPIA controller are fulfilling the minimization equation describe in (9), in fact, the f_{min} that the NPS reaches is $f_{min}=49.5$ Hz and $\Delta f_{min}=0$ Hz. The maximum *ROCOF* is -0.144 Hz/sec and the steady-state frequency is $f_{ss}=49.836$ Hz.

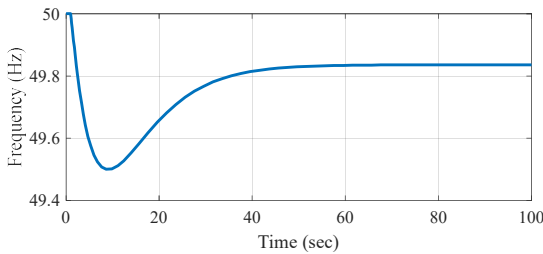


Fig. 10. Frequency response using the optimal settings for minimizing the minimum frequency deviation (Δf_{min}).

VI. CONCLUSIONS

The frequency response indicators are deteriorating in low inertia scenarios. Creating a sensitivity analysis of those

indicators can help to prevent the minimum inertia in the power system in order to ensure frequency stability when a disturbance occurred. The optimal setting of FAPIA controller parameters obtained from the optimization solution fulfils the requirements of the pre-set frequency values for the two objective functions. The optimal setting of the FAPIA controller depends only on the pre-defended steady-state value and the pre-defended frequency limits in the dynamic response. Furthermore, using the two objective function allows ensuring that the two frequency response indicators (steady-state frequency and minimum frequency) do no exceeds certain pre-set values.

ACKNOWLEDGMENT

Ms Martha N. Acosta would like to acknowledge the financial support given by CONACYT (México) as well as the support of Universidad Autónoma de Nuevo León, Mexico, and University of South-Eastern Norway, Norway.

REFERENCES

- [1] F. Sanchez, J. Cayenne, F. Gonzalez-Longatt, and J. L. Rueda, "Controller to Enable the Enhanced Frequency Response Services from a Multi-Electrical Energy Storage System," *IET Gener. Transm. Distrib.*, Nov. 2018.
- [2] H. Chamorro, F. Gonzalez, K. Rouzbehi, R. Sevilla, H. Chavez, and V. Sood, "Innovative Primary Frequency Control in Low-Inertia Power Systems Based on Wide-Area RoCoF Sharing," *IET Energy Syst. Integr.*, Feb. 2020.
- [3] T. Krechel, F. Sanchez, F. Gonzalez-Longatt, H. Chamorro, and J. L. Rueda, "A Transmission System Friendly Micro-grid: Optimising Active Power Losses," in *13th IEEE PES PowerTech Conference*, 2019.
- [4] A. J. Veronica, N. S. Kumar, and F. Gonzalez-Longatt, "Design of Load Frequency Control for a Microgrid Using D-partition Method," *Int. J. Emerg. Electr. Power Syst.*, vol. 21, no. 1, Feb. 2020.
- [5] F. Gonzalez-Longatt, "Effects of Fast Acting Power Controller of BESS in the System Frequency Response of a Multi-Machine System: Probabilistic Approach," in *International Conference on Innovative Smart Grid Technologies (ISGT Asia 2018)*, 2018.
- [6] L. Saarinen, P. Norrlund, U. Lundin, E. Agneholm, and A. Westberg, "Full-scale test and modelling of the frequency control dynamics of the Nordic power system," in *2016 IEEE Power and Energy Society General Meeting (PESGM)*, 2016, vol. 2016-Novem, pp. 1–5.
- [7] H. R. Chamorro, I. Riaño, R. Gerndt, I. Zelinka, F. Gonzalez-Longatt, and V. K. Sood, "Synthetic inertia control based on fuzzy adaptive differential evolution," *Int. J. Electr. Power Energy Syst.*, vol. 105, pp. 803–813, Feb. 2019.
- [8] H. R. Chamorro, A. C. Sanchez, A. Pantoja, I. Zelinka, F. Gonzalez-Longatt, and V. K. Sood, "A network control system for hydro plants to counteract the non-synchronous generation integration," *Int. J. Electr. Power Energy Syst.*, vol. 105, pp. 404–419, Feb. 2019.
- [9] A. J. S. J. Veronica, N. S. Kumar, and F. Gonzalez-Longatt, "Robust PI controller design for frequency stabilisation in a hybrid microgrid system considering parameter uncertainties and communication time delay," *IET Gener. Transm. Distrib.*, vol. 13, no. 14, pp. 3048–3056, Jul. 2019.
- [10] B. Sri Revathi, P. Mahalingam, and F. Gonzalez-Longatt, "Interleaved high gain DC-DC converter for integrating solar PV source to DC bus," *Sol. Energy*, vol. 188, pp. 924–934, Aug. 2019.
- [11] F. Gonzalez-Longatt, J. Rueda, and E. Vázquez Martínez, "Effect of Fast Acting Power Controller of Battery Energy Storage Systems in the Under-frequency Load Shedding Scheme," in *International Conference on Innovative Smart Grid Technologies (ISGT Asia 2018)*, 2018.
- [12] F. Gonzalez-Longatt, F. Sanchez, and Rujiroj Leelarui, "Unveiling the Character of the Frequency in Power Systems," in *IEEE-PES GTD Grand International Conference & Exposition Asia 2019 (IEEE-PES GTD Asia 2019)*, 2019.
- [13] F. Gonzalez-Longatt, J. L. Rueda, and E. Vázquez Martínez, *Effect of Fast Acting Power Controller of Battery Energy Storage Systems in the Under-frequency Load Shedding Scheme*. Loughborough University, 2018.
- [14] T. Steihaug, "The Conjugate Gradient Method and Trust Regions in Large Scale Optimization," *SIAM J. Numer. Anal.*, vol. 20, no. 3, pp. 626–637, Jun. 1983.
- [15] R. H. Byrd, J. C. Gilbert, and J. Nocedal, "A trust region method based on interior point techniques for nonlinear programming," *Math. Program. Ser. B*, vol. 89, no. 1, pp. 149–185, 2000.