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Automatic Tuning of PID Controllers

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Summary:

Proportional Integral and Derivative (PID) controllers are the most frequently used controllers in the process industry over the years. The performance is a factor of adequate tuning. The knowledge of the process is critical to the tuning process as well selecting the right PID form and the tuning algorithm. Since there can be as many as possible PID loop in an industry, manually performing this task will take a lot of time and cause down time that can results to poor product quality as well as loss of income. Thus, there is a need for tuning to be done automatically, and this form the basis of this thesis, automatic tuning of PID controller.

It's therefore pertinent to adjudge the right tuning rule as well as selecting the right model for the process and thereafter if there are any changes in this model, to readjust the parameters used for the PID algorithms to get the right PID tuning parameter on real time basis. The standard form of the PID is used in the simulation and the relay feedback experiment by Åstrom is considered for the parameters adjustment, and the variant of this method developed by Schei is also investigated and the two methods compared.

Implementation and evaluation of these methods were done with the quadruple tank and air heater processes. The relay experimentation is an easy process that ensured recursive parameter calculation based on identified point on the Nyquist plot and this method can be seamless automated with just a push point from the Operator.

Preface

This thesis title ‘Automatic Tuning of PID Controller’ is carried out in partial fulfillment of the requirement for the award of Master of Science degree in Industrial Information Technology and Automation at the University of Southeast Norway, Porsgrunn Campus.

The task description in appendix A form the basis of the work carried out for the fulfilment of the thesis objectives. The simulation is performed using MATLAB and Simulink software and the codes and supporting documentation are described in the appendices.

I will like to express my gratitude to God Almighty for His sufficient grace and favor for the period of the study. Also, I sincerely appreciate the effort of my supervisor, David Di Ruscio (Ph.D.) for sharing his knowledge and time in ensuring that the thesis work is successfully executed. Moreover, the support of my colleagues and lecturers at the department are highly appreciated.

Finally, I appreciate my wife, children and siblings for their encouragement, contribution and motivation for the entire period of the study,

Porsgrunn, 15th May 2018.

Olalekan Olusoji Ige

Contents

Abbreviation

A-H	Åstrom, Hang and Wang
As	Åstrom without noisy process
Aswn	Åstrom with noisy process
DF	Describing Function
GM	Gain Margin
IAE	Integrated Absolute Error
IE	Integrated Error
IMC	Internal Model Controller
ISA	Instrument Society of Automation
LTI	Linear Time Invariant
MATLAB	MATrix LABoratory
MIMO	Multiple Input Multiple Output
MRAS	Model Reference Adaptive Scheme
OLS	Ordinary Least Squares
PD	Proportional and Derivative
PE	Persistence Excitation
PI	Proportional and Integral
PID	Proportional Integral and Derivative
PM	Phase Margin
SISO	Single Input Single Output
Sc	Schei without noisy process
Scwn	Schei with noisy process

Abbreviation

SIMC	Simple Internal Model Controller
SSEP	Sum of Prediction Error Squared
STR	Self-Tuning Regulator
TITO	Three input three output
TV	Total Variance
WLS	Weighted Least Squares
Z-N	Ziegler Nichols

1 Introduction

Proportional Integral and Derivative (PID) controllers are widely used in virtually all area of industrial automation and control. The PID controllers has three terms which are the proportional term P, integral term I and the derivative term D. The P gives a proportional control to the input signal, I and D terms gives control that is proportional to the integral and derivative time of the error respectively.

They provide a good response to process control but may not give optimal performance under some circumstances. Hence there is a need for PID parameters (the proportional gain, integral and derivative time) readjustment in case of process offset or the dynamics of the process changing or varying. The process of adjusting the PID parameters is referredred to as tuning and this can be done manually or automatically. A large industry may have many PID and to tune these manually will take time and cause down time and thus the need to develop an adaptive or a controller with an auto tune capability.

Based on the context of the thesis, PID tuning rules discussed in chapter 3 are general PID auto tuning and will be referred to as PID tuning algorithms (most of which cannot be easily automated) while the PID tuning rules discussed in chapter 4 are easily automated and as such will be referred to as PID auto tuning.

Auto tuning is the process whereby the PID controllers are tuned automatically by the operators or users. Other terms like adaptive controllers and gain scheduling will be used to mean almost the same things but with little variations. Adaptive controllers continuously adjust their parameters to accommodates for changes in process parameters/dynamics or model because of disturbances which can be external or internal. Gain scheduling is adjusting the parameters of PID controllers based on non-linearity's of the process such as changes in operating conditions. [1].

In some context, adaptive tuning requires the prior knowledge of time scales for determining suitable sampling interval whereas automatic tuning is specified on demand by the operator without a prior information about the process dynamic. Mostly both adaptive tuning and auto tuning means relatively the same things based on the applied algorithm and principles.

In auto tuning, the process is disturbed on demand, whereas adaptive tuning does not involve operator intervention before changing the PID parameters as demand by the changes in process dynamics. The parameters of adaptive controllers are uninterruptedly adjusted to cater for a disturbance in the process. Adaptive controllers are widely used for controller parameters adjustment on both model and non-model based auto tuning.

Most common auto tuning procedure is using a relay and gain scheduling, and MRAS and STR are common used schemes for adaptive controllers. In recent years other techniques are been employed in fine tuning controllers and among them is predictive controllers, expert system, and pattern recognition.

The choice of the PID auto tuning for the thesis work is based on the relay auto tuner described by Åstrom and Hågglung in [2]. This is a non-model based auto tuning in which an operator initiates the auto tuning process and the relay is used to obtained sustained oscillation in other to capture the critical point for the parameters estimation as obtained by Ziegler and Nichols method. An extension of this method presented by Schei in [3] will also be explored.

Irrespective of the name or terms employed, the major objectives are designing a controller that will be able to

- Handle changes in process dynamic
- Handle disturbance variation
- Gives operator choice of changing the parameters [4]
- Adapt to the process dynamic

The implementation of the adaptive controllers with auto tuning function can be done by [4]

- Define the closed loop response
- Determined the control law with adjustable parameters
- Determined the mechanism for parameters adjustment- on demand, online or offline-push button, etc.
- Implement the above steps

The choice of adaptive controller types depends on the process dynamics, and the base line is parameters or model variability. A knowledge of parameter estimation or techniques for determining the process dynamics is essential for controller tuning. This will be discussed in chapter 2.

1.1 Earlier work

There has been extensive work on auto tune PID controllers that date back to the era of Ziegler and Nichols in the 1940s. They presented two famous methods of tuning PID controllers, Z-N closed and open loop methods. The ultimate period and gain are determined by a step response in closed or open loop experiment and this information is used to determine the controller parameters. Rivera et al presented the Internal Model Controller (IMC PID) in 1986 and Skogestad [5] improved on this as Simple Internal Model Controller (SIMC). These are characteristically a model-based computation and mostly the available models are not perfect or there is always a deviation from the derived model and the real plant, thus there is a need for improved methods. This method has been improved upon by a lot of researchers to include rules based and adaptive tuning methods. Nishikawa et al used the transient response to improve on disturbance effect on the model parameters determination.

These aim of these approaches (non-model based/heuristic) is to create a disturbance on the system to throw the system into an oscillation that will be within the limit of the system stability. This limit is the point on the Nyquist curve that intersects with the negative real axis and the point is called the critical point. The critical point is the ultimate gain after the sustained oscillation and the ultimate period for the oscillation. This is method may pose a danger to process equipment as the sustained oscillation may be too much, also these are difficult to perform in real time.

Åstrom [4] defined adaptive controller as a controller that have a mechanism for adjusting the parameters and describes four types which are gain scheduling, model-reference adaptive control, self-tuning regulators and dual control. Schei discussed the relay auto tuning by generating a limit cycle with a relay such that control system oscillates at a frequency that is essential to decide the critical point of the control system stability [3].

Åstrom et al, [6] employed the use of different relay types, transient, biased and parasitic relay to refine the original Åstrom experiment. This was successful for the process with oscillatory dynamics and extended to dead-time compensators and multivariable controllers.

Josefin [7] extended the application of the relay auto tuner to TITO system and employed three different versions in industrial application. The study further improved the Relay Auto tuning with Normalized Time Delay and this improved the noise susceptibility of the controller [7].

Luyben in 1987 introduces the use of relay-feedback to fit a typical transfer function [8] and Schei in his paper demonstrated auto tuning of PID based on transfer function model with the use of relay experimentation.

In [9], Finn Haugen demonstrated that the Åstrom relay auto tuning method can also be archived by turning the PID controller into a relay making the proportion and the integral constant very high and the derivative part set to zero. And this method can be applied to processes, like the air heater used for experimentation in this thesis, which does not accept negative control signal. The results obtained from this is achieved with the anti-windup function de-activated.

The Ziegler-Nichols approach suffers a deficit in that the system can be thrown out of the stability limit (the oscillations are produced by gradually increasing the proportional gain in a closed loop system) but the A-H approaches is safe as the oscillation is bounded within a bounded limit (control amplitude).

Nyquist, Nichols and Bode plot and charts are essential background in studying system stability and performance monitoring.

1.2 Aim and Objectives

The main objective of this thesis investigates the various algorithm used for automatic tuning of PID controllers. The specific objectives of the study are to:

- Perform literature review on recursive parameter estimation for PID auto tuning
- Perform literature review on PID tuning algorithms
- Investigate Åstrom relay feedback experimentation
- Investigate Schei relay feedback experimentation
- Implement Åstrom and Schei relay experimentation on simulated air heater and quadruple tank
- Execute the relay experimentation on real air heater

1.2.1 Task Description

- Perform a literature research on algorithms for automatic tuning of the PID controller
- Perform a literature research on recursive system identification based methods for PID controller parameters calculation
- Perform a simulated experiment of one or a few algorithms for automatic tuning of PID controllers.
- A laboratory experiment of the auto tuning method on quadruple tank process and/or the air heater

2 System Identification and Parameter Estimation

The PID controller parameters is based on the model or the dynamic system parameters. Therefore, effective use of a PID controller depends on the accuracy of the system model and ideal system dynamic representation. Identification of plant dynamics and the parameter of PID controllers can be determined in several ways. The most common method is manually adjusting the parameters of a system and studying the response. But this is time consuming and mostly in accurate. Hence the need to perform the parameters adjustment automatically is inevitable.

Since all mathematical models derived from system identification stages differs or may vary from the physical system, there is a need for parameters estimation and this call for adaptive control strategies. [10]. Adaptation in this context means calculating the parameter online at every time step in a recursive manner.

The major steps in system identification are [4]

- Selection of model structure
- Real experiment design
- Parameter Estimation and
- Validation

Many process models can be approximated with a first or second order linear system with a time delay term as given in equations (3-20) and (3-21) and this will be the basis of the system identification discussion. The thesis is based on an established model that has been validated but there is still need for parameters estimation to correct modelling errors and disturbance influence.1.1

The choice of parameters estimation is a determinant of the model types used. Also, the data properties must be one that can be excited persistently. It can be shown that a pulse signal is not a PE signal, a step is a PE of order one, a sinusoid is a PE of order 2, and a periodic signal of period n is a PE of order n. [4].

2.1 System Identification

System identification is defined as using observed data (input and output) to construct a mathematical model of a plant. [11]. Since system identification is done automatically in an adaptive system, selection of model structure, parameterization and recursive computation is essential keys to successful implementation of such system. Some useful definition regarding system identification will be discussed in the following section.

2.1.1 Model structure

The model structure is an important part of system identification and paramount for correct parameter estimation. Basically, the following types of model can be identified:

- Linear / nonlinear state space model which can be in the continuous or discrete form
- Input/output (polynomial) models which can be linear, nonlinear, continuous or discrete
- Transfer function model

The above listed types can be classified as shown in the following section.

2.1.1.1 White Box model

This is a model based on the first principle like Newton's law and always results to differential equations. The ordinary differential equation of the air heater is given by equation (5-1)

2.1.1.2 Black Box model

The model structure and its parameter is completely unknown and can be estimated with only input and output data. This is also known as a parametric model and described the system in terms of a differential equation and transfer functions.

The following types of black box can be identified: [12].

- Transfer function model

The differential equation can be converted to a transfer function model as shown in section 5.1 for the air heater and the transfer function given by equation (5-7).

- Polynomial models [11]

The following types of polynomial models can be identified:

  - Autoregressive, AR model: this is represented by equation (2-1) where the term A(q) represent the Auto regressive part.

$$A(q)y_k = e_k \tag{2-1}$$

  - Autoregressive eXogenous, ARX model: this is shown in equation (2-2), where the term B(q) represent the eXogenous (extra input) part

$$A(q)y_k = B(q)u_k + e_k \tag{2-2}$$

  - Autoregressive Moving Average, ARMA model: this represent the moving average of the white noise and the term C (q) in equation (2-3)is the Moving Average part.

$$A(q)y_k = C(q)e_k \tag{2-3}$$

  - Autoregressive Moving-Average with eXogenous variable, ARMAX Model is as shown in equation (2-4).

$$A(q)y_k = B(q)u_k + C(q)e_k \tag{2-4}$$

  - Output-Error Model: the auto regressive and the eXogenous part are the same as shown in equation (2-5)

$$A(q)y_k = B(q)u_k + A(q)e_k \tag{2-5}$$

  - Box-Jenkins Model: the model structure separates the input and noise path as shown in equation (2-6).

$$A(q)D(q)y_k = D(q)B(q)u_k + A(q)C(q)e_k \tag{2-6}$$

- State Space Model: The state space model is given in general forms in (2-7)

$$X_{k+1} = AX_k + BU_k + CV_k \tag{2-7}$$

$$Y_k = DX_k + Eu_k + FV_k$$

2.1.1.3 Grey Box model

The model structure is partially known from the first principle, the rest is reconstructed from data.

2.1.2 Static and Dynamic Models

The static model is determined by the information in the process characteristic curve that gives the steady state relation between input and output signal. This is a starting point in the model identification and it's mostly done with the open loop experimentation of the process. The dynamic models, on the other hand, give the input and output relationship during transients (closed loop experimentation) and can only be applied to linear time-invariant (LTI) systems.

2.1.3 Transient and Frequency Response

The response from signals like step, pulse and impulse are terms transient response while those from signal line sinusoid are term frequency response. The Z-N open and closed loop experiments are transient and frequency responses respectively.

2.1.4 Step Response

In an open loop system, a step input will be applied to the system and the response recorded. But in a closed loop system, the controller will be in manual mode and the control variable changed rapidly by increasing or decreasing, (this is done after the system is at rest). The process variable is recorded and scaled by the change in control variable. In both cases, it's advisable to repeat the process for different step changes and at different operating conditions to capture a wider range of the process dynamics. The a one parameter model

Another model structure is a two-parameter model for which the process gains and the average residence time (capture the time behavior) can be the parameter of interest. For a better approximation of the model, the number of parameters can be increased to three to give a Three-parameter model. This is characterized by three parameters as follow:

- the gain K
- the time constant, T
- the dead time or time delay

Other methods for a more precise model are four parameter and methods of moment as discussed in [1]. The step response is employed for the air heater model (which is a three parameters model) validation and the details is contained in section (3-2).

2.2 Parameter Estimation

Parameters estimation is an integral part of system identification and essential to auto tuning of PID controllers. The parameters of process change dynamically during adaptive control (auto tuning), so there is a need for an estimation method that updates the parameters recursively. There is need to validate this automatic estimation. The choice of input signals plays a major role in parameter estimation and requires some knowledge of the process.

In offline estimation, already processed input/output data are used to estimate the model parameters whereas in online parameter estimation parameters of a model are evaluated with both past and current data that are made available as each time instant during the operation of the process. It involves the use of recursive algorithms as discussed in section 2.2.3 and 2.2.4.

Recursive computation can be done in the in the least square sense (minimizing the sum of square errors) for the method for system identification and parameter estimation as well. The following section will explain the least squares method and also the recursive computation of some methods.

2.2.1 Ordinary Least Squares Estimates, OLS

The principle of least square was formulated in 1809 by Karl Gauss and used to determine the orbits of planet and asteroids. The same principle can be extended to process data in which the unknown parameters of the process model can be determined from observing the input and the output data. The objective is to minimize the sum of the squares of errors (the difference between the observed and computed values), multiply by a number that measures the degree of precision. [4]. It's essential to choose the order of the polynomial to avoid under or over fitting.

Let Y is a set of observed variables, X a set of regressors variables and B a vector or matrix of unknown parameters as given in equation (2-8).The objective function, J becomes minimizing the error, E as shown in equation (2-9). And minimizing equation (2-9) with respect to the regressor variable will give equation (2-10).

$$Y = XB + E; E = Y - XB \tag{2-8}$$

$$J = E^T E = (Y - XB)^T (Y - XB) \tag{2-9}$$

$$\frac{\partial J}{\partial B} = -Y^T X + B^T X^T X = 0 \tag{2-10}$$

The ordinary least square estimate of B is given in equation (2-11) by solving equation (2-10)

$$B_{ols} = (X^T X)^{-1} X^T Y \tag{2-11}$$

2.2.2 Weighted Least Squares Estimation

In section 2.2.1, it's assumed that all the measured parameters have an equal amount of confidence, but in reality, this is not always the case. An example may be the feedback sensors in a control loop, some sensors may be reliable than others and thus the need to put a varying degree of confidence in each sensor. The degree of confidence is term weight function and instead of minimizing the sum of squares of the errors, the weighted sum of error squared is minimized. Therefore, the objective function is as shown in equation (2-12) and minimizing this will gives equation (2-13). The weighted least square estimate of B is given in equation (2-14) by solving equation (2-13).

$$J = E^T W^{-1} E = (Y - XB)^T W^{-1} \big((Y - XB)\big) \tag{2-12}$$

$$\frac{\partial J}{\partial B} = -Y^T W^{-1} X + B^T X^T W^{-1} X = 0 \tag{2-13}$$

$$B_{wls} = (X^T W^{-1} X)^{-1} X^T W^{-1} Y \tag{2-14}$$

This requires that the measurement noise, R be a nonsingular matrix and invertible, which mean that each of the measurement y, must be noisy. [13].

2.2.3 Recursive least square estimate (RLS)[1]

In general, for online parameters estimation, there is need to measure continuously and update the estimate of the parameters with each measurement. However, this can be a problem for a large data over a large computational period. Thus, instead of performing the least squared estimate from start until the time instant, again and again, the computation can be made recursively. In section 2.2.1 and 2.2.2, if the parameters are time varying. Given P as the estimator error covariance as given in equation (2-15), the least square estimates of B, $\dot{B}$ as a function of time is given in equation (2-16)

$$\begin{aligned} P^{-1}(t) &= X^T(t)X(t) = \sum_{i=1}^{t} x(i)\, x^T(i) \\ &= \sum_{i=1}^{t} x(i)\, x^T(i) \; + \; x^T(t)x(t) \\ &= P^{-1}(t-1) + x(t)x^T(t) \end{aligned} \tag{2-15}$$

$$\begin{aligned} \dot{B}(t) &= \dot{B}(t-1) + K(t)\left(y(t) - x^T(t)\dot{B}(t-1)\right) \\ K(t) &= P(t)x(t) = P(t-1)x(t)\big(I + x^T(t)P(t-1)x(t)\big)^{-1} \\ P(t) &= P(t-1) - P(t-1)x(t)\big(I + x^T(t)P(t-1)x(t)\big)^{-1} \\ &= \big(I - K(t)x^T(t)\big)P(t-1) \end{aligned} \tag{2-16}$$

2.2.4 Kalman Filter

The Kalman filter is a great tool for analyzing and solving estimation problems. The linear standard Kalman filter is used for linear process model whereas Extended Kalman Filter, Unscented Kalman Filter, etc. are used for the non-linear process model. Its work by propagating the covariance and mean of the state through time. The Kalman filter for states estimate is optimal several different senses. The following steps are involving in Kalman filter algorithm:

- A mathematical model of the dynamic system is derived as shown in equation (2-17)

$$x_{k+1} = Ax_k + Bu_k + w_k \tag{2-17}$$

---

[1] X(t) and X(t-1) mean X at present time instant and X at previous time instant respectively. The notation X_t and X_{t-1} also mean the same things as present and previous time instant.

$$y_k = Cx_k + Du_k + v_k,$$

$$W = E[w_k w_k^T],$$

$$V = E[v_k v_k^T]$$

- Describe the propagation of the state mean and covariance of the system with time
- Discretization of the state mean and covariance as states above
- Update of the discrete form at every time step [13]

W and V are covariance's matrices which are used to tune the Kalman filter. The discrete time filter for a linear process can be implemented with the following steps [14]

- Initialize the Kalman filter for time k=0, using the initial state or known state of the system as given in equation (2-18)

$$\begin{aligned} \hat{x}_k^+ &= \hat{x}_0^+ = E(x_0) \\ P_k^+ &= P_0^+ = E[(x_0 - \hat{x}_0^+)(x_0 - \hat{x}_0^+)^T] \end{aligned} \tag{2-18}$$

- Compute the Kalman filter gain (K_f) as shown in (2-19)

$$\begin{aligned} P_{k+1}^- &= AP_k^+ A^T + W \\ K_{fk} &= P_{k+1}^- C^T (CP_{k+1}^- C^T + V)^{-1} \end{aligned} \tag{2-19}$$

- Compute the predictor, a priori state and output estimate as given in equation (2-20)

$$\begin{aligned} \hat{x}_{k+1}^- &= A\hat{x}_k^+ + Bu_k \\ \hat{y}_k^- &= C\hat{x}_{k+1}^- + Du_k \end{aligned} \tag{2-20}$$

- Compute the corrector, a posterior state estimate using the output measurement y_k as given in equation (2-21)

$$\hat{x}_{k+1}^+ = \hat{x}_{k+1}^- + K_{fk}(y_k - \hat{y}_k^-) \tag{2-21}$$

- Update the covariance of the state as given in equation (2-22)

$$P_{k+1}^+ = \left(I - K_{fk}C\right)P_{k+1}^+\left(I - K_{fk}C\right)^T + K_{fk}VK_{fk}{}^T \tag{2-22}$$

- Repeat the step all over again for the next time interval, k+1.

If the Kalman filter gain is assumed to be steady then the algorithm reduces to a single equation as shown in equation (2-23).

$$x_{k+1}^+ = A\hat{x}_k^+ + Bu_k + K_f(y_k - C\hat{x}_k^+ + Du_k) \tag{2-23}$$

2.2.5 Relay Methods

The relay experimentation as discussed in section 4.1 can be employed to determined critical point for parameter estimates for PID auto tuning. Different configuration and modification will give different points on the Nyquist curve which will give the point of interest to identify the gain and the time delay parameter in equation (3-20) and (3-21). This will only give an estimate and only valid around the ultimate frequency for an ideal relay and at different frequencies as given by the modification of the relay during the experimentation. The experimentation will recursively calculate the critical parameters as discussed in in section 4.

3 PID Controllers Tuning Algorithms

PID controllers are widely used in the industry and still gaining acceptance irrespective of the new method of automatic control based on Model predictive control and rule based control. PID controllers are used in the industries as a standalone controller or part of direct digital control package or at the lower level of a distributed control system. The PID controllers require adequate tuning of the controller parameters (proportional gain, integral and derivative time) for optimum performance. This has been a lot of challenges to the operators over the years and new a lot of research that has led to successful algorithms have been done over the year.

The design method that results to the algorithm used differ based on the dynamics of the process. A proportional and Integral (PI) controller is mostly described by two parameters and a PID by three or four parameters. The objective of the tuning process is to derive these parameters that will give the optimal performance of the controller.

The foremost tuning rule starts from Ziegler-Nichols process reaction curve and the ultimate gain methods. Thereafter, they have been a lot of researches that have resulted in better tuning rules for PID controllers. Section 3.2 will discuss few of these rules among which are, Simple internal model, Tyres-Luyben, Cohen-Coon, Relaxed Ziegler-Nichols, Good gain and Relative time delay error methods. Rehearses are ongoing on developing new rules on the subject matter as shown in the recent paper [15].

The performance of the PID controllers has to be measured based on some benchmarks. The different tuning rules offer some advantages and also disadvantages over the other. This can be subjective to the yard stick of performance indicator. This will be discussed in section 3.3, stability analysis and section 3.4, performance criteria.

The thesis is based on an Air heater process section 1.2 and this is a first order plus time delay process, therefore only PI was employed and the standard / ISA algorithm discussed in section 3.1.1.1 was used.

3.1 PID Controllers Basic

In the process and all other industries, the major aim is to keep track of the output as well as maintain the process parameters as steady state or desired value. In other to achieve this aim, a feedback/feedforward control loop is employed as shown in Figure 3-1. Whereas feed-forward control gives a perfect control in case of an exact model, in real life experiences there will be an unknown disturbance, model imperfection and this call for feedback control. PID controllers provide such a feedback which eliminate steady state error and future disturbance through the integral and derivative action respectively. [1]. The basic principle of a feedback system as stated in [1] is that the controller decreases the control effort when the process variable increases and increases the control effort otherwise. This is for a negative feedback system as the manipulated variable (the control effort) moves in the opposite direction to the process variable. For a positive feedback system, the reverse is the case. For a perfect control of both feedback and feed-forward control, the controllers' parameters must be tuned adequately. PID's controller has three terms as defined as follows:

- The proportional term which gives proportional control
- The integral term gives a control action that is proportional to the time integral of the error (set point minus the output) and keeps this error minimum (zero) at steady state.
- The derivative term gives a control action proportional to the time derivative of the error and keeps track of future errors.

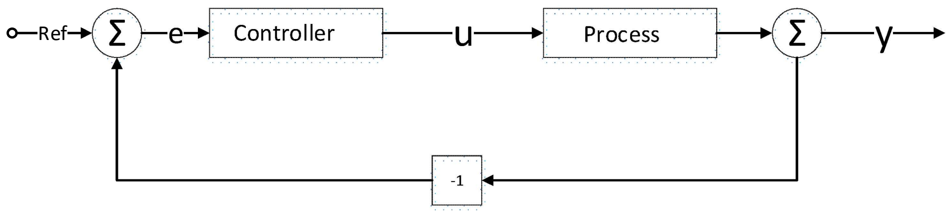


Figure 3-1: Implementation of feedback control in a closed loop.

3.1.1 PID Algorithms

The major algorithms used for PID design are in three forms, which are standard, parallel and classical. These algorithms contain several variation or modification. The major difference between these three algorithms is the way the controller gain is specified. The parallel form has a proportional gain that affects only the proportional part while the other two forms have a controller gain that affects all the three terms. [16]

For better performance, the following are taking into consideration [4]

- The derivative part is mostly applied to the process output
- Proportional part act only on a fraction of the reference part
- The integral action is kept within the saturation action of the control variables (anti windup)
- Bump less or smooth transfer from manual to automatic (or when there are parameter changes)

3.1.1.1 Standard or Non-interactive form of PID Controller

The standard form is also called the standard or ISA algorithm. In some text, it's called the expanded form and it's the form used in MATLAB [17] and in [1]. As shown in equation (3-1) and the equivalent transfer function form in equation (3-2), the controller parameters of interest are the proportional gain K_p, integral time T_i, and the derivative time T_d. The term 'e' is the control error, which is the difference between the set point and the process output variable. It's thus the sum of the terms vis: the proportional, the integral and the derivative terms. The application of the respective terms is as discussed in section 3.1.1. The algorithm internal structure is as shown in Figure 3-2. The standard PID can be represented in time and frequency domain as given in equations (3-1) and (3-2) respectively.

$$u(t) = K_p \left[e + \frac{1}{T_i} \int e.dt + T_d \frac{de}{dt} \right] \tag{3-1}$$

$$h_c(s) = \frac{u(s)}{e(s)} = K_p \left[1 + \frac{1}{T_i s} + T_d s \right] \tag{3-2}$$

Where K_p is the proportional constant, T_i is the integral time and T_d is the derivative time as shown in Figure 3-2. The output of the controller is $u(t)$ and the input e(t) in time domain respectively. And the input and output are u(s) and e(s) in frequency domain respectively. The Laplace transfer function of the controller is given as $h_c(s)$.

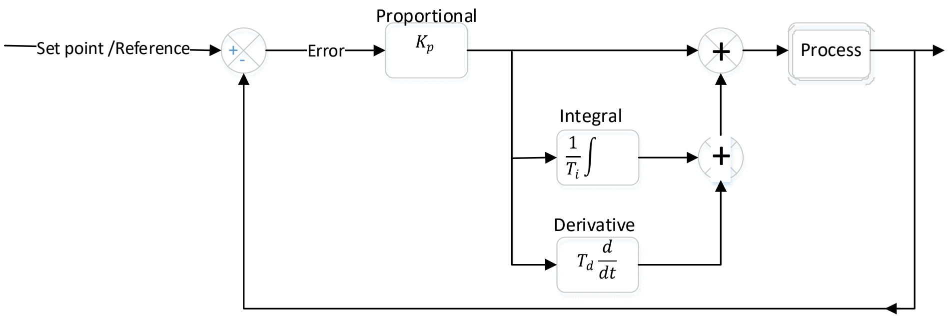


Figure 3-2: The ideal or ISA PID algorithm. It's also called the non-interactive, expanded. In this form the proportional gain affects all the three parts.

3.1.1.2 Parallel / Ideal form of PID Controller

The parallel algorithm is simple to understand, but difficult to tune by the traditional tuning method like Z-N and Cohen-Coon. The controller has a gain factor that affect only the proportional part as oppose to other algorithm that has a gain that affect three terms. However, this can be converted to an equivalent standard form for which the parameter values as given in equation (3-5)(3-7).

$$u(t) = K_p^p \times e + T_i^p \int e.dt + T_d^p \frac{de}{dt} \tag{3-3}$$

$$h_c(s) = K_p^p + \frac{T_i^p}{s} + sT_d^p \tag{3-4}$$

Where $K^p{}_p$ is the proportional constant, T_i^p is the integral time and T_d^p is the derivative time for the parallel form. The superscript 'p' is used to indicate the parallel form of the PID.

$$K_p^p = K_p; \; T_i^p = \frac{K_p}{T_i}; \; T_d^p = K_p T_d \tag{3-5}$$

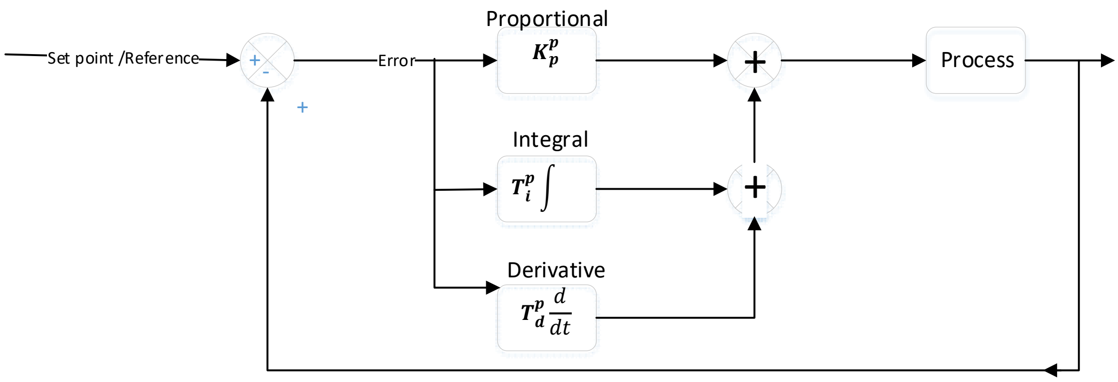


Figure 3-3: The Parallel form of PID algorithm, the gain affects only the proportional part as compared to the standard form where the proportional gain is affect both the integral and derivative parts.

3.1.1.3 Series/Classical / Interacting form of PID Controller

This is also called the series, cascade, real or interactive PID controller, in this form the PI and the PD element operated in series. The controller's parameters interact with each other. The integral time does influence the derivative terms and verse visa. The differential and the transfer function equation of the controller is given in equation (3-6) and (3-7) respectively. Figure 3-4 shows the internal structure of the controllers. The interacting and non-interacting form are the same for a P, PI or PD only controllers, they only differ when the three terms are used at the same time. If the derivative time is much smaller than the integral part, the two forms are equivalent. The choice of the forms to use depends on the manufacture and the operator needs to understand this for better tuning as to ensure optimal performance.

$$u(t) = K_c\left[e + \frac{1}{T_i^s}\int e.dt\right] \times \left[1 + T_d^s\frac{de}{dt}\right] \tag{3-6}$$

$$h_c(s) = K_c\left[1 + \frac{1}{T_i^s s}\right] \times [1 + T_d^s s] \tag{3-7}$$

Where K_c is the proportional constant, T_i^s is the integral time and T_d^s is the derivative time. The superscript 's' is used to indicate the series form of the PID.

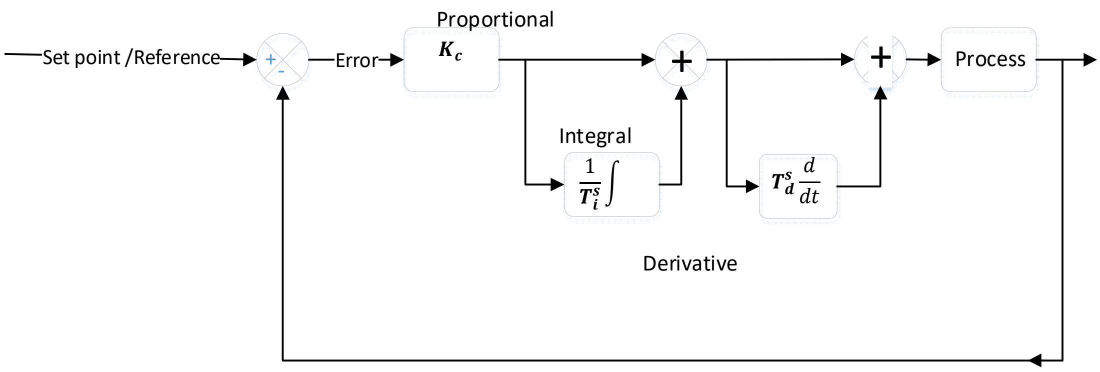


Figure 3-4: Series or interacting PID form. It's also called the cascade real form. The PID parts interact with each other, the integral part interacts with the derivative part and vice versa.

The series or cascade form of equation (3-7) is equivalent to the ideal form of equation (3-2) where the relationship between the controller parameters is as given in equation (3-8).

$$K_p = K_c\frac{T_i^s + T_d^s}{T_i^s} \; ; \; T_i = T_i^s + T_d^s \; ; \quad T_d = \frac{T_i^s \, T_d^s}{T_i^s + T_d^s} \tag{3-8}$$

Given the ideal form of the controllers' parameters their equivalent series parameters can be determined if only if the condition given in equation (3-9) holds. [18]

$$T_i \geq 4T_d \tag{3-9}$$

Then the following conversion as given in equation (3-10) is established

$$K_c = \frac{K_p}{2}\left(1 + \sqrt{\left(1 - \frac{4T_d}{T_i}\right)}\right); \; T_i^s = \frac{T_i}{2}\left(1 + \sqrt{\left(1 - \frac{4T_d}{T_i}\right)}\right); \tag{3-10}$$

$$T_d^s = \frac{T_i}{2}\left(1 + \sqrt{\left(1 - \frac{4T_d}{T_i}\right)}\right)$$

The derivative time is often much smaller that the integral time and the two forms are then equivalent as shown in equation (3-11) .

$$K_c\left[1 + \frac{1}{T_i s} + T_d s\right] \cong K_c\left[1 + \frac{1}{T_i^s s}\right] \times [1 + T_d^s s] \tag{3-11}$$

3.2 PID Controllers Tuning Rule

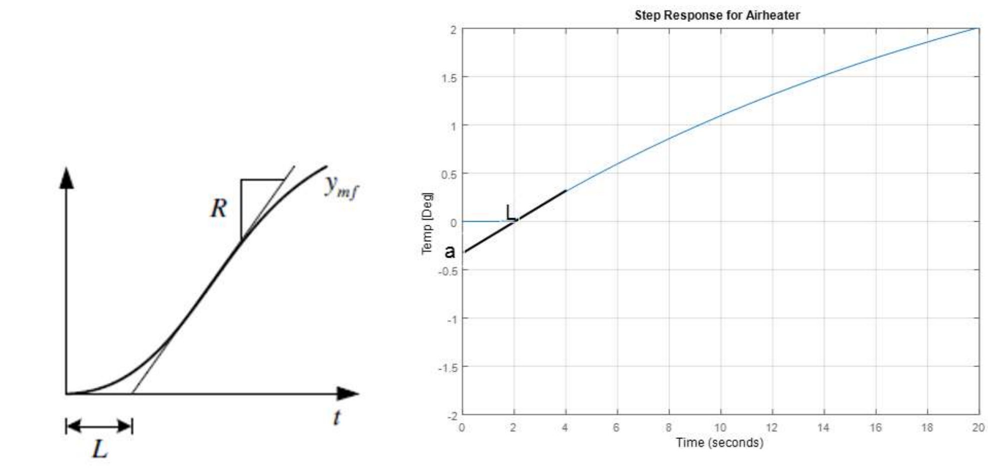
Tuning is the process of finding the optimal gains for the P, I and D parts to get an acceptable response from the control system. A good design should enable parameter changes to enable improvement on the system performance. The acceptable response is a function of performance criterial as discussed in 3.4. The tuning process can be heuristic (based on experimentation and simulation) and non –heuristic (analytical approach). Christer Dalen proposed a semi heuristic approach in [15]. Other methods like optimization method and pole placement do exist but will not be discussed in this report. Most of the heuristic tuning rules stem from the ultimate gain and period, and process reaction curve as defined by Ziegler and Nichols method (ZN) in the 1940's. The simplest tuning rules based on model reduction by half rule will be discussed in section 3.2.6 under Simple Internal Model Controller (SIMC). This method is simple and is based on approximate model of first and second order with time delay or inverse response [18].

3.2.1 Ziegler-Nichols Open loop method

The Z-N tuning method is also known as the process reaction curve and was developed by Ziegler and Nichols in 1942. This method is based on the characteristic of the open loop step response of the process as shown in Figure 3-5. This is equivalent to modelling a process by an integrator and a dead time as given in equation (3-12). This method is aggressive and oscillatory (poor stability) but has good disturbance response for integrating process as shown in [9]. With a step response experiment a first order with time delay can be approximated by equation (3-12).

$$h_p(s) = \frac{a}{sL}e^{-sL} \tag{3-12}$$

Where 'a and L are derived from step response as shown in Figure 3-5 (a) [4]. The tangent at the point of maximum slope of the step response is extended to both the vertical and horizontal axes to gives a and L respectively. The relationship between the maximum slope R, dead time L, applied step change U (which is usually unity) and a is given by equation (3-13).



A B

Figure 3-5: Process reaction curve: The left figure (A) shows L, the dead time, and R, the reaction rate, which is the max rate of change of temp with respect to time [9]. The right figure (B) shows the step response of the air heater.

$$a = \frac{LR}{U}; \quad R = max\frac{dy}{dt} \quad ; \quad L = t_1 - \frac{y_1}{R} \tag{3-13}$$

Table 3-1 gives the PID setting for Z-N open loop method where a is equivalent to LR/U as given in Figure 3-5 where U is the applied step change, L is the dead time and R is the maximum rate of change of the process output.

Table 3-1: Z-N open loop method.

Controller Type	Kp	Ti	Td
P Controller	1/a		
PI Controller	0.9/a	3L	
PID Controller	1.2/a	2L	L/2

3.2.2 Ziegler-Nichols' Ultimate Gain and Period

This method is also called the frequency response method and it is based on step response experiment on an established closed loop of the process. The PID controller is set to a P terms only controller by setting the integral time large (infinity) and the derivative term to zero. The P-term is increased until a sustained oscillation of the process is obtained without the control signal reaching is lower and upper limit. The ultimate gain and period are given by Bode stability criterion that gives open loop stability at the gain/phase cross over frequency. This is the point at which the Nyquist curve of the system intersects the negative real axis.

The value of the proportional gain that cause the sustained oscillation is the ultimate gain K_{cu} and time for the sustained oscillation is labeled the ultimate period P_{u}. The step response method in section 3.2.1 often gives higher gain Kp than the frequency response method. [1]. The PID parameters are calculated as presented in Table 3-2.

Table 3-2: Z-N Ultimate gain and Period method.

Controller Type	Kp	Ti	Td
P Controller	$0.5K_u$		
PI Controller	$0.4K_u$	$0.8\ P_u$	
PID Controller	$0.6K_u$	$0.5\ P_u$	$0.125\ P_u$

3.2.3 Good Gain method

Good Gain method is an alternative to Ziegler-Nichols closed loop method discussed in 3.2.2. It was proposed by F. Haugen in 2010 [19] and presented in detail in his paper Good Gain method for simple experimental tuning of PI controllers. [20]. As against Z-N closed method, the good gain reduced the process upset during the tuning as it does not bring the process to marginal stability during tuning. It's also gives more stable and robust controllers as compare to Z-N. [20]. It's ensure good stability at reasonable response time. The procedure is as follows:

- Set the controller to manual mode and brings the system to near set point by controlling the process manual.
- In automatic control, disable the Integral and the derivative part of the controller and put it only on P-mode starting with small value of Kp between 0 and 1
- Apply small step changes to the process with (as not to bring the control signal to saturation as this point will not give a good results),
- Increase the Kp value until there is significant overshoot and a small trace of undershoot
- The value of Kp in the step above that cause the overshoot is the good gain, K_{pg} of the controller and the time interval between the overshoot and undershoot is called the T_{out}

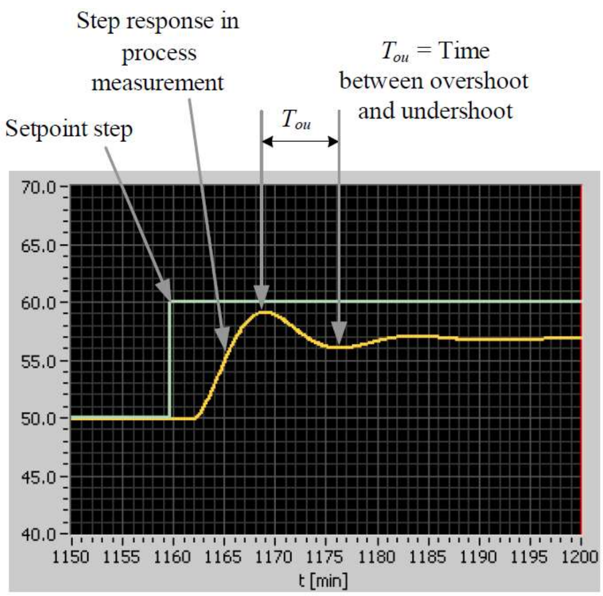


Figure 3-6: The plot showing the reading of the gain that cause first significant overshoot and T_{out} which is the time between the overshoot and undershoot. [20]. The controller is set in P only mode and the proportional gain increased to achieve this.

The controller parameter can be found in relation to Good Gain method as given in equation (3-14).

$$T_i = 0.5T_{out};\ \ K_p = 0.8K_{pg};\ \ ;\ \ T_d = 0.125T_{out} \tag{3-14}$$

Where T_{out} and K_{pg} are the time interval between the overshoot and undershoot, and the value of K_p that cause the overshoot as shown in Figure 3-6.

However, the method is not suitable for a process with double integrator, an integrator only process and time constant without time delay process. [20]

3.2.4 Cohen-Coon Method

Like Z-N method, is based on the loop response of the process as shown in Figure 3-7 . A step change is applied after the process has reached a steady state under manual mode, the time constant τ and delay td are evaluated as shown in Figure 3-7 [21]. The tuning formula is presented in table xxx. The relationship between z, k, B A, is given in equation (3-15).

$$k = \frac{B}{A};\ \ z = \frac{t_d}{\tau} \tag{3-15}$$

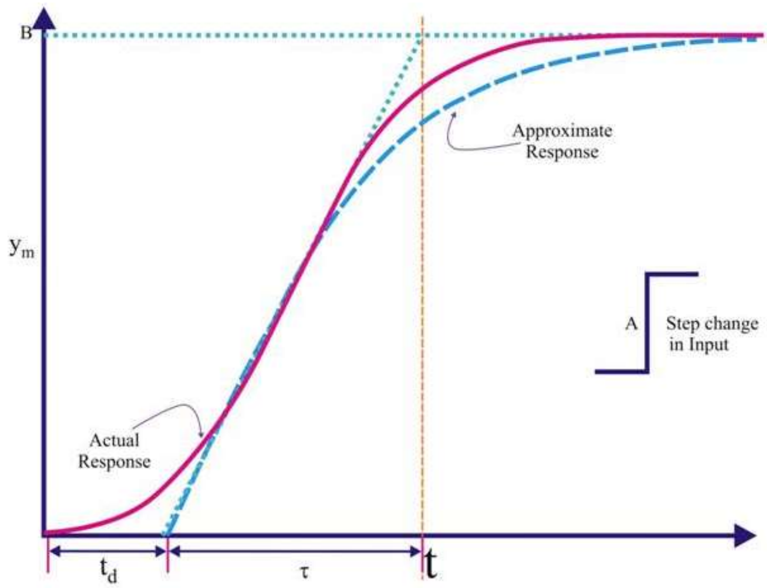


Figure 3-7: Process reaction curve for the Cohen-Coon method showing the dead time and time constant.

Table 3-3: Cohen-Coon method

Controller Type	Kp	Ti	Td
P Controller	$\frac{1}{k.z}\left(1+\frac{z}{3}\right)$		
PI Controller	$\frac{1}{k.z}\left(0.9+\frac{z}{12}\right)$	$t_d\frac{30+3.z}{9+20.z}$	
PID Controller	$\frac{1}{k.z}(1.33+0.25z)$	$t_d\frac{32+6.z}{13+8.z}$	$t_d\frac{4}{11+2.z}$

3.2.5 Relaxed Ziegler and Nichols (R-ZN)

The R-ZN is an improvement on both Z-N and SIMC tuning rules and was proposed by Finn Haugen and Bernt Lie in [22]. It's modified Z-N closed loop tuning formula based on SIMC tuning formula for an integrator plus time delay process. The parameter setting is as given in equation (3-16) and with a more relaxed (enhanced relaxer) as given in equation (3-17). The parameter E_r is a relaxation parameter chosen by the user and is always greater than or equal to unity. Equation (3-17) and (3-14) are equivalent when E_r is equal to 1. The relaxation parameter affects the gain and the integral time in inverse proportion such that an increase in the proportional gain will result to reduce integral time giving a good tradeoff between disturbance response and response time respectively.

$$K_p = 0.320\mathrm{Ku};\ T_i = \mathrm{Pu} \tag{3-16}$$

$$K_p = \frac{2}{\pi(1 + \mathrm{E}_r)}\mathrm{Ku};\ T_i = \frac{1 + \mathrm{E}_r}{2}\mathrm{Pu} \tag{3-17}$$

3.2.6 Simple Internal Model Controller (SIMC)

The SIMC method which is also called the Skogestad method stem from the Internal Model Control tuning rules (by Rivera et al.1986). The SIMC like the IMC is a model based tuning method where the controller parameters are expressed as a function of the process model parameters. The procedure involves the derivation of first- or second order plus delay model and using this to derive controller settings. The controller setting gives a PI setting if the starting model is a first order and a PID setting if it's a second order model. [5]

The method applies model reduction by half rule. The rule stated that any process with nth order can be approximated by a first or second order system with time delay. The first order plus time delay is equivalent to an integrator plus delay process as shown in equation (3-19), for a large time constant. The second order model with time delay, equation (3-21) is equivalent to first order with time delay equation (3-20) if T_2 is mush lesser than T_1 or equal to zero.

$$h_p(s) = k\frac{e^{-\tau s}}{s} \tag{3-18}$$

$$h_p(s) = k\frac{e^{-\tau s}}{1+T_1 s} \approx k'\frac{e^{-\tau s}}{s},\ k' = \frac{k}{T_1}\quad \text{for a large } T_1 \tag{3-19}$$

$$h_p(s) = k\frac{e^{-\tau s}}{1 + T_1 s} = k\frac{1 - \tau s}{1 + T_1 s} \tag{3-20}$$

$$h_p(s) = k\frac{e^{-\tau s}}{(1 + T_1 s)(1 + T_2 s)} = k\frac{1 - \tau s}{(1 + T_1 s)(1 + T_2 s)} \tag{3-21}$$

Generally, if the model does not have a dominant second order dynamic, then it's can be approximated with a first order model as given in equation (3-20). Also, a first order time delay system with a dominant time constant can be approximated with an integrator plus time delay model of equation (3-18). However, a system with dominant second order dynamics can only be approximate with second order model as given in equation (3-21). [18]. Given an nth order system as given in equation (3-22) this can be reduced to the form of equation (3-20) or (3-21) by setting the time constant and time delay as given in equation (3-23) and (3-24) respectively.

$$h_p(s) = k\frac{1-\tau s}{(1+T_1 s)(1+T_2 s)\ldots\ldots\ldots(1+T_n s)} \tag{3-22}$$

$$T_1 := T_1 + \frac{1}{2}T_2\ ;\ \tau := \tau + \frac{1}{2}T_2 \ldots + T_n \tag{3-23}$$

$$T_1 := T_1;\ T_2 := T_2 + \frac{1}{2}T_3\ ;\ \tau := \tau + \frac{1}{2}T_2 \ldots + T_n \tag{3-24}$$

τ is the time delay and $1 - \tau s$ is the time delay approximation, T_1, T_2 …. T_n are time constants. The SIMC PID controller setting for a PID[2] is given by equation.

$$K_p = \frac{T_1}{k(T_c + \tau)};\ T_i = min[T_1, c(T_c + \tau)]\ ; T_d = T_2 \tag{3-25}$$

Where the respective terms in equation (3-25) are as defined in section 3.1.1.1[3] and T_C is a user specified time constant for set-point response. In the original IMC setting the integral time is set to the dominant time constant, and in SIMC a smaller integral time was suggested to improve on the disturbance response. The parameter c, determined the value of the integral time and a value of 4 was suggested by Skogestad. A more conservative value for the parameter c is suggested in other variant of this method for better disturbance response performance.

3.2.7 Tyreus-Luyben Method

The Tyreus-Luyben method was proposed by Bjorn Tyreus and William Luyben in 1992. [23]. This method provides an improvement on IMC by using classical frequency response analysis. This method optimally selects a tradeoff between the integral time and proportional gain of a PI controller. The tradeoff is based on the fact, that there is a minimum reset time for the integrator time below which reasonable closed loop damping confidents cannot be achieved and there must be an optimum controller gain for this to occur. The method is similar to Z-N closed loop method and the experimentation steps are the same for finding the ultimate gain and period. The setting for PI and PID controller are shown in Table 3-4.

Table 3-4: Tyreus-Luyben method.

Controller Type	Kp	Ti	Td
PI	$K_u/3.2$	$2.2P_u$	
PID	$K_u/3.2$	$2.2P_u$	$P_u/6.3$

3.2.8 Relative Time Delay Error Method (RTDE)

An important and crucial process model is an integrator plus time delay process, this type of process is common in industries. A first order plus time delay system can also be approximated with an integrator plus time delay process as shown in equation (3-19) if the time constant is much larger than the delay (a lag dominant process). An example is the temperature control system for which the Air Heater laboratory process described in section 5.1 is a typical of this

[2] A first order approximation will result in a PI controller while a second order will result in a PID controller.

[3] The standard or Idea PID algorithms is used.

process. Therefore, it is necessary to examine a typical tuning rule for these processes. David [24] proposed the RTDE method that gives an improvement on SIMC discussed in section 3.2.6 for time lag dominant systems.

The PI parameters are chosen in such a way as to ensure a maximum delay error,$d_{\tau max}$ for stability margin.

$$\delta = \frac{d_{\tau max}}{\tau};\ \bar{c} = \alpha\beta;\ \beta = \frac{\bar{c}}{a}(\delta + 1);\ \alpha = \frac{a}{(\delta+1)};$$

$$a = \frac{\tan^{-1}(\bar{c}\sqrt{f})}{\sqrt{f}};\ f = \frac{1+\sqrt{1+\frac{4}{\bar{c}^2}}}{2} \tag{3-26}$$

The PI parameter are given in term of time delay error ratio parameter $\delta, metho\quad product\, parameter\ \bar{c},$ which is equal to 3.28 for the original ZN method [24] and the integrator gain velocity k' as shown in equation (3-27).

$$K_p = \frac{\alpha}{k'\tau} = \frac{a}{k'\tau(\delta+1)};\ \text{and}\ T_i = \beta\tau = \frac{\bar{c}}{a}(\delta + 1)\tau \tag{3-27}$$

This method was extended in [25] to double integrating plus time delay system where a PD and PID controller was designed for such a system based on relative time delay error. Consider a PI controller with an integrating plus time delay system will have a loop transfer function as given in equation (3-28) also a PD controller for a double integrating plus time delay system will have a loop transfer function as given in equation (3-29)

$$h_o = \frac{K_p K}{T_i}(1 + T_i s)\frac{e^{-\tau s}}{s^2} \tag{3-28}$$

$$h_o = \frac{K'_p K}{T_d}(1 + T_d s)\frac{e^{-\tau s}}{s^2};\ K'_p = K_p T_d \tag{3-29}$$

Comparing equations (3-27), (3-28) and (3-29) will give a PD controller tuning formula as given in equation (3-30).

$$T_d = \beta\tau\ ;\ K_p = \frac{\alpha}{k'\tau K_d} \tag{3-30}$$

3.2.9 A Semi-Heuristic Process-Reaction Curve PID Controller Tuning Method

A method that is based on both heuristic and analytical method is proposed by David and Dalen in [15]. This method is based on both the Z-N open loop method and the RTDE method. The open loop reaction curve shown in Figure 3-5 is used to derive the RTDE parameters as given in equation (3-27).

3.2.10 Åstrom Relay method

Åstrom modified the transient response experimentation with a relay in a feedback loop with the process. The major aim of this method is to be able to automate the Z-N method in other to capture the ultimate gain and period of the process. Detail of the process are given in chapter 4.

3.3 Stability Analysis

To understand the basis of PID tuning and for effective use of the tuning rule, the principle of stability in the control theory is important. Given a linear continues time system as given in equation (3-31), the system stability is defined given the following conditions for the system matrix A [18]

- The real of the eigenvalues are negatives which means they are in the left half of the complex plane, then the system is said to be stable.
- The system is marginally stable if one or some of the eigenvalues lies on the imaginary axis
- A system with an integrator(s) will have one or more of its eigenvalues located at zeros
- An unstable system will have one or more of its eigenvalues located at the right half plane

For a linear discrete time, system, the following conditions holds

1. The system is stable if the eigenvalues of the system matrix A is located inside the unit circle in the complex plane.
2. An integrator(s) exist in the system if one or more of the eigenvalues have magnitude of one.

$$\dot{x} = Ax + Bu: \qquad y = Cx \tag{3-31}$$

In frequency domain or transfer function representation of the system as given in equation (3-32), the above condition also holds if the poles of the transfer function satisfy the above listed conditions.

$$h_p(s) = C(SI - A)^{-1}B \tag{3-32}$$

In equation (3-32) the s is the Laplace operator A, B and C as given in equation (3-31)

The condition for stability can be study with different tools like Nyquist stability criterion, Nichols plot, Bode Plot, finding the gain margin and phase margin.

3.4 Performance Criteria

The PID tuning are based on experimentation on either the model or the real system. There is possibility of uncertainty in the model, external disturbance or process dynamic and thus the PID tuning parameters does not guarantee a time invariant capability for the system. Based on this problem, performance and robustness criterial are introduced to provide a benchmark for the tuning process in other to get a PID with stable performance over a wide range of system uncertainties.

The performance indication is set to ensure that the values of the controller parameters satisfies the underline objectives of a robust and stable system. A good controller will ensure that the system performance is within acceptable limits, to give the desired closed loop response and provide a good robustness to model uncertainty and noise. In [9] three major measures used to compare the performance of tuning methods are:

- Set point tracking and disturbance response

- Robustness against parameter changes in the model, considered as model imperfection or model variance from the real one.
- The simplicity and fastness of the tuning procedure, which is relative as this, depend on the individual operators' skills and familiarity with the controller.

3.4.1 Transient and Frequency Response Analysis

The closed and open loop frequency response measures of a system can be used to measures the robustness of the system under consideration.

- Step Response
  Open and closed loop can be used can be used to judge the criticality of process variation and the need of a controller. For a process as given by equation (3-33), with 'a' having three different values as given in Figure 3-8 and Figure 3-9 [4], the values of the disturbance can shift the system from stable to non-stable margin. Therefore, it is essential to design a controller to maintain the system stability at all times irrespective of varying disturbance. Figure shows that for the system at a= 0.01, the system is approach stability whereas for the other cases the system is unstable.(3-22)

$$H_0\,(s) = \frac{1}{(s+1)(s+a)} \tag{3-33}$$

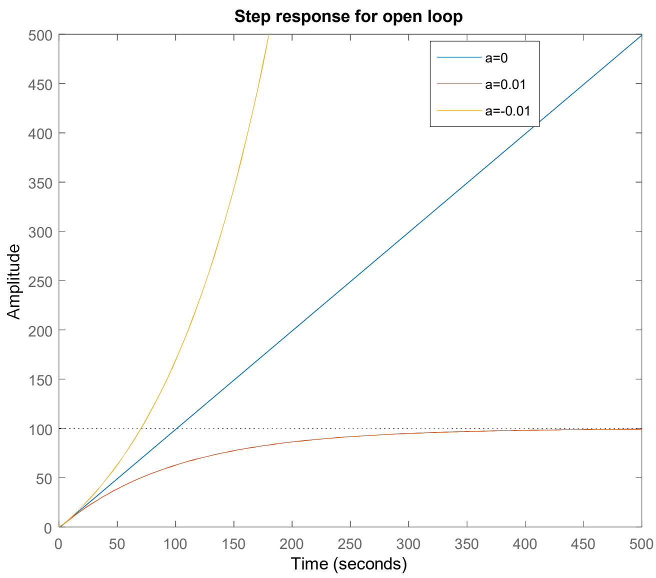


Figure 3-8: Open loop step response for the system in equation (3-33)

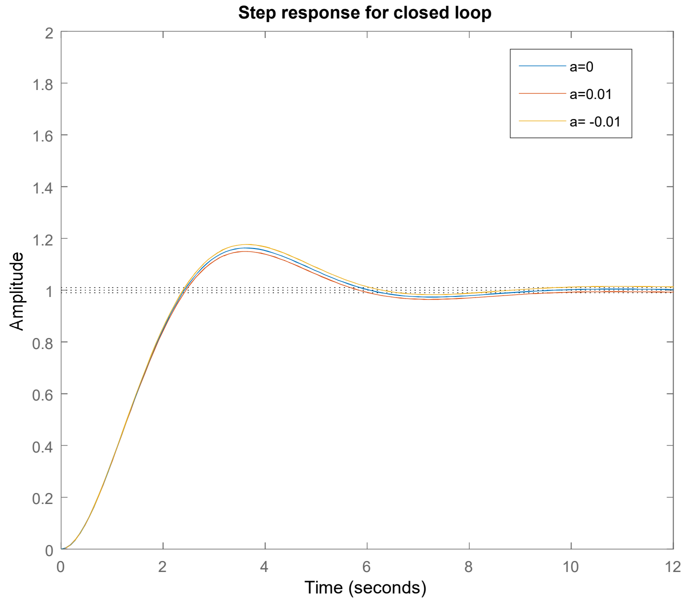


Figure 3-9: Closed loop step response for the system in equation (3-33)

The performance of the control system may be analyzing from the step response of the system as shown Figure 3-10, with the following criteria as define in [1], [26].

- The time for the system to rise from 10% to 90 % of its steady state value as shown in Figure 3-10.
- The overshoot O_s is the ratio of the first peak minus the steady value, and the steady state value.
- Steady state error is the value of the control error at steady state
- Settling time is the time for the response to reach a specified percentage of the steady value, usually within 2% of the input step.
- Decay ratio is the ratio between two consecutive peaks for a step response
- Time to peak which is the time the step response reaches its peak value

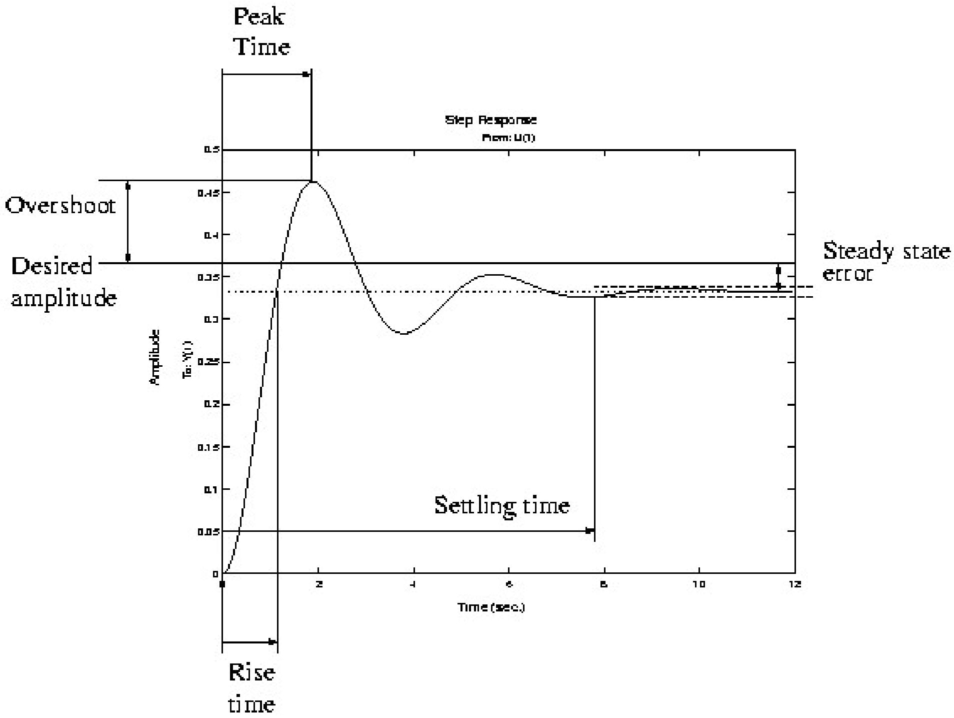


Figure 3-10: Closed loop step response showing performance indexes [26].

3.4.2 The sensitivity indexes

Robustness to process variation and measurement noise can be capture with the sensitivity function. The sensitivity index and complementary sensitivity index gives an indication of the sensitivity of the closed loop system to disturbances. As discussed in section 3.4.1 the responses of a system can be improved by providing a feedback as in a closed loop, however this may not provide an optimal result and the performance of the loop can be measures with sensitivity index as follows:

- Sensitivity Index – the sensitivity function can be defined as shown in equation (3-34) and this should be small at low frequency for robust design.

$$S(s) = \frac{1}{1 + h_o\,(s)} \tag{3-34}$$

- Complementary Sensitivity Index - the complementary function is given by equation (3-35) and this provides information about the set point tracking of the controller and should be as closed to one as possible. $h_o\,(s)$ is the open loop transfer function of the system.

$$T(s) = \frac{h_o\,(s)}{1 + h_o\,(s)} = 1 - S(s) \tag{3-35}$$

- Maximum Sensitivity Index – the maximum sensitivity function can be defined as shown in equation (3-34) as the maximum of the sensitivity index and determined the robustness of the controller

$$M(s) = max|S(s)| \tag{3-36}$$

3.4.3 Stability margins and cross-over frequencies

The following four frequency domain specification are importance tools in analyzing the performance and robustness of the control system closed loop dynamic. [17]

- The phase crossover frequency, ω_{180}

This is the frequency at which there is a phase shift of -π. This is given by equation (3-37).

$$\angle h_0(j\omega_{180}) = -\pi \tag{3-37}$$

- The Gain Margin (GM)

The gain margin is the system gain change that will results in a marginally stable system at phase crossover frequency. This shown in equation (3-38).

$$GM = \frac{1}{|h_0(j\omega_{180})|} \tag{3-38}$$

- The gain crossover frequency, ω_c

This is the frequency at which the system gain is unity. This shown in equation (3-39).

$$|h_0(j\omega_c)| = 1 \tag{3-39}$$

- The Phase Margin (PM)

This is the amount of phase shift that can be tolerated before the system will become unstable as shown in equation (3-40).

$$PM = \angle h_0(j\omega_c) \ + \pi \tag{3-40}$$

- Maximum time delay error ratio

The phase margin is an indication of the amount of delay that can be accommodated before the system will become unstable. The maximum time delay ratio is also a measure of the performance of the system as given in equation (3-41)

$$\delta = \frac{d_{\tau max}}{\tau} = \frac{PM}{\omega_c \tau} \tag{3-41}$$

The rule of thumb is that a well tune controller should have GM between 1.7 and 4.0 and PM range of 30° to 45°. [17]

3.4.4 Bode Plot and Bode Stability Criterion

The plot can be used to display the frequency response characteristic of a transfer function model of a process. The plot is generated by plotting the magnitude and phase as a function of the frequencies on separate plane. From the plot stability margins and cross over frequencies as described in section 3.4.3 can be obtained.

The Bode stability criterion stem from the stability conditions as given in section 3.3 that a system is stable if and only if all roots of the characteristic equation lie to the left of the imaginary axis in the complex plane with an exception of a single pole at the origin and that there is only single phase and gain cross over frequencies, then the closed loop system is stable if the open loop gain at the cross over frequency is less than one otherwise its unstable. [17].

The bode plot from the air heater process is shown in Figure 3-11.

The following relation also can be obtain from the Bode plot:

- A system is stable if the phase cross over frequency is greater than the gain cross over frequency. Also if both gain and phase margin are positive then the system is stable.
- A system is said to be marginally stable if both the gain and phase margins are zero, or if the phase and the gain cross over frequency are equal.
- A system is unstable if phase cross over frequency is less than gain cross over frequency or if either one/both of the margin is negative.

Table 3-5 gives a summary of the performance indication that can be capture from the frequencies response analysis.

Table 3-5: Frequency and stability relationship

	Stable	Marginally Stable	Unstable
Poles	All negative poles	One or more poles are zeros	All positive poles
Frequency margin	$\omega_c < \omega_{180}$	$\omega_c = \omega_{180}$	$\omega_c > \omega_{180}$
Gain margins	$\lvert L(jw)\rvert < 1$	$\lvert L(jw)\rvert = 1$	$\lvert L(jw)\rvert > 1$

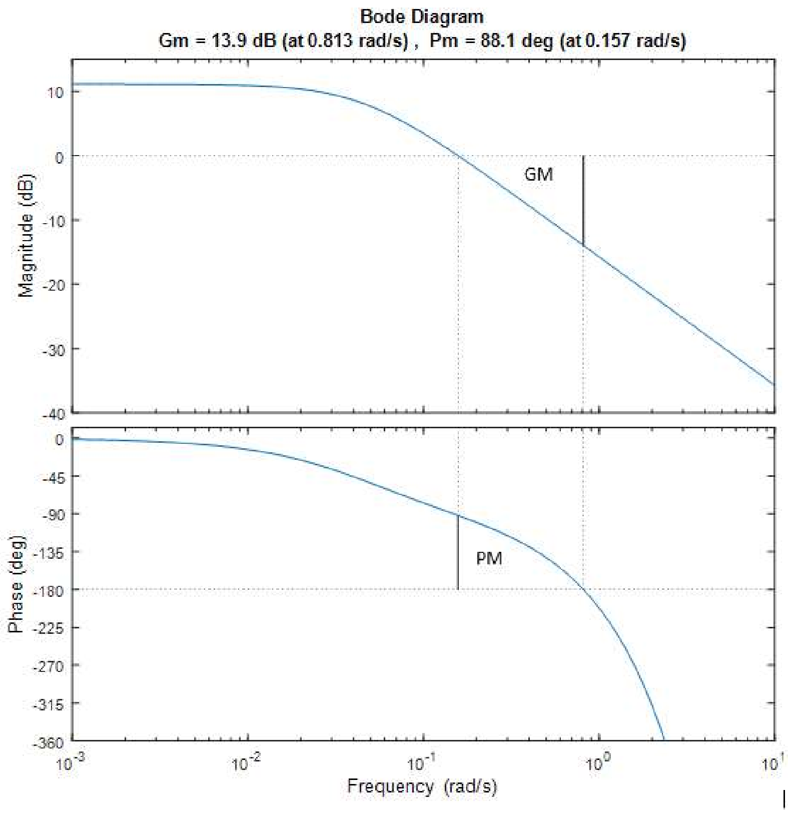
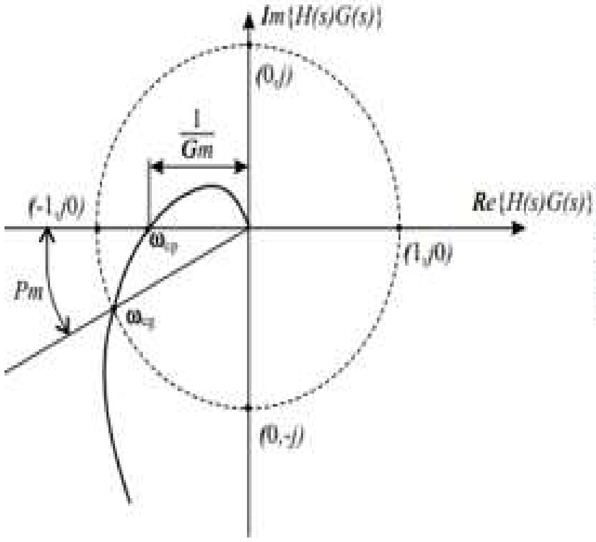


Figure 3-11: Bode Plot of the air heater (using bode function in MATLAB)

3.4.5 Nyquist Plot and Nyquist Stability Criterion

If the system has more than one phase and gain cross over frequencies (i.e. the plot crosses the -180 degrees more than once) or the open loop is unstable then Bode cannot used to analysis the system stability. The Nyquist plot can handle such a situation and it's the same as bode plot with magnitude and the phase plotted on a single plane. The stability criterion is that if open-loop Nyquist plot of a feedback system encircles the point (-1, 0) as the frequency varies from negative infinity to positive infinity, then the closed loop response is unstable. Alternatively, the number of unstable closed-loop poles is equal to the number of unstable open –loop poles plus the number of encirclements of the point (-1, j0) of the open loop transfer function of the feedback system. The right plot in Figure 3-12 shows the margins from the plot while the left figure shows the Nyquist plot for the air heater open loop transfer function. The air heater as given in equation (5-10) is stable as the contour on the Nyquist plot does not enclose the -1+0j point.



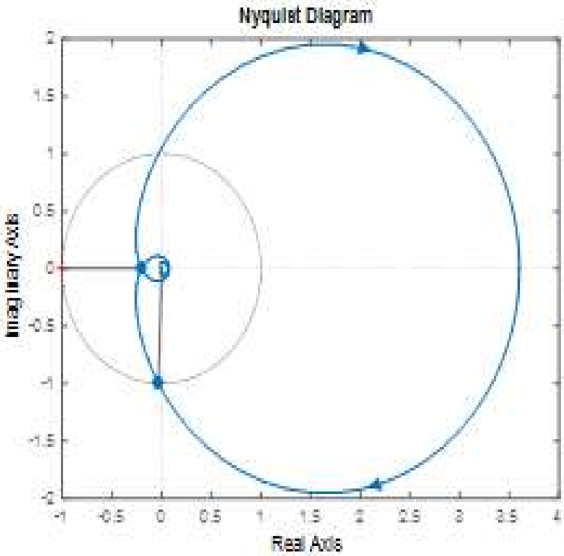


Figure 3-12: Nyquist plot showing margins [27] and the left plot show the Nyquist plot for the air heater transfer function.

3.4.6 Integrated Absolute Error Index

The Integrated Absolute Error, IAE, or set point tracking can be used as a criterion to determine the offset between the set point and the output. The drift from desired values is cause by load disturbance and IAE can also be used to measured disturbance response of the controller. The IAE index is define as the summation of the absolute error over a given period and is calculated as given in equation (3-42)(3-22).

$$IAE = \int_{t_1}^{t_2} |e| dt \qquad (3\text{-}42)$$

The IAE for set point tracking is designated as IAE_S and for disturbance response as IAE_D. The lower the IAE index the better control performance response of the system. Other index like Integral error (IE), Sum of prediction error squared (SSEP) and Integrated squared error are also used and provide almost the same bench mark for comparison.

3.4.7 Total Variance Index

The total variation of the control effort is given by the total variance index and is calculated as shown in equation (3-43)

$$TV = \sum_{1}^{\infty} |u_{k+1} - u_k| \tag{3-43}$$

4 PID Auto-Tuning Methods

PID auto tuning simplified the task of operators manually tuning each PID loop in a process plant. PID auto tuning saves time and reduces production lost by eliminating down for manual tuning of PIDs. Based on the context of this thesis the PID tuning rules as discussed in chapter 3 will be refer as PID tuning rules in offline mode. This chapter will present tuning rules that can be apply online and easily automated and these rules will be referred to PID auto tuning. PID auto tuning can be achieved in major three ways among which are gain scheduling, model base adaptive tuning and relay feedback experiment. The Relay feedback auto tuning method presented by Åstrom and Hågglund will be presented [2] [6]. An extension of this method presented by Schei [3] will be presented as well.

The aim of the auto tuning is to generate a point in the Nyquist curve that can be used to identify the varying system parameters (for changing system dynamics) as discussed in Section 2.2.5. In simple terms the relay auto tuning is a process of bringing the process to oscillation by interchanging the controller with a relay or in a feedback loop with the controller. The relay can be modified in different ways to get better result for the parameter estimation, one of such is the used of relay with hysteresis. The amplitude and frequency of this sustained oscillation is used to tune the controller parameters. This method can easily be employed in a closed loop system and can be designed to use just a push button to start the process.

The procedure for auto tuning consist of three major steps as reported in [1], these are

- The generation of process disturbance/oscillation intentionally by the operator
- Automatic evaluation or computation of the disturbance dynamics
- The usage of the disturbance response to re-evaluate the controller parameters

In the relay feedback experiment, the amplitude of the oscillation is controlled by the amplitude of the relay. [28] The major aim is to generate a limit cycle at the critical point of the process. Other point of identification is also possible with the modification of the relay. Limiting the oscillation in this way will present the process going into uncontrolled oscillation, which can damage process equipment. The major problems with this method is that if there is a noisy system that can cause hysteresis, then the identified frequency is not the ultimate and further modification and calculation need to be exploited.

There are two major methods for analyzing relay excitation of the process:

- Limit Cycle analysis
- Describing functions

Relay has been in used since the fifties as amplifier but was applied to adaptive control in the sixties. Relay auto tuning has the following advantages: [6]

- It’s easy to automate by switching from the controller to the relay during auto tuning operation
- The method does not introduce loop instability
- Little priory knowledge of the process is necessary
- With the correct selection of the relay parameter the process error can be minimize during the tuning.

Åstrom Relay is designed in such a way that whenever the process value (or error) passes a preset point, the process is actuated by the relay as shown in Figure 4-2 [29]., an oscillation within the limit cycle of the relay amplitude is generated. Schei relay is connected as shown in

Figure 4-4, and when the relay is operational the input to the system will varied between the set point plus relay amplitude and set point minus relay amplitude. The experiment is done in closed loop to keep the process output within limited bounds.

Åstrom method required that the system is brought to rest before the experiment is performed whereas Schei make used of established control loop, thus any input during the experimentation will be sufficient to trigger the relay into action.

The system is automated as shown in the block diagram of the implementation in Appendix C, where the operator can select through a knob either the PID or relay mode.

4.1 Ultimate Period and Gain from Relay Experimentation

The section will give the mathematical derivation of the system gain and period during relay experimentation. The derived frequency and period depends on the configuration of the relay used in the experiment. An ideal relay will give the approximate ultimate period and gain whereas other point of interest can be determined as discussed in Section 4.4.

4.1.1 Limit cycle Oscillation

Limit cycle is a path for which the energy will constant with no loss or gain in a system. The non-linearity of the relay causes the process to reach limit cycle oscillation quite rapidly. Debabrata and Bhattacharjee demonstrate in their paper [30] that linear systems under memory effects can cause limit cycle oscillation as well. The principle of limit cycling is a key input to the describing function analysis. The method of determining the period and amplitude of oscillation will be explained as described in [4]. The non-linearity in a relay as shown in Figure 4-1 will generate persistence excitation that will make the output of the linear element to goes to oscillation as follows:

Given a state space of the process as given in equation (4-1) a limit cycle is generated if the condition in equation (4-5) is true

$$\dot{x} = Ax + Bu;\ \text{y} = Cx \tag{4-1}$$

Assume the process is oscillating with a limit cycle of period T, and t_k is the time for relay to switches at time k, therefore the time to change from one state to another, $\Delta t\ is\ T/2$ which is half the limit cycle period as given in equation (4-2)

$$\frac{x_{k+1}-x_k}{\Delta t} = Ax_k + Bu_k; \quad \Delta t = t_{k+1} - t_k = \frac{T}{2} \tag{4-2}$$

Recall that the output of the relay, d is the input to the process, therefore resolving equation(4-2) will gives equation (4-3), since the limit cycle is symmetric, the input at time k and k+1 have the relationship given in equation (4-4)

$$x_{k+1} = \phi x_k + \gamma \text{d}$$
$$\text{Where } \phi = \left(I + \frac{T}{2}A\right) \text{ and } \gamma = \frac{BT}{2} \tag{4-3}$$

$$x_{k+1} = -x_k \tag{4-4}$$

Putting equation (4-4) into equation (4-3) and solve for the output in equation (4-1), will gives equation (4-5) which must be true for a limit cycle to occurs.

$$C(I + \phi)^{-1}\gamma = 0 \tag{4-5}$$

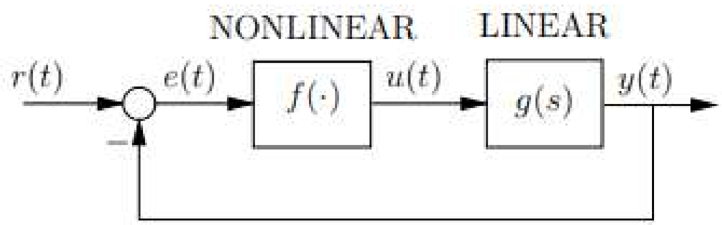


Figure 4-1: A nonlinear element f (.) such as a relay will drive the linear element g(s) into oscillation.

4.1.2 Describing function analysis (DF)

The describing function was developed to investigate the limit cycles behavior in nonlinear systems. It gives a reliable estimate of limit cycle behavior. Describing function can be defined as the ratio of the fundamental component of the relay output to the sinusoid input. The sinusoid input can be given as described by equation (4-6):

$$e(\theta) = a \sin\theta, \theta = \omega t \tag{4-6}$$

The Fourier series expansion of the periodic output signal is given by equation (4-7):

$$f(\theta) = a_0 + \sum_{n=1}^{\infty}(a_n \cos n\theta + b_n \sin n\theta) \tag{4-7}$$

Where the fundamental component is given by equation (4-8):

$$a_0 = \int_0^{\pi} e(\theta)d\theta \tag{4-8}$$

The harmonics components are given in equation (4-9) and (4-10):

$$a_n = \int_0^{2\pi} e(\theta)\cos(n\theta)d\theta \tag{4-9}$$

$$b_n = \int_0^{\pi} e(\theta)\sin(n\theta)d\theta \tag{4-10}$$

Since the output of the non-linearity is assumed to be sinusoid, the term a_0 and a_n are equal to zero because the output is symmetrical about the origin. Equation (4-7) becomes:

$$f(\theta) = \sum_{n=1}^{\infty} b_n \sin(n\theta) \tag{4-11}$$

$$b_n = \frac{2}{\pi}\int_0^{\pi} dsin(n\theta)d(\theta) \quad (4\text{-}12)$$

Solving equation (4-12), gives equation (4-13) for odd harmonics and zeros for even harmonics:

$$b_n = \frac{4d}{n\pi} \quad (4\text{-}13)$$

Thus equation (4-8) becomes which is the output of the relay:

$$f^r(\theta) = \sum_{n=1}^{\infty} \frac{4d}{n\pi}\sin(n\theta) \quad (4\text{-}14)$$

And the describing function N(a) is given by the ratio of equation (4-14) to (4-6) at the fundamental component:

$$N(a) = \frac{output\ of\ relay}{input\ of\ relay} = \frac{f^r(\theta)}{e(\theta)} = \frac{4d}{a\pi} \quad (4\text{-}15)$$

The system will be show continues cycling or marginal stability if the open loop transfer function is zero, (Zeros poles for the feedback system.). This means that the system input/output amplitude and phase are the same for the closed loop. For this condition equation (4-16) is established:

$$1 + N(a)G(iw_u) = 0;\ G(iw_u) = -\frac{a\pi}{4d} \quad (4\text{-}16)$$

The ultimate gain, Ku and ultimate period, and ultimate frequency are as given in equation:

$$Ku = \frac{4d}{a\pi};\ Pu = \mathrm{Tu};\ w_u = \frac{2\pi}{Tu} \quad (4\text{-}17)$$

Where Tu is the period of the sustained oscillation at approximately cross over frequency:

4.2 Åstrom Relay Method

Åstrom uses a simple relay in a feedback loop to achieve the same aim as Nichols and Ziegler tuning rules. The critical point of a process determined by Z-N by increasing the proportional gain indefinitely was achieved by Åstrom with the use of relay experimentation with just a fractional perturbation signal of the process range. The amplitude of the relay is a small amount of the control effort usually between 2% to 10% of the control effort [29]. This method ensure that the system oscillation is achieved without the risk of making the system goes into definite instability. As shown in Figure 4-2, the control input into the system is the relay output. This varies between positive and negative values of the relay amplitude, h, as given in equation (4-18) and (4-19) and displayed in Figure 4-3. Some process like the Air heater process does not accept a negative control value and as such the Åstrom relay tuner cannot be implemented on such a system. The system gains and frequency is calculated as given in equation (4-17).

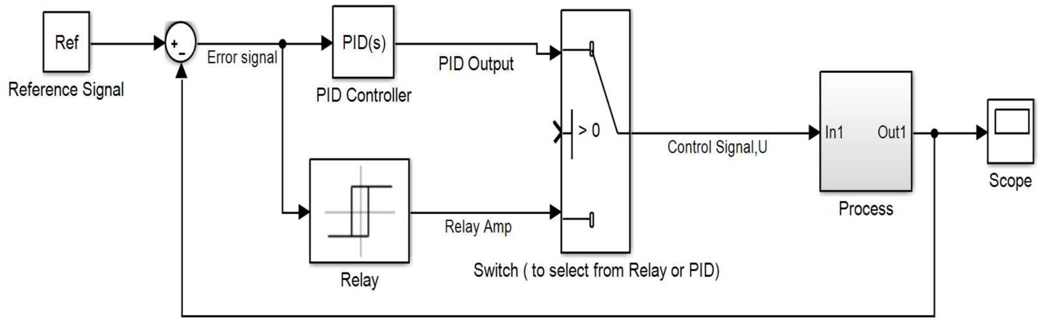


Figure 4-2 Åstrom Relay Auto Tuner showing the Relay and PID controllers' connection to the Process in a feedback loop. The operator can perform auto tuning by switching from the controller to the relay in which case the input into the process will oscillate between the set point +/- the relay amplitude.

The period of oscillation is determined through zero crossing of the signal during the relay experimentation as shown in the MATLAB code in Appendix B. Amplitude is measured through the simple measurement of the maximum and minimum values of the signal during the relay experimentation. The code for this is presented in Appendix B.

With the ultimate gain and period derived as in equation (4-17), Z-N tuning algorithms as given in Table 3-2 can be applied. Other tuning algorithms can as well be applied to achieve varying results but this will not the subject of discussion for this thesis.

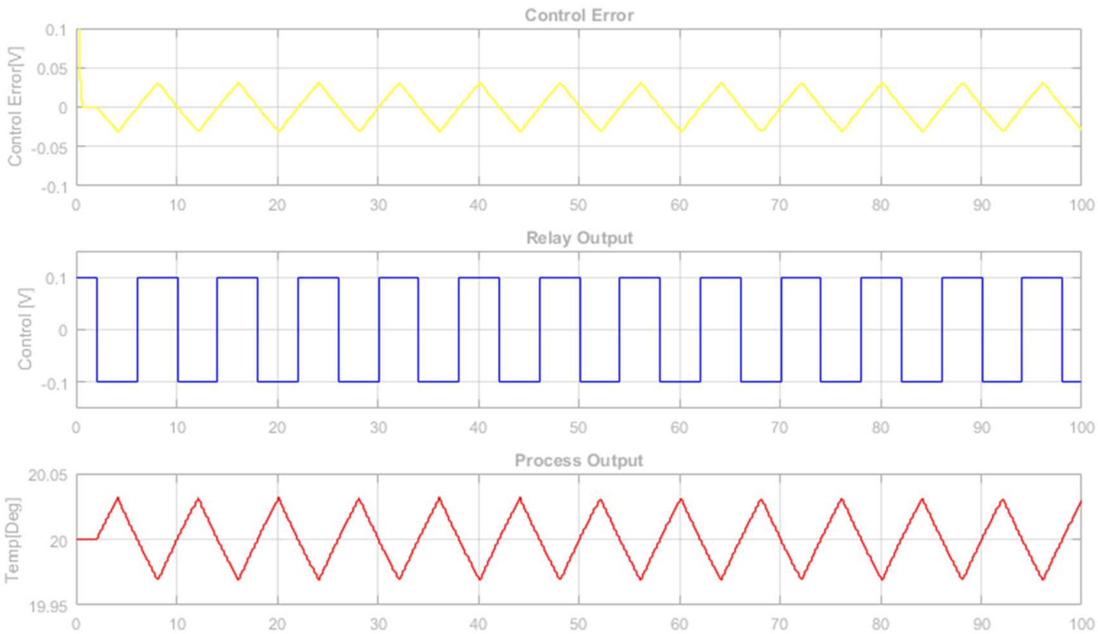


Figure 4-3: The plots from the relay experiments using Åstrom method with Air heater. The upper plot (yellow color) shows the error signal between the reference and output signal, this is used as the input to the linearizing element to get the switching action for the relay in the middle plot (blue color). The middle plot shows the output from the relay and this is the control signal to the process. The Last plot in red color shows the process output.

4.3 Schei Relay Method

Schei method is like Åstrom with the same principle of relay feedback experimentation during the tuning process. The method assume that a stable controller is in operation before the experiment and this need to be improved. The reference signal to the PID controller is varies between positive and negative values of the relay. Therefore, the control signal to the process is a function of the relay switching and thus a limit cycle will be established as discussed in section 4.1.1. It's also employed a filter in the derivative loop to help in noise situation as discussed in section 4.4. Figure 4-4 shows the block diagram of the implementation of Schei scheme and the result from simulation will be discussed in section 5.

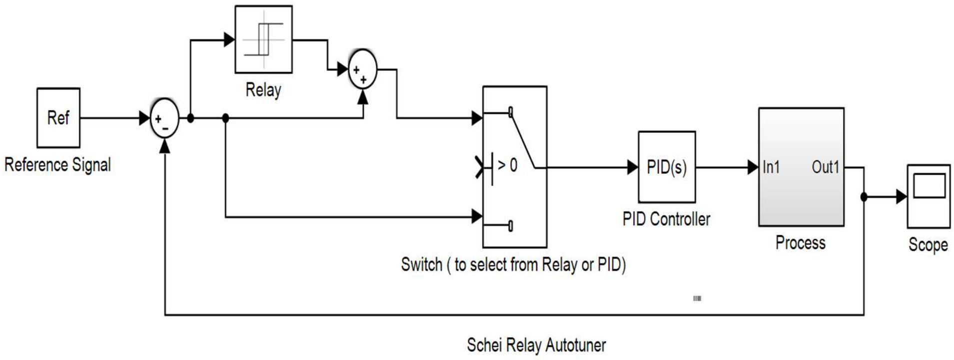


Figure 4-4: Schei Relay Auto Tuner showing the Relay and PID controllers' connection to the Process in a feedback loop. During relay experiment the reference signal varies in step by the relay output. The relay is made to switch by the error signal by the application of a linearizing element as shown in equation (4-18).

The principle is based on the near critical point estimation with automatic generated relay test signal that make the process to oscillate around the ultimate frequency with bounded amplitude. This estimation point is a function of the linearizing element in the loop.

The relay is designed to change the control effort each time the process output passes the set point (or the error crosses zero). This will create an oscillation that will becomes stable after some time as shown in Figure 4-3 and Figure 4-5 for Åstrom and Schei method respectively.

The following steps are taken for both Åstrom and Schei auto tuner

- Let the process to a steady state
- Implement relay controller
    - If process gain is positive,

$$u = \begin{cases} h & if\ e \geq 0 \\ -h & if\ e < 0 \end{cases} \tag{4-18}$$

    - If process gain is negative,

$$u = \begin{cases} -h & if\ e \geq 0 \\ h & if\ e < 0 \end{cases} \tag{4-19}$$

- After a sustained periodic oscillation, evaluate using equation (4-17),
- Use PID tuning rules as described in 3.2

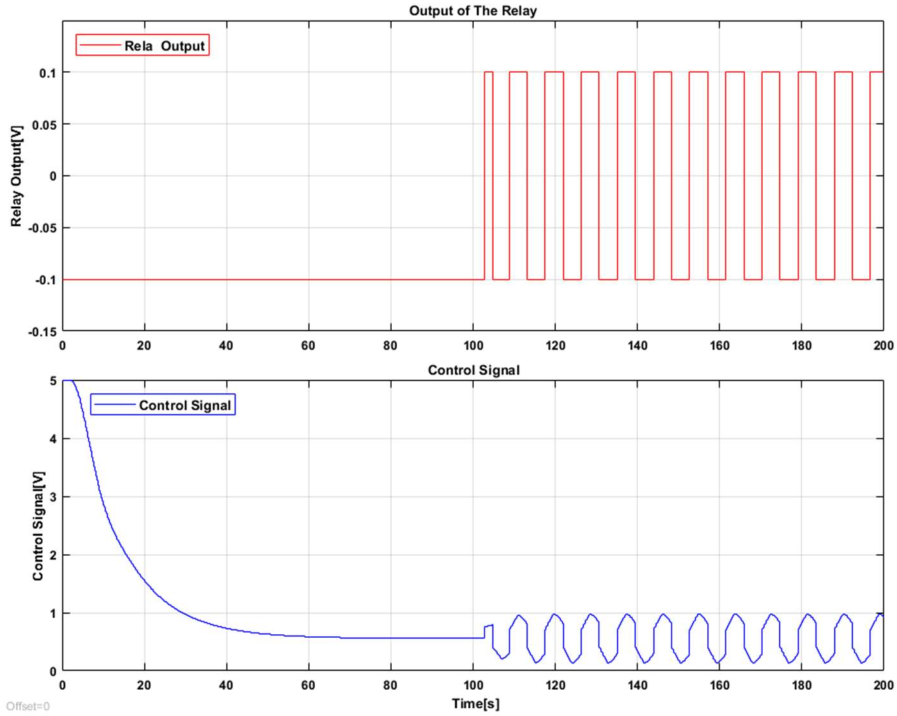


Figure 4-5: The plots from the relay experiments using Schei method with Air heater (simulator). The upper plot (red color) shows the output from the relay which is used to varies the reference signal thereby changing the error signal. The output from the PID controller is a function of the error signal and with the right perturbation of the relay, this error signal will cause a sustained oscillation in the process output which can be used to re-evaluate the process parameter.

Schei [3] auto tuner improve on the result by optimally trading between robustness and stability in choosing the PID parameters. This is done by specifying four different mode of the auto tuner based on the complementary sensitivity function as shown in equation (4-20)

$$\begin{aligned} &Mode\ 1 \quad T_i = \frac{3.0}{w_c}\ ; T_d = \frac{0.75}{w_c}, m_s = 1.1 \\ &Mode\ 2 \quad T_i = \frac{3.0}{w_c}\ ; T_d = 0, m_s = 1.1 \\ &Mode\ 3 \quad T_i = \frac{1.0}{w_c}\ ; T_d = 0, m_s = 0.9 \\ &Mode\ 4 \quad T_i = \frac{3.0}{w_c}\ ; T_d = 0, m_s = 1.3 \end{aligned} \tag{4-20}$$

4.4 Modification of the relay for noisy process and improved performance

The idea relay described in section 4.1and 4.2 perform poorly under noisy environment as shown in Figure 4-6 and can only identified the ultimate point on the Nyquist plot. Figure 4-7 shows the relay experimentation for a process with measurement noisy, the output is filtered to reduce the effect of the nose. As shown in Figure 4-7 the relay switching is erratic and cannot be use for obtaining the parameters as required. The switching makes it difficult to find the period and amplitude accurately. Modification of the ideal relay can be used to identified different point and prevent relay chattering. Several options exist for relay modification and only five of this option will be discussed in this section.

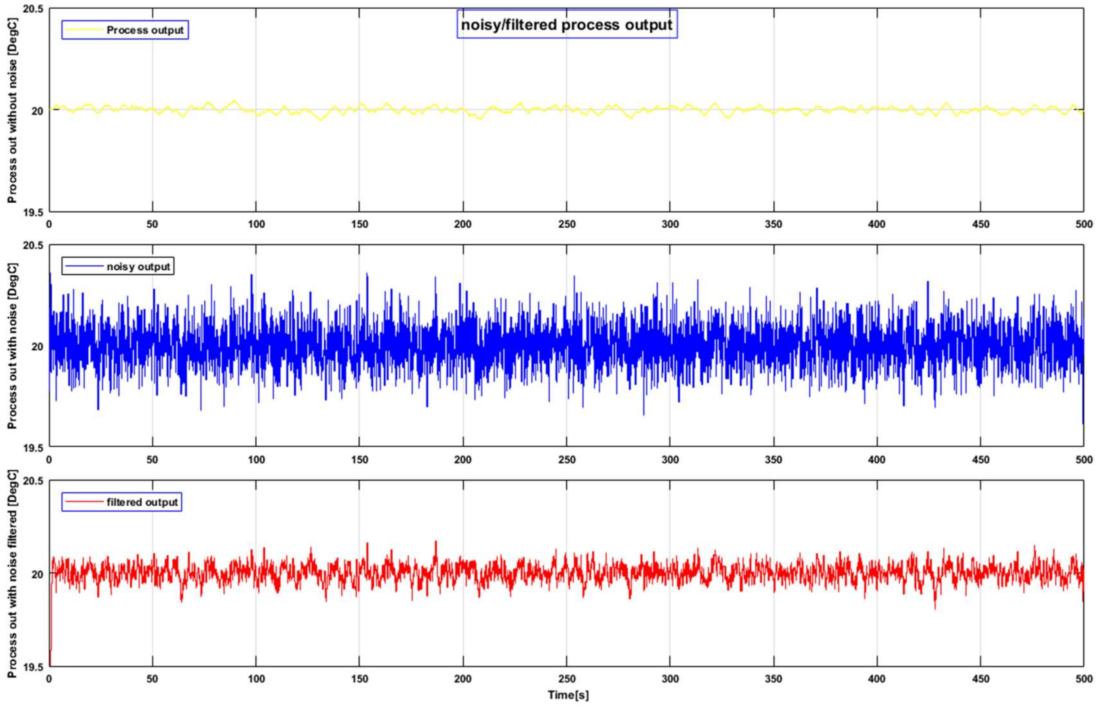


Figure 4-6: Air heater simulation with a random noise. The upper plot in yellow shows the process without noise, the middle plot shows the process with measurement noise added and the lower plot in red show the filtered process with a low pass filter.

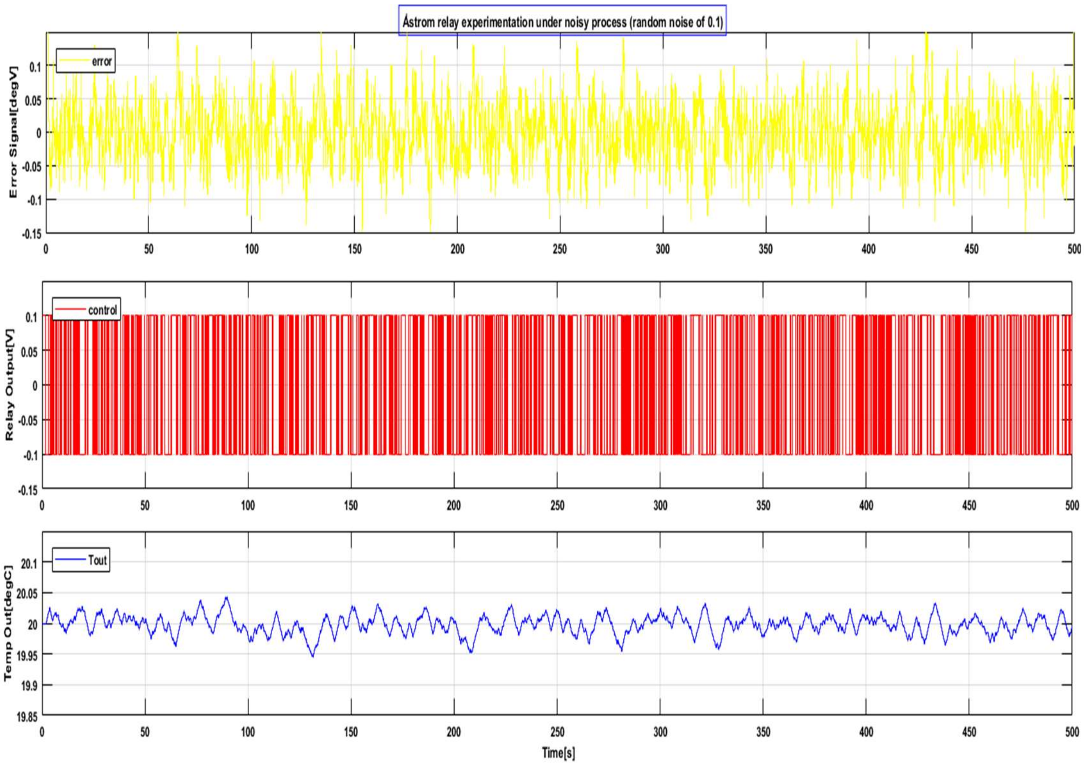


Figure 4-7: The plots from the relay experiments with noisy process using Åstrom method with Air heater. The upper plot shows the error signal between the reference and output signal, this is used as the input to the linearizing element to get the switching action for the relay in the middle plot. The middle plot show that the relay switches too fast because of the noise.

4.4.1 Symmetrical and Asymmetrical relay

Relay may be classified in general terms as asymmetrical (uneven) and symmetrical (even relay with or without hysteresis as shown in Figure 4-8 . Figure 4-8 (a, b, c, d) show relay which are symmetrical, asymmetrical symmetrical with hysteresis and asymmetrical without hysteresis respectively. Without the hysteresis Ɛ, the relay is referred to as ideal or conventional relays. [31].

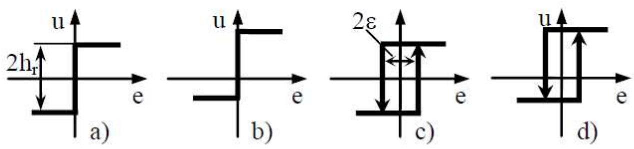


Figure 4-8: Types of relay shown (a) Symmetrical without hysteresis. (b)Asymmetrical without hysteresis. (c) Symmetrical with hysteresis. (d)Asymmetrical with hysteresis [31].

4.4.2 Relay with Hysteresis

Noise will affect the operation of an ordinary relay by making it to switch randomly and causing chattering. However, using a relay with hysteresis the noise must be larger than the hysteresis for the relay to switch. A relay with hysteresis of value Ɛ is given by equation (4-21) and switch on when the input is greater than Ɛ and switch off for input less than Ɛ.

$$u = \begin{cases} u_0 + h & if\ e \geq \mathcal{E} \\ u_0 - h & if\ e < \mathcal{E} \end{cases} \tag{4-21}$$

$$N(a) = \frac{4d}{\pi a} \frac{1}{\left(\sqrt{1 - \frac{\varepsilon^2}{a^2}} + i\left(\frac{\varepsilon}{a}\right)\right)} \tag{4-22}$$

The describing function for a relay with hysteresis is as given in equation (4-22) , with different values of the hysteresis and the relay amplitude, several points of interest can be identified on the Nyquist curve other than the critical point. [32] . The major problem is that the switching period may differs from the ultimate period. The switching oscillation period as shown in equation (4-23) is not the ultimate period.

$$\arg\big(N(a)\big) = -\pi + \arctan\left(\frac{\left(\frac{\varepsilon}{a}\right)}{\sqrt{1 - \frac{\varepsilon^2}{a^2}}}\right) \tag{4-23}$$

4.4.3 Relay with Integrator

An integrator will introduce a phase shift of 90 degree when introduced to the feedback loop as shown in Figure 4-9, instead of identifying the phase cross over frequency at frequency ω_{180} at ω_{90}.

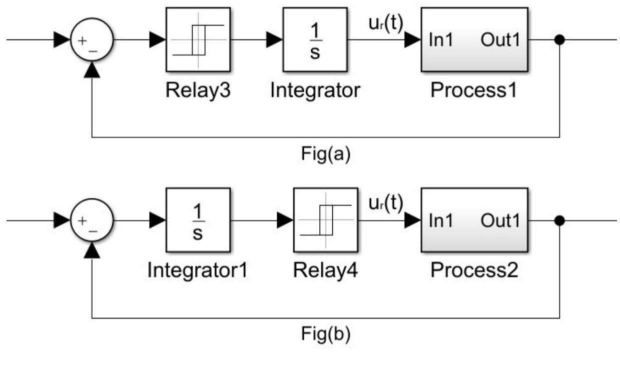


Figure 4-9: Fig(a) and Fig(b) shows the use of an integrator in conjunction with a relay to identify the parameters at frequency ω_{90}, and phase shift of -90 deg.

4.4.4 Relay with Filters

Filters may be used in conjunction with the relay to solve the problem of relay chattering due to noise. Lee and co proposed in [32], the use of low-pass filters in a noisy environment and band pass filters for process with both noise and drifts. A low pass filter will attenuate the high noise while the high pass filter will attenuate the low frequency noise part. Application of a low pass and high pass configuration as shown in Figure 4-10 will give the parameter estimation at the critical point as the effect of low and high pass filter will cancel out. [32].

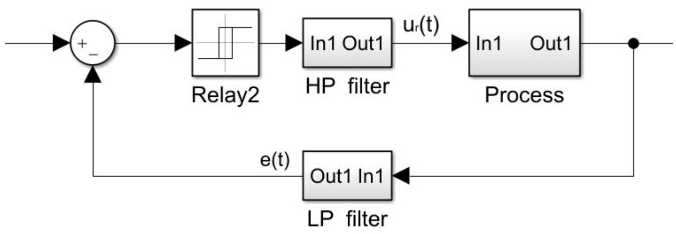


Figure 4-10: A relay with filtering implementation to prevent chattering for a noisy process.

4.4.5 Saturation relay

The describing function analysis use the ratio of the input and the output of the relay to identify the critical point. The results from this method is subjected to errors as the two functions being compare are not of the same dynamic, the input is purely a sine wave and the out a square wave. To remove this anomalies saturation relay is employed instead of the ideal relay to make the output as close as possible to the input of the relay that is to be more sine wave and less square wave. [8]. The saturation relay is characterized by the relay height h, and the slope k, and the saturation factor, ϱ, as given in equation (4-24).

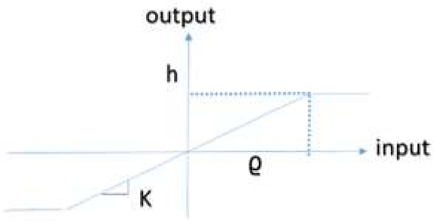
$$\varrho = \frac{h}{k}; \tag{4-24}$$

The output of the relay is given by equation (4-25)

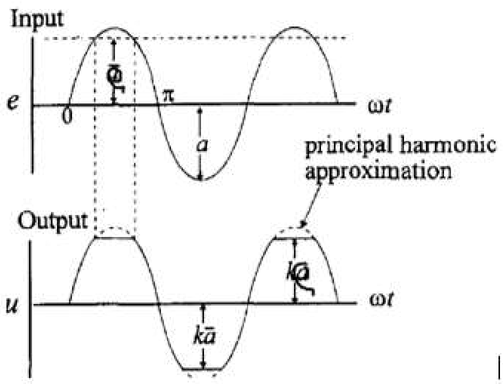
$$u = \begin{cases} k \times e \;\; if \; |e| \; \leq \varrho \\ \;h \quad if \; |e| \; > \varrho \end{cases} \tag{4-25}$$

The describing function for a saturation relay is as given in equation (4-26), if the values of the slope are too large; the saturation relay tends to behave like an ideal relay. Figure 4-11 shows the saturation relay where the relay height h, the slope k, and the factor ϱ are as given in equation (4-24) [8].

$$N(a) = \frac{2h}{\pi}\left(\frac{1}{\varrho}\sin^{-1}\left(\frac{\varrho}{a}\right) + \frac{\sqrt{a^2 - \varrho^2}}{a^2}\right) \tag{4-26}$$



(A)



(B)

Figure 4-11: The saturation relay is shown in A and the input and output of a saturation relay is shown in B. The output is more like the input (sine wave) compare to an ideal relay which is purely square wave.

4.4.6 Relay with preload

The fundamental frequency in the sustained oscillation can be boosted by adding a parallel gain to the ideal relay [33]. This will give a better estimate of the ultimate gain and period compare to an ordinary relay.

$$Ku = \mathrm{k} + \frac{4d}{a\pi};\, Pu = \mathrm{Tu};\, w_u = \frac{2\pi}{Tu} \tag{4-27}$$

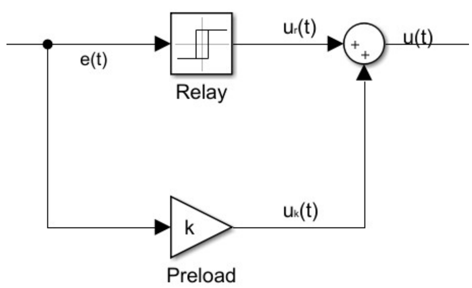


Figure 4-12: A gain K, preload in parallel with an ideal relay.

5 Simulation and experimentation study

The air heater and the quadruple tank process laboratory system were used for simulation and experimentation as discussed in section 1.2.1. A brief description of the systems will be given in the following section. Also, simulated and real experimentation on the process using the Åstrom and Schei method will be discussed. A PI controllers is the most widely controller algorithms in the industries, the D term are always de-activated [9].[4] This chapter will be based on PI controller for the simulated and real systems experimentation.

5.1 The Air Heater Process

The air heater mathematical model is given by equation (5-1). [34]. For implementation in Simulink equation (5-1) was discretized using Euler forward method to give equation (5-2)

$$\dot{T}_{Out} = \frac{1}{T_c} * (-T_{Out} + [\, K_h * u(t - T_d) + T_{env}]) \tag{5-1}$$

Where:

T_{out}- is the air temperature at the tube outlet in degree Celsius,

T_{env}- is the environmental (room) temperature in degree Celsius,

$K_{h\,[}$ °C/V] is the heater gain,

T_c is the time constant,22

$T_d[s]$ = 2 sec (is the delay due to air transportation and sluggishness in the heater),

$u[V]$ – is the control signal of the heater,

$$T_{Out(k+1)} = T_{Out(k)} + [T_s * \frac{1}{T_c} * (-T_{Out} + [\, K_h * u(t - T_d) + T_{env}])] \tag{5-2}$$

Where:

$T_{Out(k+1)}$ is the next (future) temperature out of the air heater,

T_{Out} is the present air heater temperature out of the air heater,

T_s is the sampling time.

The air heater Ordinary Differential Equation (ODE) model as given by equation (5-1) need to be transform to the transfer function model for frequency analysis in MATLAB.

The Laplace transforms of the ODE is given by equation (5-7). Equations (5-3) to (5-6) shows the steps for the transformation.

$$\mathcal{L}(\dot{T}_{out})) = \mathcal{L}(\frac{1}{T_c} * (-T_{Out} + [\, K_h * u(t - T_d) + T_{env}])) \tag{5-3}$$

[4] The D term in the PID controller do amplifies measurement noise thus causing the control signal to changes randomly.

$$sT_{Out}(s) = \frac{1}{T_c} * (-T_{Out}(s) + [\,K_h u(s) e^{-\tau s}]) \tag{5-4}$$

$$sT_{Out}(s) + \frac{T_{Out}(s)}{T_c} = \frac{K_h u(s) e^{-\tau s}}{T_c} \tag{5-5}$$

$$sT_{Out}(s) + \frac{T_{Out}(s)}{T_c} = \frac{K_h u(s) e^{-\tau s}}{T_c} \tag{5-6}$$

Simplifying equation (5-6) will give the transfer function as given in equation (5-7)

$$\frac{T_{Out}(s)}{u(s)} = \frac{K_h e^{-\tau s}}{1+T_c s} \tag{5-7}$$

Where $T_{out,}$ u, K_h and T_c have their respective definition as given for equation (5-1).

5.1.1 Ultimate Period and Gain analytically.

The mathematical model of the air heater is as given in equation (5-8), this is validated from the process reaction curve as discussed in 3.2.

$$H_p(s) = \frac{K_h e^{-\tau s}}{1+T_c s} \tag{5-8}$$

The open loop transfer function of the air heater process with a pure P controller is given by equation (5-9)

$$H_o(s) = H_p(s) H_c(s) = K_p \frac{K_h e^{-\tau s}}{1+T_c s} \tag{5-9}$$

Equation (5-9) is equivalent to equation (5-10) in frequency response

$$H_o(jw) = H_p(jw) H_c(jw) = K_p \frac{K_h e^{-\tau s}}{1+T_c s} \tag{5-10}$$

This can be written in polar form as equation (5-11)

$$H_o(jw) = H_p(jw) H_c(jw) = K_p \frac{K_h e^{-\tau s}}{1+T_c s} \tag{5-11}$$

The ultimate gain and frequency can be find at the cross over frequency ω_{180} as given in equation (5-12)

$$K_c = K_p = \sqrt{1 + T_c^2 \omega_{180}^2}; \quad (\tau\omega_{180} + \tan^{-1} T_c \omega_{180}) = \pi \tag{5-12}$$

With the value of the air given as derived from step response discussed in section 2.1.4, Tc is 22 and τ $2s$.

$$K_c = K_p = \sqrt{1 + 22^2 \omega_{180}^2}; \quad (2\omega_{180} + \tan^{-1} 22\omega_{180}) = \pi \tag{5-13}$$

Solving equation (5-13) gives K_c and ω_{180} as 17.92 and 0.8133rad/s respectively. Also, the period of oscillation is calculated to be 7.72s. This is the basis for Z-N method and the gain is always too much that it can damage some process equipment if used on real process, therefore Åstrom devised the relay experimentation method that only uses a fractional part of the ultimate gain to re-evaluate the process parameter.

5.1.2 Åstrom Relay Experiment on simulated Air heater[5]

Åstrom relay experiment was performed on the simulated air heater with and without noise. Without noise the ideal relay switched as expected and also with noise of 0.01, the relay work as expected. But with much bigger value of 0.1, the switching of the relay was erratic. The following simulation was carried out to see the effect of noise and the modification of the relay

1. Process without noise and ideal relay
2. Process with noise (0.1) and ideal relay
3. Process with noise (0.1) and relay with filter
4. Process with noise (0.1) and relay with hysteresis
5. Process with noise (0.01) and relay with hysteresis
6. Process with noise (0.01) and relay with hysteresis and filter
7. Process with noise (0.1) and relay with hysteresis and filter

The result of the above simulation is presented in Table 5-1. The calculated PI parameters and the critical gain and period with different values of filter time constant is documented. Figure 5-1 and Figure 5-2 shows the output of the relay experimentation for a process with small noise. The relay switching is not erratic but with small amplitude, increasing the noise make the relay switching to be erratic as shown in Figure 4-6 and Figure 4-7 in Section 4.4.

Integral Absolute Error (IAE) and Sum of Squared Error (SSE) are the two criterial used for performance indication. This is because the simulations are done in Ordinary Differential Equation (ODE) and to perform frequency analysis in Simulink block will be more tedious, notwithstanding the frequency analysis was perform using the transfer function of controller and process in using MATLAB function 'margin'.

[5] The noise is added to the measurement output as a random noise

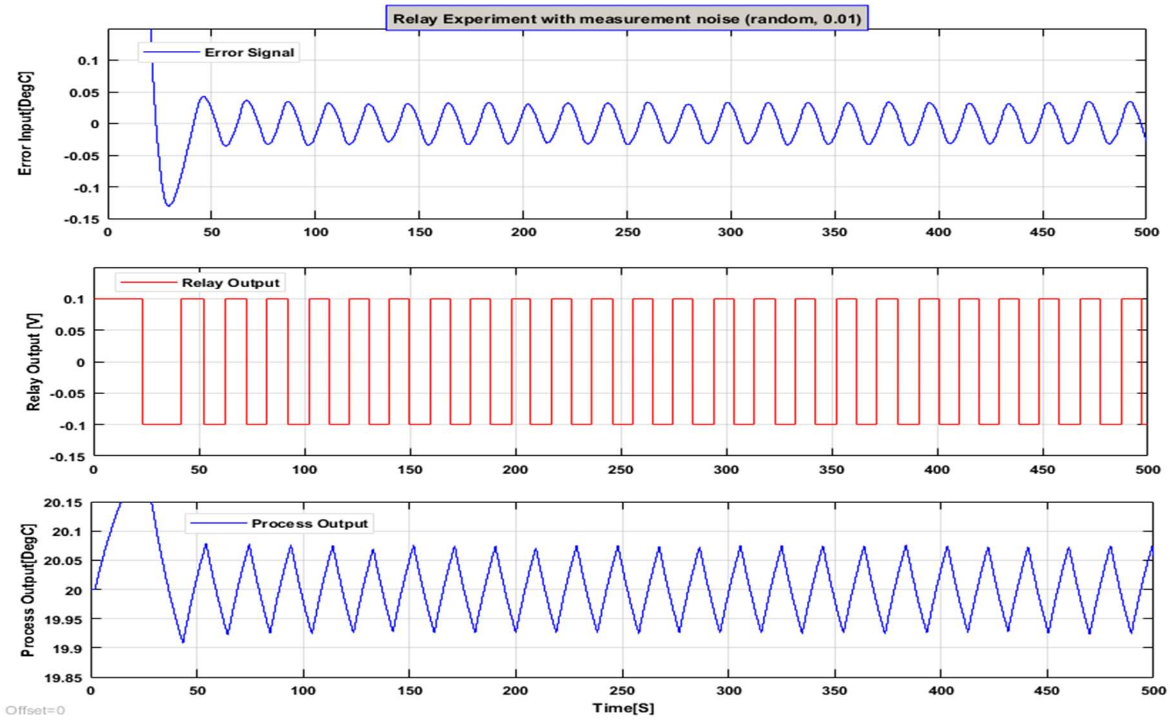


Figure 5-1: Åstrom relay experimentation for a noisy signal of random noise (0.01). The upper middle and lower plot shows the error signal, relay output and process output respectively.

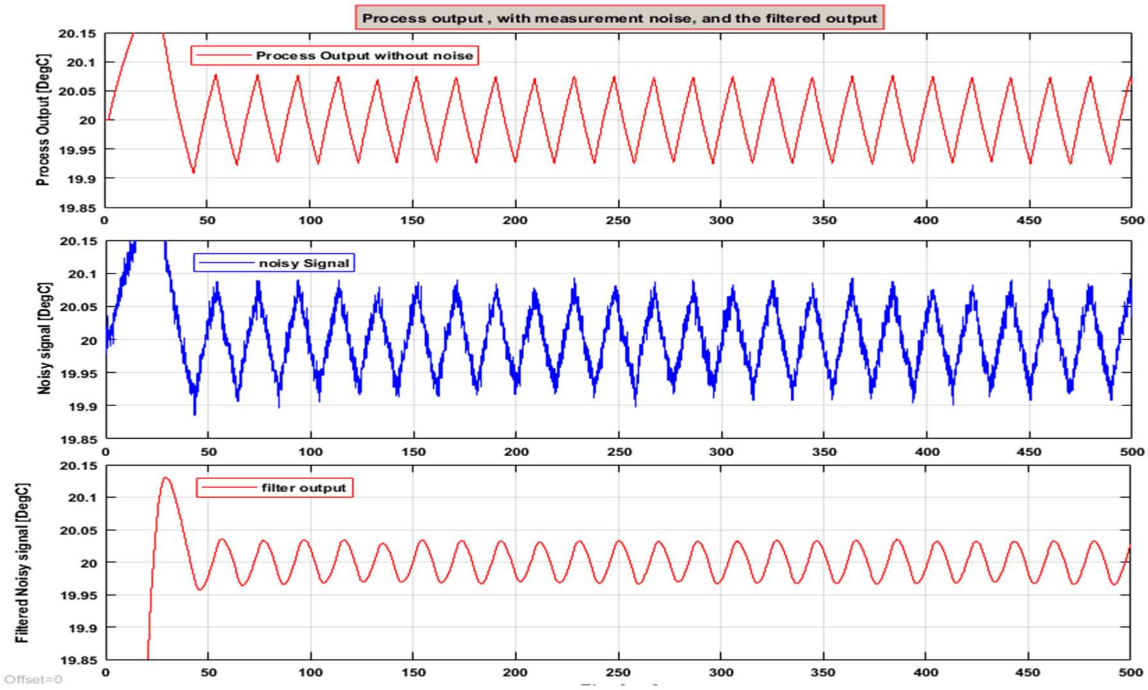


Figure 5-2: Åstrom relay experimentation for a noisy signal of random noise (0.01). The upper middle and lower plot shows the process without noise, noisy and filter output respectively.

5.1.3 Results from Åstrom relay experimentation on simulated process

From Figure 5-3, the PI parameters derived from the relay with hysteresis and filter (No7 in Table 5-1) gives the best performance and this will be used as a bench mark for the comparison with the Schei method. It converge faster, give less overshoot and with less steady state error as compared to others as shown in Figure 5-3. Also from the frequency analysis in MATLAB, as presented in Table 5-2, it gives stability performance better than the other in relation to the bench mark giving in section 3.4.3. It should be noted that the phase and gain margin are wide because of the filter element in series with the relay. Also the hysteresis act as a delay element in the process.

Table 5-1: PI parameters derived from Åstrom experimentation with the amplitude of oscillation, critical gain and period with different relay modification.

S/N	Experiment	Output Filter Constant	Pu [s]	Wcu [rad/s]	Kc Ultimate gain	Amplitude of oscillation, a [degC]	Ti[s]	Kp
1	Åstrom	N/A	8	0.786	4.089	0.03129	6.667	1.831
2	Åstrom_ noise (0.1)	0.5	1.4	4.489	8.194	0.01554	1.167	3.687
		0.8	0.6	10.473	10.06	0.01265	0.5	4.528
3	Åstrom_ noise (0.1)+Relay with filtered	0.5	0.2	31.42	5.466	0.0233	0.1667	2.49
		0.8	0.6	10.47	-9.73[6]	-0.0131	0.5	4.378
4	Åstrom_ noise (0.1) _FC +Relay with hysteresis(0.2)	0.5	37.2	0.169	1.0452	0.1256	31.00	0.4562
		0.8	26.5	0.237	1.255	0.1014	22.08	0.5649
5	Åstrom_ noise (0.01) _FC +Relay with hysteresis(0.2)	0.5	61.5	0.102	0.6072	0.2097	51.25	0.2733
		0.8	60.1	0.105	0.6126	0.2078	50.08	0.2757
6	Åstrom_ noise (0.01) +Relay hysteresis(0.2)+ Relay filtered	0.5	62.5	0.100	0.5908	0.2155	52.08	0.2658
		0.8	60.1	0.105	0.6134	0.2076	50.08	0.276

[6] Due to the eractic switching of the relay

7	Åstrom_ noise (0.1) +Relay hysteresis(0.2)+ Relay filtered	0.5	37.2	0.169	1.029	0.1237	31.00	0.463
		0.8	26.5	0.237	1.261	0.101	22.8	0.5674

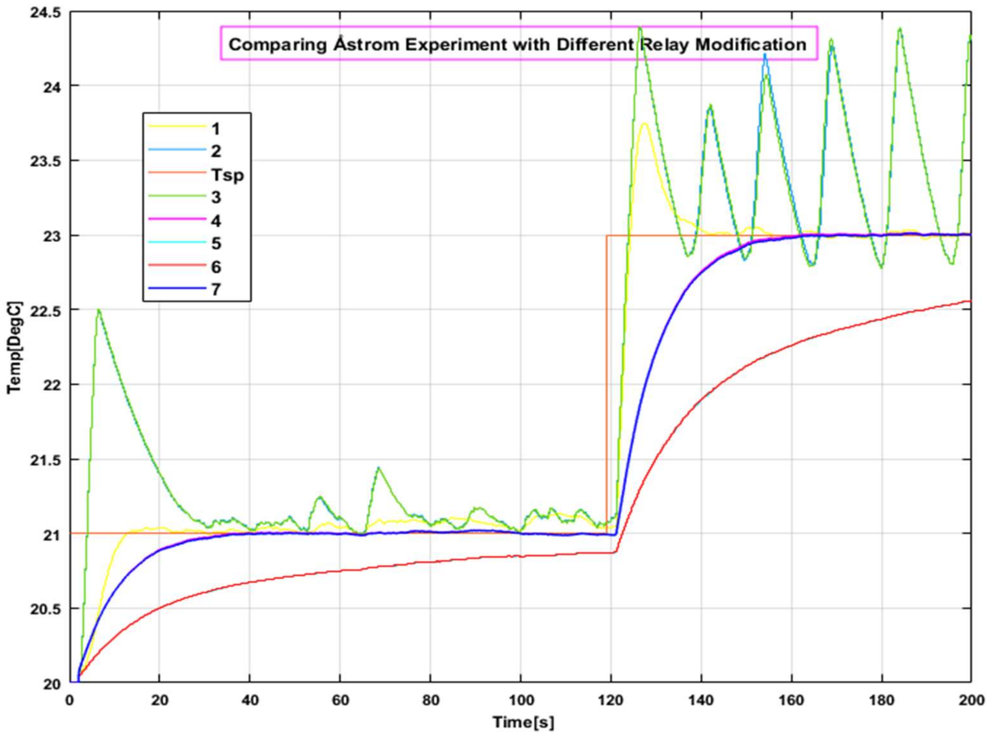


Figure 5-3: The comparison of the PI parameters estimated during Åstrom relay experiment on simulated air heater

Table 5-2: Stability analysis in frequency domain[7]

S/N	Experiment with output filter time constant of 0.8 (except no 1)	GM [dB]	PM [deg]	W_{180} [rad/s]	Wc [rad/s]
1	Åstrom	2.9040	67.2926	0.8764	0.2964
2	Åstrom_ noise (0.1)	10.9092	116.8090	0.9093	0.0680
3	Åstrom_ noise (0.1)+Relay with filtered	10.9358	116.8381	0.9129	0.0680
4	Åstrom_ noise (0.1) _FC +Relay with hysteresis(0.2)	9	110.4443	0.8313	0.0805
5	Åstrom_ noise (0.01) _FC +Relay with hysteresis(0.2)	18.2285	inf	0.8212	NAN

[7] The MATLAB function MARGIN was used with the transfer function equivalent of the controller and the process

6	Åstrom_ noise (0.01) +Relay hysteresis(0.2)+Relay filtered	18.2086	inf	0.8212	NAN
7	Åstrom_ noise (0.1) +Relay hysteresis(0.2)+Relay filtered	8.9603	110.2495	0.8313	0.0809

5.1.4 Schei Relay Experiment on simulated Air heater

The Schei relay experimentation was carried out using simulated process without noise and with noise. The results are shown in Table 5-3 which include the critical gain and period, amplitude of oscillation and the derived PI parameters. Different relay modification was tried and relay with hysteresis gives the desired relay switching response. In particular, it was noted that the relay does not switch when its output was filtered. Figure 5-4 shows the relay output and control signal while Figure 5-5 shows the process output and error signal from the Schei relay experiment. Figure 5-6 shows the period and frequency of oscillation during the Schei relay experimentation. The indicate margin is calculated from MATLAB using the transfer function of the controller and process.

Table 5-3: PI parameters derived from Schei experimentation with the amplitude of oscillation, critical gain and period with different relay modification.

[8]Schei Exp	Pu [s]	Kc	Amplitude of oscillation, a	Ti [s]	Kp	GM [dB]	PM [deg]	W_{180} [rad/s]	Wc [rad/s]
Schei	8.4	1.22	0.1044	7	0.549	9.66	112.15	0.8732	0.0775
Schei _ noise (0.1) +Relay hysteresis(0.1)	10.8	0.93	0.1362	9	0.4206	12.45	125.75	0.8593	0.0517

[8] For Schei method the relay does not switch with a filter in series with it as the output is filtered out

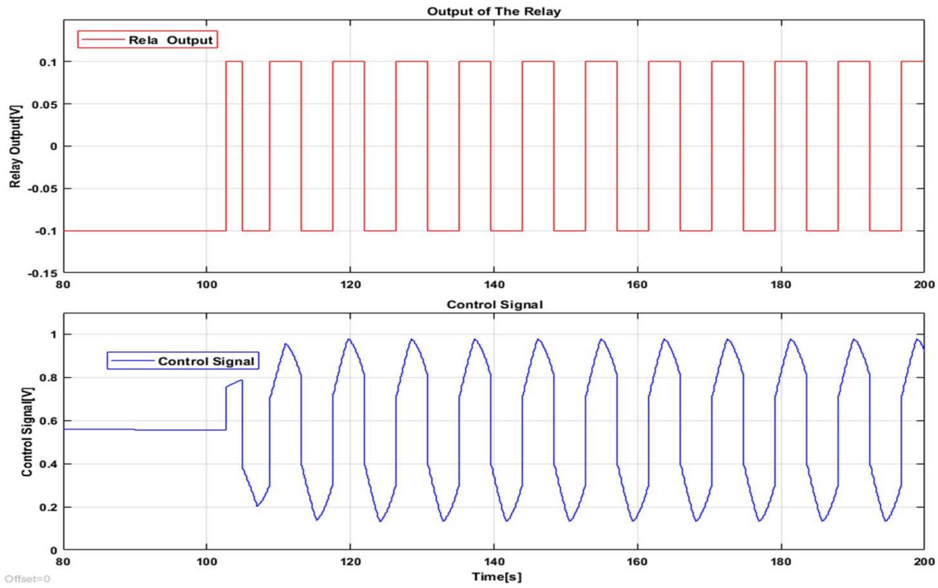


Figure 5-4: The upper plot show the relay output and the lower plot shows the control signal during Schei experimentation for noisy process. The experimentation is initiated around 110 seconds.

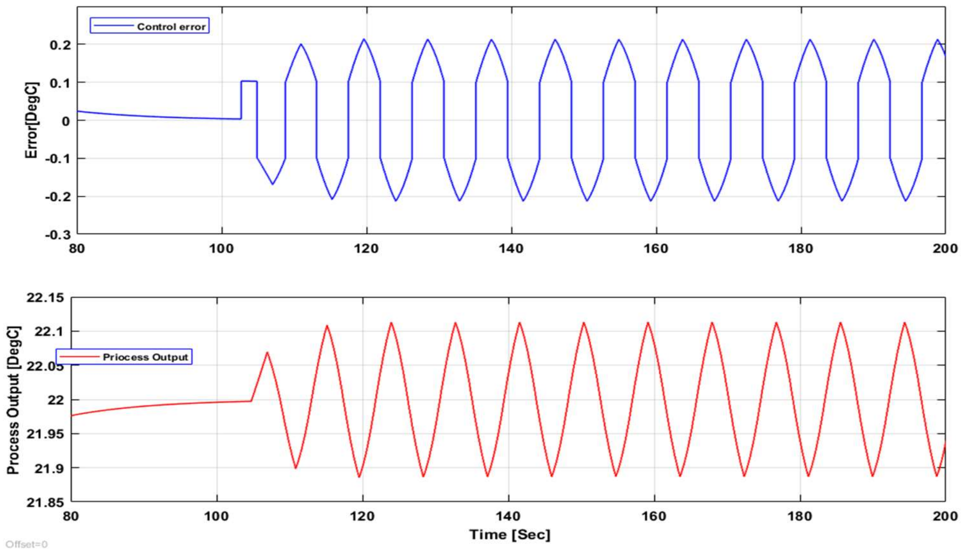


Figure 5-5: The upper plot show the control error and the lower plot shows the process output during Schei experimentation for noisy process. The experimentation is initiated around 110 seconds.

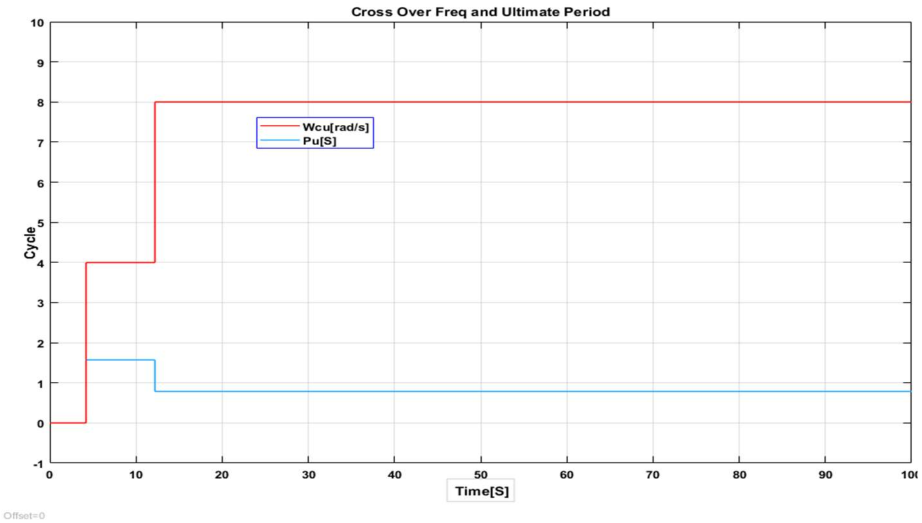


Figure 5-6: The frequency and period during Schei Relay experimentation.

5.1.5 Comparing the performance of Åstrom and Schei Relay Experiment on simulated Air heater

The performance criteria as discussed in section 3.4 will be used in this section to judge the response of the system with the PI parameters derived with the relay experimentation under the following selected cases:

- Åstrom without noise (As)
- Åstrom with noise process using relay with hysteresis and filtered (Aswn)
- Schei without noise (Sc)
- Schei with noise process using relay with hysteresis (Scwn)

The performance of the controller is a function of the user requirement. A process that need fast response will require a small rise time and a process with a delicate equipment will required a small overshoot. The results of the comparison for both process with noise and with noise is presented in Table 5-4 and Table 5-5.

Figure 5-7, Figure 5-8, Figure 5-9, Figure 5-10 and Figure 5-11 shows the results of using the PI controllers' parameter derived from AS, Aswn, Sc and Scwn respectively.

Based on the criterial presented in Table 5-4 and Table 5-5, an ideal relay parameter estimation give a better PI performance for a non-noisy process and a relay with hysteresis and filtered gives a better parameter estimation for a noisy process. A PI controller derives from Åstrom relay experiment gives less percentage overshoot and but more steady state error than the Schei relay experiment PI controllers. Moreover for set point tracking the Åstrom relay experimentation is better than Schei while the later gives better performance for disturbance rejection.

Table 5-4: Performance tracking for a process without noise using different PI parameters.

	IAE	SSE	IAE_d	%overshoot	Rise Time(s)
As	9.07	113.7	10.08	0%	15.6

Aswn	11.6	109.8	22.33	0.05%	48.4
Sc	3.54	107	7.07	1.24%	12.6
Scwn	5.944	116.5	11.92	0.81%	18.3

Table 5-5: Performance tracking for a process with noise using different PI parameters.

	IAE	SSE	%overshoot	Rise Time(s)	Steady state error
As	-4.604[9]	123	0.19%	12.17	0.19
Aswn	11.1	119.5	0.09%	43.7	0.01
Sc	3.473	116.3	1.24%	12.17	0.01
Scwn	5.864	126.3	0.81%	18.3	0

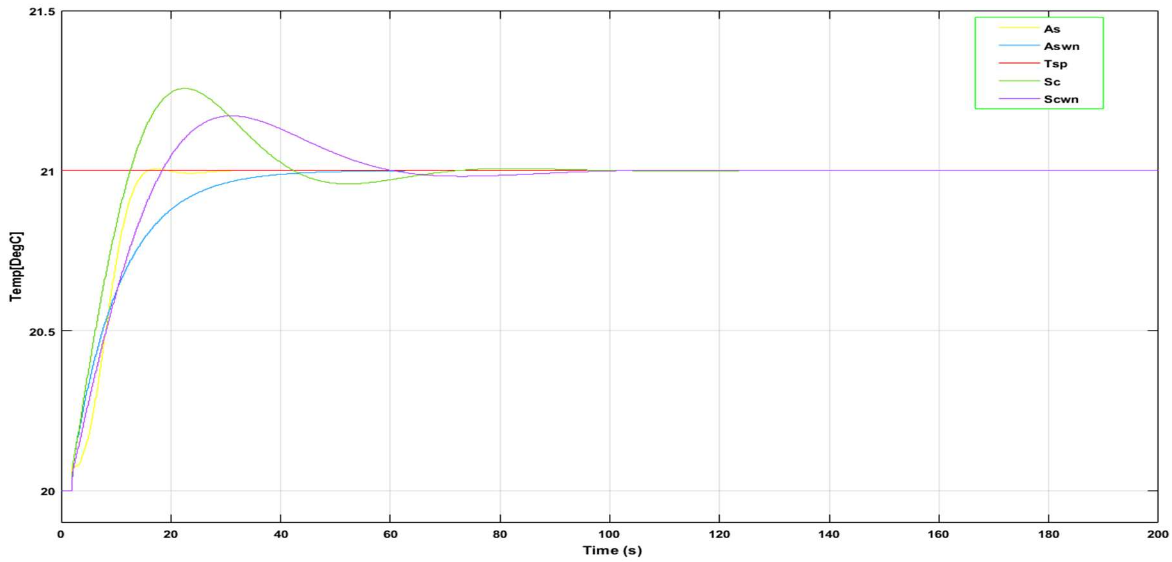


Figure 5-7: The plot of the different PI parameters derived from the relay experiment (Astrom and Schei with and without noise) to control the simulated process without noise

[9] The negative sign means that the process does not converge within the time frame of comparisons

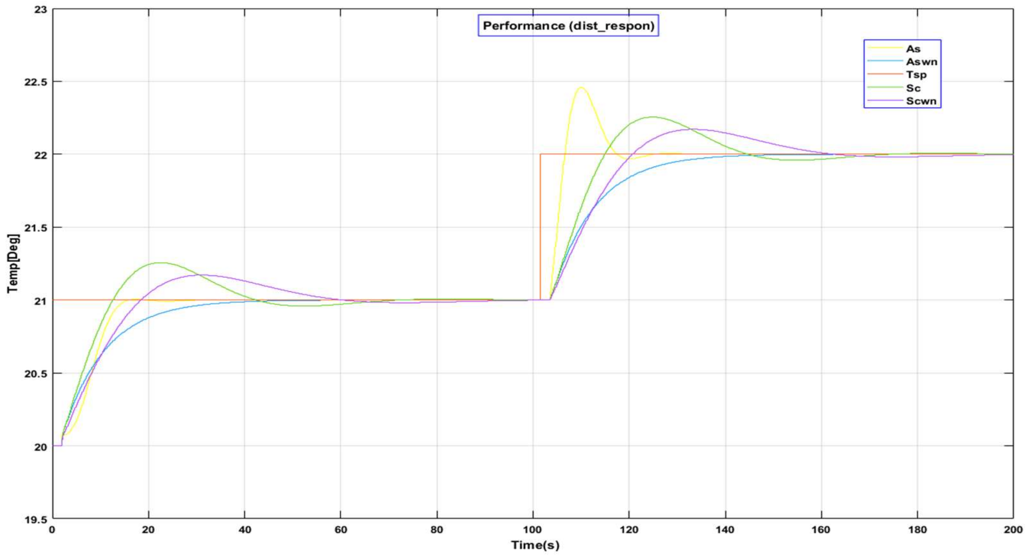


Figure 5-8: The plot of the different PI parameters derived from the relay experiment experiment (Astrom and Schei with and without noise) to control the simulated process without noise with a step change around 100s.

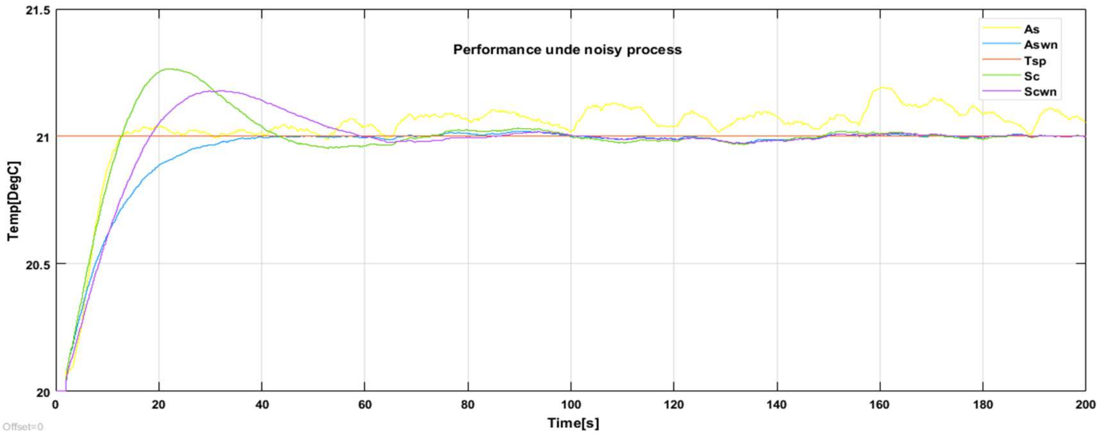


Figure 5-9: The plot of the different PI parameters derived from the relay experiment (Astrom and Schei with and without noise) to control the simulated noisy process

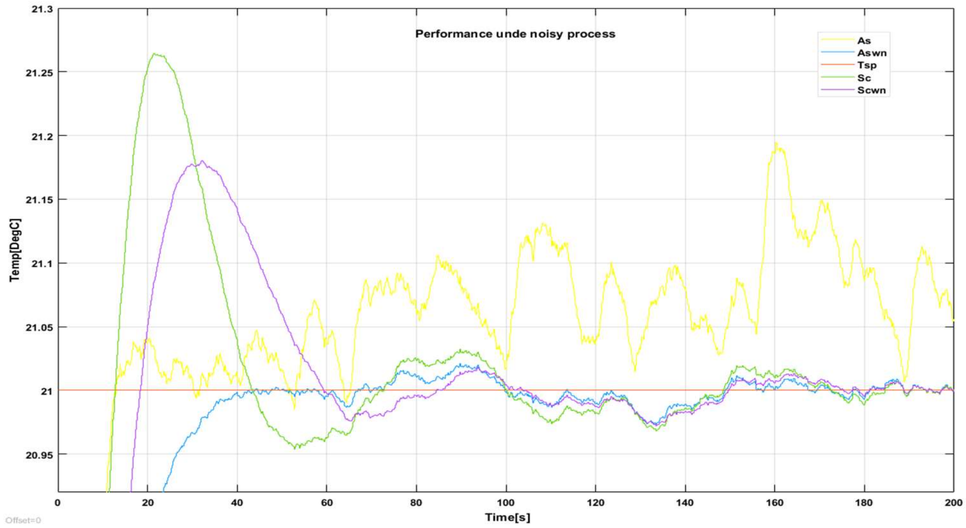


Figure 5-10: The plot of the different PI parameters derived from the relay experiment (Astrom and Schei with and without noise) to control the simulated noisy process with reduced scale to have a better view of the effect of noise.

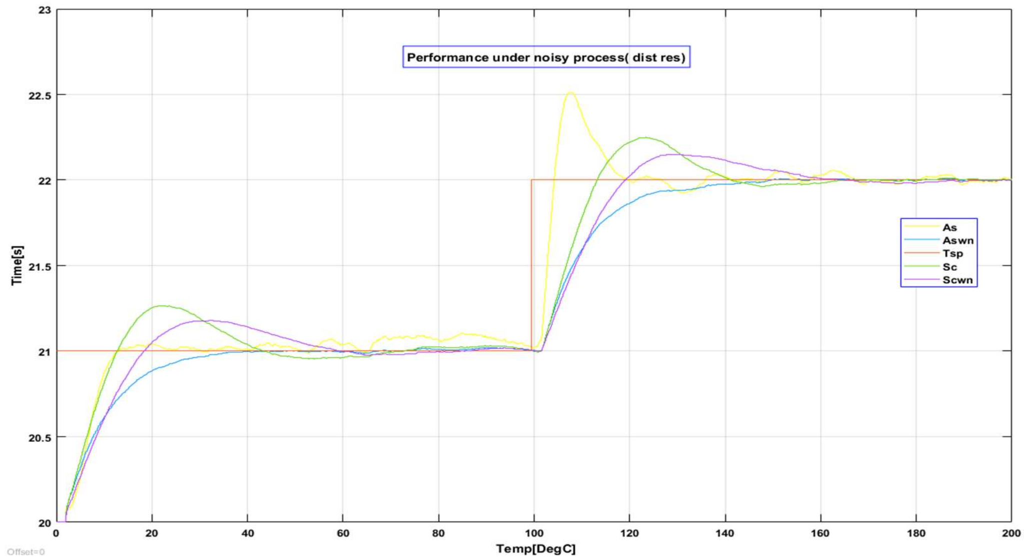


Figure 5-11: The plot of the different PI parameters derived from the relay experiment (Astrom and Schei with and without noise) to control the simulated noisy process noise with a step change around 100s.

5.1.6 Schei Relay Experiment on Real Air heater[10]

The Schei relay auto tuning was used on the real air heater using the PI parameter derived in section 5.1.4 as the starting point. The relay experiment was performed after a steady state process output was achieved, this is to improve on the PI parameter. Note that a real time spacer was used during the experiment and as such all-time relater parameter has to take into account the real time spacer multiplier. Figure 5-12 shows the actual process output and the filter output. A low pass filtered of first order and time constant of 0.8 was used. The filtered was so chosen so that the experiment will be a representatives of a real process noisy

[10] Real Time space was used in the Simulink environment and this was set to 10. Therefore all-time dependent parameter need to be divided by 10.

environment and the dynamic of the process will still remain as such. Figure 5-13 shows the relay switching and the control signal. The relay experimentation was started around 350 second, the frequency counter was not effective because of the random noise and as such does not give an accurate PI parameters. The measurement was average over the period of time as shown in Appendix D, to get a conservative figure for the controller implementation. Alternatively, the measurement was done manually to cross check the derived parameters as shown in Figure 5-14. The derived parameters are Kp 1.8 and Ti is 15s.

The control signal is kept within the bound by the PI controller and such for a good performance of the auto tuner, a fairly good controller parameters must be the starting point. More plots from the experimentation are presented in Appendix D.

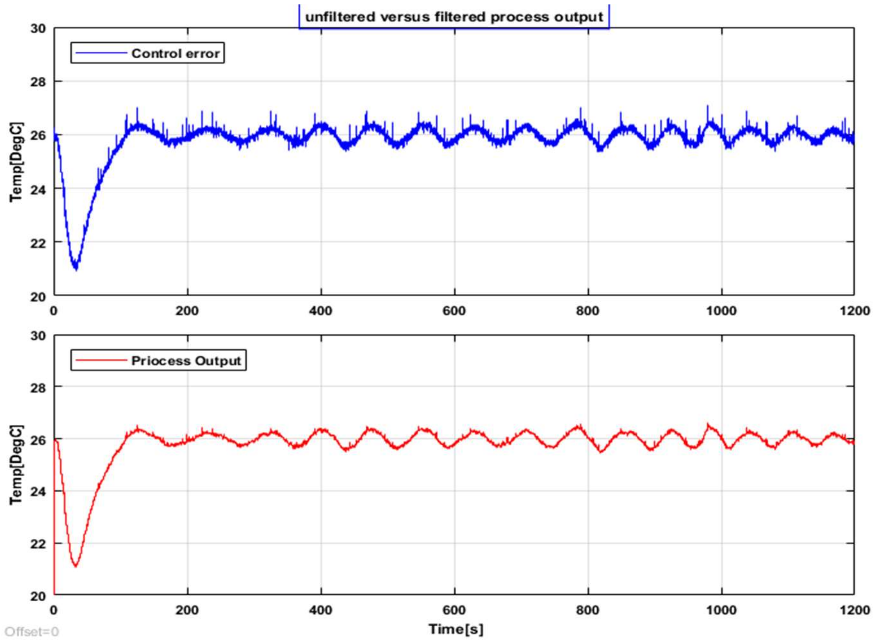


Figure 5-12: Real air heater; upper plot show the unfiltered output while the lower plot show the filter output

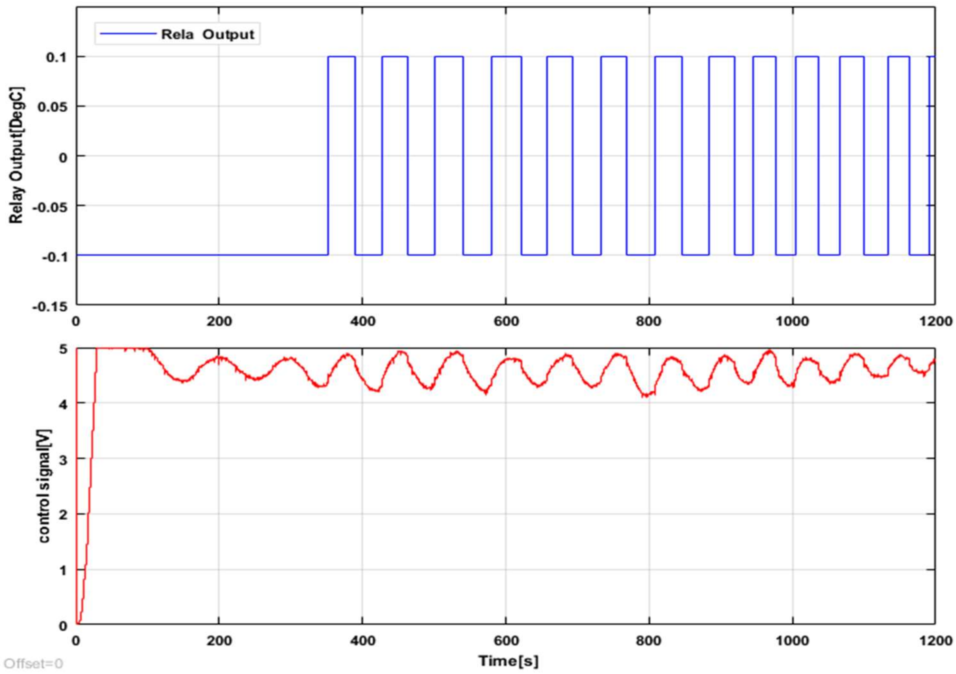


Figure 5-13:Real air heater; upper plot show the relay output while the lower plot show the control signal during the relay experiment

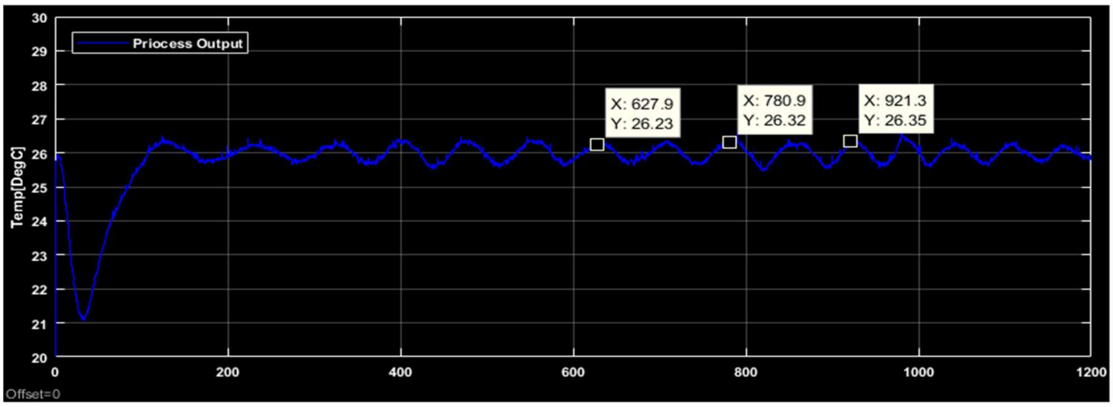


Figure 5-14: Process output from the real air heater used to cross check the calculated parameter from the auto tuner

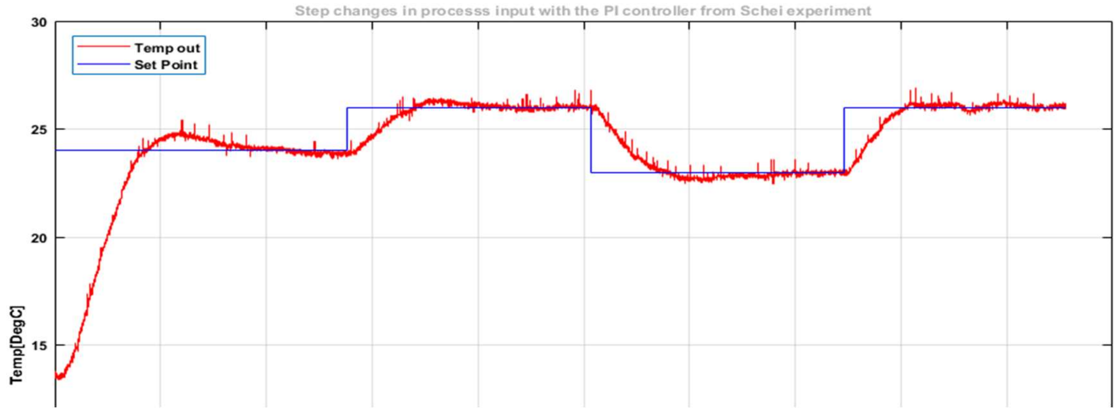


Figure 5-15: Process output and reference plot with the derived PI controller controlling the real air heater with step changes at interval. Notice the time axis has been cut off from the plot.

5.2 The Quadruple tank Process

The quadruple tank is a multivariate laboratory process that consist of four tank system with two pumps to control the level in the lower two tanks through two valves as shown in Figure 5-16. The control input to the system are the control voltages to the two pumps while the output are the level from tank 1 and 2. [35].

The mass balance equation for the system is derived from mass balances equation and Bernoulli's law equation (5-14) and can be given in state space model as shown equation (5-15).

$$\begin{aligned}
\dot{h}_1 &= \frac{-a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1}v_1 \\
\dot{h}_2 &= \frac{-a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2}v_2 \\
\dot{h}_3 &= \frac{-a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2)k_1}{A_3}v_2 \\
\dot{h}_4 &= \frac{-a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_2)k_1}{A_4}v_1
\end{aligned} \tag{5-14}$$

Where $\dot{h}_1$ is the rate of change of water level h_i for tank i, a_i is the cross-section of the outlet hole in tank i, v_i is the pump voltage. 'g' is the acceleration due to gravity, A_i is the cross-section of tank i. The constant γ_1 is a valve parameter that measure the diversion of flow from the valve to the tanks, and the range is from 0 to 1. The model parameters is as given in Table 5-6. The linearized model, linearization point and the derived state space model are given in Appendix B . The non minimum phase characteristic as presented in [35] and [36] will be used for the simulation study.

$$\frac{dx}{dt}=\begin{bmatrix}-\frac{1}{T_1} & 0 & \frac{A_3}{A_1T_3} & 0\\ 0 & -\frac{1}{T_2} & 0 & \frac{A_4}{A_2T_4}\\ 0 & 0 & -\frac{1}{T_3} & 0\\ 0 & 0 & 0 & -\frac{1}{T_4}\end{bmatrix}x+\begin{bmatrix}\frac{\gamma_1k_1}{A_1} & 0\\ 0 & \frac{\gamma_2k_2}{A_2}\\ 0 & \frac{(1-\gamma_2)k_1}{A_3}\\ \frac{(1-\gamma_2)k_1}{A_4} & 0\end{bmatrix}u \tag{5-15}$$

$$y=\begin{bmatrix}k_c & 0 & 0 & 0\\ 0 & k_c & 0 & 0\end{bmatrix}x$$

Where, T_i are time constant as given in

$$T_i=\frac{A_i}{a_i}\sqrt{\frac{2h_i^o}{g}} \tag{5-16}$$

$$i=1,\dots,4.$$

Table 5-6: Parameter of quadruple tank

parameter	Value
A_1, A_3	28 [cm^2]
A_2, A_4	32 [cm^2]
a_1, a_3	0.071 [cm^2]
a_2, a_4	0.057 [cm^2]
g	981[cm/s^2]

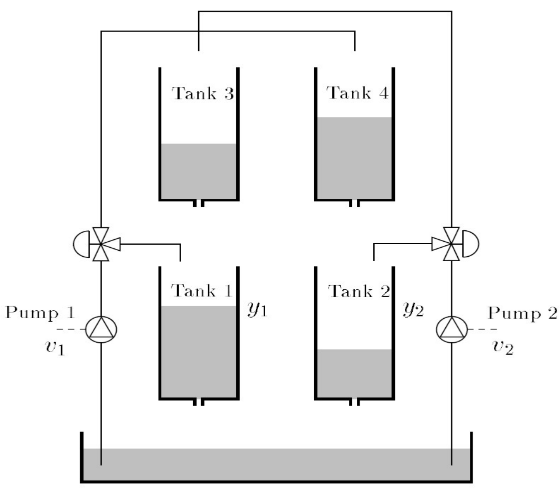


Figure 5-16: Schematic diagram of the quadruple tank process [35]

For the experimentation the control is decentralized with two separate controller as shown in Figure 5-17. This is a case of multiple input and multiple output (MIMO) system as presented in [7]. The quadruple will be consider as two SISO processses whereby each tank will have an independent loop without pairing.

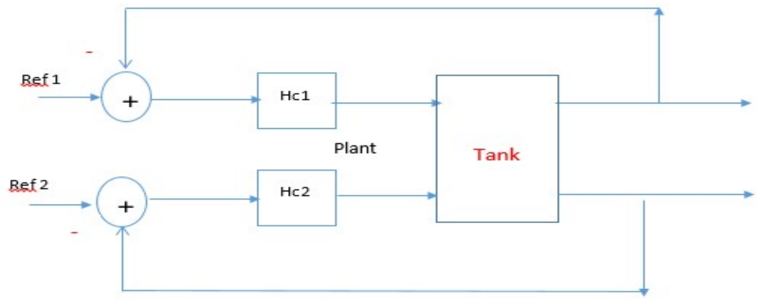


Figure 5-17: The block diagram of the two input and two output system with decentralized PI control, Hc1 and Hc2.

5.2.1 Åstrom relay experiment on simulated quadruple tank

The experimentation was performed on the tank as follows:

- PI control on tank 1 and tank 2 put in manual mode (experiment1)
- Thereafter, the experiment was performed for the two controller as two SISO system. The result PI parameter derived is given in table (experiment2).

The simulation plots from the experimentation using PI on tank 1 and manually controlling tank 2 is shown in Figure 5-18. The plot shows that the experiment converge less than 50 second. Figure 5-19 shows the plot from controlling the tank with the derived PI controller on tank 1. There is different step changes on both tank reference value during the simulation and the controller was able to adjust with less overshoot with these step changes. Also the control to tank 2 was manually controlled to acts as disturbance on the system and the controller handle the disturbance accordingly. Relay with hysteresis was used to achieve the best result.

The ultimate period and gain derived were used in Z-N closed loop formula to calculate the PI parameters as presented in Table 5-7.

Figure 5-20 shows the plot from relay experimentation carried out with the system controlled with two different controllers as in a case of two SISO system. The derived PI parameters from these experiment is used to control the two SISO system and the plots shown in Figure 5-21. The frequency, period, calculated proportional gain and integral time are shown Figure D-1 4 and Figure D-1 5 in Appendix D.

Table 5-7: The parameters derived during Åstrom relay experimentation on the quadruple tank

Controllers	Experiment 1		Experiment 2	
	Kp	Ti	Kp	Ti
Tank 1	5.495	8.417	5.614	8.833
Tank 2	Manual control		5.715	11.33

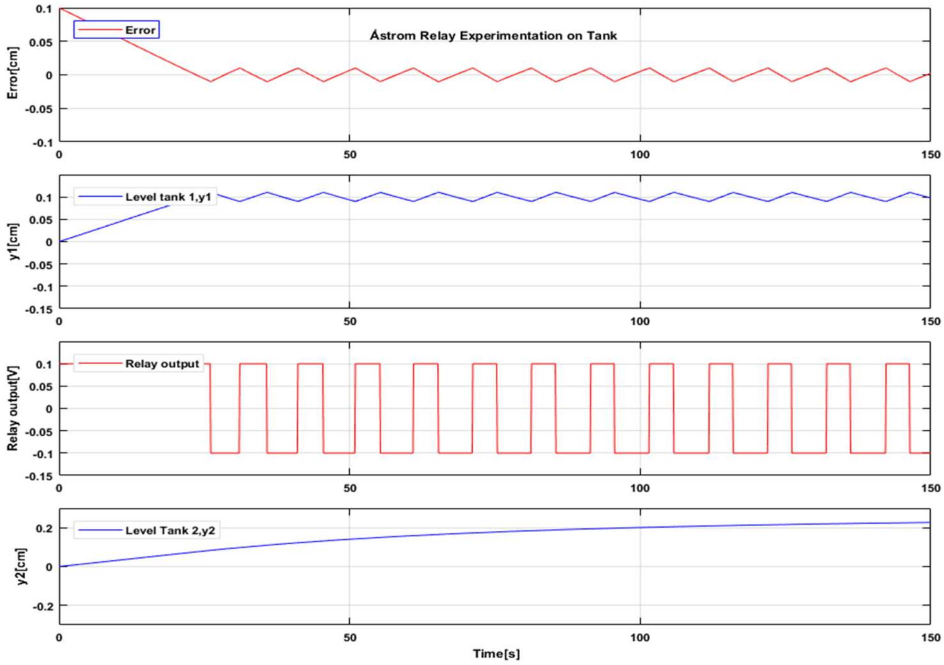


Figure 5-18: Process error and relay switching during Åstrom experimentation on quadruple tank (closed loop control on tank 1 and open loop control on tank 2).

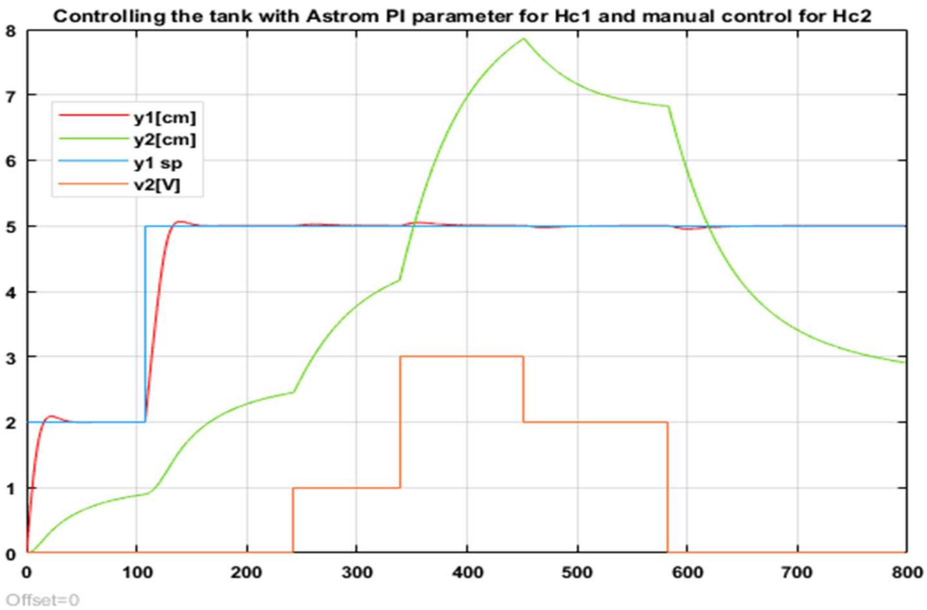


Figure 5-19: Controlling the tank with the PI parameters derived during the experiment (PI on tank 1 and manual control on tank 2). Step changes for the reference and control input for both tank 1 and 2 respectively were made at intervals.

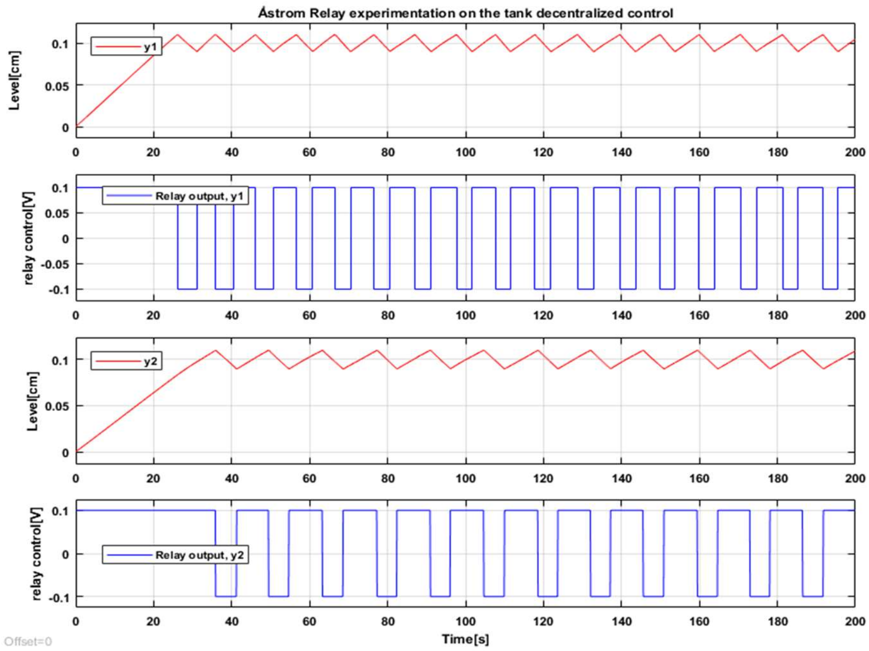


Figure 5-20: Process output and relay switching during Åstrom experimentation on quadruple tank (closed loop control on both tanks).

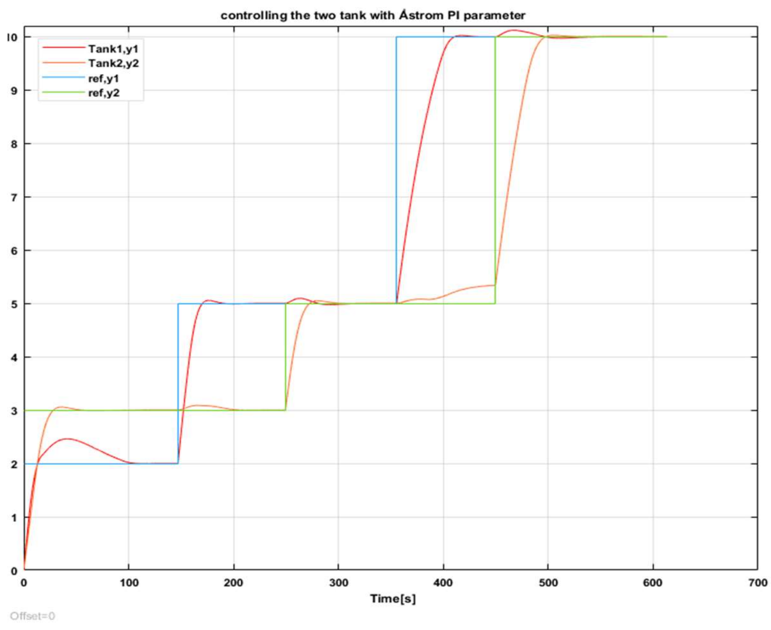


Figure 5-21: Controlling the tank as two separate SISO system with PI derived from Åstrom relay experimentation

5.2.2 Schei relay experiment on simulated quadruple tank

The relay experiment was performed on the quadruple tank as two SISO system with the PI parameters derived from the Åstrom relay experiment in other to improve on the performance. Based on the fact that no proper paring of input and output variables was done, the tuning process was difficult and take longer time and correct relay biased to achieved. A relay hysteresis of 0.1 was finally used and resulting parameters presented in Table 5-8. Tank two relay switching was irregular and the internal working structure of the tank pairing is required to improve on the relay structure to achieve a better result. Figure 5-22 show the plot of the error and control signal while Figure 5-23 show the plot of the relay and process output during the experiment.

Table 5-8: The parameters derived during Schei relay experimentation on the quadruple tank

Controller	Kp	Ti[s]	Amplitude of oscillation, [cm]	Period of Oscillation[s]	Ultimate gain	Wc[rad/s]
Tank1	0.5714	16.58	0.1003	19.9	1.27	0.3157
Tabk2	0.5681	46.58	0.1009	55.9	1.262	0.1124

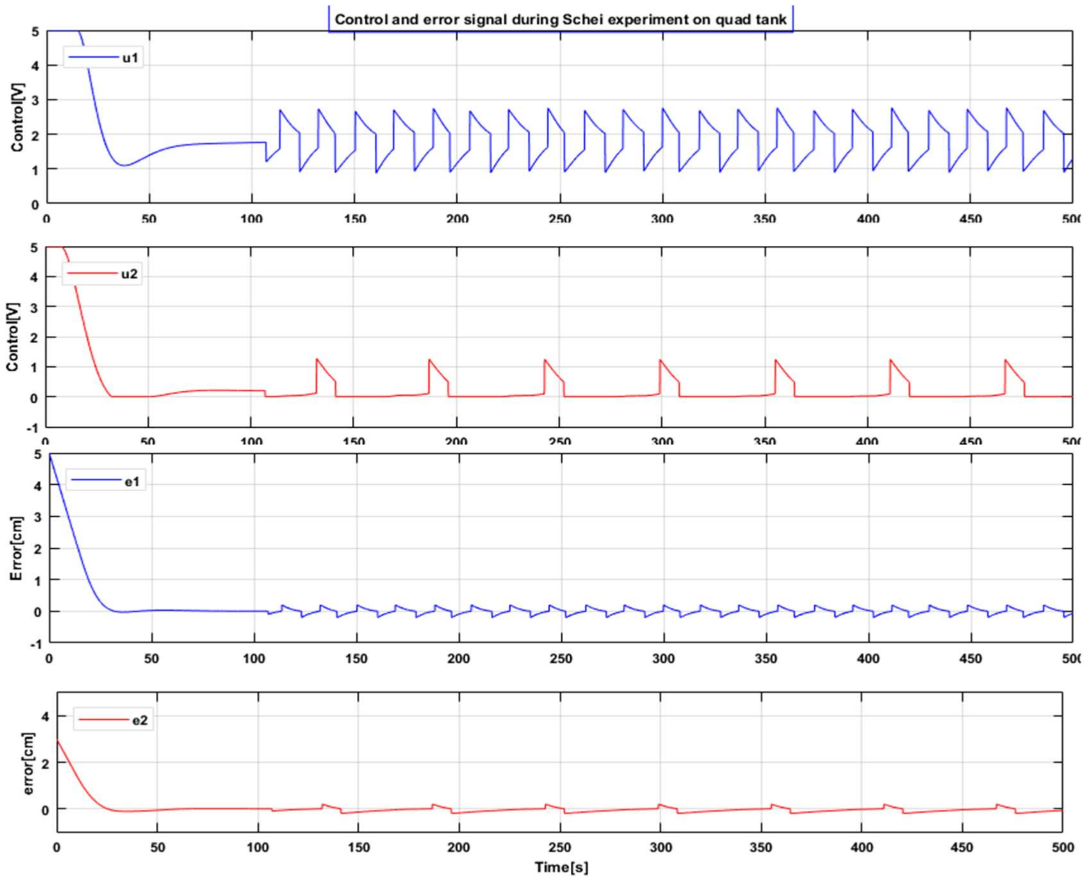


Figure 5-22: The plot of simulation from the Schei relay experimentation showing the control signal (u1 and u2) and the error signal (e1 and e2) for both tanks.

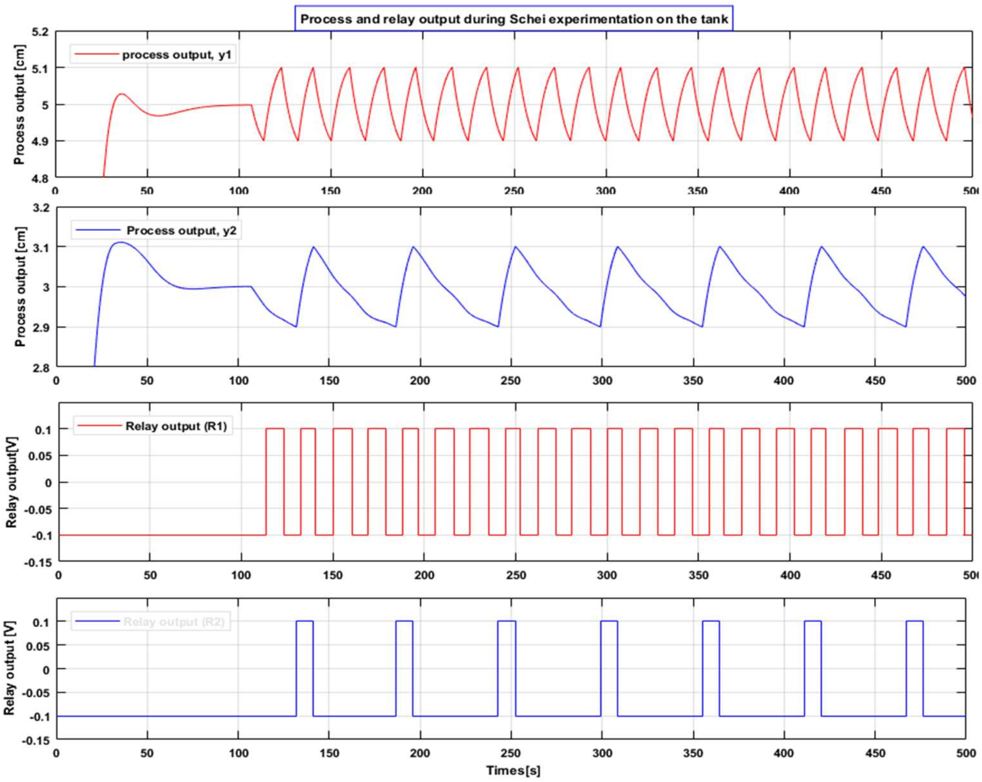


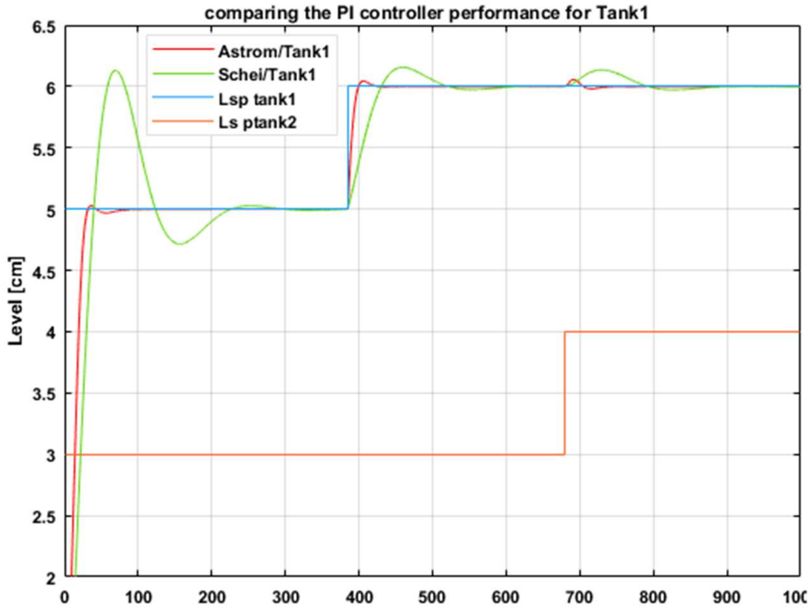
Figure 5-23: Figure 5-24: The plot of simulation from the Schei relay experimentation showing the process output (y1 and y2) and relay output (R1 and R2) for both tanks.

5.2.3 Comparing the performance of Åstrom and Schei Relay Experiment on simulated quadruple tank

The performance criteria as discussed in section 3.4 will be used in this section to review the response of the system with the PI parameters derived with Åstrom and Schei relay experimentation on the quadruple tank system. Based on the derived parameters as presented in Table 5-7 and Table 5-8 , the tank was controlled with the PI controllers and with different step changes on both tanks as shown in Figure 5-25. Step changes on one tank acts as a disturbance on the other tanks. The performance of the controllers is presented in Table 5-9 and from this it can be inferred that Åstrom gives the best performance in terms of stability and robustness measured subject to the fact that the initial PI parameters for Schei was not perfectly paired.

Table 5-9: Performance tracking for quadruple tank with different PI parameters.

	Method	IAE	SSE	%overshoot	Rise Time(s)	Steady state error
Tank1	Åstrom	62.33	1990	0.56%	32	0
	Schei	52.79	3636	22.6%	41.1	0.002
Tank2	Åstrom	27.87	568.6	3.7%	25.1	0
	Schei	12.73	1758	21.7%	53.3	0.002



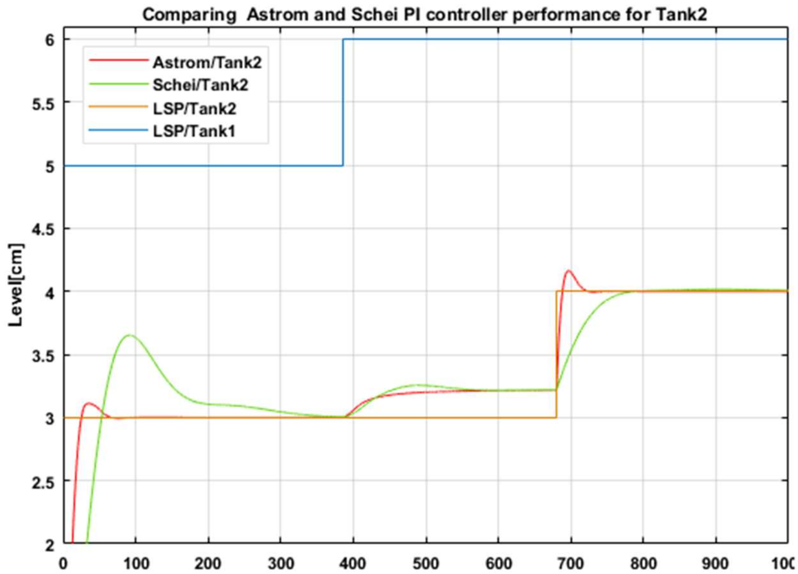


Figure 5-25: comparing Åstrom and Schei PI parameter with step changes on both reference-tank1 and tank2- to show the effect of cross coupling of the tanks on the controllers. (Tank 1 on the upper plot and tank 2 on the lower plot).

6 Discussion and Result

A PID loop must be robust even if there is a change in the dynamic of the process. The PID parameters must be able to cover a wide range of process dynamic. The phase and gain margin must be wide enough to accommodate this changes before the system become unstable. Bode, Nyquist, and other MATLAB function like 'all margin' are effective tool employed to check the robustness of the design. The loop was implemented in time domain for the simulation and frequency analysis was done with the transfer function model.

The Åstrom relay experimentation is an effective and simple method for parameter estimation online and the method converge faster. The drawback is that it cannot be applied on some process like the real air heater. This is because the relay switch from positives value to negative value and the air heater does not accept negatives values as it does not have a cooling facilities.

The Schei relay method take much longer time than the Åstrom but it can be used for all kind of process as its employ the control action generated through the PID loop. The step back is that a well-tuned PID controller must be in operation for its implementation and thus the experimentation is a factor of the previous PID performance.

The dynamics of the system was study in relation to the relay experimentation, and random noise and the filter time constant was used as a parameters variance for both processes. From the simulation performed on both the air heater and quadruple tank process, the following can be inferred:

- The filter time constant affect the dynamic of the entire process, small time constant give good filtering but increase the dead time of the process. While a large time constant give poor filtering but at small dead time.
- The filter time constant affect the final derived controller parameter
- An Ideal relay will switch erratically under noisy condition and parameters estimation will be difficult
- The uses of a filter in series with the relay reduces the hysteresis of the relay switching.
- The value of the relay hysteresis depend on the noise value.
- A better performance is achieved with relay with hysteresis and filter.
- Because of the filter and the hysteresis in the relay it's difficult to access the performance of the controller based on the gain and phase margin. The derived margins was outside the specified range for robust design.

Several methods of recursive parameters estimation was study and since the aim of the thesis is controller tuning and not system identification or state estimation, the relay method was employed.

In the real experimentation carried out with the air heater the frequency counter design was not effective as it gave erratic values (due to the noisy nature of the real experiment), this was cross check manually to get conservative values for the PI parameters. The real experiment on the quadruple tank could not be performed due to logistics.

The implementation of the auto tuning enable the operator to perform the tuning on demand by selecting relay tuning or PID mode as shown in Figure C-1 1. It's also possible to monitor the convergence of the operation as well as the relay switching.

6.1 Åstrom versus Schei Auto tuner

The following can be inferred from the simulation study:

- The implementation of Åstrom is simpler and easier than Schei and take less time to converge during the experiment.
- For a multiple input and output system, the Schei relay experimentation become more complicated as the correct paring of input and output variables must be determined for the optimal performance of the controller in the loop. This is the case of the quadruple tank where the Schei derived parameters performed poorly compared to Åstrom'.
- A knowledge of the process dynamic change is required for Åstrom relay experimentation for the relay to switch accordingly whereas the PID in the loop for Schei relay experimentation will bring the process to the required steady state.
- Åstrom relay experimentation gives a PI performance with less percentage overshoot
- Schei relay experimentation gives a PI performance with smaller steady state error compared to Åstrom.
- Both methods are safe as only a very small amplitude of process oscillation occurred during the experiments.

7 Conclusions

The original Åstrom relay feedback auto tuning and the modification of the method by Schei have been reviewed. Their performance was accessed through simulated air heater and the quadruple tank processes.

Modification of the relay was carried out to ensure the relay switching was not erratic under noisy condition. A relay with hysteresis performed adequate for the purpose of a noisy process. The use of filter in series with the relay was tried but failed for Schei method. The dynamic of the measurement noise filter and if applicable the filter in the relay loop do affect the tuning process.

Åstrom method does not work on the real air heater as the process does not accept negative values from the relay. The Schei method was successful applied to the real air heater. Schei method perform better for disturbance rejection than Åstrom but the performance is based on a prior controller parameters before the experiment.

The auto tuning by relay experimentation is simple and easy to implement method of recursive process parameter estimation and also re-adjusting the parameters of PID controllers. It does not pose a danger of causing the process instability as well as damage to process equipment during the tuning operation. It can be used by an inexperienced operator as it does not required prior knowledge of tuning process as it done by merely pushing a button.

The control algorithm implementation is manufacturer dependent and they used different names for the same PID algorithm. Notwithstanding as an Engineer, operator or end users of PID controller, they must be conversant with different form of PID and should know the form deployed by a specific PID manufacturers. The conversion from one forms of PID to others was discussed as this is crucial for optimum performance.

The choice of the tuning rule is a subject of the optimal choice of the user and this can be decided by using Pareto optimality concept which is recommended for future work.

7.1 Further Works

There is a lot of options that could possibly improve the performance of the Relay experimentation under noisy condition and this is recommended for further investigation.

The choice of tuning algorithms is a factor that the determined the accuracy of the final PID parameters obtained from the relay experimentation, the thesis work could only exploit the Z-N algorithms for simulation and real experimentation. This is an option for future work to implement the relay experiment on different tuning algorithms. Also the degree of freedom of the controller could be extended.

The comparison of the results from the two relay experimentation is a subject of the optimal choice of the user and this can be decided by using Pareto optimality concept (a function of IAE for the input and output disturbances as a function of the sensitivity index) which is not considered within the scope of the thesis but could possibly be explored.

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Appendices

Appendix A: Thesis's Task Description

University College
of Southeast Norway

Faculty of Technology, Natural Sciences and Maritime Sciences, Campus Porsgrunn

FMH606 Master's Thesis

Title: Automatic tuning of PID controllers

HSN supervisor: David Di Ruscio

External partner: None

Task background:
Methods for automatic tuning of PID controllers are of greate interests, e.g. for online tuning of the PID controller parameters. These parameters may change with different process operations and vary with changes in process operations. Different algorithms are published in the literature. Such methods also involve some way of recursive system identification (and/or parameter identification), of a detailed linearized model or some critical model characteristics, e.g. as the critical gain and the corresponding phase crossover frequency, or similar. Other terms as adaptive control and self-tuning control are used on the subject.

Task description:

1. Perform a literature research on algorithms for automatic tuning of the PID controller. It make sense to restrict the work to the most common and promising algorithms published in the literature.
2. Perform a research on recursive system identification based methods which may be useful in correspondence with methods for automatic tuning of the PID controller.
3. Perform simulation experiments of one or a few algorithms for automatic tuning of PID controllers. Existing models may be used.
4. A laboratory experiment of the autotuning method is of interest. The recently developed quadruple tank process ans/or the air heater may be used.

Student category: IIA

Practical arrangements: Laboratory process available

Signatures:

Student (date and signature):

Supervisor (date and signature):

Appendix B: MATLAB CODES FOR SIMULINK BLOCK

```
%% Define parameters for the intialization
 Ts = 0.1;% sampling time
Td = 2;% Time delay
n = Td / Ts;% for array creation
Kh=3.6;
Tt=23;
 u0 = zeros(n, 1);
```

PI algorithm

```
function [uk, ek]= PI_Temp(Ts, Kp, Ti, Tsp, Tout, ek_1, uk_1)
 ek = Tsp - Tout; % error
 umin = 0; % [V]
umax = 5; % [V]
g0 = Kp * (1 + (Ts / (2 * Ti)));
g1 = -Kp * (1 - (Ts / (2 * Ti)));
 uk = uk_1 + g0 * ek + g1 * ek_1; %deviation form using trapezoidal method
 if (uk < umin) % Anti wind up implementation
     uk = umin;
 elseif (uk > umax)
     uk = umax;
end
end
```

Linear element Block

```
function zf = fcn(Tout,Tsp,S)
er=Tsp-Tout;
% zf=0
 if er>=0.1 && S==0 % With hyteresis on the relay
    zf=1;
    elseif er<=-0.1
    zf=-1;
else
    zf=0
end
%
% if ek>=0.2
%     zf=1;
% elseif ek<=-0.2
%     zf=-1;
% else
```

```
%     zf=0
% end
```

Amplititude Calculation

```
function [p, o, y_tmp, Ymin]= fcn(m, n, y, y_temp, Ymin_hold)
% Y = main data for period calculation
% y_temp = to store three values to check maxima
% k = index to shift from y0 to y1 in Y
o = n;
Ymin = Ymin_hold;
y_tmp = y_temp;
if m>=3
    if y_temp(1) > y_temp(2) && y_temp(2) > y
        if n == 0
            Ymin(1,1) = y_temp(2);
            Ymin(2,1) = m - 1;
            o = 1;
        elseif n == 1
            Ymin(1,2) = y_temp(2);
            Ymin(2,2) = m - 1;
            o = 0;
        end
    end
end
p = m + 1;
y_tmp(1) = y_temp(2);
y_tmp(2) = y;
```

AStrom Tuner Block

```
function [a,pu,Wc,Nr,Kp_n,Ti_n] = fcn(Y,Tsp,Ar)
 a=Y(1,1)-Tsp;                              % amplitude of oscillation
Ts=0.1
A=abs(Ar)                                   %Relay amp
Nr=(4*A)/(pi*a);                        % Altimate gain
pu= abs((Y(2,2)-Y(2,1)))*Ts            %Altimate Period
Kp_n=0.45*Nr;                          %New Kp using Z-N
Ti_n=pu/1.2;                                %New Ti
Wc=2*pi/pu
```

Filter Block

```
function Toutf = fcn(Tf,Tout,Toutf_1)
Ts=0.1;
```

a=Tf*(Tf+Ts);

Toutf=a*Tout +(1-a)*Toutf_1;

Toutf_1=Toutf;

Air Heater

```
function [ud, U]= delay_fun(u, uk, n) % air heater delay implementation in ODE form
ud = u(1);
U = u;
for i=1:n-1
    U(i) = u(i+1);
end
U(n)=uk;
function Tout_1=airheater_Ode(ud, Ts, Te, T_out, Kh, Tt) % Air heater in ODE form
Tout_1 = (Ts/Tt)*(-T_out + Kh*ud + Te) + T_out;
```

Quadruple Tank [36]

```
% The quadruple tank linearized model state space matrices for minimum
% phase case
% Parameters for the quadruple tank level process
% Modified from David Di Ruscio
A1=28; A3=28; A2=32; A4=32; % cross section of tank[cm^2]
a1=0.071; a3=0.071; a2=0.057; a4=0.057; % cross section of outlet hole[cm^2]
kc=0.50; % ratio of voltage and level[V/cm]
g=981; %acceleration due to gravity
%linearised operating point (nominal value for minimul phase case)
h10=12.4; h20=12.7;h30=1.8; h40=1.4;u10=3.0; u20=3.0;
k1=3.33; k2=3.35;
g1=0.7; g2=0.6; %linearised operating point (nominal value for minimul phase case)
% State space matrix
T1=(A1/a1)*sqrt(2*h10/g); T2=A2*sqrt(2*h20/g)/a2; T3=A3*sqrt(2*h30/g)/a3;
T4=A4*sqrt(2*h40/g)/a4;
A=[-1/T1,0 ,A3/(A1*T3),0;0 ,-1/T2,0 ,A4/(A2*T4);0 ,0 ,-1/T3 ,0;0 ,0 ,0 ,-1/T4]
B=[g1*k1/A1 ,0;0 ,g2*k2/A2;0 ,(1-g2)*k2/A3;(1-g1)*k1/A4,0]
D=[kc, 0,0,0;0,kc,0,0]
```

Results

A = [-0.0159 ,0 ,0.0419 ,0; 0 , -0.0111, 0 , 0.0333; 0, 0, -0.0419, 0; 0,0 , 0, -0.0333]

B=[0.0833, 0; 0, 0.0628; 0 ,0.0479; 0.0312 ,0]

D =[0.5000, 0; 0, 0; 0, 0.5000; 0, 0]

Appendix C: SIMULINK BLOCK IMPLEMENTATION

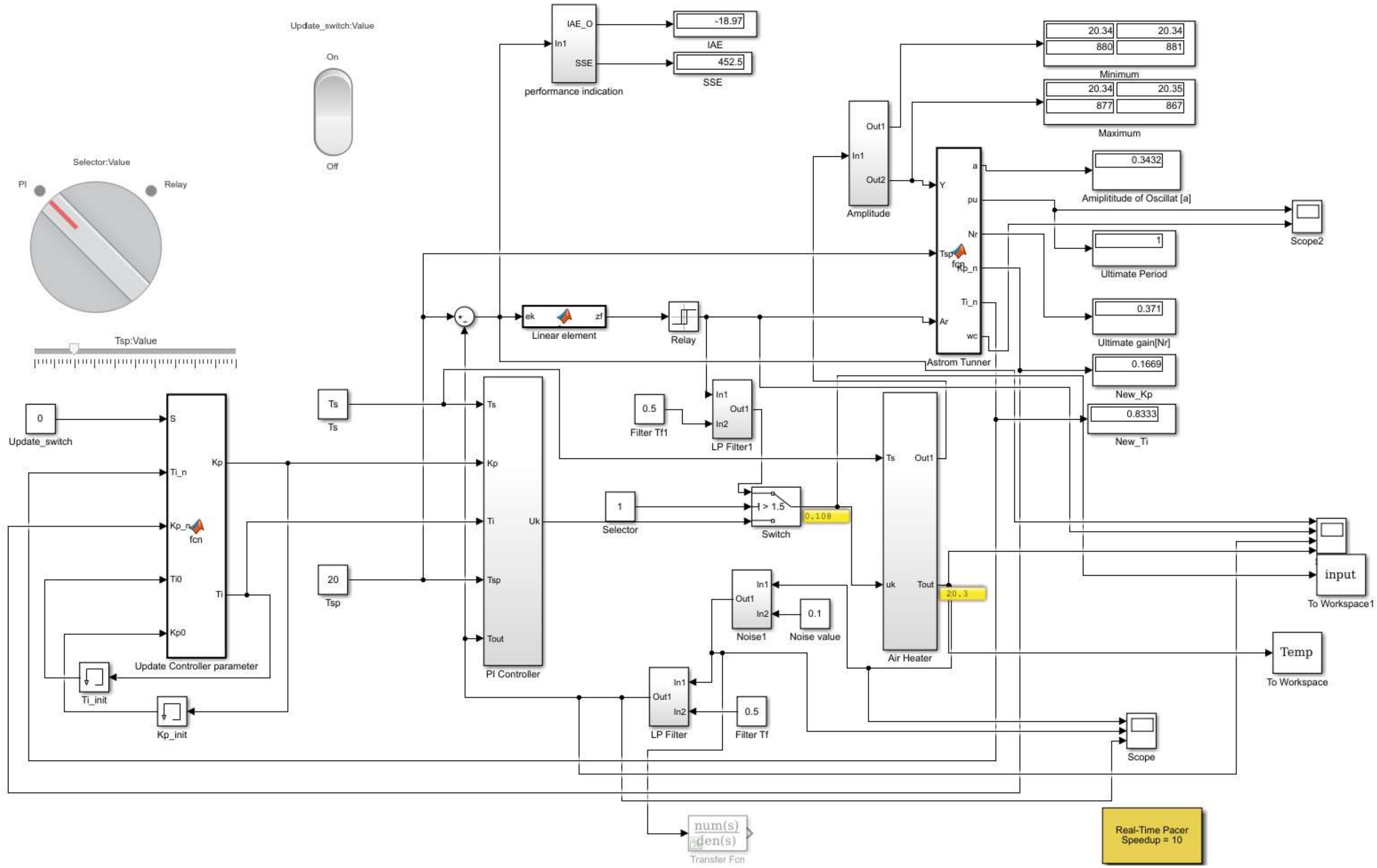

Figure C-1 1: **ASTROM BLOCK**

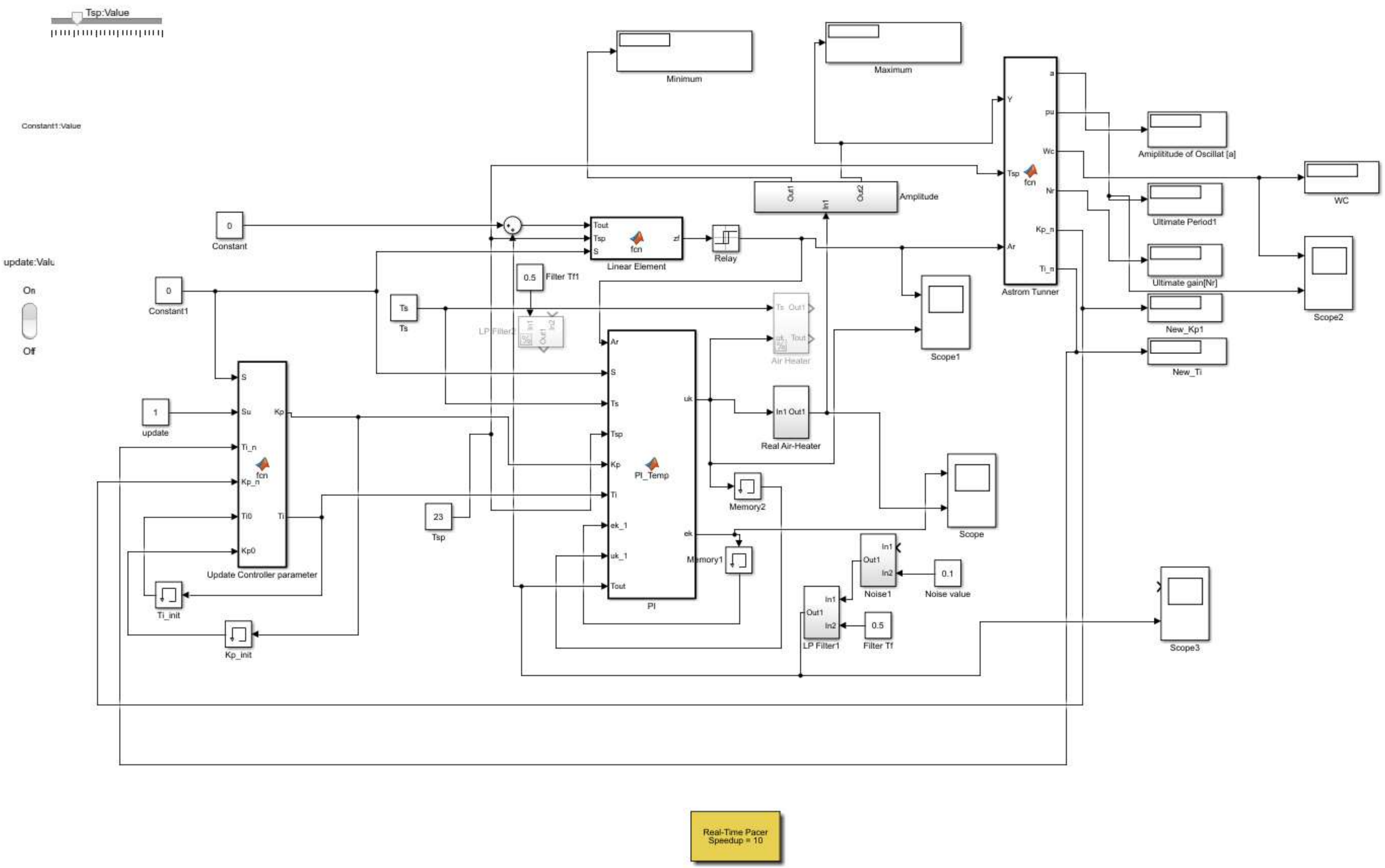

Figure C-1 2: **SCHEI BLOCK**

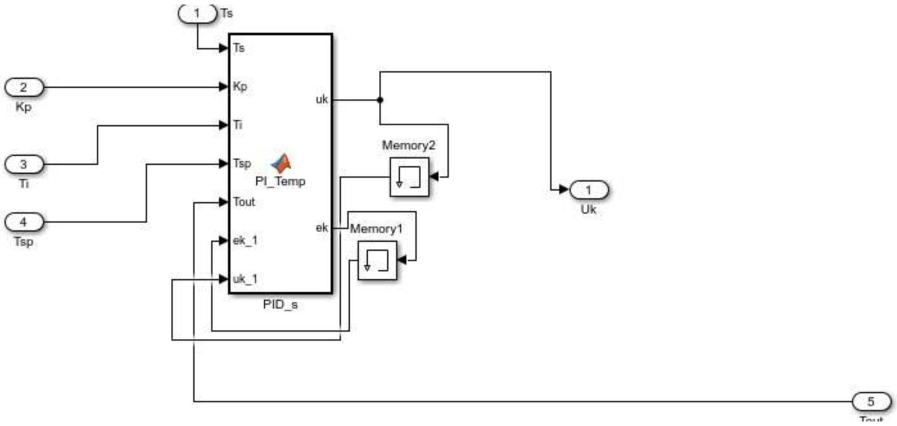

Figure C-1 3: **PI CONTROLLER BLOCK**

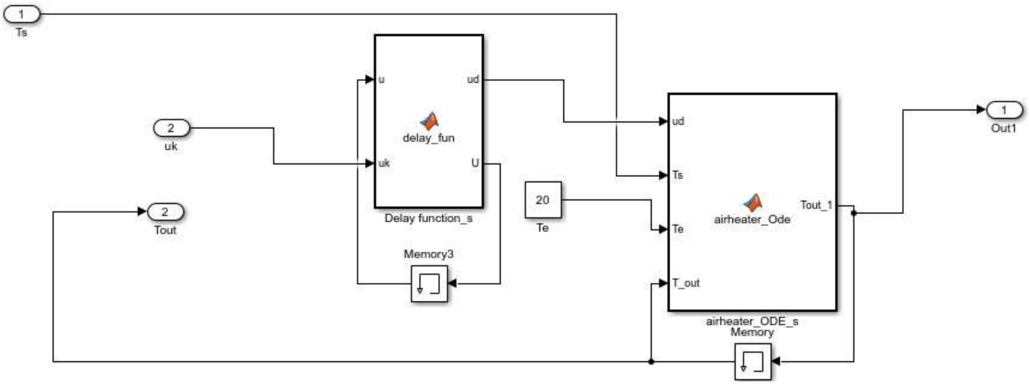

Figure C-1 4: **AIR HEATER BLOCK**

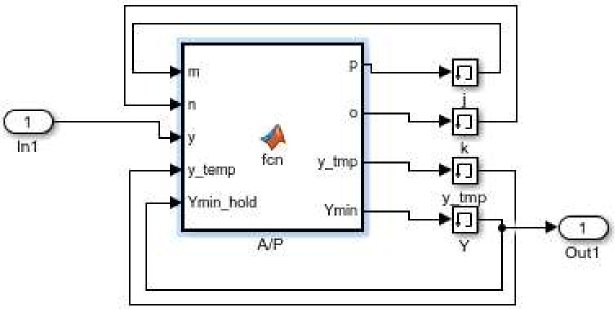

Figure C-1 5: **AMPLITITDE CALCULATION BLOCK**

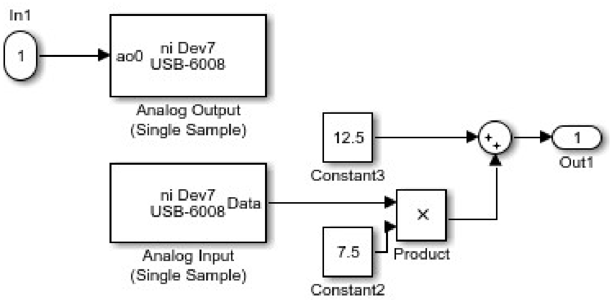

Figure C-1 6: **Real Air heater connection block (DAQ assistance)**

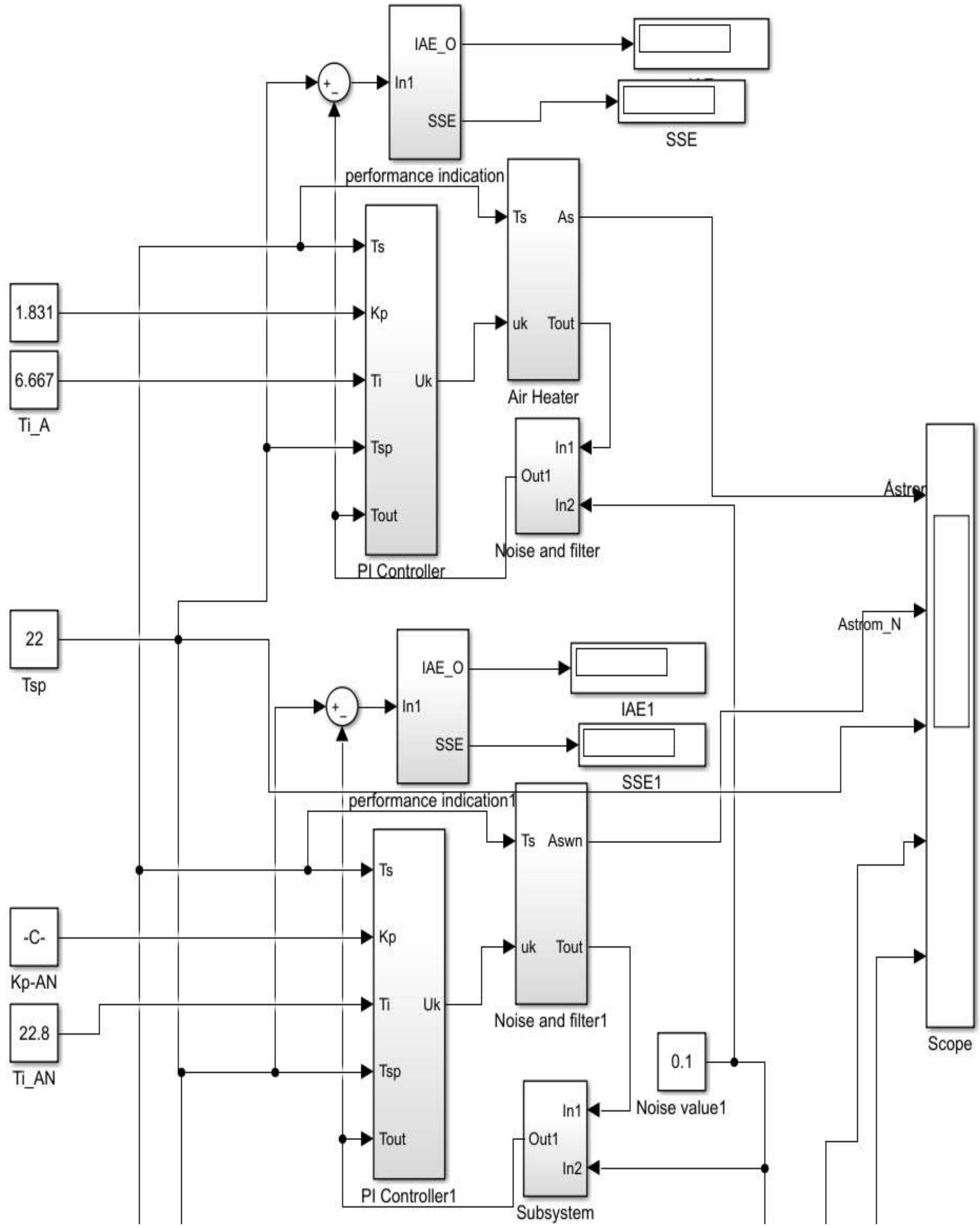

Figure C-1 7: **Comparing the PI parameters for different simulation**

Appendix D: Plots from Relay Experimentation

Plots from Schei relay experiment on Real Air heater

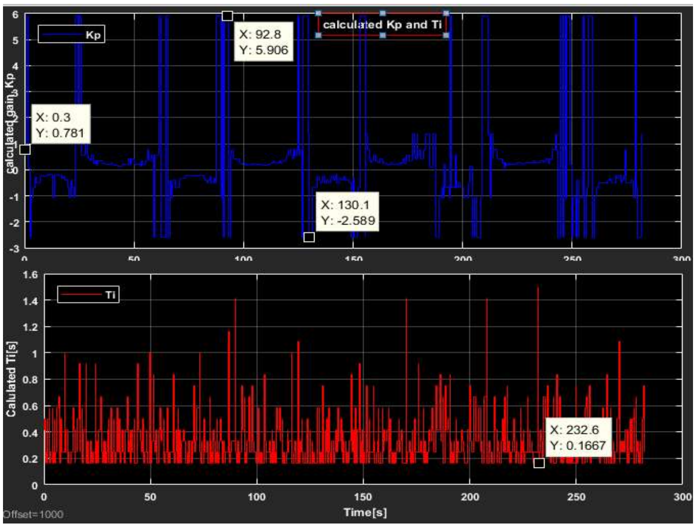

Figure D-1 1 The upper and the lower plots show the calculated Kp and Ti respectively during the relay experiment on real air heater.

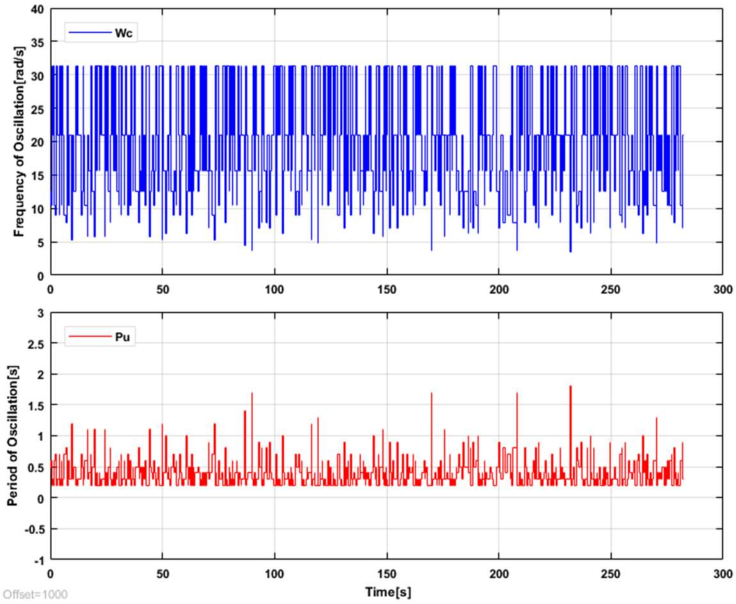

Figure D-1 2: The upper and the lower plots show the frequency and the period of oscillation respectively during the relay experiment on real air heater.

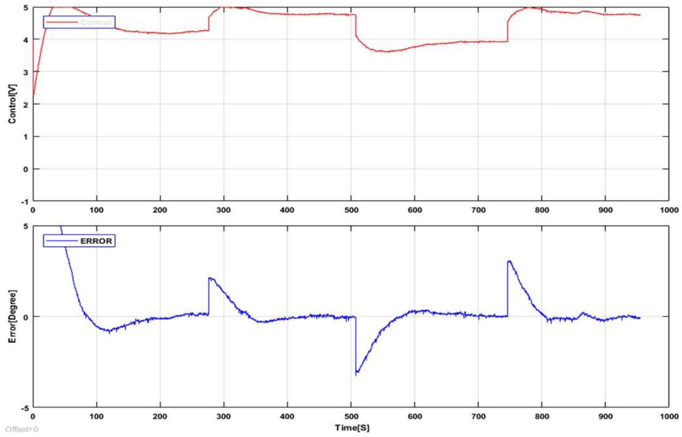

Figure D-1 3: The upper and the lower plots show the control voltage and the error signal respectively during the relay experiment on real air heater

Simulation plots from Åstrom relay experiment on the quadruple tank

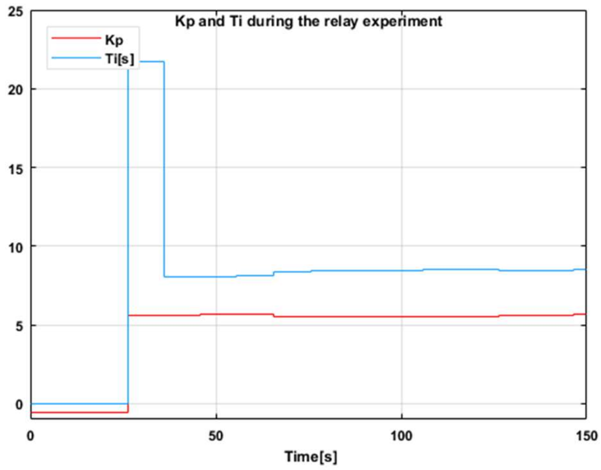

Figure D-1 4: The Proportional gain and integral time convergence during Åstrom relay experimentation on the quadruple tank (PID on tank 1 and manual on tank 2)

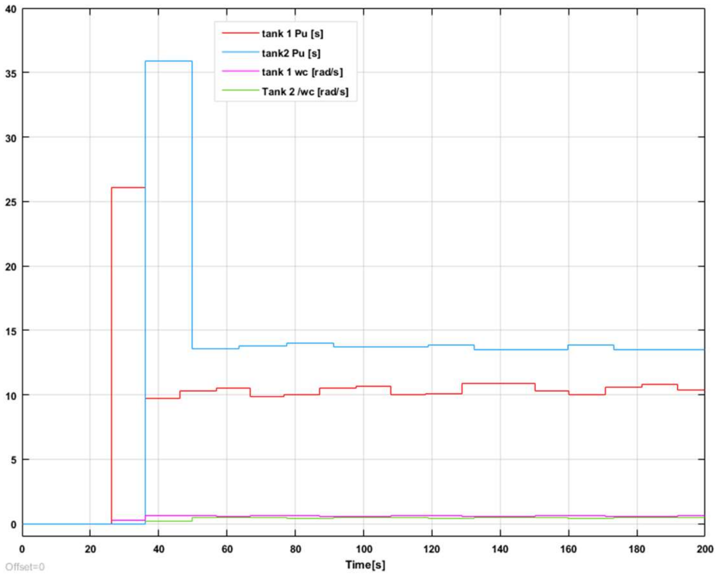

Figure D-1 5: The period and frequency of oscillation during Åstrom relay experimentation on the quadruple tank (as two separate SISO)

Appendix E: Nyquist Plot from Relay Experimentation simulated Air Heater

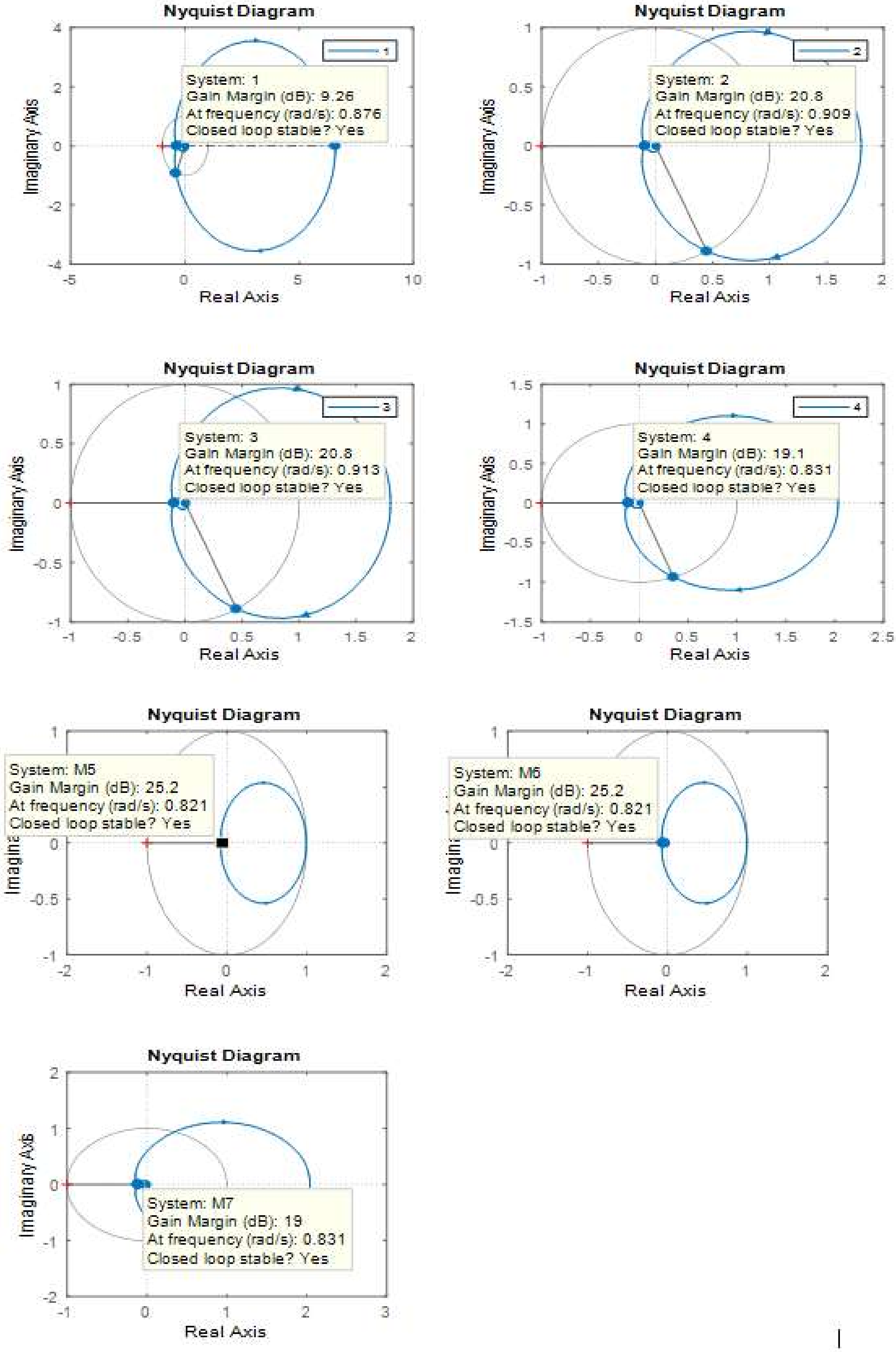

Figure E1 1: Nyquist plot from the result presented in Table 5-1.None of the curve enclosed the 0-j point, so they are stable.

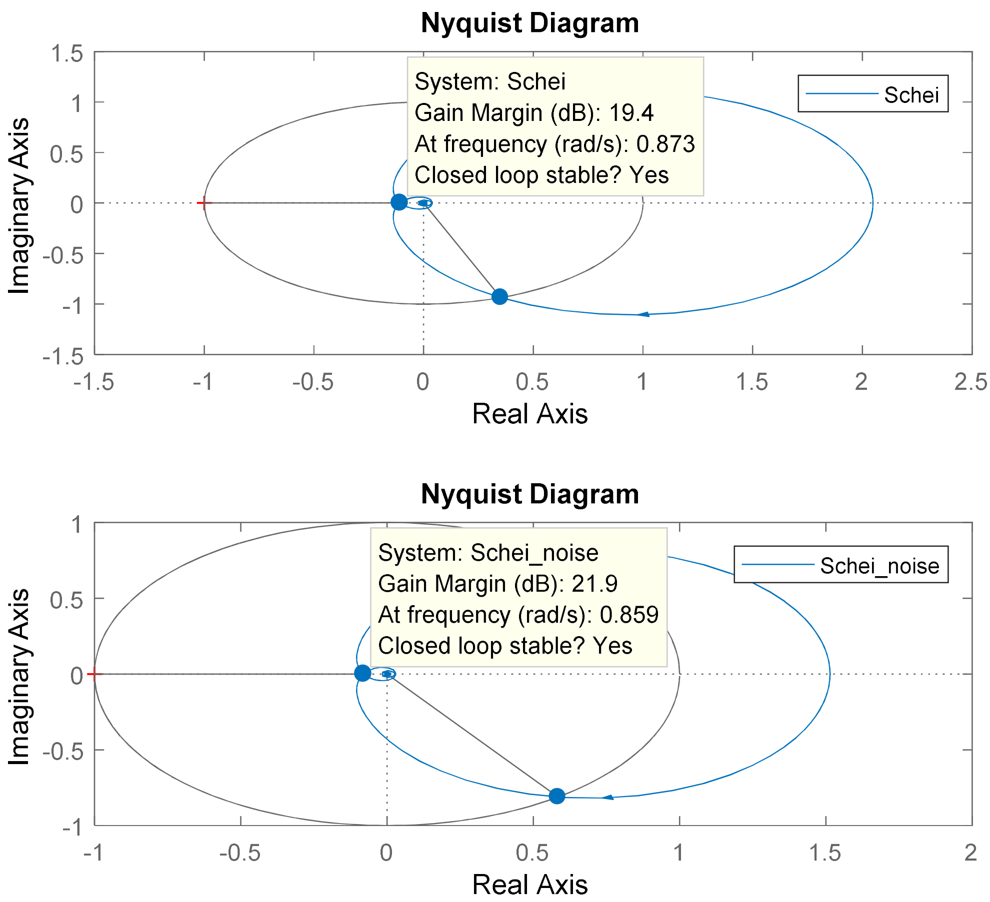

Figure E1 2: Nyquist plot from the result presented in Table 5-8.