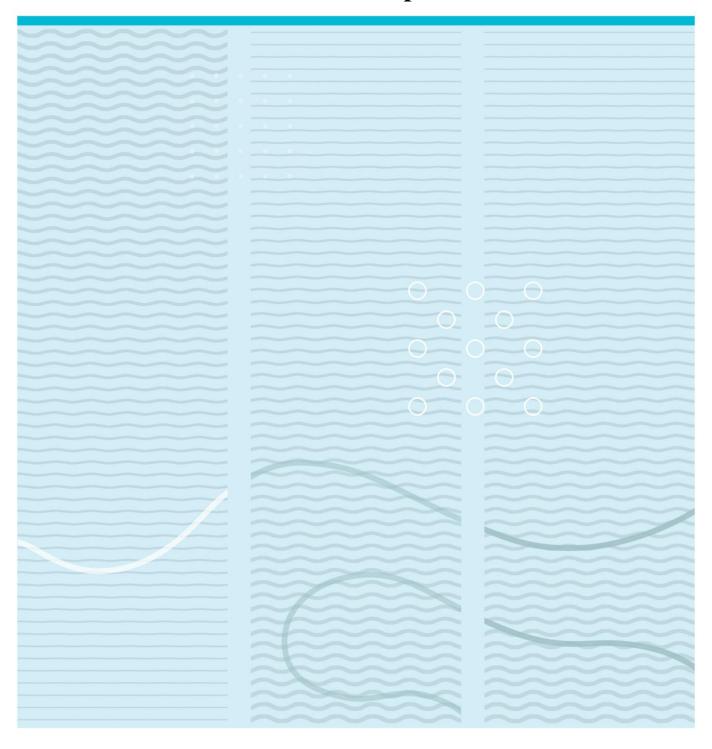


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# Super-imposition methods to enhance the projector resolution: Simulations and experiment



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This thesis is worth 30 study points

#### **Summary**

This work investigates different methods to enhance the resolution of projectors beyond its native resolution. I developed two novel algorithms which prioritize darker and brighter details in an image. Detailed simulation, visual image quality analysis, quality metric assessment and measurement results of the new methods along with previously proposed methods are described in this thesis.

Chapter 1 gives a brief introduction of projectors, its resolution definition and techniques that focus on improving resolution without upgrading its spatial light modulator resolution. It also looks into different metrics for quantitative quality assessment comparison of images and then defines the objective of this work. Chapter 2 provides details of challenges involved in enhancing the resolution through superimposition techniques in general and defines the problem as a linear system of equations. Chapter 3 describes methods which will be simulated and measured in this work. This includes two novel techniques which focus on prioritizing darker and brighter details in an image. Chapter 4 presents results from MATLAB simulations of all the techniques. The resulting superimposed images from all the methods have been accessed visually for image quality and artefacts as well as using MSSSIM quality metric. Chapter 5 describes the experimental setup where all the methods were tested using projector setup provided by Barco, Fredrikstad. Chapter 6 presents results from the measurements performed and its visual assessment. Chapter 7 compares the performance of the methods. Iterative techniques are found to be the best in terms of MSSSIM as well as visual assessment but they need higher computational resource compared to other techniques. Prioritizing dark pixels gave result similar to a filter with dark details present and brighter details absent. Chapter 8 concludes that using optomechanical actuator improves the quality of the image projected. It also concludes that results from prioritizing dark pixel indicate that prioritizing certain properties in an image may be considered for future work.

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# Foreword

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# **1** Introduction

#### **1.1 Projectors**

A projector is an optical device that projects an image (or moving images) onto a surface, commonly a projection screen. The basic components of a projector are the light source, spatial light modulator (SLM) and projection lens. Projection lens projects the light produced by the light source and modified by the spatial light modulator according to the digital picture input to the screen. The quality of a projector can be defined by the range of colour, its resolution, contrast ratio, sharpness, aspect ratio, refresh rate, cost and the maximum brightness of the image it can project. These qualities are determined by the type of technology used and market factors.

This work is mainly focused on improving the resolution. After a brief discussion about other parts in the projector, we will focus on parts of projectors that directly contribute to the resolution of the projected image.

#### 1.1.1 Light source

Light source used in a projector impacts the color range of output image and the maximum brightness of the projector. Ultra-high-performance (UHP) lamps such as Xenon short-arc lamps and Mercury-vapor lamps have been the traditional light sources used in most commercial projectors. [1] The disadvantage of this type of light source is its low lifetime, high warm-up/ cool down time and high-power dissipation [2]. Apart from these, another limitation is in the range of colors which can be projected. [3] Light Emitting Diodes (LEDs) are attractive light sources for low brightness projectors

because of its long lifetime (greater than 50,000 hours), high efficacy, ease of electronic control and excellent color range. For high brightness projectors, LEDs suffer from power dissipation issue and has to be designed with good cooling system [3]. Moreover, they emit light over a broad angle and area, making it difficult to combine many LEDs for high brightness projectors.

Laser-based displays were not commercially viable due to the large size and high cost of laser sources, but recent developments in visible laser technology are expected to give momentum to its development [4]. The use of lasers will enable wider range of color which is not easily possible in case of other light sources. Solid-state lasers also has a long lifetime similar to LEDs.

#### 1.1.2 Spatial light modulator (SLM)

Spatial light modulator changes the brightness of light in spatial plane to match the input image information. Along with color wheel or color filter, they can define color and brightness of each pixel projected on the wall. Although SLM is partly responsible for range of color, increasing it is actually limited by the source of the light used [3], [4]. However, they play prominent role in determining the resolution of the projector as they define the smallest possible detail that can be projected on the wall. In systems that do not use superimposition, the resolution of the projectors is equal to the number of individually addressable SLM cells. The most commonly used types of SLM are digital micromirror device (DMD) (or digital light processing (DLP)) based projectors and active pixel matrix in case of liquid crystal based projectors (LCD). For this work, we use DLP based projector hence the underlying technology of DMD is described in the following section.

#### 1.1.3 Digital micromirror device (DMD)

The DMD chip is a matrix of micromirrors each of size around 12  $\mu$ m x 12  $\mu$ m [5] or smaller. Two such mirrors are shown in figure (1). Each mirror is a combination of optomechanical and electro-mechanical elements. The mechanical component allows the mirror to move in ON and OFF state. In the ON state, the mirror reflects the light forward to the projection lens making that pixel bright on the wall. While in OFF state, it reflects the light to an optical sink thus making that pixel dark on the screen. The electrical component controls the mechanical movement of the mirror and the switching between the two states. The switching time of micromirrors of around ~15 $\mu$ s which makes displaying multiple images within the temporal sensitivity of human visual system possible [5]. This fast switching speed is one of the enabling factors for the concept of increasing the resolution using superimposition of multiple frames.

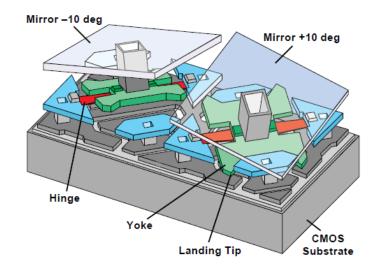


Figure 1: Two micromirror in a DMD chip [5]

Colors can be displayed in two different ways. First way is to use three different DMD chip for each red, green and blue light source to project each part of an image and then synchronize them to get color images. The other way is to use one DMD chip, white light source and a color wheel which will rotate to produce red, blue and green part of the image in synchronization with the switching of mirrors. The intensities of each part is controlled by pulse width modulation [6], [7].

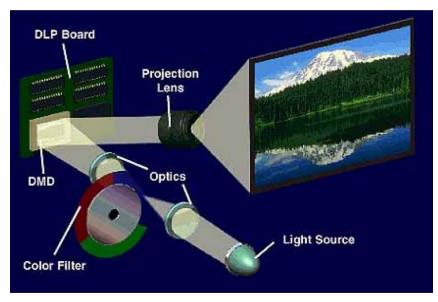


Figure 2: DMD chip with colour wheel system [8]

#### **1.2 Resolution**

As mentioned in earlier section, resolution of a projector can be defined by the number of SLM pixels inside it. It can also be defined in terms of the smallest detail a projector can project perfectly. Both statements are logically possible definitions in case of a normal projector without superimposition techniques. But the definition of resolution of projectors that employ superimposition and techniques like them becomes complex. Definition of resolution can be divided into two parts. One it is based on the smallest detail a projector can project, and second is based on the ability to project as many of those details as covered in the projection area. For example, if we compare a native 4K (2840 x 2160 pixels) projector with superimposition projector with subframes of 2K (1920 x 1080 pixels) resolution, then the smallest detail projected perfectly by both the projectors will be of the same size. But unlike a native 4K projector which can project 4K number of small details, superimposition based projectors will not be able to do so. Hence, measuring the resolution of such projectors is difficult since the number of details it can project perfectly varies not just with the superposition technique but also with type

of image content and image processing algorithms. Hence for this work, even if the aim is to increase the resolution, measurements will be based on image quality metrics (discussed later) and visual inspection.

There is a significant difference in resolution between image produced by commercial image sensors and the capability of the display systems such as projectors and high dynamic range (HDR) monitors to display them. Typically, the resolution of the image sensors is more than double the resolution of the display systems, hence the images need to be down scaled to be displayed [9]. The resolution of projection displays is limited by the resolution of the system's Spatial light modulator (SLM). Increasing the resolution of SLM is expensive. Thus, the cost of increasing resolution in display systems is considerably higher than that of the image sensors [10]. Increasing the resolution capability of projectors and other display systems while keeping the native SLM is a challenging problem on which many researchers have focused recently [9]–[29].

#### **1.3 Previous attempts to increase resolution**

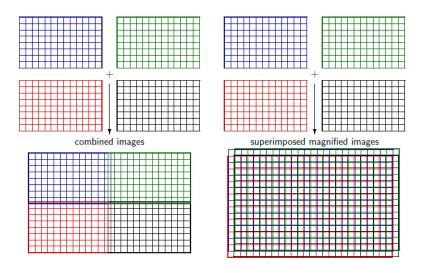


Figure 3: Different ways to increase resolution using multiple projectors [9]

#### 1.3.1 Multiple projectors

One of the approach is to use multiple projectors of lower resolution to produce a higher superimposed image with greater resolution [22]–[24]. There are mainly two ways of achieving this, one is by dividing image into tiles and placing them side by side to produce a larger image as shown in figure 3 (a). Another approach is dividing the high-resolution image into lower resolution subframes. These subframes are then projected on top of each other with a shift less than the size of one pixel of SLM and are thus super imposed on each other as shown in figure 3(b). As compared to a tiled image projection, a superimposed projection enables more scalability [23]. The main drawback of these multi-projector systems is the dependence of image properties on several projectors. Malfunction of any one of them could lead to poor performance as well as complex configuration problems in alignment, color and brightness of the system, as these would be combined function of all the projectors involved [24], [27].

Damera-Venkata et. al first proposed achieving higher resolution through multiple projectors [24]. This was then explored by other authors including detailed definition and theoretical study of resolution and properties [10], [26], [29] [22], [28].

#### 1.3.2 Superimposition using time multiplexing

Allen et. al. [25] came up with another approach using an opto-mechanical actuator (or optical actuator). This approach is essentially doing similar superimpositions as in the previously described method but using a single projector. A high-resolution image is first divided into multiple subframes. These subframes are then projected one at a time but with a shift that is less than the width of a pixel of SLM. These shifts are done using an optical actuator which is able to shift fast enough so that multiple subframes are projected such that human visual system (HVS) comprehends it as a single image [4]. The high cut off temporal frequency also known as critical flicker frequency (CFF) defines the frequency above which HVS does not notice changes and observes image as a single image. CFF also depends on factors such as the adaptation luminance, spatial extent of flickering pattern, and retinal region at which pattern is projected [14]. This is known to vary from 60 Hz to about 15 Hz as the luminance decreases [30].

Berthouzoz et. al. extended the idea of Allen et. al. discussed earlier by generating the optimal subframes using iterative technique [9]. Similar work was also proposed by Barshan et. al. where they use piezo-electric actuator to move glass component into two different positions to achieve sub-pixel shifts [27].

#### 1.3.3 Superimposition without using time multiplexing

These techniques do not use time multiplexing, instead they either use multiple sources or lens to create shift. Sajadi et. al proposed a technique in 2012, to produce spatiotemporal superimposition, but using lens system and two sets of SLM [20]. Sajadi et. at. proposed another technique in 2013 where two lenses were placed between the light modulator panel and the projection lens were used to produce the shifts instead of a opto-mechanical actuator. In this work, they use single image which is used in both shifts. [17]. Heide et al. came up with another approach in 2014 where two SLMs were cascaded on top of each other such that light from one SLM goes through another SLM which causes subtraction of light instead of addition as seen in all the techniques described so far.

Table 1: Different subframe generation t	techniques and hardware ty	pes
--	----------------------------	-----

Technique	Hardware type	Time multiplexing	Subframe generation technique
Niranjan Damera- Venkata and Chang, 2007	Multiple projectors	No	Iterative/ fast filter banks
Allen and Ulichney, 2005	Opto-mechanical actuator	Yes	Downscaled superimposed
Berthouzoz and Fattal, 2012	Opto-mechanical actuator	Yes	Iterative with multiple subframes
Sajadi et al., 2012	Multiple SLM with optical pixel sharing	Yes	Special technique based on edge detection
Sajadi et al., 2013	System of two lens producing optical shift and overlay	No	Single subframe iterative technique
Barshan et al., 2015	Opto-mechanical actuator	Yes	Iterative/ fast filter banks similar to Niranjan Damera- Venkata and Chang

#### 1.4 Description of image quality metrics

To be able to quantify closeness of resultant image with the original image is an important part when investigating various techniques for superimposition. The best way to compare visual data is to do a subjective assessment, where the images are judged by number of unbiased individuals. This kind of assessment is not convenient to perform. It is usually time consuming and often expensive. Hence, it is necessary to have a good objective assessment criterion [17].

#### 1.4.1 Mathematical metrics

One of the simplest ways of measuring closeness between high resolution and the superimposed image is by using mathematical metrics. In these techniques, the numerical difference between the original image and the superimposed image, or in other words the error between the two is considered for judgement. Some examples of these metrics are: Mean Square Error (MSE) and different versions of Signal to Noise Ratio (e.g. Peak Signal to Noise Ratio (PSNR)). While these metrics appear logically best placed to serve as quality metrics, this does not necessarily represent visual similarity. They do not often correlate with subjective assessment of the image [5]. This is because

even if the numerical values between the two data sets are close, they still might be different visually. A common example of this can be a blurred image, even if the blurred and the original image are close numerically, visually they are not as close as the metric suggests.

#### 1.4.2 Human visual system (HVS) inspired metrics

These metrics consider structural similarity not just numerical similarity. They aim to compare elements that are important to HVS to give more correlated results to subjective assessments. They compare elements like the luminance, contrast, and structure of images. SSIM (MSSSIM), ESSIM, SRSIM, Feature-SIM (FSIM), DCTex, VIF and VSNR are some of the widely used metrics of this type. Hansen et. al. investigated the metrics for evaluating superimposed images and concluded that MSSSIM performs better since it is better at detecting loss in single-pixel details as well as it could rate superimposed images higher than the downscaled versions. [17]

#### 1.4.3 Objective of this work

This work is devoted to developing better subframe generation techniques and comparing them. We recreate several techniques implemented earlier as well as develop two new techniques. The aim of the work is not only to find optimal technique for subframe generation but also to examine different possibilities based on understandings from previous research and intuition. All the techniques have been simulated in MATLAB and then tested in a projector with DMD based SLM and opto-mechanical actuator. Measurements are also done using high speed camera.

## 2 Description of problem for additive superimposition

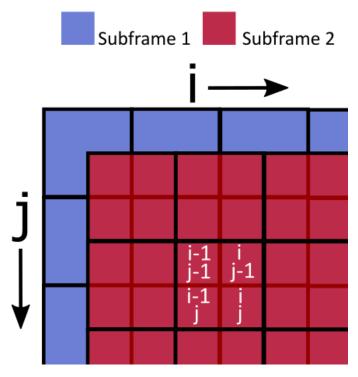


Figure 4: Pixel grid of two subframe

In the configuration where two subframes are shifted by half pixel along the diagonal, each pixel in superimposed image is dependent on two subframes. If P(i, j) is any pixel in the high-resolution input image, defining it perfectly is possible in the configuration using two subframes. (*Sub*1 and *Sub*2) Considering i and j to be even numbers,

Superimposed pixel, S(i, j) = Sub1(i/2, j/2) + Sub2(i/2, j/2) (2.1)

The cost of defining this pixel in superimposed image perfectly will have consequences in six other neighboring pixels. Three of them (i-1, j-1), (i, j-1), (i-1, j) are dependent on Subframe 1 (sub1) and three others, (i+1, j+1), (i+1, j), (i, j+1), are dependent on Subframe 2 (sub2). Once the values of Sub1(i/2, j/2) and Sub2(i/2, j/2) are fixed, one of these pixels can still be defined perfectly using other subframe but defining each will affect another three pixels in superimposed image and so on. Hence, in additive superimposition defining one pixel perfectly will impact not just the neighboring pixels but other pixels throughout the superimposed image. If the neighboring pixels in superimposed image are close in value to each other, then even if some of the pixels could not be defined exactly, it will still give result close to the original image. Thus, it could be said that the dependence is proportional to the spatial frequency of the content. Finding out the best possible way to assign the values to get the maximum pixels defined perfectly is the challenge for subframe generation techniques.

Another important consideration in this type of setup is the maximum brightness of each subframe. If we compare a normal projector with a single subframe of this setup, the subframe is going to be projected only half the time. Thus, the highest brightness achievable by the subframe is only half the maximum brightness of the projector. This is also an important difference between using opto-mechanical actuator and other methods of superimposition described previously such as using multiple projectors or lens systems for shifts.

#### 2.1 Different configurations of opto-mechanical actuator

There are different possible configurations for the number of subframes and pixel shifts. Sajadi et. al. in 2013 have considered different shifts possibilities (shown in figure 4) and concluded that none of the pixel shift perform better than half pixel shifts in both horizontal and vertical direction [15]. Considering higher number of subframes and other shifts is only logical if there is possibility of better performance, otherwise it will only add complexity to the process of subframe generation. Hence, the same configuration (two subframes diagonal shift) is used for this research.



Figure 5: Different pixel shifts considered by Sajadi et. al [17]

#### 2.2 Formulation of linear system of equations

The superimposition of subframes can be defined as a linear system of equations both in the case of opto-mechanical actuator and multiple projectors [17], [22], [24].

If P be the high resolution image, Sub1 and Sub2 are two subframes. Superimposed image S, is given by:

$$S(i, j) = Sub1(x_1, y_1) + Sub2(x_2, y_2)$$
 (2.2)

 $(x_1, y_1)$  and  $(x_2, y_2)$  are the position of pixels in subframes responsible for pixel P(i, j)in superimposed image. They are dependent on the direction of the pixel shift used. Figure (6) shows the two directions and table 1 shows the relation between pixel in the input image to the relevant pixel in the subframes. This will be used to identify the relevant pixels for each pixel in input image.



Figure 6: Different configurations of half pixel shifts

	Up-left configuration		Up-right configuration	
	Even	Odd	Even	Odd
Subframe1	$x_1 = i / 2$	$x_1 = (i+1)/2$	$x_1 = i / 2$	$x_1 = (i+1)/2$
	$y_1 = j / 2$	$y_1 = (j+1)/2$	$y_1 = j / 2$	$y_1 = j / 2$
Subframe2	$x_2 = i / 2$	$x_2 = i / 2$	$x_2 = i / 2$	$x_2 = i / 2$
	$y_2 = j / 2$	$y_2 = j / 2$	$y_2 = j / 2$	$y_2 = (j+1)/2$

Table 2: Identifying relevant subframe pixels for different configuration

# **3** Techniques for generation of subframes

Among the techniques described below, there are two novel techniques developed during this work, four of them have been suggested by earlier research while the rest are based on simple intuitive upgrades to previous techniques. The methods are described below in four different sections according to their principles.

#### 3.1 Iterative techniques

Iterative techniques described first have been used by number of researchers mentioned in the previous section.[12], [17], [24] As discussed earlier, there is a relation between each pixel of the superimposed image. One of the best way to solve this problem of dependencies is by using iterative methods. The major drawback of this could be the computation time and resource required.

#### 3.1.1 Single subframe approach

Behzadi et al. used single subframe to minimize computation time and resources. This admittedly does not use the technique to its full potential and produces the sub-optimal subframes, but it will still be able to improve the resolution of the resulting image in comparison to the downscaled version. Importantly, it could be used practically as a lowcost solution. In this approach, the problem is taken as constraint linear least square and the aim is to reduce the error between the resulting superimposed image and the original high resolution image.

Considering the problem as systems of linear equations like equation (3.1),

$$AI = I_T \qquad (3.1)$$

Where I is the subframe which is unknown, A is a spare matrix which defines the linear equations and  $I_T$  is the given high-resolution image. The paper readdresses the problem as quadratic programming problem using the bound  $0 \le I \le 1$ . Then proceeds to solve it using Gaussian Belief Propagation (BP) solver, which does not guarantee convergence but is faster than other algorithms like Jacobi or Gauss-Seidel. The limit  $0 \le I \le 1$  is enforced using Lagrange multipliers updated using gradient ascent at each iteration. The objective of the iterations is to minimize equation (3.2)

$$\min \frac{1}{2} \mathbf{I}^{\mathrm{T}} \mathbf{A}^{\mathrm{T}} \mathbf{A} \mathbf{I} \mathbf{I}^{\mathrm{T}} \mathbf{A}^{\mathrm{T}} \mathbf{I}_{\mathrm{T}}$$
(3.2)

After considering the Langrage multipliers and defining  $\mathbf{J} = \mathbf{A}^{\mathrm{T}}\mathbf{A}$  and  $\mathbf{h} = \mathbf{A}^{\mathrm{T}}\mathbf{I}^{\mathrm{T}}$ , the objective function can be redefined as equation (3.3).

$$\frac{1}{2}I^{T}JI - I^{T}h - I^{T}\gamma + (I-1)^{T}\lambda$$
(3.3)

This technique does not use time multiplexing, hence to adjust the issue of brightness, either the bound must be changed from  $0 \le I \le 1$  to  $0 \le I \le 0.5$  or equation (3.1) should be re-written as

$$AI = 2 \times I_T (3.4)$$

#### 3.1.2 Two subframe iterative approach

The aim of extending the same to two subframe was to examine the potential of the superimposition setup, since iterative technique was the most reliable way of checking the maximum yield of the system.

The same approach can be extended to create two subframes.

$$I_{1} + I_{2} = I$$

$$AI_{1} + BI_{2} = I_{T}$$

$$\begin{bmatrix} A B \end{bmatrix} \begin{bmatrix} I_{1} \\ I_{2} \end{bmatrix} = I_{T}$$

$$* AT = I_{T}$$
(3.5)

A and B are two sparse matrices which depends on the size of the image.

Solving equation (3.5) just like in the case of single subframe case, using Gaussian Belief Propagation(BP) solver, the two subframes can be calculated. Similar adjustment for time multiplexing has to be implemented here as well.

Various algorithms are available to solve constrained linear least square such as the Jacobi or Gauss-Seidel. According to Behzadi et al. Gaussian Belief Propagation (BP) solver is faster and suits applications where minimum computations are preferred [13][31].

#### 3.2 Pick and place techniques

These are based on native technique which is used by Said in 2006 for mathematical analysis of subframe generation techniques. [29]The other two techniques following

naïve technique is simple intuitive modifications to naïve technique intended for better results.

In these techniques, the pixels for the subframes are chosen based on their spatial position. If a high-resolution input image is available, then there are four possible choices as shown in the figure (7). Subframe 1(red) is decided from set of P1, P2, P3 and P4 while Subframe 2 (blue) is decided from P4, P5, P6, and P7. If high resolution input image is not available the input image is scaled first to the double resolution of SLM and then used.

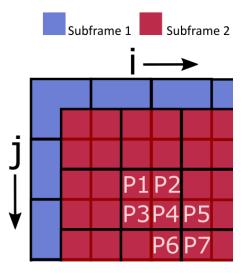


Figure 7: The pixel grid showing two subframes overlapped in original image

#### 3.2.1 Naïve technique

As the name suggests this is the one of the simplest technique. It considers pixels which are row wise odd numbered in the input image in one subframe while even numbered in the other. In figure (7), subframe 1 would be either P1 or P3 (odd) and Subframe 2 would be either P4 or P6 (even).

#### 3.2.2 Picking mean technique

Naïve technique considers only one of the four different possibilities. The aim of Picking mean technique is to increase the dependency in all four pixels in the input image. This technique considered mean value of the four pixels. That is Subframe 1 = Mean (P1, P2, P3, P4) and Subframe 2= Mean (P4, P5, P6, P7).

#### 3.2.3 Picking minimum and maximum technique

This technique also considers one out of four possibilities but it takes exact values from the original image and considers darkest and brightest pixel out of the four possibilities. This technique is much more content dependent than the Naïve technique. This technique considers least value in one subframe while maximum value in the other subframe. That is Subframe 1 = Minimum (P1, P2, P3, P4) and Subframe 2 = Maximum (P4, P5, P6, P7).

#### 3.3 Other techniques

#### 3.3.1 Downscaled superimposed

This technique was first suggested by Allen et. al. to show the effectiveness of superimposition setup [4]. In this technique, the downscaled version of image is used as both the subframes. Even this simple technique, eliminates screen door effect which is seen in normally projected images. The effect where thin black lines are seen on the projection screen due to gap between pixels that do not generate light is called screen-door effect. These gaps (also known as dead-zones) are created because the pixel fill-factor is never 100 % between adjacent pixels on the projector's physical image plane. [32] Thus, there is some benefit of superimpositioning using even this simplest technique.

#### 3.3.2 Gaussian downscaled technique

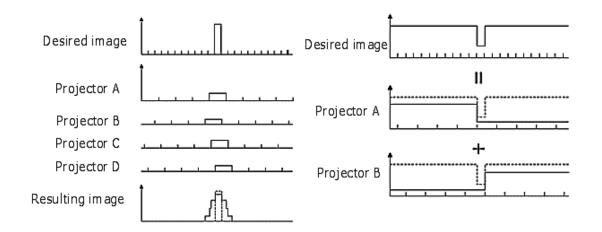
This technique is also a simple upgrade to the Naïve technique. The original image is first passed through gaussian filter and thereafter steps are similar to the Naïve technique. This was done after observing noise in the result of the Naïve technique that resulted in loss of details.

#### 3.3.3 Gaussian sharpened technique

In this technique, in addition to the Gaussian filter, the input image is passed through a sharpening filter as well. The sharpening filter was added because the Gaussian technique resulted in blurry result.

#### 3.4 Dark priority technique

This is one of the new methods that was developed as a part of this thesis. As described earlier in chapter 2, all the pixels in the high resolution image cannot be addressed perfectly in superimposition setup. Hence it can be interesting to have a technique which is able to priorities which pixels to translate perfectly. All the method described above do not have capability to do so. Iterative technique tries to fit all the pixels perfectly while the rest of techniques do not give any importance to the content of the image. The algorithms described in this section were developed with the view to priorities dark pixels first.



#### 3.4.1 Why prioritising black pixel could give better results

# Figure 8: Figure showing projecting black pixel (right) and white pixel (left) using multiple projectors [23]

Okatani et al. [11] highlighted an interesting fact about superimposing techniques. It is possible to create a perfect black pixel of higher resolution using multiple lower resolution projectors while it is much difficult to make a white pixel. In figure 8 (a), white pixel is created using multiple low-resolution projectors, the result is not perfect while in figure 8(b) perfect black pixel is created using two projectors.

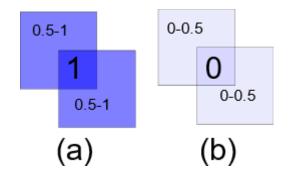


Figure 9: The cases for bright and dark pixel

Extending this concept to the opto-mechanical actuator setup, the additional factor to be considered is the brightness of each subframe. To create a fully white pixel, each relevant pixel in subframes must be projected to its maximum brightness which is half of the maximum possible (say 0.5 in a normalised case). This limitation is because of projection time of each subframe which is half of maximum possible for two subframe system. Now the neighbouring pixels can be defined accurately if they are brighter than half brightness value. That is if their value is between 0.5 to 1 and any details below this value will be ignored or projected wrongly creating an error of at the most 0.5 each pixel. This is indicated in figure 9(a). On the other hand, if we consider black pixel as shown in figure 9(b). Both the subframe would have to be minimum brightness possible (say 0). Now, if the neighbouring pixel can be filled to produce result between 0 to 0.5 (once again limited by maximum brightness of the subframe). The maximum error for this case is also 0.5. In both the cases, mathematically the error is the same however they are different visually. In the case of white pixel, the dark pixel could wrongly be displayed as a grey pixel and in the case of black pixel white pixel will be displayed as grey. It is difficult to say which one could get better results visually.

Hence, the aim of this approach of prioritising dark pixel over bright is to see if that gives better quality visually.

#### 3.4.2 Version 1

The image is first converted from RGB to YCbCr format and the operations are done only on the luminescence channel (Y). This helps to decrease the computation time as well as helps isolate brightness of image better. The values for the other two channels are picked while the pixels for the luminescence channel is picked. Image is first analysed and all the pixels are sorted from dark to bright along with the positions in the image. Then taking the darkest pixel, equivalent subframe pixels are calculated and then placed in the superimposed grid. Keeping half of the pixel value in each subframe. That is,

 $Subframe1 = Darkest \_ pixel \_ value / 2$  (3.6)

Subframe2 = Darkest \_ pixel \_ value / 2 (3.7)

Such that, *darkest \_ pixel \_ value = subframe1 + subframe2*.(3.8)

#### Algorithm:

- 1. Convert image into YCbCr format.
- 2. Check luminescence channel (Y) for unique values and sort them from dark to bright.
- 3. Initialize subframe1 and subframe 2 matrixes as a double data-type in MATLAB, mark all the pixels by keeping unrealistic value like 300 for an image data range of 0 to 255.
- 4. Go to the location of darkest pixel(s) find the equivalent subframe 1 and subframe
  2 and place half the value of that pixel in each such that.
  darkestpixelvalue = subframe1 + subframe2 (3.9)
- 5. Go to location of next darkest pixel, find the equivalent subframes, check if any of them are already filled. (if they have unrealistic value as initialized they are not filled). If both are unfilled place half the value of pixel in each. If one of them is filled, fill the other one with the difference.

For example, if subframe 1 is already filled, put the difference in subframe 2 such

that

Subframe2 = value of the pixel - Subframe1 (3.10)

If both the subframes are filled ignore this pixel.

- 6. After all the values of pixel are checked, convert the subframes back to RGB format.
- 7. Since the subframes are projected half the time their brightness is half of the maximum possible brightness of projector, hence the brightness of the subframes is doubled for all the channels.

#### 3.4.3 Version 2

This version of dark priority technique has been developed after looking at the results from the previous version. The noise observed in the previous version was as a result of the step where one of the subframes are already defined and the difference between the pixels were kept. Hence, in this version of the technique, this step is ignored and instead replaced by value of pixel/2. For example, if subframe 1 is already filled, Subframe 2

would be value of the pixel/2. This is done to get as close as possible to the real value of the pixel.

#### Revised algorithm:

- 1. Convert image into YCbCr format.
- 2. Check luminescence channel (Y) for unique values and sort them from dark to bright.
- 3. Initialize subframe1 and subframe 2 as a double in MATLAB and mark all the pixels by keeping unrealistic value like 300 for an image data range of 0 to 255.
- 4. Go to the location of darkest pixel(s) find the equivalent subframe 1 and subframe
  2 and place half the value of that pixel in each such that.
  darkestpixelvalue = subframe1 + subframe2 (3.11)
- 5. Go to location of next darkest pixel, find the equivalent subframes, and check if any of them are already filled. (If they have unrealistic value as initialized they are not filled). If both are unfilled place half the value of pixel in each. If one of them is filled, fill the other one with difference.

For example, if subframe 1 is already filled, put half of the value of the pixel in

subframe 2 such that

Subframe 2 = value of the pixel / 2 (3.12)

If both the subframes are filled ignore this pixel.

- 6. After all the values of pixel are checked, convert the subframes to RGB.
- 7. Since the subframes are projected half the time their brightness is half of the maximum possible brightness of projector, hence the brightness of the subframes is doubled for all the channels.

#### 3.5 Dark and bright priority combination technique

This is another novel technique developed in this thesis after observing the simulation results from dark priority techniques. In this technique, first subframes are generated using dark priority algorithm and then prioritizing bright pixels. Then the mean value of the subframes are taken as subframes for superimposition. This was developed after results from dark priority technique showed the result resembled that of dark pixel filter. If the same is done for bright pixel, then combining both could potentially combine the details from both.

#### Algorithm:

- 1. Convert image into YCbCr format.
- 2. Check luminescence channel (Y) for unique values and sort them from dark to bright.

- 3. Initialize subframe1 and subframe 2 as a double in MATLAB and mark all the pixels by keeping unrealistic value like 300 for an image data range of 0 to 255.
- 4. Go to the location of darkest pixel(s) find the equivalent subframe 1 and subframe
  2 and place half the value of that pixel in each such that.
  darkest \_ pixel \_ value = subframe \_ dark \_ 1 + subframe \_ dark \_ 2 (3.13)
- 5. Go to location of next darkest pixel, find the equivalent subframes, check if any of them are already filled. (if they have unrealistic value as initialized they are not filled). If both are unfilled place half the value of pixel in each. If one of them is filled, fill the other one with difference.

For example, if subframe 1 is already filled, put half of the value of the pixel in subframe 2 such that

 $Subframe \_ dark2 = value \_ of \_ the \_ pixel / 2 (3.14)$ 

If both the subframes are filled ignore this pixel.

- 6. Go to the location of brightest pixel(s) find the equivalent subframe 1 and subframe 2 and place half the value of that pixel in each such that. brightest \_ pixel \_ value = subframe \_ bright \_1 + subframe \_ bright \_ 2 (3.15)
- 7. Go to location of next darkest pixel, find the equivalent subframes, check if any of them are already filled. (if they have unrealistic value as initialised they are not filled). If both are unfilled place half the value of pixel in each. If one of them is filled, fill the other one with difference.

For example, if subframe 1 is already filled, put half of the value of the pixel in subframe 2 such that

 $Subframe\_bright2 = value\_of\_the\_pixel/2$  (3.16)

If both the subframes are filled ignore this pixel.

- 8. Take mean of Subframe\_bright and Subframe\_dark and assign to Subframes. Subframe1 = (Subframe\_dark\_1+Subframe\_bright\_1)/2 (3.17) Subframe2 = (Subframe\_dark\_2+Subframe\_bright\_2)/2 (3.18)
- 9. After all the values of pixel are checked Convert the subframes to RGB.
- 10. Since the subframes are projected half the time their brightness is half of the maximum possible brightness of projector, hence the brightness of the subframes is doubled for all the channels.

# 4 Simulation results

Simulations were done using MATLAB with three test images. The subframes were generated. The subframes were scaled to mimic the projection by repeating each pixel four times in a 2x2 block. Such that each block can be considered as a pixel when projected. One of the subframe was then shifted and superimposed on the other. The resulting images were inspected both visually as well as measured using metrics MSSSIM. Remarks based on visual inspection are presented here while MSSSIM values are discussed in later section. All the visual and MSSSIM results can be found in Annex of this thesis.

#### Test images:

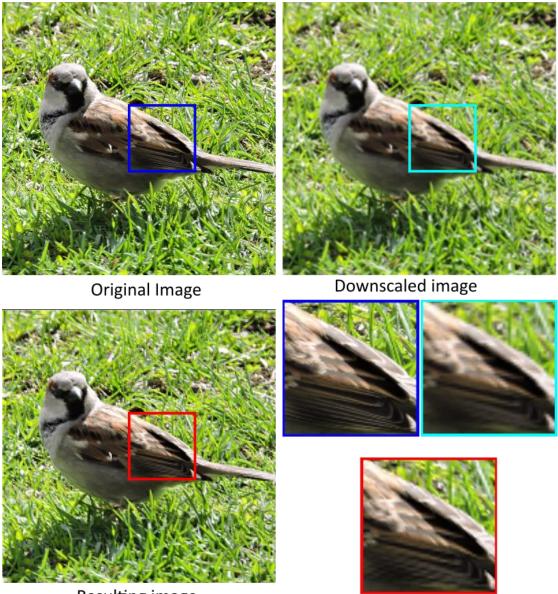


Figure 10: Three test images used for simulation

#### 4.1 Iterative methods

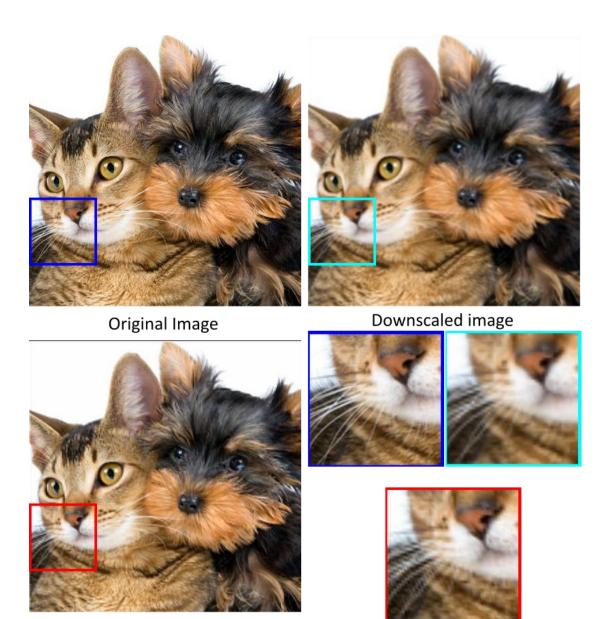
#### 4.1.1 Single subframe approach

One subframe iterative technique was simulated in MATLAB using Gaussian Belief Propagation technique for maximum of 20 iterations. After every iteration, error was calculated and then, error between each iteration was compared. If the change in error was constant or the change was less than 0.01, the process was stopped and the solver was said to have converged. The technique was applied to each channel one by one.



Resulting image

Figure 11: Simulation results for test image 1 for single frame iterative technique



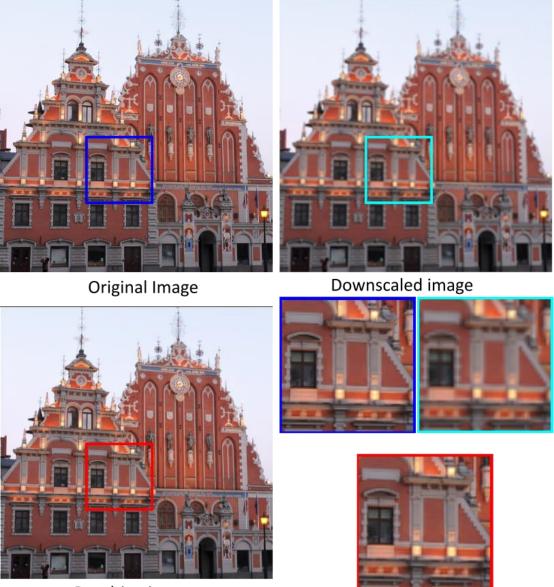
**Resulting image** 

#### Figure 12: Simulation results for test image 3 for single subframe iterative technique

As shown in the magnified part of the figure (11) and (12), the superimposed images are sharper than the downscaled image but is not sharper in comparison to the original image. It has details missing in the downscaled image but not well presented. Such as in figure (11), the feathers in the lower left part is completely missing in downscaled version, it is present in the superimposed image but not accurately. This inaccuracy can be better observed in figure (12) where the whiskers in the magnified part is present but with by zig zag shaped inaccuracy.

#### 4.1.2 Two subframe iterative method

The two subframe iterative method was simulated in MATLAB using Gaussian Belief Propagation algorithm taking 20 iterations as maximum and difference of 0.01 as criteria of convergence. It was applied 1 channel at a time.



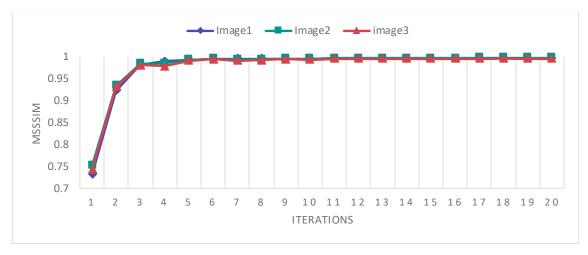
**Resulting image** 

#### Figure 13: Simulation results for test image 2 for two subframe iterative technique

The superimposed images observed were close to the original image apart from the noise which is shown in the magnified figure (13). There are small tile like rectangular artifacts which are the only difference between the original and the superimposed image. The performance of the technique is similar in all the test images.

The quality of iterative techniques is also dependent on the iteration and convergence value selected. Hence, another experiment was conducted where the quality of image was measured using MSSSIM after every iteration. The aim of this experiment was to see if it is possible to achieve good result with less computation and to determine if it can be practical to use this technique.

The experiment was conducted on all three images and iteration was taken from 1 to 20. Results are based on 1 to 20 iterations.



#### Figure 14: Chart showing the result of iteration test

It seems from the graph that fifth iteration is enough to obtain satisfactory result out of this technique. Still the time taken for it to do five iterations is significant. The result after 5<sup>th</sup> iteration and 20<sup>th</sup> iteration for test image 2 is shown below in figure (15).

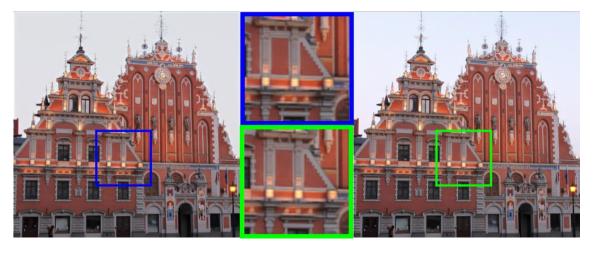
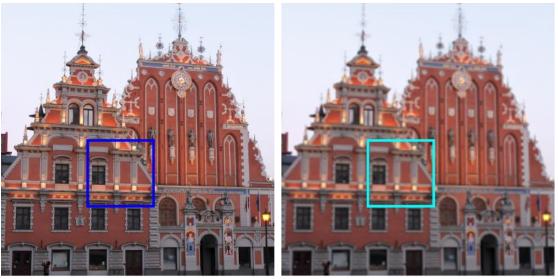


Figure 15: Results from 5 iteration (left) and 20 iterations (right) of test image 2

There does not seem to be significant difference after the fifth iteration.

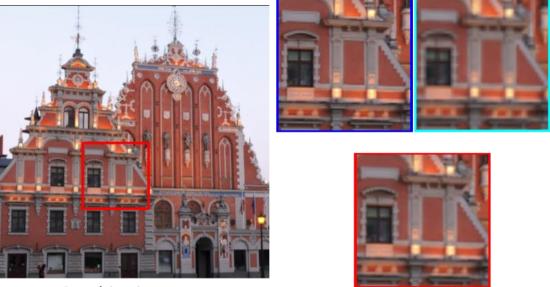
# 4.2 Pick and place techniques

#### 4.2.1 Naïve technique



Original Image

Downscaled image

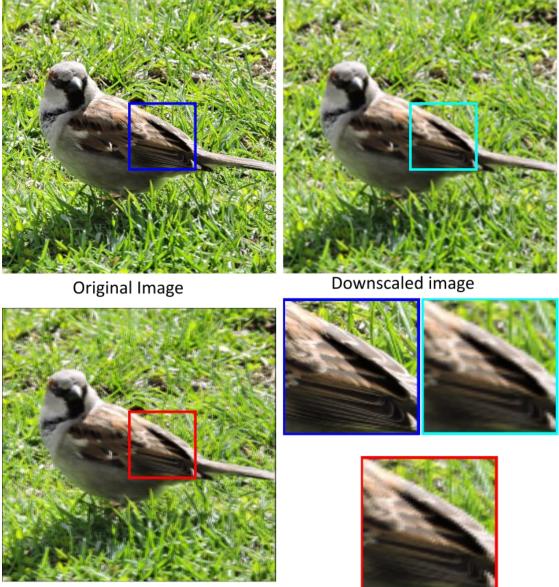


**Resulting image** 

Figure 16: Simulation results for test image 2 for Naive technique

The resultant image has details which are missing in downscaled image and it is much sharper. The missing details could be identified in test image 2 shown in figure (16). The magnified part shows the edge of the roof is detailed in much better way in the resultant image than in the downscaled version. The resultant images are blurrier than the original in all the test images.

#### 4.2.2 Picking mean technique

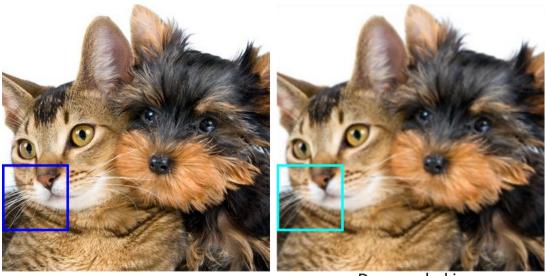


Resulting image

Figure 17: Simulation results for test image 3 for Picking mean technique

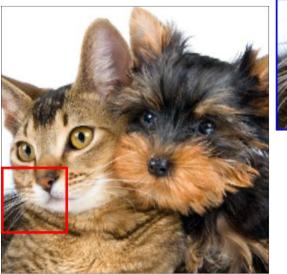
The resulting images seem blurrier than the downscaled version. The magnified part of test image 1 shown in figure (17) shows there are little details which are missing in the downscaled version like the feathers in the left bottom part of magnified image seems to have captured some details missing in downscaled version but it seems much blurrier in other areas of the same image.

4.2.3 Picking minimum and maximum technique



Original Image

Downscaled image



Resulting image

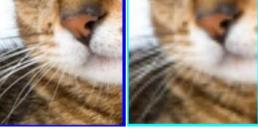




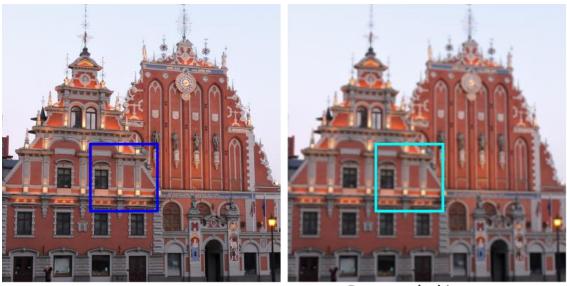
Figure 18: Simulation results for test image 3 for Picking minimum and maximum

The resulting image is sharper than the downscaled image and but there are artefacts as shown in the magnified part of figure (18). The whisker of the cat is sharper but its shape is distorted when compared to the original image.

# 4.3 Other techniques

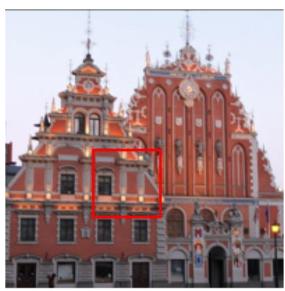
### 4.3.1 Downscaled superimposed technique

This technique does not show improvement in simulation but in real projection setup screen door effect is eliminated which will be shown in later section. As shown in the figure (19), the quality is similar to that of normal downscaled image.



**Original Image** 

Downscaled image



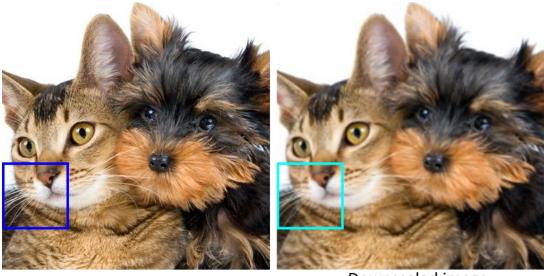
**Resulting image** 





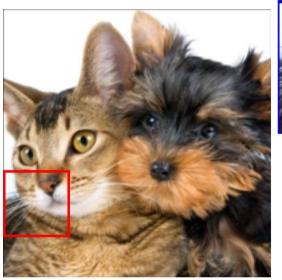
Figure 19: Simulation results for test image 2 for downscaled superimposed technique

### 4.3.2 Gaussian naive technique



**Original Image** 

Downscaled image



Resulting image

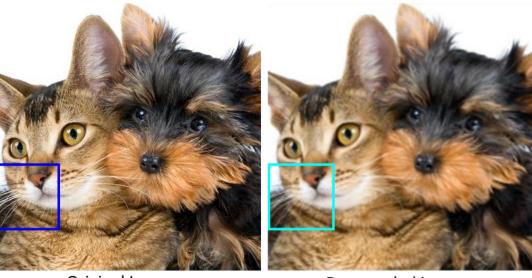




### Figure 20: Simulation results for test image 3 for Gaussian naïve technique

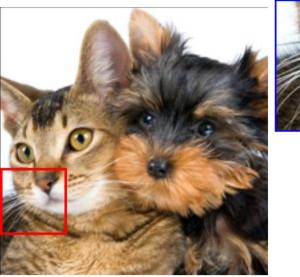
The resulting image has similar amount of blurriness but the details are represented much better. As in the case of magnified part of figure (20) the whiskers of the cat in the resulting image does not have distortions as in the downscaled version.

### 4.3.3 Gaussian sharpened technique



Original Image

Downscaled image



**Resulting** image



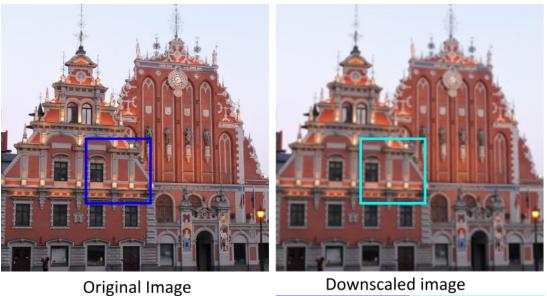


Figure 21: Simulation results for test image 3 for Gaussian sharpened naive technique

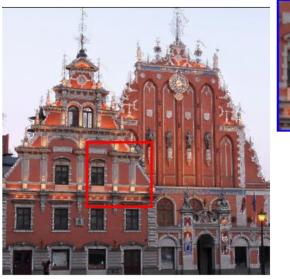
The resulting image shown in figure (21) is similar to the Gaussian naive version but it is slightly sharper. The artifacts in the downscaled version is absent.

# 4.4 Dark priority techniques

### 4.4.1 Version 1

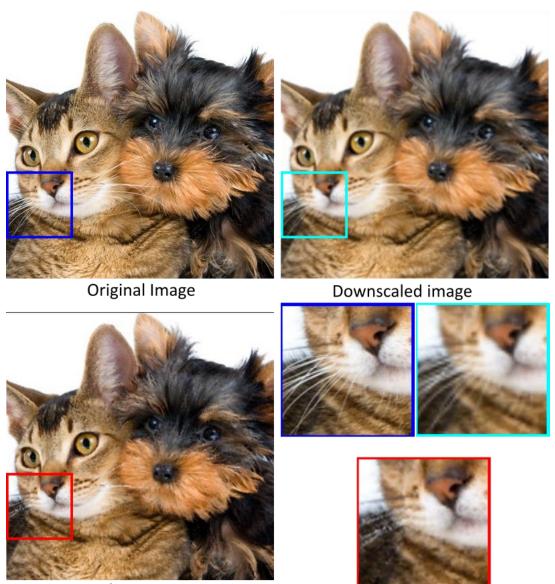


Original Image



Resulting image

Figure 22: Simulation results for test image 2 for dark priority technique (version 1)

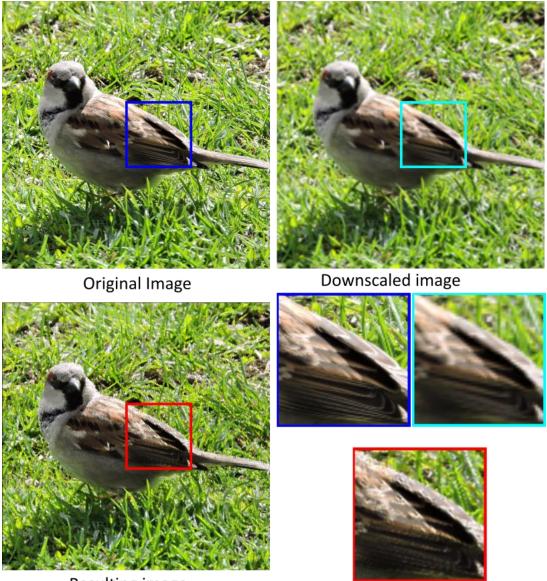


**Resulting image** 

#### Figure 23: Simulation results for test image 3 for dark priority technique version 1

The images are sharper than downscaled version but they also have noise. This can be seen in the magnified part of figure (22) where the all the white details seem to have noise. The resulting images have better representation of darker pixels and some brighter details are absent. This can be observed better in figure (23). The magnified part of the image shown does not have white whiskers but all the darker details around it are present.

The noise is noticeable more in the figure (23) but it is present in all the images if inspected closely. It is most noticeable in the case of test image 2 in the boundaries between the building and the horizon. More details about the origin of this noise is presented in discussion.

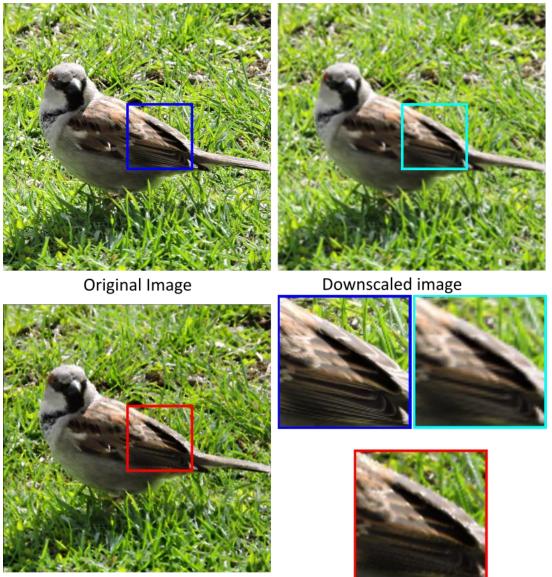


Resulting image

Figure 24: Simulation results for test image 1 for dark priority technique version 1

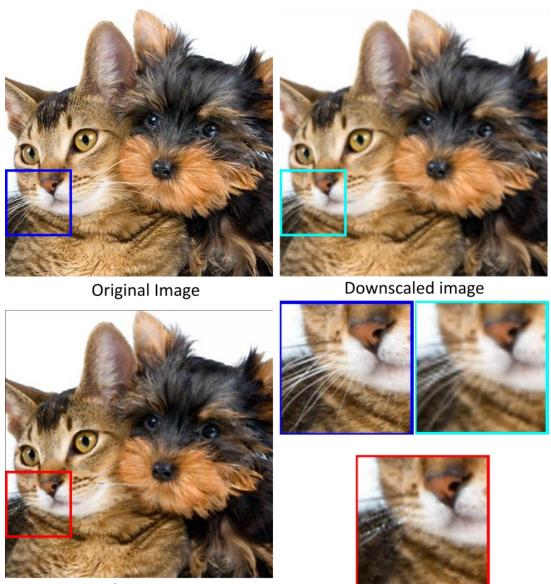
The technique performs better where there are less bright pixels in the original image. For instance, in the case of test image 1 shown in Figure (24) where the magnified area does not have bright pixels in the original image, the details are much better but it still has some noise.

# 4.4.2 Version 2



Resulting image

Figure 25: Simulation results for test image 3 for dark priority technique version 2

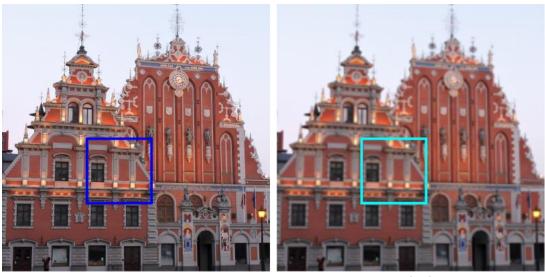


Resulting image

Figure 26: Simulation results for test image 3 for dark priority technique version 2

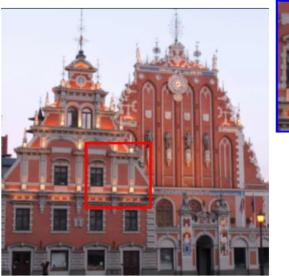
The noise which was present in the previous version is no longer there. The resultant images are much better in when they do not have or have less number of bright pixels in the original image. This can be seen in figure (25), where magnified part shows feathers of the bird with much better accuracy. The resultant image however misses brighter details which is prominently seen in test image 3 shown in figure (26) where the white whiskers of the cat is missing.

# 4.5 Dark and bright priority combination technique



Original Image

Downscaled image



Resulting image





Figure 27: Simulation results for test image 2 for dark and bright priority combination technique

The resulting images have details which are represented but better than downscaled version. The magnified part of the figure (27) shows details of the rood area is better presented in the resulting image than downscaled version. The images are however blurrier than the original image.

# 5 Measurements with Barco F70 4k/UHD projector

## 5.1 Equipment used

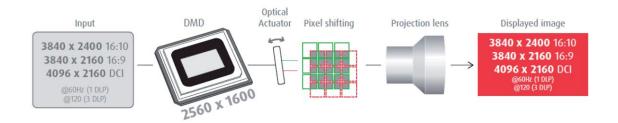


Figure 28: The working model of Barco F70 4k/UHD projector [31]

### 5.1.1 Barco F70 4k/UHD projector

The Barco F70 4k/UHD projector has an optical actuator in its hardware. The optical actuator could shift in up-right direction with frequency of 120Hz. It could also be used without optical shift in 60 Hz refresh rate. The default firmware had been designed to work for a video input which would have to be 120 frames/second. This was challenging task. Another challenge was to use a computer which can output the video at that rate. Another option was to use a different firmware which could enable two inputs through display port and HDMI port. Each port could be connected to a subframe and then the projector would then shift between the two inputs. The latter option was used for this project.

### 5.1.2 Phantom M310 high speed camera

This camera is required to view and capture individual subframes. This camera can capture images up to 3200 frames per second with 1280x800 resolution. It is possible to increase the frame rate by decreasing the resolution. The resolution and framerate are limited by throughput of 3.2 Gpx/s. [33]

### 5.1.3 Nikon D5100 camera

A DSLR camera was used to capture the superimposed image in higher resolution and in better quality than the high-speed camera. The resolution of Nikon D5100 is 4928 × 3264.

### 5.2 Preparation of subframes

The experiment was done with test image 2 and test image 3 for all the techniques discussed earlier. The original images were of size 400 x 400 but framed with black background and made of size 2560x1600 to fit the native resolution of the projector. There were two modes in which the projector could work. First one required a video input with subframe 1 and 2 alternating with a frame rate of 120 Hz. The projector would shift the input with the same rate hence there would be no need for synchronization. This also required a computer which could display that video in its frame rate. The second mode of operation required inputs from two different ports; the display port and HDMI port. One subframe would be displayed through each port.

First attempt was made using option of video input, input videos were generated using MATLAB with required framerate, but during implementation even with required hardware to display the video, the video failed to be displayed at the given framerate of 120 Hz. Then the decision was made to switch to the second mode of operation. For which firmware had to be updated in the projector.

### 5.3 Experimental setup

The experiment was conducted in laboratories in Barco Fredrikstad. The equipment were arranged as shown in the schematic in figure (29). The two cameras are placed in the either side in front of the screen such that it does not block the area where subframes are projected. Both the camera is connected to a laptop each. DSLR camera is controlled by software provided by the manufacturer called "Camera control pro 2". High speed camera was controlled by phantom CV 2.8 which is software provided by the manufacturer. All the settings regarding the camera like the required resolution and speed of the camera could be controlled using the camera. [34]

It is essential to synchronize the camera with the projector and ensure that the subframes are captured at the right interval. If not synchronized, the captured image can be transition between the two subframes which would have different brightness and color. This synchronization was enabled using a trigger input in the camera which was connected to the trigger output of the projector.

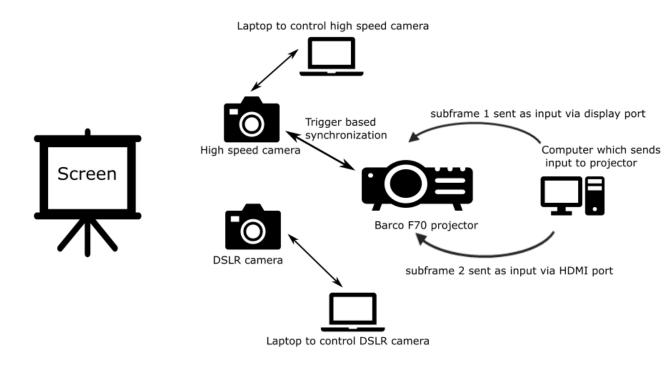


Figure 29: Schematic for experimental setup

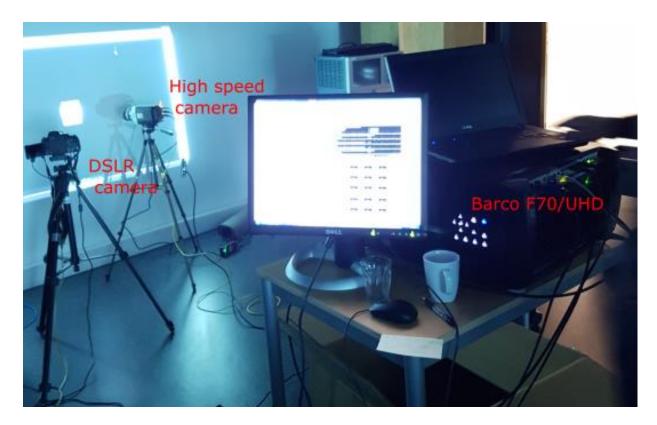


Figure 30: Arrangement of equipment

Figure (30) shows the actual experimental setup which shows DSLR camera, High speed camera and Barco F70/UHD projector.

# 6 Results from measurement

### 6.1.1 Comparing the subframes



Figure 31: Comparison between experimental results and simulation results

The figure (31) shows two subframes and result from simulation (top) and actual setup (bottom) for two frame iterative technique. They are visually similar apart from the noise and brightness level of the two images. The noise is because of the high-speed camera. The result was similar for all the other techniques, that the simulated and the captured images were closely resembling each other.

### 6.1.2 Screen door effect

Downscaled image was used to observe screen-door effect. It was first projected without shifting optical actuator and second it was shifted with the optical actuator. Both results were captured with the Nikon D5100 camera.

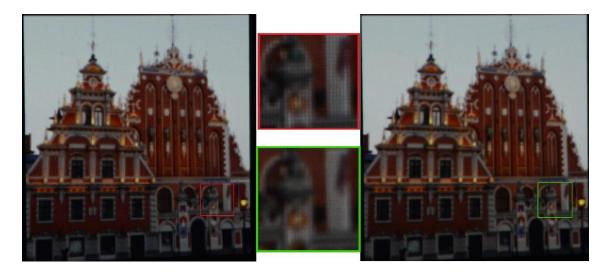


Figure 32: Screen door effect observed for test image 2, with and without superpositioning

The screen door effect can be seen in figure (32), where the left side shows the image without shift and superposition. The magnified part shows small black boxes which are there due to the screen-door effect. The right-side image is the same image when optical actuator is shifting the image. Here the screen-door effect is not present making picture visually smoother.

### 6.1.3 Results of dark priority based results



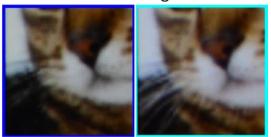
Result of version 2



Result of version 1



Result of dark and bright combined





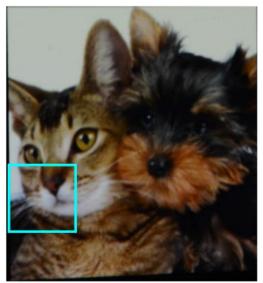
#### Figure 33: Results from measurement for dark priority based techniques

Dark priority version2 brings out dark parts of the image and is less noisy and smoother than the version with halo effects. This is similar to what was seen in the simulations. This can be seen in the three magnified parts shown in figure (33). The white whiskers of the cat can only be seen in the result from dark and bright priority combined technique.

### 6.1.4 Results of iterative techniques



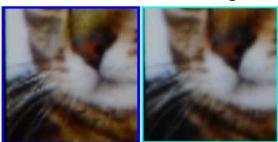
Result of single frame iterative



Result of downscaled image



Result of two frame iterative





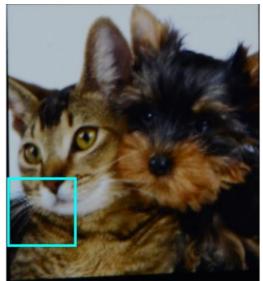
#### Figure 34: Results from measurement for iterative techniques

The difference is not remarkable between the two iterative techniques as shown in figure (34). Because of the noise, most of the small details are not visible. But the noise in the whisker of the cat present in simulation of one subframe technique is still present in the magnified part while the same part is smooth in the result of two subframe technique.

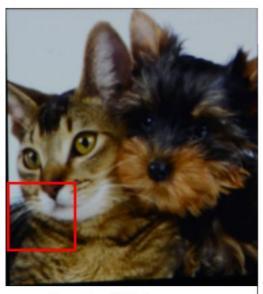
### 6.1.5 Results for pick and place techniques



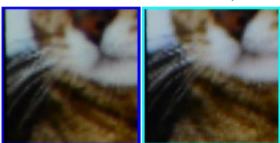
Result of Pick min. and max.



Result of Naive technique



**Result of Pick mean** 





### Figure 35: Result for techniques based on pick and place techniques

Result of the Pick mean technique is blurrier than the result of Picking minimum and maximum as well as the Naïve technique. There are small distortions like the one seen in simulation but it is less than the one seen in the Naïve technique as shown in figure (35).

6.1.6 Results for techniques based on downscaled image



Result of Gaussian naive

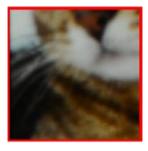


**Result of Naive** 



Result of Gaussian sharpened

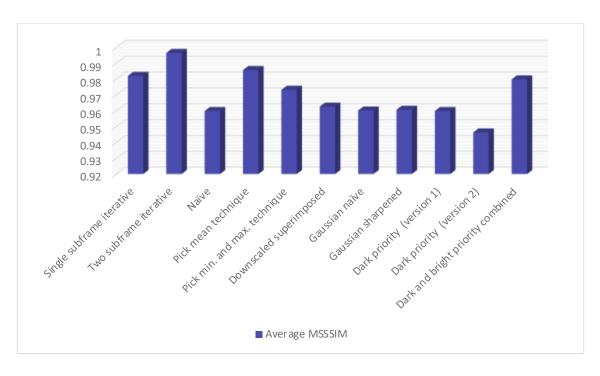




### Figure 36: Result for Gaussian naive, Naive, and Gaussian sharpened technique

The images for these techniques are close to each other but due to camera noise, it is hard to identify differences between them as shown in figure (36).

# 7 Discussion



### 7.1 Comparing different techniques based on simulation

# Figure 37: Chart showing the average MSSSIM values of three test images from simulation of different techniques

According to the MSSSIM values, the best technique is two subframe iterative technique. This is an expected result and it completely in line with the visual inspection. It is followed by Pick mean technique which is not in agreement with the visual assessment. It produced blurry details. This result highlights the weakness of the MSSSIM metrics, even though MSSSIM does much more than compare the error between the two images, it fails to recognize the blurriness observed visually. Next best technique is Single subframe iterative followed closely by Dark and bright priority combination. Among the Pick and place techniques even though Naïve technique scored average in MSSSIM, it was sharper visually and produced most of the details even if with distortions.

### 7.1.1 Iterative methods

Iterative techniques produce the best results among the techniques implemented in this work. The major downside of this technique is the computation time and complexity. In comparison to other techniques the computation time for this technique is much higher.

The experiment to see if less iteration could give good result showed that fifth iteration would give good enough improvement but this still takes considerably longer time that other techniques. While highest time taken to produce subframes among all the other techniques was around 20 seconds for dark and bright priority combined technique, the single frame iterative technique for the same image was around 600 seconds with the same hardware. Although, Sajadi et. al. do claim to produce results using the same technique within few seconds, this is using special hardware. [17]

#### 7.1.2 Pick and place techniques

The Pick and place techniques can produce good results with less computation time but they do not consider the dependency among the pixels in the resulting image created by superimposition setup. This produces noise and is often impossible to predict in which part of the image they are more prominent.

Picking actual values from the input image like in the case of Naïve approach and Picking minimum and maximum produce better result visually than Picking mean value although the MSSSIM value is higher for the later. This shows in image data it is important to get the exact values for the pixels from input image to achieve better visual experience and the metrics misses this in the assessment. When compared with downscaled image which would be what a native projector would project, these techniques still produce better results.

#### 7.1.3 Other techniques

The downscaled superimposed image was close to the standard downscaled image visually. They do not appear to be significantly better than the downscaled version in simulations but this will be different in actual setup because of screen-door effect. This is discussed further in later sections. The use of Gaussian filter further decreases the noise in the Naïve result but introduces blurriness, thus the increase in average MSSSIM value to 0.96 from 0.95. Gaussian sharpened technique makes the image slightly sharper in the visual inspection but does not improve the MSSSIM value.

### 7.1.4 Dark priority techniques

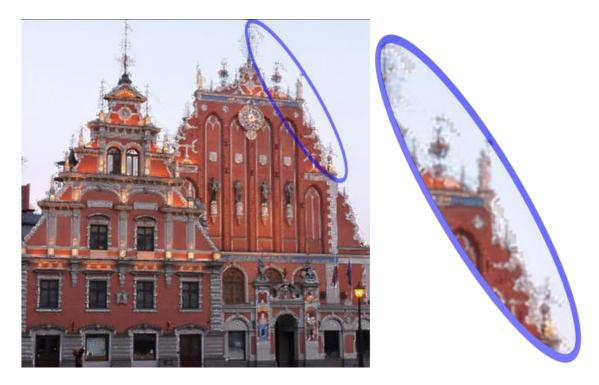


Figure 38: Halo effects seen in simulation result of test image 2 for dark priority technique version 1

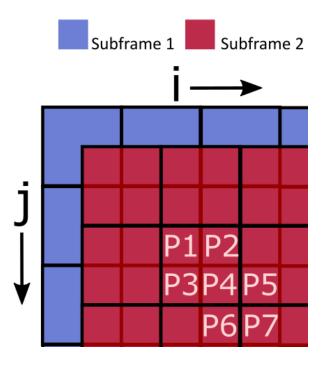


Figure 39: Pixel grid of two subframe

The most prominent noise observed in simulations of Dark priority Version 1 technique were the halo like noise shown in figure (38). They are there because of part of algorithm

which deals with cases when one of the two relevant subframes are already filled with a darker pixel. Since only one of the subframe is not available to define the intended pixel, the difference between the actual pixel in input image and the pixel value in the filled pixel of the subframe is chosen for the subframe. If P4, shown in figure (39), be the pixel in the original image that is to be filled and subframe1 and subframe2 are its equivalent pixels in the superimposed image and subframe2 is already filled then,

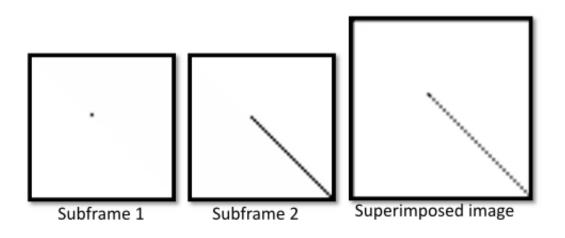
#### subframe1 = P4 - subframe2 (7.1)

This is done to maximize the chances of transferring exact values from the original image to the superimposed one.

If the next pixel in the superimposed image to be placed is P5, P6, or P7. They will have only one subframe to be defined as well. This will mean it will again go back to the same part,

$$subframe2 = P5 - subframe1(7.2)$$

$$subframe2 = P5 - P4(7.3)$$



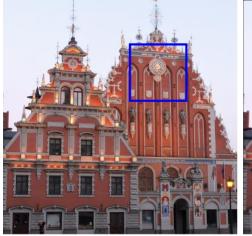
#### Figure 40: The subframes and the result of dark priority technique for a black dot input

P5-P4 is large value (more than 0.5 for normalized case) there will alternate shift between the two values. This is shown in the figure (40). The difference between white background and black dot is more than 0.5 which is why there is a halo effect.

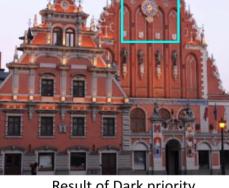
Looking at the results from both versions of Dark priority algorithms, although the one with halo effect appears sharper, it does not necessarily have any details which is missing in the version without it.

Moreover, it adds artefacts which are not present in the original image. This makes the version less effective than the one without halo effects.

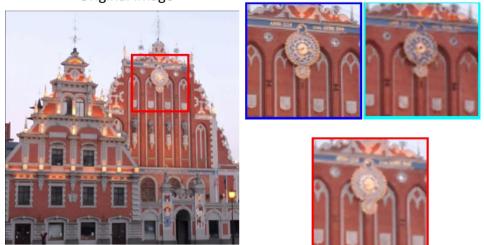
Dark priority technique results can be compared to that of a filter. It passes darker details while brighter pixels are missed. The principle can be extended to other properties and can be used to priorities other properties in the image as well.



**Original Image** 



Result of Dark priority



Result of bright priority

Figure 41: Result of prioritising dark pixel and bright pixels

The idea of combining the results from prioritizing dark pixels with bright pixels came after observing the results from both. (shown for test image 2 in figure (41)) They both act as filters and one has details missing in the other. But as observed in the results combining the result as a mean value adds blurriness to the image even though it does preserve all the details.

### 7.2 Discussion from the measurements

The results produced during the simulations are similar to the one produced in the measurements in the actual setup apart from the noise due to camera.

### 7.2.1 Iterative methods

Iterative techniques did not stand out as much as it did in the simulations, due to noise which took away possibilities to see small details which iterative techniques are excellent at preserving. Even so, the results were devoid of blurriness and artifacts seen in other techniques made this one best among the techniques.

### 7.2.2 Pick and place techniques

The results were similar to simulations, the blurriness of Pick mean method was prominent when compared to Naïve and Pick minimum and maximum technique.

### 7.2.3 Other techniques

The screen door effect which was present in native projector was absent in the downscaled version, this shows the advantage of using opto-mechanical actuator over native projectors. It was difficult to find clear advantage of using Gaussian filter and sharpening filter due to the difference being marginal and because of noise.

### 7.2.4 Priority based techniques

The halo effects seen prominently in simulations were not so prominent in the actual setup. When comparing the results from both the versions, even though the noise which was the issue behind the second version was not prominent but still there were no details present in version 1 which was absent from version 2. This means that there was no clear advantage of using the version 1. Moreover, the artifacts seen in the simulations may have become less prominent in the actual measurement because of noise and may resurface in other cases.

The Dark and bright combination technique gave similar result as in the simulations.

# 8 Conclusion

#### 8.1 My contribution

After a literature review of all the techniques for enhancing resolution of an image without increasing the capacity of SLM, a comparative study of eight available techniques for subframe generation which could be adapted to be used in a opto-mechanical actuator based systems were examined by simulations and experiments. The simulations were done on three test images using MATLAB. The results have been compared visually and with MSSSIM. I found that iterative techniques produce the best results while other technique also improve the quality of the images when compared with the downscaled version. Therefore, using opto-mechanical actuator with proper subframe generation technique can be a low-cost technique to enhance the resolution of the image. In addition, it could also fill the gap left by unavailability of SLMs of required format and size. Among the techniques, iterative techniques show the potential of superimposition setup and none of the techniques discussed in this thesis come close to utilizing the potential of superimposition setup.

I also developed two novel techniques for subframe generation to explore the possibilities of prioritizing a characteristic in an image such as brighter details or darker details for additive superimposition setup. The first technique prioritizes darker pixels in an image. The result from this technique indicated that darker details were clear in the superimposed image while brighter details were ignored in some cases which was comparable to the result from a dark filter. In the second technique, bright pixels were prioritized separately and then combined with subframes obtained from first technique using mean values of the subframes. The resulting superimposed image had both the details but slight blurriness was observed due to the averaging operation.

Preparation were made for two modes of operation for Barco F70 4k/UHD projector. Video was generated with proper frame rate in MATLAB for first mode as well as still images of individual subframes were generated for second mode. Then all the techniques were tested experimentally using Barco F70 4k/UHD projector, a high speed camera and a high resolution DSLR camera setup using second mode of operation where still images were given as input using HDMI and Display port.

## 8.2 Future work

In a superimposition setup, it is expected that resulting image will have losses in detail in one form or another. In previously reported techniques, it is not possible to be selective on which property to preserve and which could be ignored. Dark priority is the first technique which is able to priorities and save details of choice. Even though only one property (brightness in the detail) has been shown in this work it is possible to extend the techniques for other properties such as certain frequency in the image or even certain part of the image depending on function.

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# 9 Annexes

# 9.1 Annex 1: Results from simulation

### 9.1.1 MSSSIM results

### Table 3: MSSSIM values for 3 test images

Techniques	Test image 1	Test image 2	Test image 3
Single subframe iterative	0.9802	0.9834	0.9825
Two subframe iterative	0.9963	0.9974	0.9959
Naïve	0.9515	0.9648	0.9630
Pick Mean	0.9832	0.9874	0.9866
Pick Minimum and Maximum	0.9667	0.9748	0.9782
Downscaled image (superimposed)	0.9547	0.9659	0.9671
Gaussian Naïve	0.9519	0.9639	0.9644
Gaussian Sharpened naive	0.9532	0.9641	0.9645
Dark priority version 1	0.9487	0.9646	0.9662
Dark priority version 2	0.9259	0.9502	0.9627
Dark and bright combination	0.9737	0.9847	0.9809

# 9.1.2 Test Image 1



Figure 42: Single subframe iterative



Figure 43: Two subframe iterative



Figure 44: Naive



Figure 45: Pick mean



Figure 46: Pick minimum maximum



Figure 47: Downscaled (superimposed)



Figure 48: Gaussian Naive



Figure 49: Gaussian Sharpened Naive



Figure 50: Dark priority version 1



Figure 51: Dark priority version 2

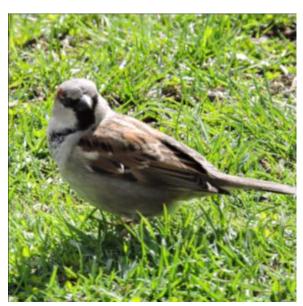


Figure 52: Dark and bright combination

# 9.1.3 Test Image 2

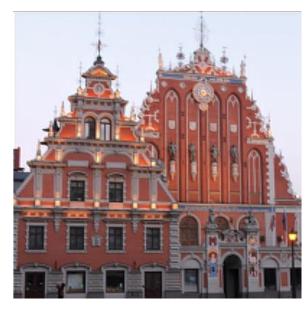


Figure 53: Single subframe iterative



Figure 54: Two subframe iterative



Figure 55: Naive



Figure 56: Pick mean



Figure 57: Pick minimum maximum



Figure 58: Downscaled (superimposed)



Figure 59: Gaussian Naive



Figure 60: Gaussian Sharpened Naive



Figure 61: Dark priority version 1



Figure 62: Dark priority version 2



Figure 63: Dark and bright combination

## 9.1.4 Test image 3



Figure 64: Single subframe iterative



Figure 65: Two subframe iterative



Figure 66: Naive



Figure 67: Pick mean



Figure 68: Pick minimum maximum



Figure 69: Downscaled (superimposed)



Figure 70: Gaussian Naive



Figure 71: Gaussian Sharpened Naive



Figure 72: Dark priority version 1



Figure 73: Dark priority version 2



Figure 74: Dark and bright combination

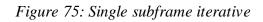
### 9.2 Annex 2: Results from measurement



9.2.1 Results from high speed camera for test image 2

Subframe 1

Result





Subframe 1

Subframe 2

Result

Figure 76: Two subframe iterative



Subframe 1

Subframe 2

Figure 77: Naïve



Subframe 1



Result

Figure 78: Pick mean

Subframe 2



Subframe 1

Subframe 2

Figure 79: Pick minimum maximum



Subframe 1

Subframe 2

Figure 80: Gaussian naive

Result



Subframe 1





Subframe 2

Result





Subframe 1

Subframe 2

Figure 82: Downscaled superimposed



Subframe 1

Subframe 2

Result

Figure 83: Dark priority version 1



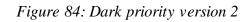
Subframe 1



Subframe 2



Result





Subframe 1

Subframe 2

Figure 85: Dark and bright priority combination

# 9.2.2 Results from high speed camera for test image 3



Subframe 1



#### Figure 86: Single subframe iterative



Subframe 1

Subframe 2

Figure 87: Two subframe iterative



Subframe 1





Figure 88: Naïve



Result



Subframe 1



Subframe 2

Figure 89: Pick mean





Result



Subframe 1



Subframe 2



Result

Figure 90: Pick minimum maximum



Subframe 1





Result

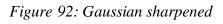
Figure 91: Gaussian naive



Subframe 1



Subframe 2



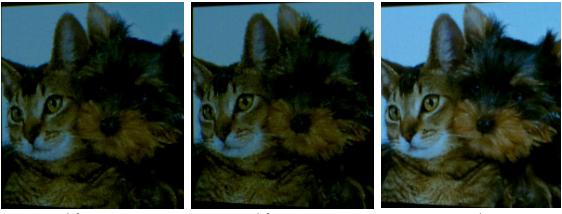


Subframe 1

Subframe 2



Figure 93: Downscaled superimposed



Subframe 1

Subframe 2

Result

Figure 94: Dark priority version 1



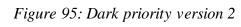
Subframe 1



Subframe 2



Result





Subframe 1

Subframe 2

Figure 96: Dark and bright priority combination

#### 9.2.3 Results from DSLR camera for Test image 2



Figure 97: Two subframe iterative



Figure 98: Single frame iterative

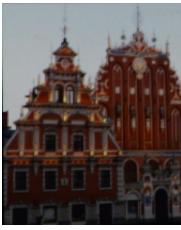


Figure 99: Naive



Figure 100: pick mean





Figure 102: Gaussian



Figure 103: Gaussian sharpened

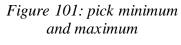




Figure 104: Dark priority version 1



Figure 105: Dark priority version 2

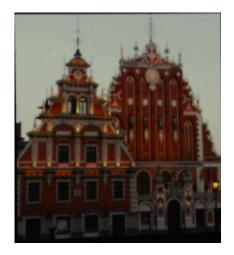


Figure 106: Dark and bright priority combination



Figure 107: Downscaled superimposed

#### 9.2.4 Results from DSLR camera for Test image 3



Figure 108: Two subframe iterative



oframe Figure 109: Single frame iterative



Figure 110: Naive



Figure 111: pick mean



Figure 112: pick minimum and maximum



Figure 113: Gaussian



Figure 114: Gaussian sharpened



Figure 115: Dark priority version 1



Figure 116: Dark priority version 2



Figure 117: Dark and bright priority combination



Figure 118: Downscaled superimposed