



Prediction of void fraction and minimum fluidization velocity of a binary mixture of particles: Bed material and fuel particles

Cornelius Emeka Agu^{a,*}, Christoph Pfeifer^b, Britt M.E. Moldestad^a

^a Department of Process, Energy and Environmental Technology, University of South-Eastern Norway, 3918 Porsgrunn, Norway

^b Department of Material Sciences and Process Engineering, University of Natural Resources and Life Sciences, 1190 Vienna, Austria

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ABSTRACT

For operational control and design of a fluidized bed reactor containing different types of solid particles, the bed void fraction and minimum fluidization velocity are vital parameters. This paper demonstrates a method for predicting the void fraction and minimum fluidization velocity of different binary mixtures of particles with improved accuracy. A new model for predicting the void fraction is presented. This model is non-linear and continuous, and it is developed by introducing a packing factor and establishing a mass balance between the solid phases in the packing environment. The results show that the model can accurately predict the void fraction of a binary mixture where the particles are well mixed, partially mixed or segregated. Using this void fraction model and the Ergun equation of pressure drop, the minimum fluidization velocity can be predicted with mean errors of 15.2% for a mixture of two inert materials and 7.0% for a mixture of biomass and inert particles.

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1. Introduction

Some fluidized bed applications involve different types of solid particles. The difference in properties between these particle types may influence the bed behaviour. For example, in a bubbling fluidized bed biomass gasifier with sand as the bed material, the difference in density, size and shape between biomass and sand particles often leads to particle segregation [1]. Particle segregation in a biomass gasification reactor can also be influenced by devolatilization of the fuel particles and formation of bubbles around the particles [2,3]. For a bed of coarse particles characterized with large exploding bubbles, the quality of the fluidization can be improved by adding some amount of fine particles of the same material [4]. Due to the size difference between the fine and coarse particles, the void fraction of the mixture is lower than that of the coarse particles, resulting in flow of smaller bubbles in the fluidized bed. In addition to changes in bubbling behaviour, the difference in properties between different solid types in a bed also influences the minimum fluidization velocity of the bed. For operational control and design of a reactor containing two or more solid phases, the bed minimum fluidization velocity is a key parameter.

The minimum fluidization velocity of a bed of particles of the same size and density can be predicted using different correlations. Most of these correlations [5–7] were derived from the Ergun [8] equation but are independent of the bed void fraction. For a binary mixture of

particles, similar correlations for predicting minimum fluidization velocity are also available [9–11]. Moreover, there are other models, which involve interpolations between the minimum fluidization velocities of the pure components [12,13]. Li et al. [14] and Asif [15] however, showed that the minimum fluidization velocity of a binary mixture can be predicted with a better accuracy by using a model that considers the void fraction. One major challenge in this approach is prediction of the bed void fraction at minimum fluidization condition. For a completely mixed binary system, the Westman [16] model can be used to predict the bed voidage with a good accuracy when the density difference between the solids in the mixture is very small [14]. In a bed where segregation occurs due to density difference, the Westman [16] model is inappropriate [15]. The void fraction of a completely mixed bed of two solid phases can also be predicted using other models classified as 2-parameter [17,18], compressible [19] and 3-parameter [20] models. These models are linear, and according to Chan and Kwan [21], their accuracies depend on the size ratio between the two size classes in the mixture. Moreover, each of these models comprises a set of two equations, which are solved separately to determine the mixture void fraction based on the maximum value in the solution set. The models are thus discontinuous over the entire range of mixture composition.

This study therefore presents a new model for predicting the void fraction of a binary mixture and how it can be used to improve the prediction of minimum fluidization velocity of the mixture independent of particle types. The proposed model is analytically developed based on the mass balance between two solid phases in a packing environment. In developing the model, it is assumed that the smaller particles first fill the available void without changing the volume occupied by the

* Corresponding author.

E-mail addresses: cornelius.e.agu@usn.no (C.E. Agu), christoph.pfeifer@boku.ac.at (C. Pfeifer), britt.moldestad@usn.no (B.M.E. Moldestad).

larger particles in static conditions. The excess of these particles then occupies the space above the larger particles. On the basis that there is a limit to which solids can be packed in a given space, a packing factor is introduced. The packing factor compares the actual mass of smaller particles contained in the void of larger particles with the mass that would have occupied the maximum available void space. An expression for the packing factor is derived as a function of mass fraction of the smaller particles, the particle size ratio, and an interaction parameter between the two particle types in the mixture. The interaction parameter also depends on the size ratio as well as the density ratio between the particles. With the correlation proposed for the interaction parameter, which is obtained using some data in the literature, prediction of the mixture void fraction from the proposed model contains no adjustable parameter. For different binary mixtures, the results of the model are compared against experimental data in the literature.

2. Minimum fluidization velocity and mixture properties

Similar to pure solid components, the minimum fluidization velocity of a binary mixture of solids is generally obtained from curves of pressure drop against the superficial gas velocity. The measured minimum fluidization velocity depends on the procedure employed (i.e. whether the pressure drop is measured at increasing or decreasing gas velocity) and on the distribution of solids at the fixed bed condition [22]. The pressure drop curve at increasing gas velocity usually reviews the transition between the initial and full fluidization condition of the bed due to segregation effects. For this reason, several authors [23,24] reported the onset of full fluidization as the true minimum fluidization considering that the whole bed is capable of being fluidized beyond this gas velocity. However, for a well-mixed system, the difference between the initial and full fluidization velocities is insignificant [14]. To avoid the initial bed effect associated with increasing gas velocity procedure, the minimum fluidization velocity can be measured based on decreasing gas velocity procedure. Despite the measurement procedure, the reported minimum fluidization velocity for a given binary mixture often lies between those of the pure components of the mixture. Marzocchella et al. [23] concluded that neither of the initial and full fluidization velocities is related to the minimum fluidization velocities of the individual solids in the mixture. This means that the mixture minimum fluidization velocity is a weighted average of the pure component values [22]. This section presents the models for predicting the average minimum fluidization velocity of a binary mixture of particles and the corresponding bed void fraction. The average minimum fluidization velocity lies between the initial and full fluidization velocity, and it can be obtained from the pressure drop curve at the intersection of two extrapolation lines drawn through the fixed bed and fluidized bed conditions as noted in the literature.

2.1. Models for minimum fluidization velocity

At minimum fluidization, the required superficial gas velocity can be obtained from the force balance between the bed weight and the upward force exerted by the fluid on the particles. Using the Ergun [8] equation, the minimum fluidization velocity of a bed of mono-sized particles can be computed from

$$\frac{1.75}{\varphi_s \varepsilon_{mf}^3} \left(\frac{\rho_f U_{mf} d_s}{\mu_f} \right)^2 + \frac{150(1-\varepsilon_{mf})}{\varphi_s^2 \varepsilon_{mf}^3} \left(\frac{\rho_f U_{mf} d_s}{\mu_f} \right) = Ar \quad (1)$$

$$Ar = \frac{d_s^3 \rho_f (\rho_s - \rho_f) g}{\mu_f^2} \quad (2)$$

here, U_{mf} and ε_{mf} are the superficial fluid velocity and bed void fraction at minimum fluidization condition, respectively. While d_s is the particle diameter, φ_s is the particle sphericity, and ρ_f and ρ_s are the fluid and

particle densities, respectively. For a binary mixture, different correlations are derived from Eq. (1) for predicting the mixture minimum fluidization velocity, where the particle diameter, density and sphericity are replaced with their equivalent average properties. Some of these correlations are given in Table 1.

As shown in Table 1, there are different expressions for the average bed properties. For the methods based on the Ergun [8] equation, it can be shown briefly that the volume-average particle density and the surface-to-volume mean diameter are appropriate.

Considering a bed containing different types of particles with d_{si} , φ_{si} and ρ_{si} the particle diameter, sphericity and density of each particle type, respectively, the total specific surface area, a of the bed can be expressed as

$$a = \sum \left(\frac{6}{\varphi_{si} d_{si}} \alpha_i \right) \quad (3)$$

giving the hydraulic diameter of the bed as

$$D_h = \frac{4\varepsilon_m}{\sum \left(\frac{6}{\varphi_{si} d_{si}} \alpha_i \right)} \quad (4)$$

where α_i is the solid volume fraction of the individual particle type and ε_m is the mean void fraction averaged over the bed height. With $1 - \varepsilon_m = V_b/V$ and $\alpha_i = (V_{bi}/V_b)(V_b/V)$, it can be shown that

$$\alpha_i = \frac{\rho_{sm}}{\rho_{si}} x_i (1 - \varepsilon_m) \quad (5)$$

$$\rho_{sm} = \frac{1}{\sum \frac{x_i}{\rho_{si}}} \quad (6)$$

where x_i is the mass fraction of each type of particles and ρ_{sm} is the mean density of the solid mixture. Using Eqs. (4) and (5), a modified Ergun [8] equation can be expressed as

$$\frac{\Delta p}{\Delta L} = 150 \frac{\mu_f U_0 (1 - \varepsilon_m)^2}{\varepsilon_m^3 \left(\rho_{sm} \sum \frac{x_i}{\rho_{si} \varphi_{si} d_{si}} \right)^{-2}} + 1.75 \frac{\rho_f U_0^2 (1 - \varepsilon_m)}{\varepsilon_m^3 \left(\rho_{sm} \sum \frac{x_i}{\rho_{si} \varphi_{si} d_{si}} \right)^{-1}} \quad (7)$$

Comparing Eq. (7) with the Ergun [8] equation derived for a bed of mono-sized particles, the equivalent mean particle size (Sauter mean diameter) d_{smeq} for a bed of different types of particles is given by

$$\frac{1}{d_{smeq}} = \rho_{sm} \sum \frac{x_i}{\rho_{si} \varphi_{si} d_{si}} \quad (8)$$

From the definition of particle sphericity, φ_s , as the ratio of surface area of a sphere to surface area of a particle of the same volume as the spherical particle [4], it means that d_{si} is the volume-equivalent spherical particle diameter of the individual solid in the mixture. Hence, the average volume-equivalent spherical particle diameter, d_{sm} of the mixture can be obtained as

$$\frac{1}{d_{sm}} = \rho_{sm} \sum \frac{x_i}{\rho_{si} d_{si}} \quad (9)$$

and the average mixture particle sphericity φ_{sm} as

$$\varphi_{sm} = \frac{d_{smeq}}{d_{sm}} \quad (10)$$

For a spherical particle, $\varphi_s = 1$, and if all the particles are spherical, $\varphi_{sm} = 1$. The particle sphericity can be found experimentally or computed from the particle geometry if well defined [4].

Table 1
Correlations for predicting the minimum fluidization velocity in binary mixtures.

Correlation	Application	Reference
$U_{mf} = U_{mf2} \left(\frac{U_{mf1}}{U_{mf2}} \right)^{y_1^2}$	All binary mixtures	Cheung et al. [12]
$U_{mf} = \frac{1}{\sum \left(\frac{y_j}{U_{mfj}} \right)}$	All binary mixtures	Rincon et al. [13]
$U_{mf} = \frac{d_{sm}^2 (\rho_{sm} - \rho_f) g}{1650 \mu_f}$; $\rho_{sm} = \sum (x_i \rho_{si}); k = 20d_{s2} + 0.36$	Biomass - inert mixture	Rao & Bheemarasetti [29]
$d_{sm} = k^{1/2} d_{s2} \left(\frac{d_{s1} \rho_{s2}}{d_{s2} \rho_{s1}} \right)^{x_1/x_2}$; $Re_{mf} = (C_1^2 + C_2 Ar_m)^{1/2} - C_1$; $C_1 = 25.65 (\varphi_{s1}^{0.25} \varphi_{s2}^{0.15})$; $C_2 = 0.056 (\varphi_{s1}^{-0.045} \varphi_{s2}^{0.025})$; $\rho_{sm} = \sum (x_i \rho_{si})$; $d_{sm} = d_{s1} d_{s2} \left(\frac{x_1 \rho_{s2} + x_2 \rho_{s1}}{x_1 \rho_{s2} d_{s2} + x_2 \rho_{s1} d_{s1}} \right)$	Biomass - inert mixture	Si and Guo [30]
$Re_{mf} = \{30.28^2 + [0.046(1-x_1) + 0.108x_1^{1/2}] Ar_m\}^{1/2} - 30.28$; $\rho_{sm} = \frac{1}{\sum \frac{x_i}{\rho_{si}}}$; $d_{sm} = \rho_{sm} \sum \frac{x_i}{\rho_{si} d_{si}}$	Biomass - inert mixture	Paudel and Feng [31]
$Ar_m = 914.2 \varphi_{sm}^2 Re_{mf}^2 + 14.838 Re_{mf}^2$; $\varphi_{sm} = \sum (x_i \varphi_{si})$; $\rho_{sm} = \sum (x_i \rho_{si})$; $d_{sm} = d_{s1} d_{s2} \left(\frac{x_1 \rho_{s2} + x_2 \rho_{s1}}{x_1 \rho_{s2} d_{s2} + x_2 \rho_{s1} d_{s1}} \right)$	Biomass - inert mixture	Kumoro et al. [32]

$$Ar_m = \frac{d_{sm}^3 \rho_f (\rho_{sm} - \rho_f) g}{\mu_f^2} \text{ and } Re_{mf} = \frac{\rho_f U_{mf} d_{sm}}{\mu_f}$$

The mixture density and particle diameter given in Eq. (6) and Eq. (8) are described as the volume-average particle density and the surface-to-volume mean diameter, respectively. Hence, to obtain the minimum fluidization velocity of a bed of different particle types, ρ_s , d_s and φ_s in Eq. (1) are replaced with the corresponding values for the mixture.

In addition to the correlations given in Table 1, U_{mf} of a binary mixture of particles can also be obtained directly from Eq. (1) when ε_{mf} of the mixture is known. For a completely mixed binary system, the bed void fraction can be obtained from the pure component values using the Westman [16] equation.

$$\left(\frac{v - y_S v_S}{v_L} \right)^2 + 2G \left(\frac{v - y_S v_S}{v_L} \right) \left(\frac{v - y_S - y_L v_L}{v_S - 1} \right) + \left(\frac{v - y_S - y_L v_L}{v_S - 1} \right)^2 = 1 \quad (11)$$

here, y_S and y_L are the volumetric fraction of the smaller and larger particles, respectively, and v_S and v_L are the respective specific volume, where

$$y_j = \frac{\rho_{sm} x_j}{\rho_{sj}}; j = S, L \quad (12)$$

$$v_j = 1/\alpha_j; \varepsilon_m = 1 - 1/v \quad (13)$$

The parameter G can be obtained from the correlation proposed by Yu et al. [25] or Finkers and Hoffmann [26].

2.2. Model development for bed voidage

For direct application of Eq. (1) in a binary mixture of particles, this section introduces a new model for void fraction of the bed mixture.

In a given mixture of two solid phases, we define the packing factor θ as follows:

$$|\theta| = \frac{m_*}{(1 - \alpha_1) \rho_{s2} V_0} \quad (14)$$

where m_* is the mass of the smaller particles occupying the interstices between the larger particles, and V_0 is the initial total volume occupied by the larger particles. The subscripts 1 and 2 denote the larger and smaller particles, respectively. The modulus $|\theta|$ indicates that θ can be

negative or positive. When $\theta < 0$, the bed is contracting and when $\theta > 0$, the bed is expanding. A binary mixture of particles contracts if the bulk volume of the mixture is lower than the sum of the bulk volumes of the two particle types in the mixture. Bed expansion occurs when the volume of an initially well-mixed system increases due to particle segregation. The packing factor is a measure of packing density of a binary system. The larger the value of $|\theta|$, the lower the void fraction of the mixture.

Assuming that N_1 and N_* are the respective number of particles in the packed bed, Eq. (14) can be simplified to

$$\theta = \frac{N_*}{N_1} \frac{\alpha_1}{(1 - \alpha_1)} \left(\frac{d_{s2}}{d_{s1}} \right)^3 \quad (15)$$

where

$$\frac{N_*}{N_1} \approx \frac{\alpha_*}{\alpha_1} \left(\frac{d_{s2}}{d_{s1}} \right)^{-2} \quad (16)$$

Substituting Eq. (16) into Eq. (15) and using the relation, $\alpha_i + \varepsilon_i = 1$, where ε_i is the pure component void fraction, the packing factor can be expressed as

$$\theta = \left(1 - \frac{\varepsilon_1 - \alpha_*}{\varepsilon_1} \right) \left(\frac{d_{s2}}{d_{s1}} \right) \quad (17)$$

when $d_{s2}/d_{s1} = 1$, $m_* = 0$. For Eq. (17) to satisfy this condition, the term $(\varepsilon_1 - \alpha_*)/\varepsilon_1$ must be a function of d_{s2}/d_{s1} in addition to the amount of smaller particles present in the mixture. Thus,

$$\theta = \left(1 - \left(\frac{d_{s2}}{d_{s1}} \right)^{\beta x_2} \right) \left(\frac{d_{s2}}{d_{s1}} \right) \quad (18)$$

here, β can be described as the interaction parameter between the two solid phases. When a bed contracts during solid mixing, the value of m_* is high. On the contrary, m_* is lower when the particles segregate. To account for these effects, $\beta < 0$ for a well-mixed system and $\beta > 0$ for a segregated mixture.

Furthermore, the total mass of the bed is expressed as

$$m = (1 - \varepsilon_m) \rho_{sm} (V_0 + \Delta V) \quad (19)$$

where ΔV is the total volume occupied by the smaller particles above the larger particles, and it can be obtained from

$$\Delta V = \frac{m_2 - m_s}{\alpha_2 \rho_{s2}} \quad (20)$$

here, m_2 is the total mass of the smaller particles in the bed. Substituting Eq. (20) into Eq. (19) and noting that $V_0 = m_1 / (\alpha_1 \rho_{s1})$,

$$(1 - \varepsilon_m) \left[(\alpha_2 - (1 - \alpha_1) \theta) \frac{m_1}{\alpha_1 \rho_{s1}} + \frac{m_2}{\rho_{s2}} \right] = \frac{\alpha_2 m}{\rho_{sm}} \quad (21)$$

$$\varepsilon_m = 1 - \frac{\alpha_2 m}{\rho_{sm} \left[(\alpha_2 - (1 - \alpha_1) \theta) \frac{x_1}{\alpha_1 \rho_{s1}} + \frac{x_2}{\rho_{s2}} \right]}$$

Substituting Eq. (18) into Eq. (21) and replacing the subscripts 1 and 2 with the corresponding letters, yields

$$\varepsilon_m = 1 - \frac{\alpha_s}{\left[(\alpha_s - (1 - \alpha_L) \left(1 - \left(\frac{d_{ss}}{d_{sl}} \right)^{\beta x_s} \right) \left(\frac{d_{ss}}{d_{sl}} \right) \right) \frac{y_L}{\alpha_L} + y_s \right]} \quad (22)$$

Eq. (22) can be used to predict the void fraction in a binary mixture of different particle types. As can be seen, the equation requires the solids/void fraction of the pure components and contains only one fitting parameter, β . The value of β depends on the relative difference between the properties of the different particle types in the mixture and on whether the bed is well mixed, partially mixed or segregated as shown in section 4. It should be noted that the value of ε_m predicted from Eq. (22) is the bed voidage averaged over the bed height which may differ from the local void fractions in the bed. Depending on the particle size ratio, d_{ss}/d_{sl} , the local void fraction can vary along the bed axis due to segregation effect [27]. For a mixture containing biomass particles, the higher the value of d_{ss}/d_{sl} , the wider the deviation of ε_m from the local void fraction at the segregated layers. The accuracy of Eq. (22) with a correctly assigned value of β is demonstrated in section 4.

3. Results and discussion

In this section, the validation of the proposed model, Eq. (22) and its application to predicting the minimum fluidization velocity of a binary mixture are demonstrated using different experimental data from the literature. Since it is often difficult to measure void fractions at

minimum fluidization condition, a systematic procedure in using Eq. (22) to predict the mixture U_{mf} is also highlighted.

3.1. Bed voidage of binary particle mixtures

Fig. 1 compares the void fraction at static condition predicted using Eq. (22) against the experimental data given in Marzocchella et al. [23] for a mixture of glass particles with mean diameter 500 μm and sand particles with mean diameter 125 μm at different mixture compositions. The data obtained from Tharpa et al. [28] at fixed bed condition are also shown for a mixture of 3500 μm plastic and 709 μm zirconium oxide particles. At minimum fluidization condition, the model results are compared against the experimental data obtained from Li et al. [14] and Formisani et al. [22] for different binary mixtures: two glass powders with mean sizes (385 and 163) μm and two glass powders with mean sizes (612 and 154) μm , respectively. The particle properties in these mixtures are shown in Table 2. As can be seen, the results from Eq. (22) strongly agree with the experimental data shown in both figures. With the correlation of Yu et al. [25], the Westman [16] equation also agrees well with the experimental data at the minimum fluidization condition. For the mixtures given in Li et al. [14], the Westman [16] equation and Eq. (22) predict the same results for all values of x_1 (mass fraction of the larger particles). However, for the data obtained at fixed bed condition as shown in Fig. 1(a), the Westman [16] equation does not give good predictions.

Fig. 2 compares the accuracy of Eq. (22) with that of the Westman [16] equation against the experimental data. The experimental data include those shown in Fig. 1 and those obtained from Formisani et al. [22] for a binary mixture of two different glass particles with mean diameters 499 and 271 μm . The figure shows that Eq. (22) predicts the experimental data with a very good accuracy. The mean prediction error associated with Eq. (22) is 1.5%. The prediction error using the Westman [16] equation can be as high as $\pm 15\%$ due to poor prediction of the bed voidage reported in Marzocchella et al. [23] and Tharpa et al. [28] at static conditions. However, the mean errors using the Westman [13] model are 4.0% based on the Yu et al. [25] correlation and 4.1% based on the Finkers and Hoffman [26] correlation.

3.2. Correlation for β

As can be seen in Fig. 1, β varies from one system to another. The individual value of β used in the results is obtained by fitting the experimental data to the model, Eq. (22). To successfully apply Eq. (22) without experimental data, a correlation for β is required. Analysis of some literature data obtained at the minimum fluidization condition

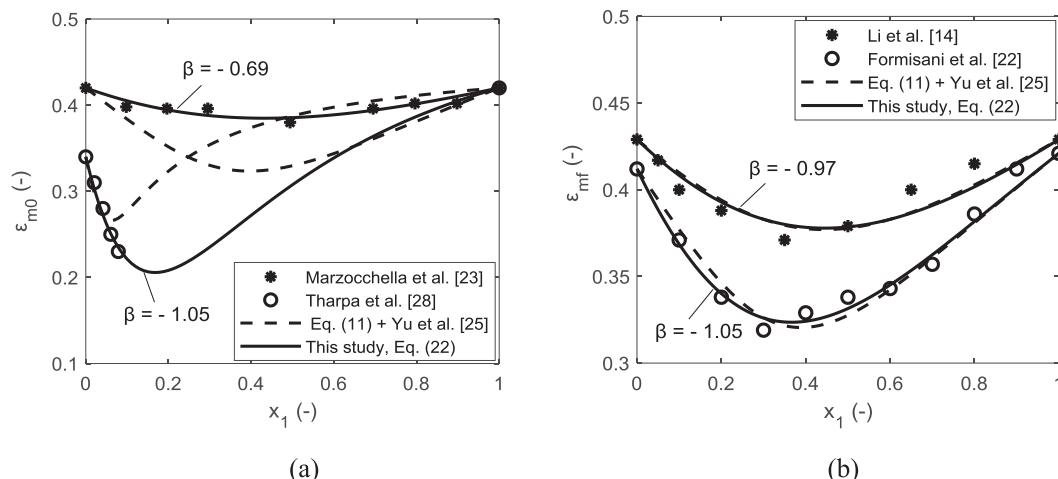


Fig. 1. Voidage variation comparing the predicted results with the experimental data obtained at (a) static bed condition (b) minimum fluidization condition.

Table 2
Properties of pairs of particles in the completely mixed binary mixtures.

Binary mixture	Particles	Shape	ρ_s (kg/m ³)	d_s (μm)	φ_s (-)	U_{mf} (m/s)	Ref.
I	Glass	Spherical	2540	500	1.0	0.225	[23]
	Sand	Spherical	2600	125	1.0	0.0212	
II	Plastic	Spherical	964	3500	1.0	0.85	[28]
	ZrO ₂	Spherical	5850	709	1.0	0.67	
III	Glass	Spherical	2520	385	1.0	0.143	[14]
	Glass	Spherical	2520	163	1.0	0.025	
IV	Glass	Spherical	2480	612	1.0	0.3148	[22]
	Glass	Spherical	2480	154	1.0	0.0232	
V	Glass	Spherical	2480	499	1.0	0.2222	[22]
	Glass	Spherical	2480	271	1.0	0.0602	
VI	H. char	Spherical	1080	775	1.0	0.227	[14]
	Glass	Spherical	2520	165	1.0	0.025	

H. char = hollow char, ZrO₂ = zirconium oxide.

shows that the absolute value of β decreases with the ratio $d_{ss}\rho_{ss}/(d_{sl}\rho_{sl})$ as represented in Eq. (23).

$$\beta = 0.623 \left(\frac{d_{ss}\rho_{ss}}{d_{sl}\rho_{sl}} \right)^{-0.61} \quad (23)$$

In the subsequent sections, β computed from Eq. (23) is used in Eq. (22) for prediction of the mixture void fraction.

3.3. Minimum fluidization velocity of binary mixtures

The results in Fig. 1 show that the voidage of a binary mixture can be predicted with a good accuracy from the void fractions of the pure components. Since accurate prediction of void fraction of a pure component at minimum fluidization condition is a challenge, we present a method where U_{mf} of the solid phases in a binary mixture are inputs to Eq. (22). As illustrated in Fig. 3, ε_{mf} of the pure components are computed from the respective U_{mf} values using Eq. (1). For a given mixture composition (mass fraction or volumetric fraction of the solid phases), the average particle properties and void fraction of the mixture are calculated from the relevant equations. From the values of ε_m , average density, sphericity and particle diameter of the mixture, the mixture U_{mf} is

computed using Eq. (1). Due to the cohesiveness of biomass particles, the minimum fluidization velocity of a pure biomass is much higher than that predicted by Eq. (1) even when the volume equivalent spherical diameter of the particle is used. Since the sphericity of most practical biomass can be as low as 0.2, using the actual sphericity of biomass in Eq. (1) will result in a much lower value of U_{mf} for the particles. Hence, for a mixture involving biomass and inert particles, $\varphi_{si} = 1$ should be used in the proposed algorithm.

3.3.1. Mixtures of two inert materials

Fig. 4 shows the predicted values of U_{mf} based on four different models at different mass fraction of the larger particles. For each of the models, U_{mf} of the different particle types are used as inputs. As shown in Fig. 4(a), the predicted results from the different models are in good agreement with the experimental data. However, the results in Fig. 4(b) shows that a combination of Eq. (1) with the Westman [16]/Yu et al. [25] equation or with the model given by Eq. (22) shows

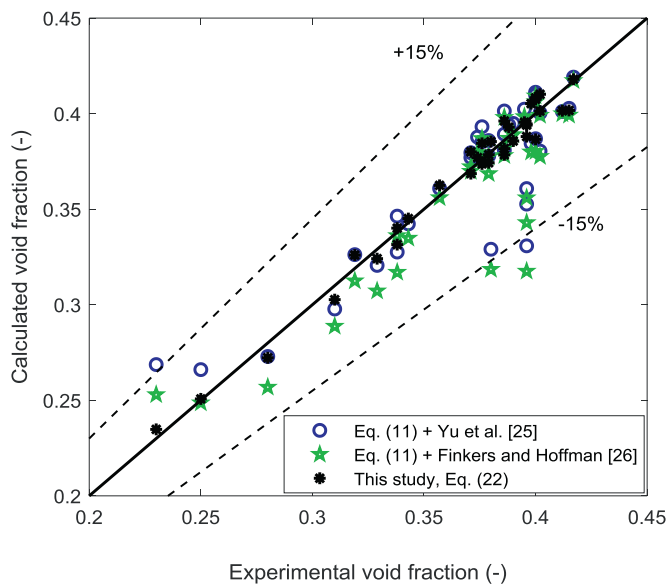


Fig. 2. Parity plot comparing the predicted void fraction with the experimental values for different beds of two inert materials.

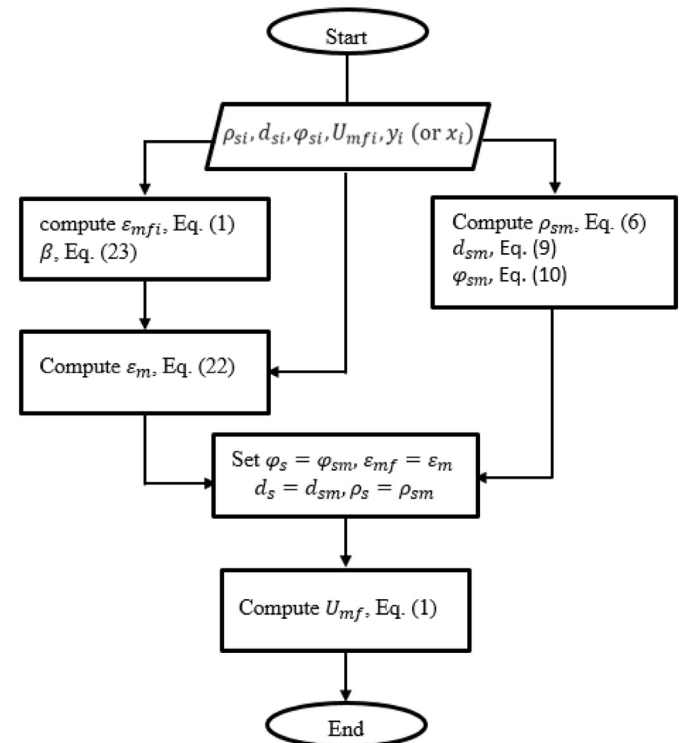


Fig. 3. Flow chart showing an algorithm for computing the minimum fluidization velocity in a bed of binary mixture of particles.

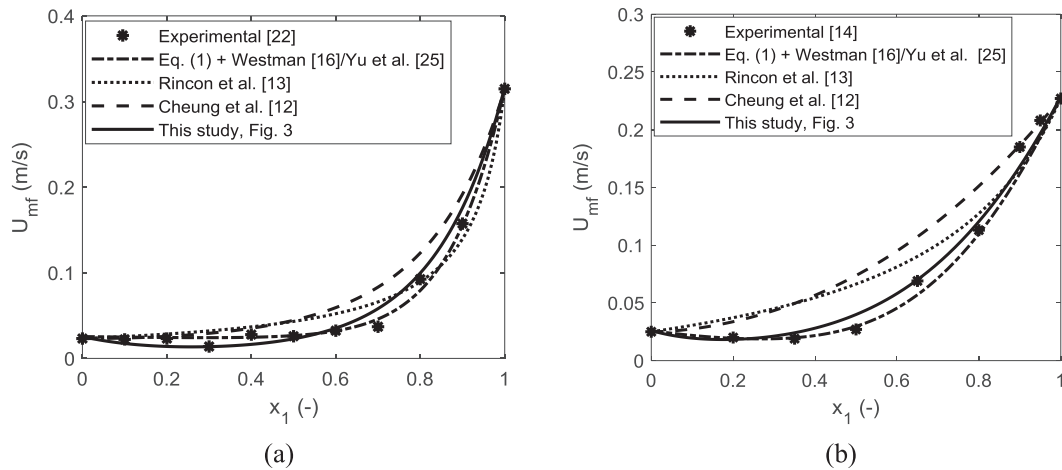


Fig. 4. Variation of minimum fluidization velocity comparing the predicted results with the experimental data for a binary mixture of particles (a) 612/154 μm glass [22] (b) 775 μm hollow char/165 μm glass [14].

a better prediction than those given by Cheung et al. [12] and Rincon et al. [13].

Furthermore, Fig. 5 compares the calculated values of U_{mf} from these four models against the experimental data obtained from different literature [14,22,23]. The result is based on the binary mixtures (I, III, IV, V and VI) given in Table 2. By using any of the four models, Fig. 5 shows that the minimum fluidization velocity of the beds can be predicted with an error within $\pm 35\%$. On average, the predictions based on the present study give the best results with mean absolute error of 15.2%, whereas those based on the Westman [16] equation with Yu et al. [25] correlation have a mean error of 15.5%. The models given by Cheung et al. [12] and Rincon et al. [13] show very high prediction errors with mean values 27.6% and 30.5%, respectively.

3.3.2. Mixtures of biomass and inert materials

Unlike the mixture of two inert materials with more or less the same particle density, a mixture of biomass and inert particles can show some degree of segregation. Hence, application of the Westman [16] equation in Eq. (1) will not be appropriate. However, this section shows that the proposed model, Eq. (22) can also be applied for prediction of minimum

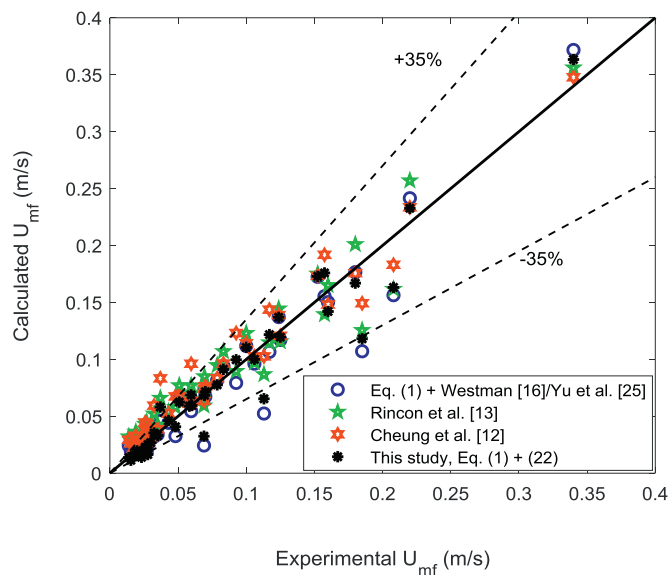


Fig. 5. Parity plot comparing the predicted minimum fluidization velocity with the experimental values for different beds of two inert materials; mixtures: I, III - VI; see Table 2.

fluidization velocity of a mixture of biomass and inert particles. To be able to predict the volume expansion in the binary mixture, a positive value of the parameter β , which can be computed from Eq. (23), is required.

Fig. 6(a) shows the void fraction computed using Eq. (22) at the minimum fluidization condition for a mixture of plastic particles with effective particle diameter 2550 μm and sand particles with particle diameter 550 μm . The plastic particles have a density of 1761 kg/m^3 and sphericity of 0.87 while the corresponding properties for the sand particles are 2664 kg/m^3 and 1.0. The experimental data are obtained from Asif [15] where water is used as the fluidizing fluid at 20 $^{\circ}\text{C}$. With $\beta > 0$, the result shows that Eq. (22) predicts the bed voidage with a good accuracy when the mass of the plastic particles is considerably high, i.e. $x_1 > 0.4$. At a lower mass fraction, the bed is partly mixed and partly segregated. Thus, Eq. (22) with $\beta = 1.35$ (computed from Eq. (23)) over predicts the mixture voidage. However, when the value of β is reduced to -0.38 , Eq. (22) predicts the voidage with a better accuracy when $x_1 < 0.4$. This result and those presented above therefore show that with $\beta > 0$, Eq. (22) gives the voidage for a well-segregated bed. With $\beta < 0$, the model provides results where there is some degree of mixing. When $\beta < 0$ and the magnitude of β is computed from Eq. (23), Eq. (22) predicts the voidage for a well-mixed bed. For prediction of β in a bed exhibiting partial mixing behaviour, a different correlation than Eq. (23) is required. In addition, a model for predicting the mixture composition at which the bed begins to segregate is also required. In spite of the error in predicting the void fraction where the bed exhibits partial segregation, Fig. 6(b) shows that the minimum fluidization velocity of the bed can be well predicted using the combination of Eq. (1) and Eq. (22) for all values of x_1 . For the result where $\beta = 1.35$ is used over the entire values of x_1 , the prediction error of the proposed model is 11.3% as against 27.5% and 27.6% errors obtained from the Cheung et al. [12] and Rincon et al. [13] models, respectively. If the value $\beta = -0.38$ is used for the compositions $x_1 < 0.4$, the proposed model predicts the minimum fluidization velocity shown in Fig. 6(b) with a better accuracy and the mean prediction error is reduced to 7.5%.

As the main aim of this study is to predict with improved accuracy the minimum fluidization velocity of a biomass-inert mixture, which often exhibits segregation behaviour, the results in Fig. 6 show that this can be achieved. The properties of different mixtures of biomass and inert particles used for this demonstration are given in Table 3 and the beds as described subsequently are fluidized with air at the ambient condition. For all computations in this section, Eq. (23) is used to predict the absolute value of β .

For the mixture of 856 μm walnut shell and 241 μm sand particles, Fig. 7(a) shows the predicted values of U_{mf} compared with the

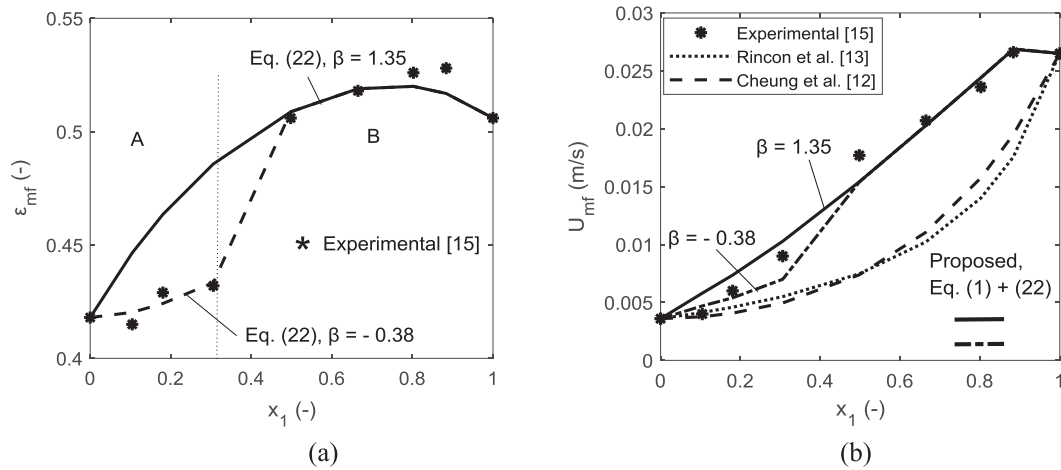


Fig. 6. (a) Variation of void fraction where A = partly mixed region and B = segregated region and (b) superficial water velocity at minimum fluidization condition, comparing the predicted results with the experimental data in a bed mixture of 2550 μm plastic and 550 μm sand particles with segregation behaviour.

experimental data. The results obtained for a mixture of 1560 μm rice husk and 350 μm sand particles are shown in Fig. 7(b). As can be seen in Fig. 7(a), the computed values of U_{mf} using the Paudel and Feng [31] model are closer to the experimental values although the model does not capture the expansion behaviour of the bed at increasing mass of biomass particles. The Kumoro et al. [32] model under predicts the bed expansion at higher values of x_1 , giving a lower value of U_{mf} for the biomass mixture. The Si and Guo [30] model gives the best prediction when $x_1 \leq 0.4$ but shows the greatest prediction error at higher mass of biomass particles. However, in Fig. 7(b), the Si and Guo [30] model gives the least prediction error for biomass mass fraction within $0.3 < x_1 < 0.8$. The Kumoro et al. [32] model over predicts the U_{mf} value at a higher mass fraction of the rice husk particles even though the experimental data were used in the model development. Unlike these two latter models, which also predict the expansion and contraction behaviour of the bed, the Paudel and Feng [31] model predicts a steady increase in U_{mf} with an increase in the amount of rice husk particles. As the models given by Si and Guo [30] and Kumoro et al. [32] consider particle sphericity, these results show that particle shape plays a significant role in prediction of U_{mf} . It should be noted that inclusion of particle sphericity in these two models also means that the models indirectly consider the bed voidage since these two properties are closely related. Moreover, the results in Fig. 7 show that by using the proposed model, U_{mf} is predicted with a better accuracy in both different bed mixtures. The results given by the proposed model is based on $\beta > 0$ where β

value is as given in Eq. (23). The results also show that the predicted U_{mf} using Eq. (1) and Eq. (22) gets better at increasing amount of biomass particles due to higher degree of segregation effect. Where there is some degree of bed contraction as shown in Fig. 7(a), the proposed model slightly over predicts the U_{mf} value due to the steady expansion behaviour predicted by Eq. (22) when $\beta > 0$ is used as demonstrated in Fig. 6(a).

In addition, Fig. 8 compares the prediction accuracy of the proposed model with those of the existing models for biomass-inert systems. The experimental data are based on different mixtures of biomass and inert particles given in the literature [31–33]; see Table 3. As shown in the figure, the Cheung et al. [12] model under predicts the mixture U_{mf} with an error as high as 40%. The accuracy of the Cheung et al. [12] model increases with increasing size ratio d_{s1}/d_{s2} and with increasing amount of biomass in the mixture. The high prediction errors shown by the models of Si and Guo [30], Paudel and Feng [31] and Kumoro et al. [32] are associated with the size ratio and density difference. The higher the values of d_{s1}/d_{s2} and $\rho_{s2} - \rho_{s1}$, the better the model accuracies. For $d_{s1}/d_{s2} < 3.5$, these models over predict the mixture U_{mf} with an error >40%. However, the method proposed in this study as described in Fig. 3 using Eq. (1) and Eq. (22) predicts the mixture U_{mf} with a better accuracy for all values of d_{s1}/d_{s2} and $\rho_{s2} - \rho_{s1}$. The mean prediction error using the proposed model is 7.0%, whereas those using the models of Cheung et al. [12], Si and Guo [30], Paudel and Feng [31] and Kumoro et al. [32] are 23.4%, 24.4%, 27.0% and 27.7%, respectively.

Table 3
Properties of particles in the biomass-inert mixtures.

Binary mixture	Particles	Shape	ρ_s (kg/m ³)	d_s (μm)	ϕ_s (-)	U_{mf} (m/s)	Ref.
VII	W. shell	Irregular	1200	856	0.78	0.553	[31]
	Sand	Spherical	2630	241	0.94	0.074	
VIII	Rice husk	Irregular	635	1560	0.18	0.642	[32]
	Sand	Spherical	2450	350	0.95	0.164	
IX	Corn cob	Irregular	1080	1040	0.71	0.608	[31]
	Sand	Spherical	2630	241	0.98	0.074	
X	M. beans	Spherical	1640	3200	1.0	1.053	[33]
	Sand	Spherical	2700	1000	1.0	0.558	
XI	M. beans	Spherical	1640	3200	1.0	1.053	[33]
	C. cinter	Spherical	1870	2800	1.0	0.918	
XII	C. stalk	Cylindrical	365	7200	0.55	1.16	[33]
	Sand	Spherical	2700	500	1.0	0.318	
XIII	C. stalk	Cylindrical	365	7200	0.55	1.16	[33]
	C. cinter	Spherical	1870	2800	1.0	0.918	

W. shell = walnut shell, M. beans = mung beans, C. cinter = CFB cinter, C. stalk = cotton stalk.

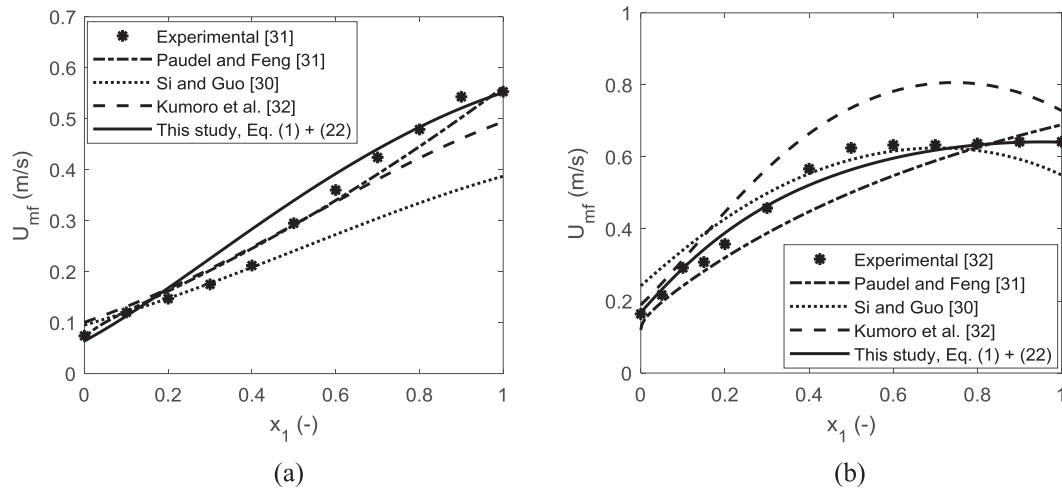


Fig. 7. Variation of minimum fluidization velocity comparing predicted results with experimental data for a binary mixture involving biomass particles (a) 856 μm walnut shell/241 μm sand particles [31] (b) 1560 μm rice husk/350 μm sand particles [32].

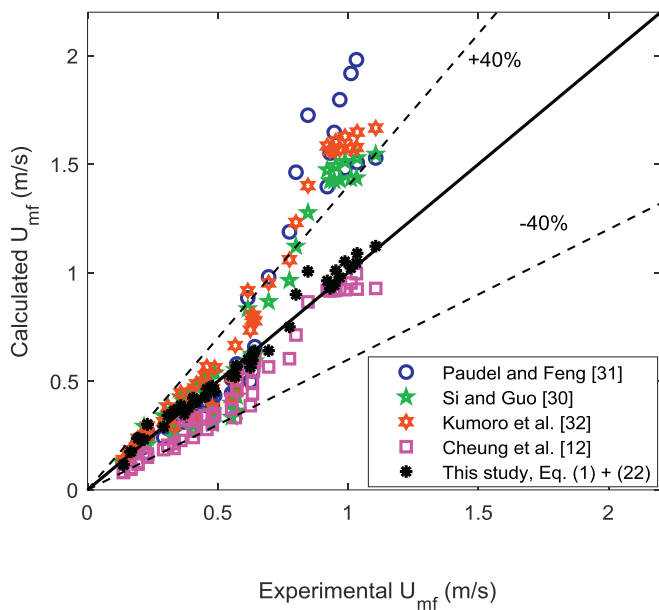


Fig. 8. Parity plot comparing the predicted minimum fluidization velocity with the experimental values for different beds of biomass and inert particles; mixtures: VII – XIII; see Table 3.

In summary, the accuracy of Eq. (22) in predicting the void fraction of a binary mixture depends on the value of the interaction parameter, β used. As shown in Figs. 2 and 6, Eq. (22) can predict the experimental data with a very good accuracy if a correct value of β is assigned. For the results shown in Figs. 4–8, Eq. (23) was used to estimate the values of β . Although the figures show that the U_{mf} values of the binary mixtures are predicted to a reasonable accuracy, the results can also be better with an improvement in the correlation for β . In its current form, Eq. (23) was derived from data of six binary pairs of solids. If a larger data set is analysed, the model for the interaction parameter can be improved.

4. Conclusion

In a binary mixture, the difference in properties between the two different particle types greatly influences the bed behaviour. For this reason, accurate prediction of minimum fluidization velocity of binary

mixtures, especially those involving biomass particles, has been a challenge. This paper presents a new model for predicting the bed void fraction and its application to predicting the minimum fluidization velocity of a binary mixture.

For prediction of the bed void fraction, the proposed model requires the void fractions of the pure components in the mixture. However, with known values of minimum fluidization velocities of the different particles in the mixture, the approach presented in this paper avoids the challenge in determining the bed voidage.

For a completely mixed system involving two inert materials, the proposed model can predict the minimum fluidization velocity with a mean error of 15.2%. For a bed mixture of biomass and inert materials, the model can predict the minimum fluidization velocity with an error of 7.0%.

Finally, for accurate prediction of the voidage and minimum fluidization velocity in a partly mixed bed of two types of particles, further work is required to establish a correlation for the binary interaction parameter as well as the mixture composition at the transition to the segregation behaviour.

Nomenclature

A	Bed cross-sectional area, m^2
Ar	Dimensionless particle Archimedes number
a	Solid specific surface area, m^{-1}
D_h	Hydraulic diameter, m
d	Diameter, m
g	Acceleration due to gravity, m/s^2
m	Mass, kg
N	Number
Re	Dimensionless Reynolds number
U	Superficial gas velocity, m/s
V	Volume, m^3
v	Dimensionless specific volume
x	Dimensionless mass fraction of a species in a mixture
y	Dimensionless volumetric fraction of a species in a mixture

Greek symbols

α	Dimensionless solids volume fraction
β	Dimensionless interaction parameter
ε	Dimensionless Void Fraction
θ	Dimensionless packing factor
μ	Dynamic viscosity, $\text{Pa}\cdot\text{s}$
ρ	Density, kg/m^3
φ	Dimensionless particle sphericity

Subscripts

b	Bed
f	Fluid
i, j	Indices
L	Particles of Larger Size
m	Mixture
mf	Minimum fluidization
S	Particles of Smaller Size
s	Solid
(zero)	Initial state or entry positionw

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