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Abstract:

Modelica is rapidly developing programing language that allows building models of dynamic systems and analysing their operations. One of such systems is power plant, that requires constant regulation, optimization and has continuously changing parameters, that makes Modelica application practically oriented. For this one of the Modelica free programing environments, OpenModelica and JModelica, that allows studying of the dynamic systems with support via Pyhton or MATLAB, will be used.

But previously basic units of modeled hydro power plant system need to be revised, their possible working regimes, studied and possibility of their realization with a help of Modelica units checked. It is also of interest to implement possible switching options between systems' parts and coordination between them, for which also different level of specification can be implemented. Both steady and dynamics states are revised from modeling and simulation points of view. Possible challenges while Modelica usage are discussed. The values that are used for model library testing are taken from Sundsbarm power plant.

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Preface

This master thesis is submitted as a part of the masters' degree education within Master Program within Energy and Environmental Technology, Telemark University Collage.

The following report is an attempt to build a model library for hydro power plant using OpenModelica language, that fully covers all the task requirements. Moreover the task focus is also to implement the developed library for special case, Sundsbarm power plant.

Due to the lack of time, problems with software and lack of complete data it was not possible to implement model library structure fully showing its extension capabilities (for example, an option of changing types and number of turbines or generators, load, switching between the blocks in case of error situations etc) . Moreover due to the same reasons the advanced optimization/control part of thesis' task was not covered.

Despite of all difficulties on thesis writing way, it would not be possible to implement it for me without great guiding, support and patience of my supervisor Professor Bernt Lie, to whom my faithful gratitude goes.

Special thanks to the thesis co-supervisor, Dietmar Winkler, who provided an invaluable help at final stage of thesis writing and its realization in OpenModelica.

Furthermore, my thanks are extended towards all master students who assisted me in my thesis preparation and to all former master students, whose theses have become a great theoretical background for my thesis.

Thanks, Tussen takk, дякую!

Porsgrunn, June 16th, 2013

Iuliia Vinnik

Nomenclature

A_c - constant cross section area of the conduit, m^2

A_s - area of surge tank, m^2

D - machine diameter, m

F_{fc} - friction force, N

F_{fp} - friction force for penstock

f_c - fanning friction factor for conduit

f_s - fanning friction factor for surge volume, m^3

$\mathrm{f_s}$ - system frequency, Hz

f_{ref} -desired frequency of network, Hz

J_a - moment of inertia for aggregate, kg m^2

h_s - height of surge tank, m

$\mathrm{H_t}$ - turbine head, m

H_p - height of penstock, m

$\boldsymbol{H}$ - resultant magnetomotive force vector at the rotor axis measured in the stator reference frame, A.turns

$H_a(H_b, H_c)$ – resultant magnetomotive force of the current flowing in the “a”(“b”, “c”) winding measured in the stator reference frame, A.turns

$\widetilde{H_d}(\widetilde{H_q})$ – magnitude of projection of H onto the “d”(“q”) axis, m^2

I_{to} - RMS value of terminal phase current at steady state, A

$\underline{I_{to}}$ - phasor of output current of generator at steady state, A

k_{fa}- friction factor for aggregate bearings

$\dot{K}_p$ - convective kinetic power upstream from the gate,W

L_c - length of conduit, m

L_p - length of penstock, m

$\dot{m}_c$ - mass flow in conduit, m/s

$\dot{m}_p$ - mass of water in penstock, kg/s

$\dot{N}$ - rotational velocity of the machine, m/s

n_p - number of poles in generator

p_a - atmospheric pressure, Pa

p_{cx} - intersection pressure between conduit, surge tank and penstock, Pa

p_{sx} - outflow pressure in surge tan, Pa

p_{px} - outflow pressure in penstock, Pa

$\mathrm{P_{in}}$ – hydraulic power transferred to turbine, W

$\mathrm{P_{out}}$ – active electric power output at terminals of generator, W

$\mathrm{P_{loss}}$ – power losses through turbine and generator, W

P_o - an active power at steady state condition, W

$\mathrm{Q_t}$ – turbine volumetric discharge, $\mathrm{m^3/sec}$

Q_o - a reactive power at steady state condition, rVA

S -droop,%

S_B - rated power of generator, MVA

u_t - valve opening, p.u.

$\boldsymbol{u_d}(\boldsymbol{u_q})$ – unit vector along the “d”(“q”)

$\boldsymbol{u_a}(\boldsymbol{u_b}, \boldsymbol{u_c})$ – unit vector along the “a”(“b”, “c”) axis

$\dot{V}_c$ - volumetric flow rate in conduit, $\mathrm{m^3/s}$

v_c - constant velocity across the cross section, m/s

v_p - velocity in penstock, m/s

$\dot{V}$ - volumetric flow rate through the machine, $\mathrm{m^3/s}$

$\dot{V}_s$ - volumetric flow rate in surge volume, $\mathrm{m^3/s}$

v_s - velocity in surge tank, m/s

V_s - phase bus voltage, V

$\mathrm{V_{tr}}$ - terminal voltage, V

$\dot{W}$ - produced mechanical power,W

$\dot{W}_e$ - mechanical power consumed to produce electric power in generator,W

Greek symbols

Δp - pressure drop over the machine,Pa

Δp_v - pressure drop inside the penstock, Pa

Δp_t - turbine gate pressure drop, Pa

δ_{eo} −electrical rotor angle, rad

$\mathrm{\eta_g}$ – overall efficiency of generator

η_t - turbine efficiency at 85% flow of maximal water flow rate

ρ_c - density , $\mathrm{kg/m^3}$

ρ_s - density in surge volume, $\mathrm{m^3}$

P_s - perimeter of the surge tank cross section area, $\mathrm{m^2}$

P_c - perimeter of the conduit cross section area, m

P_p - perimeter of the penstock cross section area, m

ω_a - angular velocity of aggregate, rad/sec

φ_o - a power angle at steady state, rad

Ψ - flux produced by the current in the generator winding, $\mathrm{mol{\cdot}m^{-2}{\cdot}s^{-1}}$

1 Introduction

1.1 Background

With increasing demand of electrical energy and its growing trade volumes the problems of electricity efficient and optimized usage is of high importance in nowadays society. Especially this question is crucial in Norway, where 96 per cent of electricity production (compared to 11 per cent in European Union) is covered by hydropower(Gonzalez David; KilincAygün; Weidmann Nicole). That is why questions of operation optimization of different parts of hydro power plants are of great research interest.

That is why different usage of modeling and simulation tools, structures and object-oriented models is highly motivated. Because of complexity of hydro power systems the problem of complete model library development can be viewed from two perspectives:

- Modeling , which gives mathematical understanding of processes that are run in each part of the hydro power plant depending on type of equipment that is installed there, emergency situation, environment conditions;
- Simulation, which focuses on implementation of modeling part within specific programming language, taking into consideration language particularities and possible ways of model realization.

The thesis focuses on building of new library for hydro power plant using open sources/freeware. It can be used further for studying of possible automation and optimization at hydro power plants. For this other Modelica tools can be used: OpenModelica is used for connecting units. Moreover it is expected that the option of changing the level of details will be also available at newly built library. The developed library is going to be tested using Sundsbarm Hydro Power Plant data as a specific case.

1.2 Previous work

The programming tools with similar problem understanding have already been presented.

One of such tools that was developed for hydro power plant operation modeling is Hydro Power Library, a Modelica® Model Library, which is more into practical issues solving. Hydro Power Library allows to analyze control strategies that to decrease the influence of noncontrollable sources of energy (for example renewable ones).

According to the web-site of developers (Modelon) the range of possible simulation tools is suitable for:

- Hydro power plant design and analysis;
- Planning of commissioning tests;
- Estimation of waterway dynamics;
- Identification of objectives for the water level control;
- Analysis of extreme working conditions of the plant.

Hydro Power Library was developed and works together Dymola, Dynamic Modeling Laboratory, which “a complete tool for modeling and simulation of integrated and complex systems for use within automotive, aerospace, robotics, process and other applications”(Dymola, 2002-2013). Dymola by itself also uses Modelica®, open modeling language, suitable for describing the problems related to power production, in which there is a possibility to develop a new library for specific case using Modelica Standard Library.

The big drawback of Hydro Power Library and Dymola is that both of them are commercial and, hence, are of limited access for educational purposes (that also means impossibility to have a look “inside” some parts of the library and see how specific commands, loops are written there). Also the case that Hydro Power Library works only with Dymola, but not with other Modelica free programming environments such as OpenModelica, which is not-commercial Modelica-based modeling and simulation source, or JModelica.org, which is also not-commercial Modelica-based source for optimization, simulation, analysis of complex dynamics systems(JModelica.org, 2009). However those softwares have some practical issues while simulation (for example difficulties with Phyton support).

However described above library is not the only one tool that is used for modeling and simulation of hydro power plants. Swedish company Solvina has also introduced their simulator like SolvSim Power Plant(Solvina) at the market, which has more user friendly interface and similar range of problems to be solved, but the software is more suitable for training either than to practical implementation of the received results.

Taking into the account everything mentioned above, it would be of great benefit to omit mentioned drawbacks in newly developing model library, at the same time keeping in mind purposes of Hydro Power Library and SolvSim.

1.3 Report structure

Chapter 2 shows the example of already developed code for study case, Sundsbarm plant, model inputs and initial conditions are stated.

Chapter 3 gives detailed model of each block of the system: penstock, turbine, head-water, tail-water system, turbine controller and electrical part (synchronous generator).

In *Chapter 4* model library structure is given. Simulation particularities of developed in Chapter 3 model in OpenModelica environment are presented. The results of newly built library running for Sundsbarm case are also presented within this chapter.

Chapter 5 is focused on discussion of problems that appeared while model implementation, possible ways of their fixing, future extension, improvements that can be done in developed library.

Chapter contains conclusions along with recommendations for future work.

Appendix A contains the task description that was agreed and signed at the commencement of the thesis.

Appendix B has the operational values of Sundsbarm hydro power plant.

Appendix C shows the code of simplified model of Sundsbarm hydro power plant where inelastic waterway and aggregate are taken into account.

In *Appendix D* the code of the model library is presented.

2 Example of Simplified Hydro Power Plant Modelling and Simulation

The previous thesis work (Shaheri, 2011) , despite of its detailed theoretical overview of the hydro power system modeling, has a huge drawback, which is the complexity of mathematical model that was used. That is why the purpose of future library, that is going to be implemented in OpenMdelica environment, is not only to cover the system demands, but to be "user friendly", structured and theoretically understandable. Ruled by the mentioned above principles and based on the approximate values and assumptions, a model for hydro power plant was developed in OpenModelica by professor Bernt Lie, Telemark University Collage. The model is simplified and it does not take into consideration compressibility of water and elasticity of penstock[1].

2.1 Case description

The purpose of any hydro power plant is to convert the kinetic energy of water into electricity. The sketch of hydropower system and typical process diagram of hydro power plant are shown on Figure 2-1 and Figure 2-2.

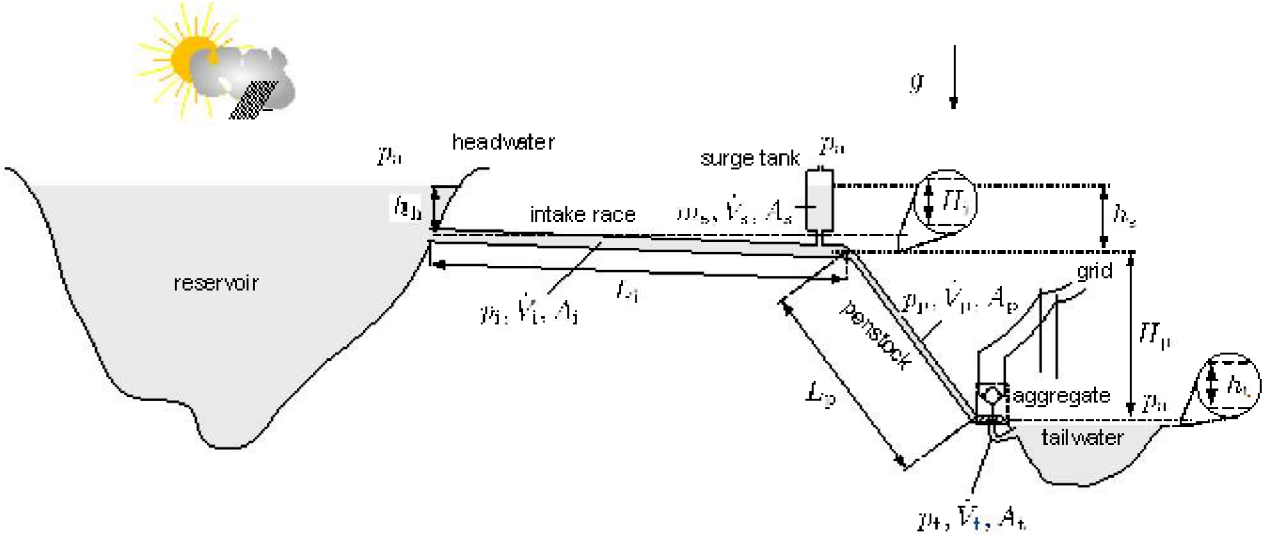

Figure 2-1 Hydropower system[1]

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[1] Bernt Lie, presentation «Optimal vannkraftproduksjon, modellering» at Skagerak Energi, January 29th, 2013

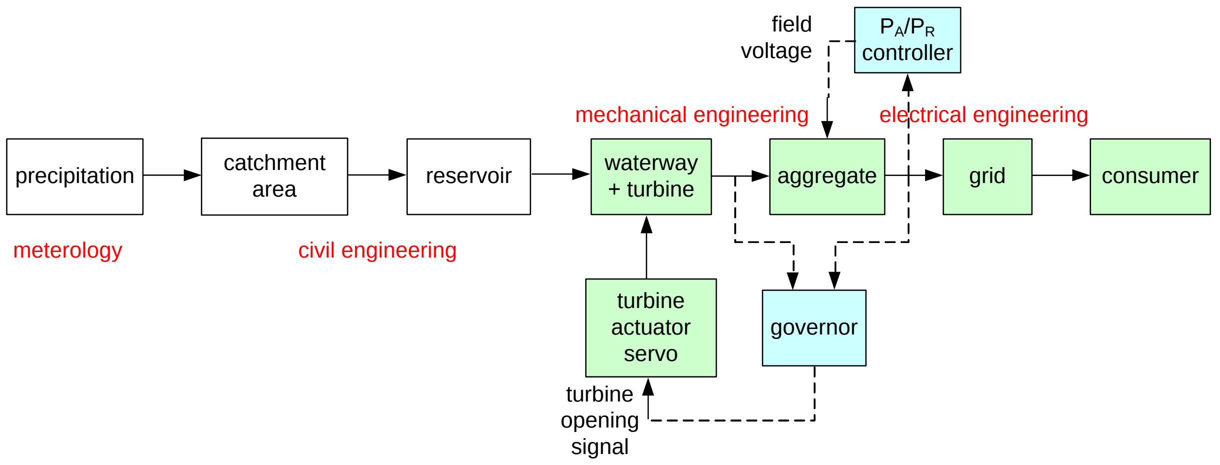

Figure 2-2 Process diagram of hydro power plant[1]

The hydro power plant can be divided into the following main parts each of which have specific requirements:

- *Waterway* consists of penstock system, reservoir, surge, and conduit. It worth to mention that waterway can be divided into three parts:
  1) From environment to reservoir
  2) From precipitation region to reservoir
  3) From reservoir to the hydro power turbine

Taking into account the complexity of the model and the metaphysics nature of the system, which is going to be mechanistic and it is of great interest to develop a switch between incompressible/non-elastic model.

- *Aggregate* (hydro-turbine and generator) is responsible for conversion of “energy from water to rotational energy of turbine”[2]. According to the task of thesis the library should solve a wide range of problems (control, optimization and faulty situations), that is why a model of this part of plant has to be detailed that to also take into account the model of three phases. Moreover the needed moving force for operating the hydro-turbine opening is high, that is why it is crucial to use servo motor that will allow not only to move vanes in turbine, but also will influence on operation dynamics.
- *Electrical grid* needs to take into account different types of loads from different types of consumers.

The operational values for simplified model of Sundsbarm hydro power plant are given in Appendix B.

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[1] Bernt Lie, presentation «Optimal vannkraftproduksjon, modellering» at Skagerak Energi, January 29th, 2013

[2] Bernt Lie, ongoing report ”Modeling and Simulation of Sustainable Energy Systems. Process System Simulation”, August 2013

2.2 Modeling part

The schematic waterway of hydro power plant is shown on Figure 2-3.

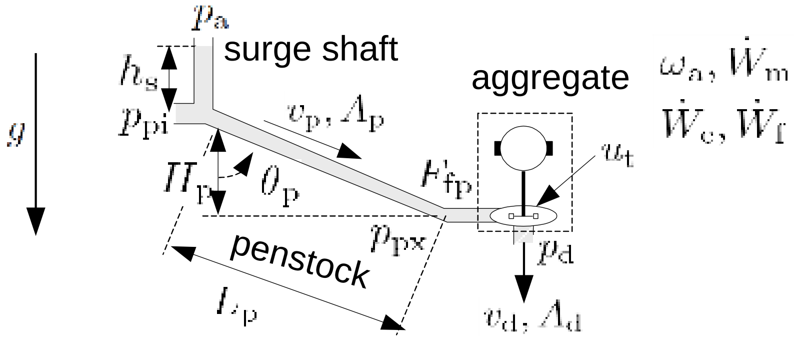

Figure 2-3 Schematic waterway of hydropower[1]

For conduit it was assumed that the water density ρ_c is constant, A_c is constant cross section area of the conduit, v_c is constant velocity across the cross section[1]. Consequently volumetric flow rate can be calculated as:

$$\dot{V}_c = A_c v_c \tag{2-1}$$

Mass flow is defined as:

$$\dot{m}_c = \dot{V}_c \rho_c \tag{2-2}$$

Following Newton's law the momentum balance can be written:

$$\frac{d}{dt} m_c v_c = (p_{ci} - p_{cx}) A_c + m_c g \cos\theta - F_{fc} \tag{2-3}$$

where

p_{cx} – intersection pressure between conduit, surge tank and penstock, that can be calculated:

$$p_{cx} = p_a + \rho g h_r \tag{2-4}$$

F_{fc} – friction force, that can be determined according to the following equation:

$$F_{fc} = f_c \frac{\mathrm{P}_c}{2} \rho_c L_c v_c |v_c| \tag{2-5}$$

where

f_c – fanning friction factor for conduit,

P_c - perimeter of the conduit cross section area,

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[1] Bernt Lie, presentation «Optimal vannkraftproduksjon, modellering» at Skagerak Energi, January 29th, 2013

L_c – length of conduit.

For the surge volume[1] with constant ρ_s , which is density of water for inelastic case, mass balance leads can be described as:

$$\frac{dm_s}{dt} = \rho_s \dot{V}_s \tag{2-6}$$

Then the momentum balance can be presented as:

$$\frac{d}{dt}(m_s v_s) = (p_{si} - p_{sx})A_s - m_s g - F_{fs} \tag{2-7}$$

where outflow pressure in surge tank is calculated:

$$p_{sx} = p_a + \frac{m_s g}{A_s} \tag{2-8}$$

where

p_a – atmospheric pressure,

A_s – area of surge tank,

Velocity v_s changes its direction under normal operation[2] that is why friction force for surge volume can be written:

$$F_{fs} = f_s \frac{\mathrm{P}_s}{2} \rho_s h_s v_s |v_s| \tag{2-9}$$

where

f_s – fanning friction factor for surge volume,

P_s - perimeter of the surge tank cross section area,

h_s – height of surge tank.

For penstock[3] the mass of water in waterway is constant.

The momentum balance is:

$$\frac{d}{dt}(m_p v_p) = \left(p_{pi} - p_{px}\right)A_p - m_p g \cos\theta - F_{fp} \tag{2-10}$$

where

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[1] Bernt Lie, ongoing report ”Modeling and Simulation of Sustainable Energy Systems. Process System Simulation”, August 2013

[2] Bernt Lie, ongoing report ”Modeling and Simulation of Sustainable Energy Systems. Process System Simulation”, August 2013

[3] Bernt Lie, ongoing report ”Modeling and Simulation of Sustainable Energy Systems. Process System Simulation”, August 2013

$$cos\,\theta_p = \frac{H_p}{L_p} \tag{2-11}$$

where

H_p – height of penstock,

L_p – length of penstock.

Friction force for penstock can be calculated by the following formula:

$$F_{fp} = f_p \frac{\mathrm{P}_p}{2} \rho_p h_p v_p^2 \tag{2-12}$$

where

f_s – fanning friction factor for penstock,

P_p - perimeter of the penstock cross section area,

h_s – height of penstock,

v_p - velocity in penstock.

Outflow pressure in penstock is:

$$p_{px} = p_a + \Delta p_v \tag{2-13}$$

where Δp_v – pressure drop inside the penstock.

The main principle equation *for manifold*[1] is based on preservation of mass in steady state, that is why, with constant density assumption:

$$\dot{V}_c = \dot{V}_s + \dot{V}_p \tag{2-14}$$

where $\dot{V}_p$ – penstock volumetric flow rate.

Using Bernoulli's equation and assuming steady state conservation of mechanical energy, *for valve* the following equation for calculation of convective kinetic power at the exit of the hydro power turbine can be applied:

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[1] Bernt Lie, ongoing report "Modeling and Simulation of Sustainable Energy Systems. Process System Simulation", August 2013

$$\dot{K}_{ht} = \Delta p_t \dot{V}_p + \dot{K}_p \tag{2-15}$$

where

Δp_t- turbine gate pressure drop,

$\dot{K}_p$ – convective kinetic power upstream from the gate.

$$\dot{K}_p = \frac{1}{2}\dot{m}_p v_p^2 \tag{2-16}$$

where $\dot{m}_p$ – mass of water in penstock.

It is also worth to mention the next statement: the flow through the valve depends on the valve opening u_t and this dependency can be expressed:

$$\dot{V}_p = F(u_t)\sqrt{\Delta p_t} \tag{2-17}$$

where

$$\Delta p_t = \frac{\dot{V}_p^2}{F_t^2(u_t)} \tag{2-18}$$

Application of energy balance to aggregate leads to:

$$\frac{d\omega_a}{dt} = \frac{1}{J_a \omega_a}(\eta_t \dot{K}_{ht} - \dot{W}_e - \frac{1}{2}k_{fa}\omega_a^3) \tag{2-19}$$

where

ω_a – angular velocity of aggregate,

J_a – moment of inertia for aggregate,

η_t – turbine efficiency at 85% flow of maximal water flow rate,

$\dot{W}_e$ – mechanical power consumed to produce electric power in generator,

k_{fa} – friction factor for aggregate bearings.

The main focus in the developed model has been put in combination of penstock and the turbine model. For this sub-models were combined[1]:

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[1] Bernt Lie, ongoing report ”Modeling and Simulation of Sustainable Energy Systems. Process System Simulation”, August 2013

$$\frac{d}{dt}(m_c\frac{\dot{V}_c}{A_c}) = (p_{ci} - p_{cx})A_c + m_c g\frac{H_c}{L_c} - f_c\frac{\mathrm{P}_c}{2}\rho_c L_c\frac{\dot{V}_c}{A_c}\left|\frac{\dot{V}_c}{A_c}\right| \tag{2-20}$$

$$\frac{d}{dt}(m_s\frac{\dot{V}_s}{A_s}) = (p_{si} - p_{sx})A_c + m_s g - f_s\frac{\mathrm{P}_s}{2}\rho_s L_s\frac{\dot{V}_s}{A_s}\left|\frac{\dot{V}_s}{A_s}\right| \tag{2-21}$$

$$\frac{d}{dt}(m_p\frac{\dot{V}_p}{A_p}) = (p_{pi} - p_{px})A_p + m_p g\frac{H_p}{L_p} - f_p\frac{\mathrm{P}_p}{2}\rho_p L_p\frac{\dot{V}_p}{A_p}\left|\frac{\dot{V}_p}{A_p}\right| \tag{2-22}$$

$$\frac{d\omega_a}{dt} = \frac{1}{J_a\omega_a}(\eta_t\dot{K}_{ht} - \dot{W}_e - \frac{1}{2}k_{fa}\omega_a^3) \tag{2-23}$$

2.3 Simulation results

The results of the simulation are graphs that demonstrate the working parameters of the system during the selected period of operation (1000seconds): hydro turbine kinetic flow (Figure 2-4), volumetric flow rate, aggregate angular velocity. The graphs are shown on Figure 2-4, Figure 2-5, Figure 2-6.

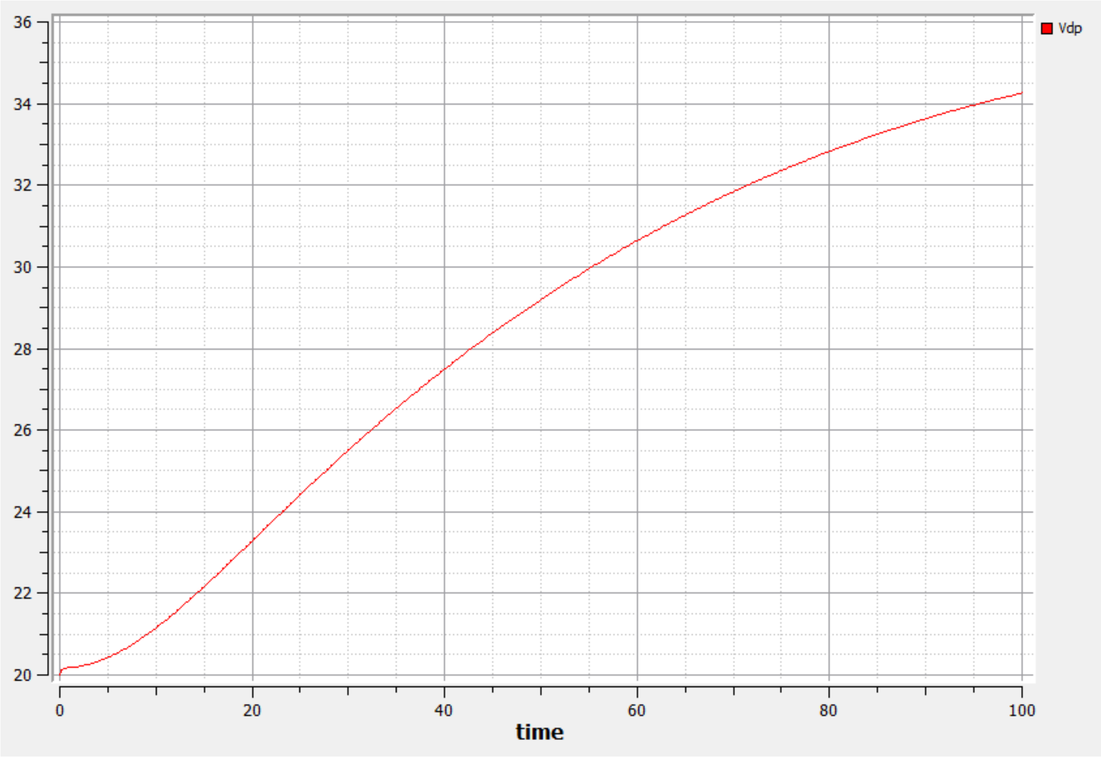

Figure 2-4 Change of penstock volumetric flow rate Vdp , m^3/sec

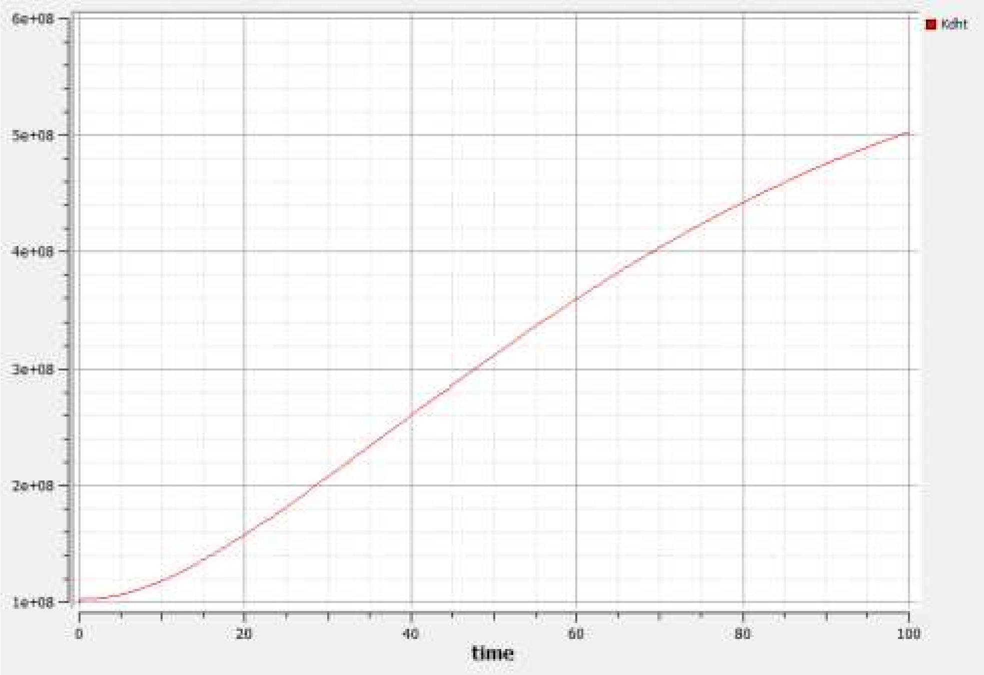

Figure 2-5 Change of hydroturbine kinetic flow Kdht, MW

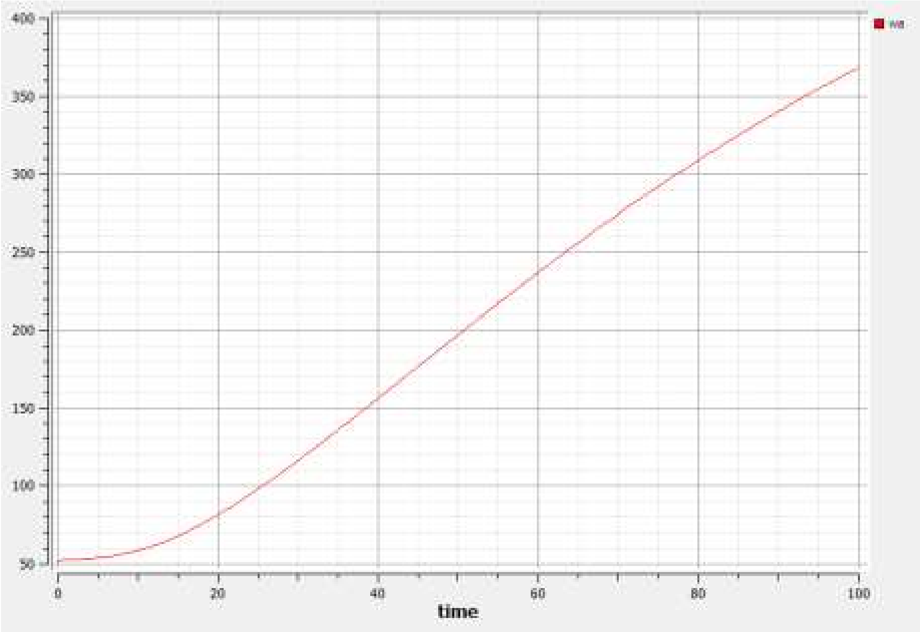

Figure 2-6 Change of aggregate angular velocity ω_a*, rad/sec*

For further model implementation and synchronisation between parts of the whole system during simulation, it is worth to put the following assumptions:

$$p_{cx} = p_{sx} = p_{px} \tag{2-24}$$

where

p_{cx} –outflow pressure in conduit,

p_{sx} – outflow pressure in surge tan

p_{px} – outflow pressure in penstock,

and surge volume level:

$$h_s = \frac{m_s}{\rho_s A_s} \tag{2-25}$$

It is worth to mention that this type of model has taken into account simplified combination of penstock and turbine.

The simulation program implemented in OpenModelica is shown in Appendix C.

First that is seen from the graphs that the calculation crashes at 100sec, probably it is related to some uncertainties among fanning friction factors. There was an attempt to udjust these values, but it was not really successful. It is also visible from Figure 2-6 that the values are increasing with time, that is also quite impossible . It is “expected” that it can be increase, but still after some time they should stabilize around 50.

It is also worth to mention that this type of model has taken into account simplified combination of penstock and turbine. And for the future library it is of great importance to show the synchronised work of the whole parts of the system and show the influence of different types of regimes that can appear and level of detail of station performance. Despite of example simplicity, it has not taken into consideration electric part of the system and its possible control, which also is a crucial source of sufficient influence on working regimes. That is why the newly appearing requirement for the extended library is not only to save the simplicity, but to be able to cover most of possible station working modes, such as short circuit or system overload.

3 Hydro Power Plant Modeling

3.1 Overview

The main purpose of detailed hydro power plant is to have a look into each part of plant separately. For sure that partially has been done in previous section, but here the look will be put into possible way to extend the library. The theoretical base for such extended hydro power model was presented in previous master thesis related to this topic (Shaheri, 2011). Some parts of model will stay the same (penstock, tailor water, control system), as it was shown at previous chapter. But the current mathematical model with possible options for extension of some its parts (turbine) will be described further.

3.2 Modeling of Hydro Power Plant Parts

3.2.1 Waterway with Penstock

As it was mentioned above the model for waterway with penstock[1] can be taken the same it was described in section 2.2, equations (2-21), (2-22), (2-23), (2-24). The above model is valid for the system *with inelastic penstock,* that is going to be used for further simulation.

For further modelling and model library specification the model of waterway way with elastic penstock can be also evaluated.

3.2.2 Turbine

Turbine can be named one of the most complicated parts of hydro power plant, not only because of different types that can be installed, but also because of different methods of its investigation and calculation.

As it was researched while work on model for case study hydro power plant rotational turbine is used. Its particularity is that “the working fluid completely fills the passageways through which it flows” (Bruce R.Munson, 2013). There are two types of rotational turbines:

- Francis turbine (the one that is used at Sundsbarm plant) is mixed-flow hydraulic turbine (Bruce R.Munson, 2013), shown on Figure 3-1;
- Kaplan turbine is “the most efficient type of turbine” at very low heads the most efficient type of turbine and is the axial flow or propeller turbine (Bruce R.Munson, 2013).

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[1] Bernt Lie, ongoing report ”Modeling and Simulation of Sustainable Energy Systems. Process System Simulation”, August 2013

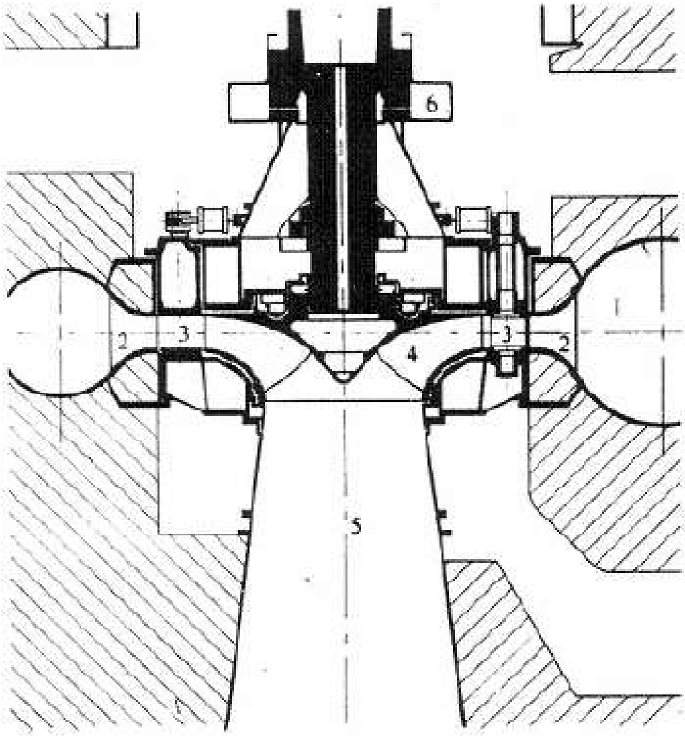

Figure 3-1 Francis turbine construction[1]: 1)Volute 2)Fixed guide vanes 3)Adjustable guide vanes 3)Runner 4)Draft tube 5)Support bearing

In Chapter 2 model of Pelton turbine, that is “a classical example of an impulse turbine” (Bruce R.Munson, 2013), was described. For the following thesis case Francis turbine is going to be evaluated.

In the example model in section 2.2.1 the turbine that was used is a Pelton one. In the remaining of the thesis Francis turbine will be used.

There are a few methods of turbine work investigation:

- Using dimensionless analysis;
- Using “velocity triangle”.

In case of *dimensionless analysis* the important parameters that can be determined by turbine are the pressure drop Δp and the mechanical power $\dot{W}$. For their calculation volumetric flow through the machine, rotational speed of the aggregate and actuator signal should be known[2]. That is why for steady state operation of the machine the following relationship must be satisfied:

$$f_j\left(\Delta p, \dot{V}, \dot{N}, \dot{W}, \rho, \mu, \beta, D, Y\right) = 0, j = \{1,2\} \tag{3-1}$$

where

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[1] Dr.Ir.Harinaldi, M.Eng, Mechanical Engineering Department of Engineering University of Infdonesia, presentation «Reaction Turbines», Mechanical Engineering Courses

[2] Bernt Lie, ongoing report ”Modeling and Simulation of Sustainable Energy Systems. Process System Simulation”, August 2013

Δp is the pressure drop over the machine,

$\dot{V}$ is the volumetric flow rate through the machine,

$\dot{N}$ is the rotational velocity of the machine,

$\dot{W}$ is the produced mechanical power,

ρ is the fluid density, μ is the fluid viscosity,

β is the fluid compressibility,

D is machine diameter,

Y is dimensionless actuator signal.

Using the set of basic dimensions $\Delta = (mass, length, time)$, the dimensions of the 9 arguments $q = (\Delta p, \dot{V}, \dot{N}, \dot{W}, \rho, \mu, \beta, D, Y)$ of function f_j can be shown as follows:

$$\begin{gathered}
[\Delta p] = mass^{1}length^{-1}time^{-2} \\
[\dot{V}] = mass^{0}length^{3}time^{-1} \\
[\dot{N}] = mass^{0}length^{0}time^{-1} \\
[\dot{W}] = mass^{1}length^{2}time^{-3} \\
[\rho] = mass^{1}length^{-3}time^{0} \\
[\mu] = mass^{1}length^{-1}time^{-1} \\
[\beta] = mass^{-1}length^{1}time^{2} \\
[D] = mass^{0}length^{1}time^{0} \\
[Y] = mass^{0}length^{0}time^{0}
\end{gathered} \tag{3-2}$$

For description of turbo machines the following sets of dimensionless numbers can be used Table *3-1*).

Table 3-1 Sets of dimensionless numbers for turbine[1]

Dimensionless numbers	Comments
$\pi_{\dot{V}} = \frac{\dot{V}}{\dot{N}D^3}$	Flow coefficient
$\pi_{\Delta p} = \frac{\Delta p}{\dot{N}^2 D^2 \rho}$	Pressure coefficient
$\pi_{\dot{W}} = \frac{\dot{W}}{\dot{N}^3 D^5 \rho}$	Power coefficient
$\pi_{Re} = \frac{\dot{N}D^2\rho}{\mu}$	Reynolds number
$\pi_{Ma} = \rho\beta D^2 \dot{N}^2$	Mach number
$\pi_Y = Y = \frac{u}{U}$	Dimensionless actuator signal

This form allows choosing a number of alternative dimensionless numbers,that come from combination of several above mentioned numbers. For turbine two dimensionless numbers are calculated, that is shown in Table 3-2.

Table 3-2 Calculation of two dimensionless numbers[2]

Alternative dimensionless numbers	Comments
$\pi_{\eta} = \frac{\pi_{\dot{W}}}{\pi_{\dot{V}}\pi_{\Delta p}} = \frac{\dot{W}}{\Delta p \dot{V}}$	Turbine efficiency
$\pi_{\dot{N}} = \frac{(\pi_{\dot{W}})^{\frac{1}{2}}}{(\pi_{\Delta p})^{\frac{5}{4}}} = \frac{\dot{N}}{\frac{(\Delta p)^{\frac{5}{4}}}{\rho^{\frac{3}{4}} \dot{W}^{\frac{1}{2}}}}$	Specific speed

For turbine model simplification it is worth to use principles of Buckingham's theorem, which allows writing similarity model:

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[1] Bernt Lie, ongoing report "Modeling and Simulation of Sustainable Energy Systems. Process System Simulation", August 2013

[2] Bernt Lie, ongoing report "Modeling and Simulation of Sustainable Energy Systems. Process System Simulation", August 2013

$$F_j\left(\frac{\dot{V}}{\dot{N}D^3}, \frac{\Delta p}{\dot{N}^2 D^2 \rho}, \frac{\dot{W}}{\dot{N}^3 D^5 \rho}, \frac{\dot{N} D^2 \rho}{\mu}, \rho\beta D^2 \dot{N}^2, Y\right) = 0 \tag{3-3}$$

from which applying “decoupling” and mathematical simplifications with regard to dimensionless numbers from which $\dot{W}$ can be found. Using Hill charts efficiency of turbine can be estimated. Further Hill chart explanation will be provided

Another method that is used for defining turbine produced mechanical power and efficiency is “velocity triangle”. The movement of water particle from the guide vanes till the moment it leaves the runner is shown on Figure 3-2.

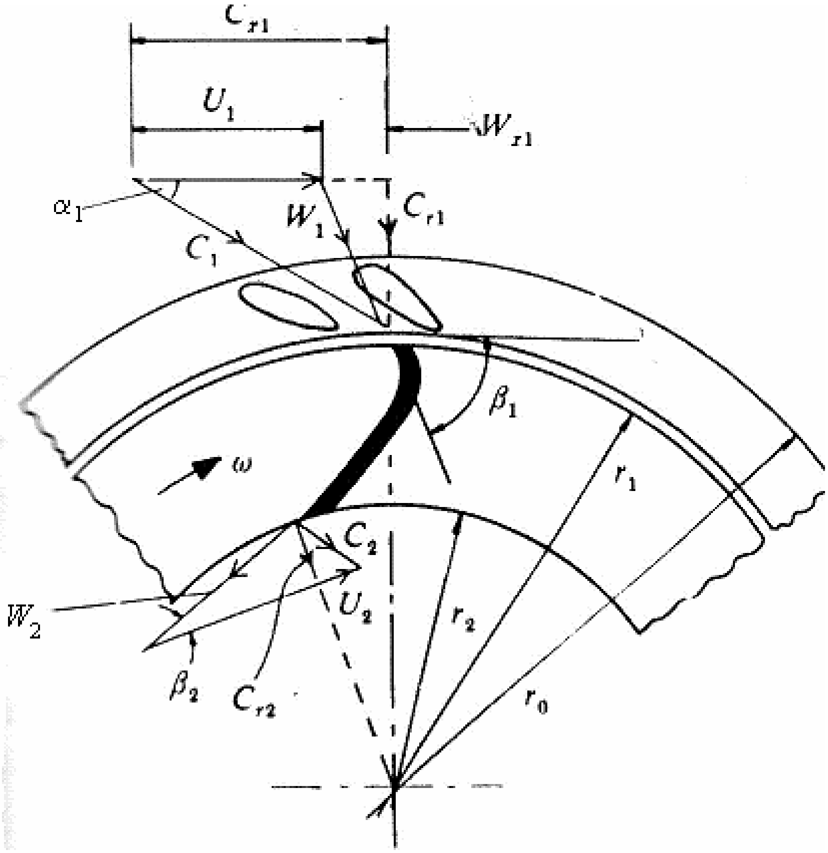

Figure 3-2 Movement of a water particle in turbine[1]

From mentioned above figure with Euler equation[2] energy given to the runner can be estimated:

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[1] Dr.Ir.Harinaldi, M.Eng, Mechanical Engineering Department of Engineering University of Infdonesia, presentation «Reaction Turbines», Mechanical Engineering Courses

[2] Dr.Ir.Harinaldi, M.Eng, Mechanical Engineering Department of Engineering University of Infdonesia, presentation «Reaction Turbines», Mechanical Engineering Courses

$$E = \frac{W}{mg} = \frac{(U_1 C_{x1} - U_2 C_{x2})}{g} \tag{3-4}$$

where

Q is flow rate through the runner, [m^3/sec]

$C_{x1} = \frac{Q}{2\pi r_1 b_1}$ and $C_{x2} = \frac{Q}{2\pi r_2 b_2}$ are flow velocities, [m/s]

b - runner height, [m].

For reaction turbines effective head, H_E, which is relative to the surface of the tailrace (Dixon, 1998), together with "kinetic, potential and pressure energies" forms the following energy balance at entry to the runner:

$$g(H_E - \Delta H_N) = \frac{p_2 - p_a}{\rho} + \frac{1}{2}c_2^2 + gz_2 \tag{3-5}$$

where ΔH_N is the loss of head due to friction in the volute and guide vanes and p_2 is the absolute static pressure at inlet to the runner, as it is stated in (Dixon, 1998).

For runner outlet it can be stated, that:

$$g(H_E - \Delta H_N - \Delta H_R) - \Delta W = \frac{p_3 - p_a}{\rho} + \frac{1}{2}c_{32}^2 + gz_3 \tag{3-6}$$

where $g\Delta H_R$is friction work in the runner, ΔW is the specific work, p_3 is the absolute static pressure at runner exit.

From *(3-4)*(3-4)and (3-5) the specific work is calculated:

$$\Delta W = \frac{p_{02} - p_{03}}{\rho} - g(z_2 - z_3) - g\Delta H_R \tag{3-7}$$

where p_{02}, p_{03} are absolute total pressures at inlet and exit runner respectively,

z_3 is the vertical distance between the exit plane of the runner and the free surface of the tailrace (Dixon, 1998).

The expression for hydraulic efficiency is the following formula and The hydraulic efficiency of turbines is shown on Figure 3-3:

$$\eta_H = \frac{\Delta W}{gH_E} \tag{3-8}$$

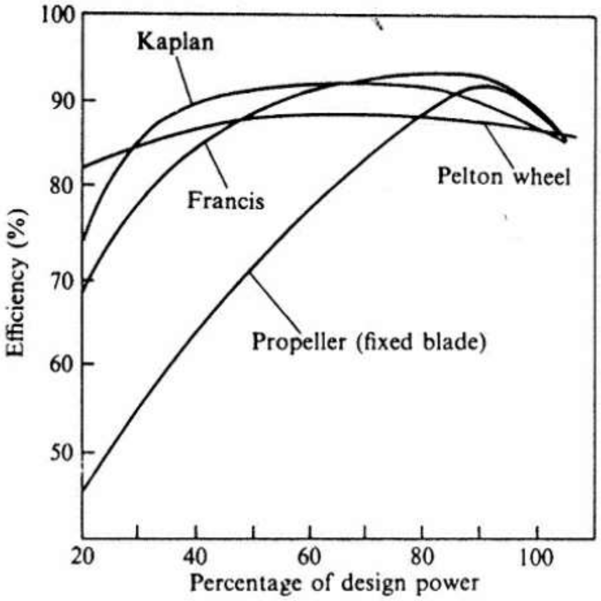

Figure 3-3 Comparison of Hydraulic Turbine Efficiencies[1]

For sure method based on basic calculations is more simple, but it gives not so accurate estimation of turbine working regimes and do not take into account the parameters that are used in dimensionless analysis.

For the turbine design it is of great value to use of “hill charts”, that provides efficiency as “a function of the turbine volumetric flow and head at design speed for a specific series of turbines” (Shaheri, 2011). As it was mentioned above hill chart is a turbine efficiency charts that are defined for different types of turbines can have different origin, for example[2]:

- From experiments that are run with investigated turbine. After that information is tabled, that allows further to use interpolation.
- From experiments that are run for turbine of similar type, and the received data is generalized by using similarity models for the turbine that is investigated. Benefit is that this method can be used for a number of different turbines.
- From computational fluid dynamics methods with a usage of detailed simulations of the turbine.

For Francis turbine the following set of design operational values, $(\dot{V}^o, H^o, \dot{N}^o, \pi_{\ \eta}, Y)$, where $\Delta p^o = \rho g H^o$ and $\dot{W}^o = \pi_\eta^o \Delta p^o\ \dot{V}^o$, form the hill chart, that is shown on Figure 3-4.

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[1] Dr.Ir.Harinaldi, M.Eng, Mechanical Engineering Department of Engineering University of Infdonesia, presentation «Reaction Turbines», Mechanical Engineering Courses

[2] Bernt Lie, ongoing report ”Modeling and Simulation of Sustainable Energy Systems. Prosess System Simulation”, August 2013

Despite of the fact that the chart shown on Figure 2.8 does not utilize the idea of similarity, the following procedure can be followed[1]:

1. The actual flow rate $\dot{V}$, the actual rotational speed $\dot{N}$ and the actual actuator signal Y should be known.
2. Hypothetical volumetric flow rate V^* is calculated using similarity principles:
3. Actual pressure drop can be calculated using dimensionless number:

$$\pi_{\dot{V}} = \frac{\dot{V}}{\dot{N}D^3} = \frac{\dot{V}^*}{\dot{N}^*D^3} \Rightarrow \Delta p = \Delta p^* \left(\frac{\dot{N}}{\dot{N}^o}\right)^2 \tag{3-9}$$

Efficiency is a dimensionless number in the similarity model. But due to the fact that large size turbines are usually more efficient than the smaller one, some deviations from the computed value are accepted.

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[1] Bernt Lie, ongoing report ”Modeling and Simulation of Sustainable Energy Systems. Prosess System Simulation”, August 2013

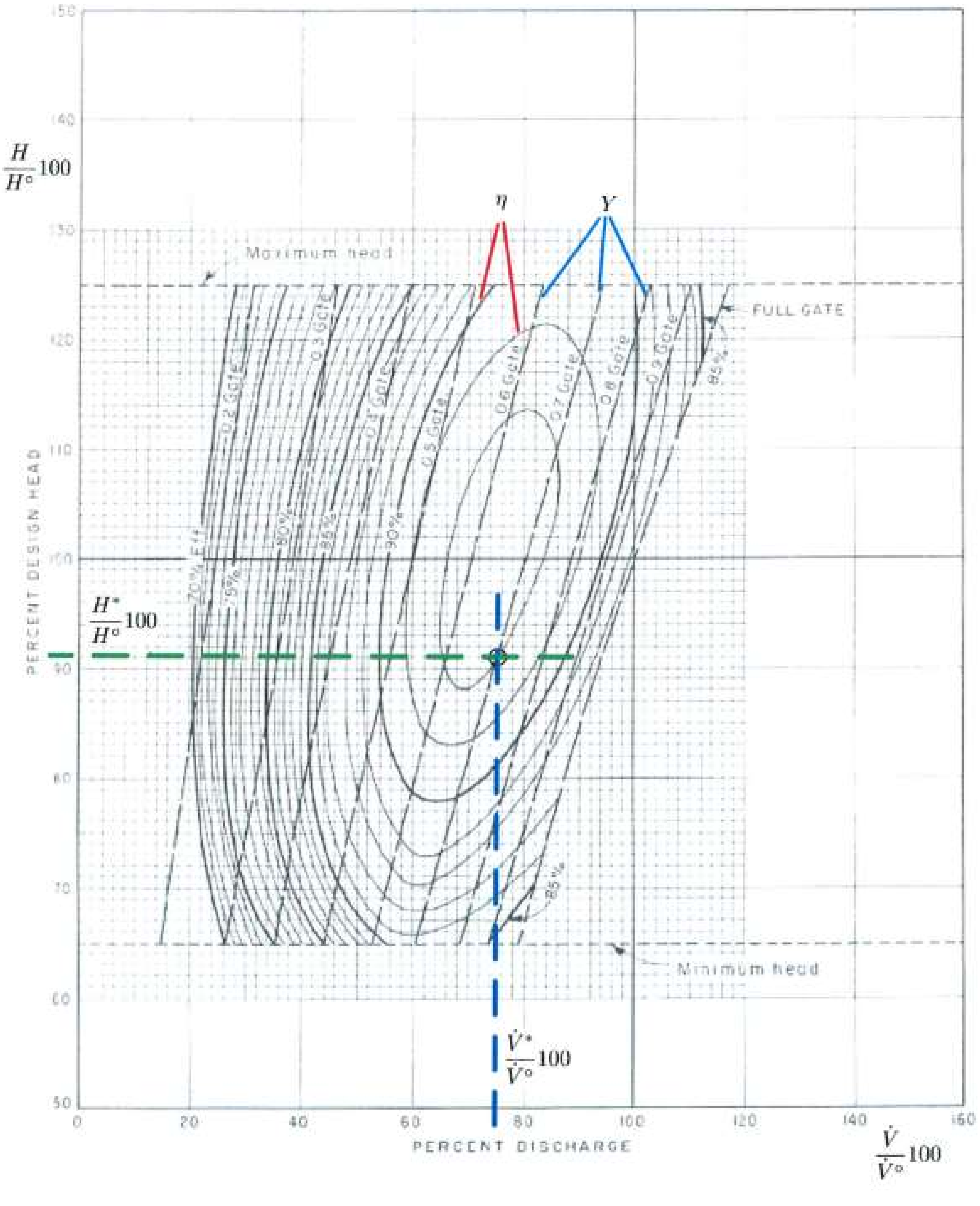

Figure 3-4 Hill chart for Francis turbine

For case study of this thesis the experience from (Shaheri, 2011) was taken into consideration and the hill chart was tabulated for specific turbine using relationship between position data (R,%, θ, degree) “of a particular point in the hill chart and corresponding values” can be calculated (Shaheri, 2011):

$$Rcos(72.04^o + \theta) = 32.2 + Q \tag{3-10}$$

$$Rsin(72.04^o + \theta) = 331.6 + (H - 60)\frac{134}{70}$$

where Q is percent discharge, H is percent head, θ is guide vane opening in percent measured in clockwise direction. Table 3-3 demonstrates θ for the range of vane openings.

Table 3-3 The angle θ for different guide vane openings (Shaheri, 2011)

θ, degrees	0	0.64	1.5	2.56	3.3	5.3	6.76	8.16	10.09
Guide vane opening, p.u	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2

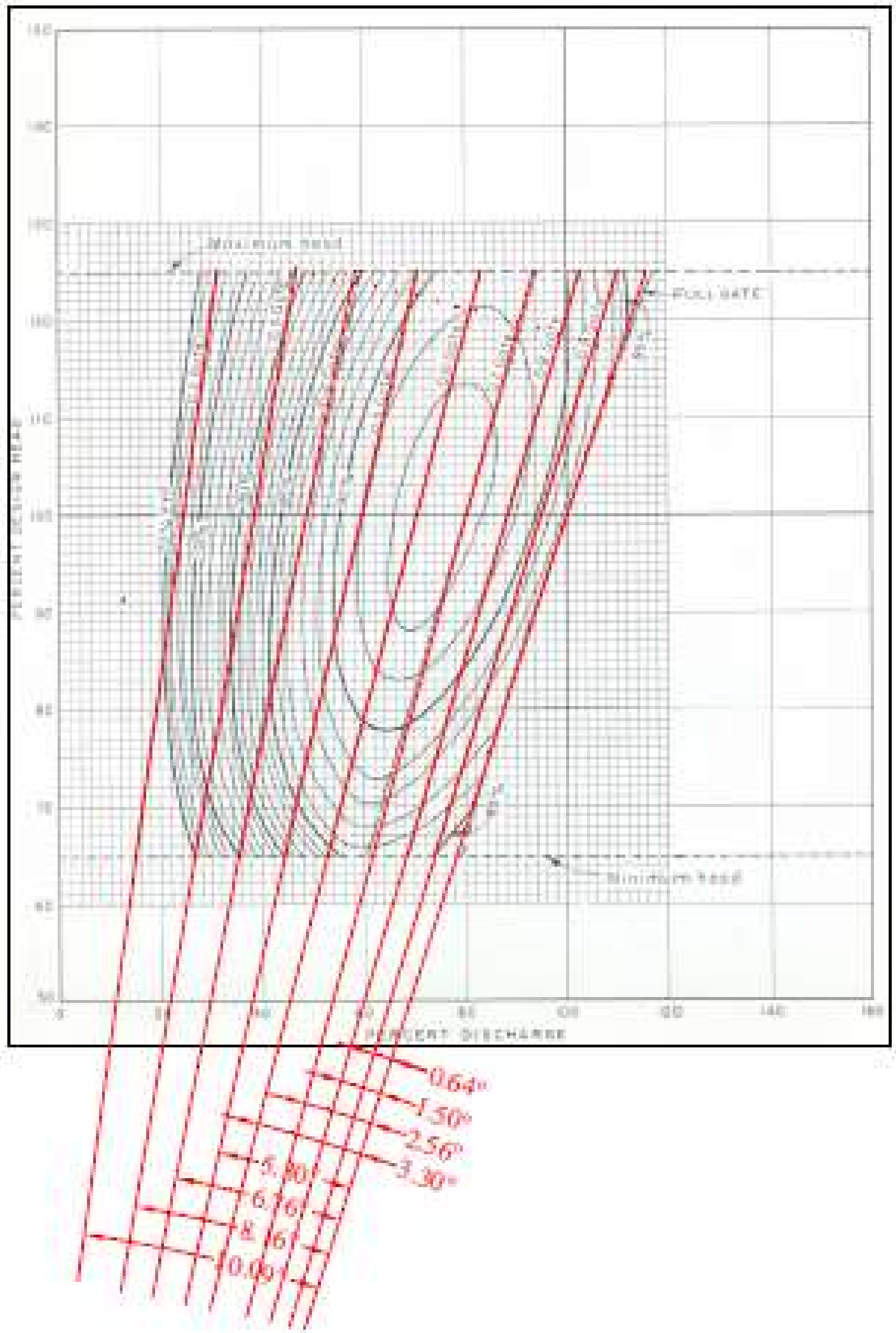

Figure 3-5 Relationship between turbime head and guide vane opening (Shaheri, 2011)

3.2.3Turbine controller

In (Shaheri, 2011) the wide explonation of turbine controller was given. The basics for the controller theory (according to (Shaheri, 2011)) lay in “swing equation” and “speed drop”.

The “swing equatio “ outcome has already been used in Chapter 2, and also is going to be applied for synchronous generator (where it is going to be explained more detailed). It is known, that the electrical frequency changes depending on “the balance between the total generated and consumed power in network” (Shaheri, 2011). The generator is used for its correction by variation of output power. Due to turbine governor this effect becomes possible.

According to (Shaheri, 2011) droop, or a regulation, can be calculated as:

$$S = -100\frac{\Delta f/f_{ref}}{\Delta P/S_B} \tag{3-11}$$

where

S is a droop,[%]

f_{ref} is a desired frequency of network, [Hz]

S_B is a rated power of generator, [MVA].

Block diagram of transient droop controller for hydropower turbines is shown on Figure 3-6 Block Diagram of Transient Droop Controller. As it is seen this system consists of two servomors: pilot one, which is responsible for relay valve operation , that is connected to the main servomotor, which is responsible for the guide vanes opening position and is “modeled as an integrator with limit on the output and also on the rate of change of the guide output in both directions (increasing-decreasing)” (Shaheri, 2011). The drawback of such governor is “the feedback for implementing the steady state droop is taken from the guide vanes position instead of generator output active power” (Shaheri, 2011).

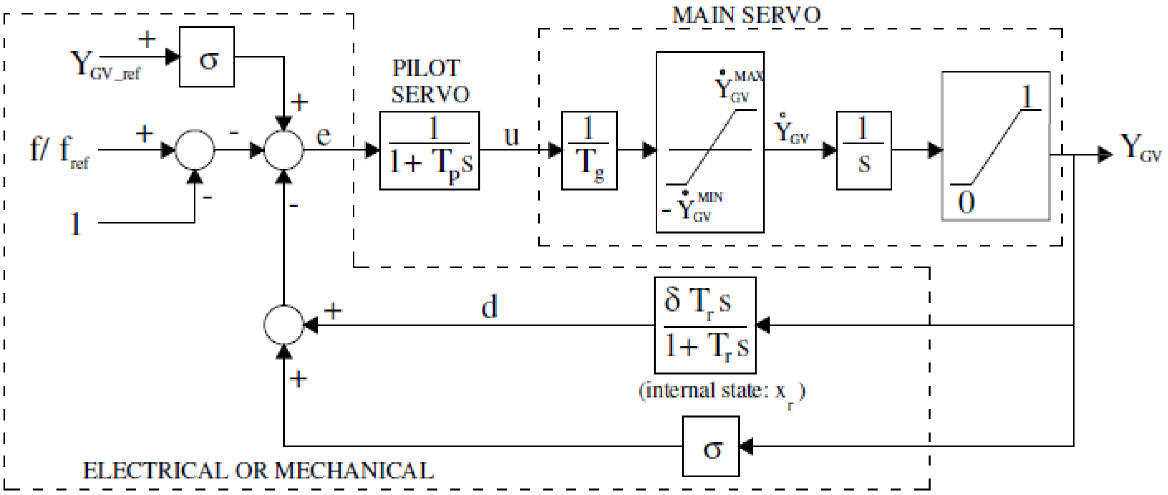

Figure 3-6 Block Diagram of Transient Droop Controller (Machowski, 2008)

From (Machowski, 2008) values for parameter of transient droop are taken:

$T_P = 0.04[sec], T_g = 0.04[sec], T_r = 5T'_w, \delta = 2.5T'_w/T_m$

where

T'_w is water starting time, [sec]

T_m is mechanical time constant.

For future simulation the transient droop controller model can be stated as in (Shaheri, 2011) and shown in Table 3-4.

Table 3-4Transient Droop Controller Model (Shaheri, 2011)

$$\mathrm{d} = \delta \mathrm{Y}_{\mathrm{GV}} - \mathrm{x}_{\mathrm{r}}$$

$$\mathrm{e} = \sigma\left(\mathrm{Y}_{\mathrm{GV}_{\mathrm{ref}}} - \mathrm{Y}_{\mathrm{GV}}\right) - \left(\frac{\mathrm{f}}{\mathrm{f}_{\mathrm{ref}}} - 1\right) - \mathrm{d}$$

$$T_r \frac{\mathrm{dx}_{\mathrm{r}}}{\mathrm{dt}} + \mathrm{x}_{\mathrm{r}} = \delta \mathrm{Y}_{\mathrm{GV}}$$

$$T_p \frac{\mathrm{du}}{\mathrm{dt}} + \mathrm{u} = \mathrm{e}$$

$$\frac{\mathrm{dY}_{\mathrm{GV}}}{\mathrm{dt}} = \begin{cases} 0, \mathrm{Y}_{\mathrm{GV}} \le 0 \text{ or } \mathrm{Y}_{\mathrm{GV}} \ge 1 \\ \dot{\mathrm{Y}}_{\mathrm{GV}}^{\mathrm{max}}, \mathrm{u}/T_g \ge \dot{\mathrm{Y}}_{\mathrm{GV}}^{\mathrm{max}} \\ -\dot{\mathrm{Y}}_{\mathrm{GV}}^{\mathrm{min}}, \dfrac{\mathrm{u}}{T_g} \le -\dot{\mathrm{Y}}_{\mathrm{GV}}^{\mathrm{min}} \end{cases}$$

Else $\frac{\mathrm{dY}_{\mathrm{GV}}}{\mathrm{dt}} = \frac{\mathrm{u}}{T_g}$

3.2.4 Electrical part: Synchronous generator

The library implementation is not possible without description of electrical part of hydropower system, which is quite complex. But for developing model library it was simplified taking into account its main part, synchronous generator, which is 12-pole. It is shown on Figure 3-7.

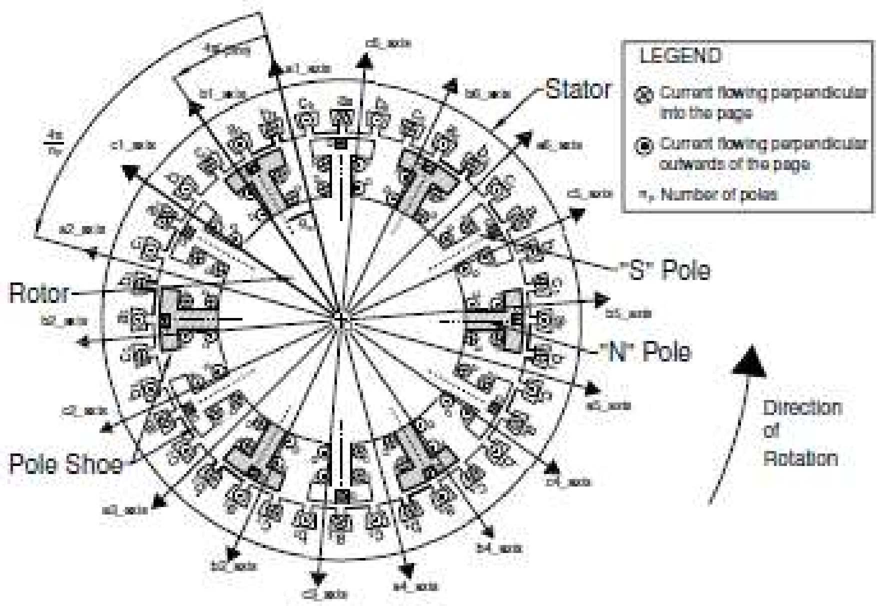

Figure 3-7 Cross-section area of a 12-poles synchronous generator (Shaheri, 2011)

It is of great importance to understand the meaning of denotations of Figure 3-7. For phase "a" stator slots are denoted as $a_1, a_2, \ldots, a_3$ (Shaheri, 2011). The same notation is applied for phases "b" and "c". Phases "a", "b" and "c" are "armature windings and are responsible for "the generator output current and producing terminal voltages" (Shaheri, 2011). "Field windings" are marked by "F". Constant direct current (DC, or so called "field current", "rotor magnetizing current" is carried through "F" and responsible for steady-state. D and Q, "Damper Windings", are "short circuited windings which stabilize the generator operation during rapid changes in operating conditions" (Shaheri, 2011). The armature of the machine is mounted on the stator and consists of a number of phase turn windings in such way that to connect the flux from the rotor winding. As the rotor turns, the variation of flux from the rotor induces a voltage in the armature windings.

The theory of synchronous generator is quite complicated. Its brief explanation will be given further in range required for the future model library.

The Inductance Matrix

According to electromagnetism theory, the flux linkage in each of the machine windings can be considered "as a linear function of the machine currents" (Shaheri, 2011). In case if "saturation and hysteresis effects can be neglected" (Machowski, 2008):

$$\begin{bmatrix}\Psi_a\\ \Psi_b\\ \Psi_c\\ \Psi_F\\ \Psi_D\\ \Psi_Q\end{bmatrix}=\begin{bmatrix}\mathrm{L}_{aa} & \mathrm{L}_{ab} & \mathrm{L}_{ac} & \mathrm{L}_{aF} & \mathrm{L}_{aD} & \mathrm{L}_{aQ}\\ \mathrm{L}_{ba} & \mathrm{L}_{bb} & \mathrm{L}_{bc} & \mathrm{L}_{bF} & \mathrm{L}_{bD} & \mathrm{L}_{bQ}\\ \mathrm{L}_{ca} & \mathrm{L}_{cb} & \mathrm{L}_{cc} & \mathrm{L}_{cF} & \mathrm{L}_{cD} & \mathrm{L}_{cQ}\\ \mathrm{L}_{Fa} & \mathrm{L}_{Fb} & \mathrm{L}_{Fc} & \mathrm{L}_{FF} & \mathrm{L}_{FD} & \mathrm{L}_{FQ}\\ \mathrm{L}_{Da} & \mathrm{L}_{Db} & \mathrm{L}_{Dc} & \mathrm{L}_{DF} & \mathrm{L}_{DD} & \mathrm{L}_{DQ}\\ \mathrm{L}_{Qa} & \mathrm{L}_{Qb} & \mathrm{L}_{Qc} & \mathrm{L}_{QF} & \mathrm{L}_{QD} & \mathrm{L}_{QQ}\end{bmatrix}\begin{bmatrix}\mathrm{i}_a\\ \mathrm{i}_b\\ \mathrm{i}_c\\ \mathrm{i}_F\\ \mathrm{i}_D\\ \mathrm{i}_Q\end{bmatrix} \tag{3-12}$$

Using values of each matrix’s “L” given in (Machowski, 2008) for two-pole machine, the following relations can be used for a general machine with n_p poles (Shaheri, 2011):

$$\begin{gathered}
\Theta_e=\Theta_m\frac{n_p}{2},\Theta_e'=\Theta_e-\frac{2\pi}{3},\Theta_e''=\Theta_e+\frac{2\pi}{3}\\
\mathrm{L}_{aa}=\mathrm{L}_s+\mathrm{L}_m\cos(2\Theta_e)\\
\mathrm{L}_{bb}=\mathrm{L}_s+\mathrm{L}_m\cos(2\Theta_e')\\
\mathrm{L}_{bb}=\mathrm{L}_s+\mathrm{L}_m\cos(2\Theta_e'')\\
\mathrm{L}_{ab}=\mathrm{L}_{ba}=-\mathrm{M}_s-\mathrm{L}_m\cos\left(2\Theta_e+\frac{\pi}{3}\right)\\
\mathrm{L}_{bc}=\mathrm{L}_{cb}=-\mathrm{M}_s-\mathrm{L}_m\cos\left(2\Theta_e'+\frac{\pi}{3}\right)\\
\mathrm{L}_{ac}=\mathrm{L}_{ac}=-\mathrm{M}_s-\mathrm{L}_m\cos\left(2\Theta_e''+\frac{\pi}{3}\right)\\
\mathrm{L}_{aF}=\mathrm{L}_{Fa}=\mathrm{M}_F\cos(\Theta_e)\\
\mathrm{L}_{bF}=\mathrm{L}_{Fb}=\mathrm{M}_F\cos(\Theta_e')\\
\mathrm{L}_{cF}=\mathrm{L}_{Fc}=\mathrm{M}_F\cos(\Theta_e'')\\
\mathrm{L}_{aD}=\mathrm{L}_{Da}=\mathrm{M}_D\cos(\Theta_e)\\
\mathrm{L}_{bD}=\mathrm{L}_{Db}=\mathrm{M}_D\cos(\Theta_e')\\
\mathrm{L}_{cD}=\mathrm{L}_{Dc}=\mathrm{M}_D\cos(\Theta_e'')\\
\mathrm{L}_{aQ}=\mathrm{L}_{Qa}=\mathrm{M}_Q\cos(\Theta_e)\\
\mathrm{L}_{bQ}=\mathrm{L}_{Qb}=\mathrm{M}_Q\cos(\Theta_e')\\
\mathrm{L}_{cQ}=\mathrm{L}_{Qc}=\mathrm{M}_Q\cos(\Theta_e'')\\
\mathrm{L}_{FF}=\mathrm{L}_F,\qquad \mathrm{L}_{DD}=\mathrm{L}_D,\qquad \mathrm{L}_{QQ}=\mathrm{L}_Q\\
\mathrm{L}_{FD}=\mathrm{L}_{DF}=\mathrm{M}_R\\
\mathrm{L}_{FQ}=\mathrm{L}_{QF}=\mathrm{L}_{DQ}=\mathrm{L}_{QD}=0
\end{gathered} \tag{3-13}$$

where $\mathrm{L}_s, \mathrm{L}_m, \mathrm{M}_s, \mathrm{M}_F, \mathrm{M}_D, \mathrm{L}_F, \mathrm{L}_D, \mathrm{L}_Q$ and M_R are positive real values of mutual inductances, Θ_m rotor angle which can “be measured from axes “a” to the nearest “N” pole in counter/clockwise direction” (Shaheri, 2011).

The conventional direction of current in the “Q” winding can be chosen differently, as it was shown in (Machowski, 2008) and (Andersson, 2012). But for the investigating case for multi-pole machine the experience of (Shaheri, 2011) will be used, where relations (3-6) were studied . The main principle that was used is that relations were explained by describing the magnetomotive forces and flux line paths for different cases and rotor angles:

- L_{aa}: Figure 3-8 demonstrates the flux paths and magnetomotive forces generated by a constant dc in the winding “a” for various rotor angles:
  1) $\Theta_m = \frac{2k\pi}{n_p}$ $(k = 0,1,\dots)$ when the poles are in one direction with magnetomotive forces, induced by “a” windings. That causes the least rotor reluctance for “a” windings, and that is why L_{aa} has its highest value.
  2) For $\Theta_m = \frac{(2k+1)\pi}{n_p}$ $(k = 0,1,\dots)$ because of the symmetry no flux is generated (Shaheri, 2011) in rotor, hence the rotor magnetic circuit has the largest reluctance that leads to minimum value of L_{aa}.

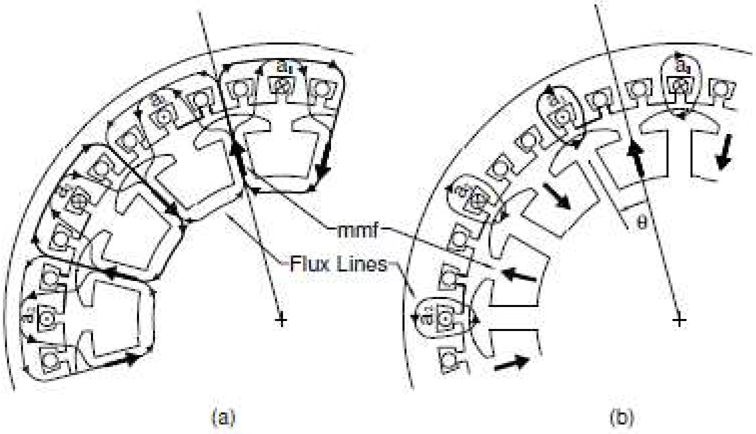

Figure 3-8 Flux lines generated by the current in "a" windings for different rotor positions (Shaheri, 2011)

- L_{ab}: the case when the flux path is induced by dc currents in the “a” and “b” windings for two different rotor angles and current directions (Shaheri, 2011) (shown on Figure 3-9):
  1) $\Theta_m = (2k\pi + \frac{2\pi}{3})/n_p$ $(k = 0,1,\dots)$ that gives equal and positive currents both in “a” and “b” windings, the highest positive linkage with the “b” winding is due to the current in “a” winding
  2) $\Theta_m = (2k\pi - \frac{\pi}{3})/n_p$ $(k = 1,\dots)$ that gives equal currents both in “a” and “b” windings, but “a” goes with positive sign, “b” – negative. For this case the flux linkage reaches maximum and negative value.

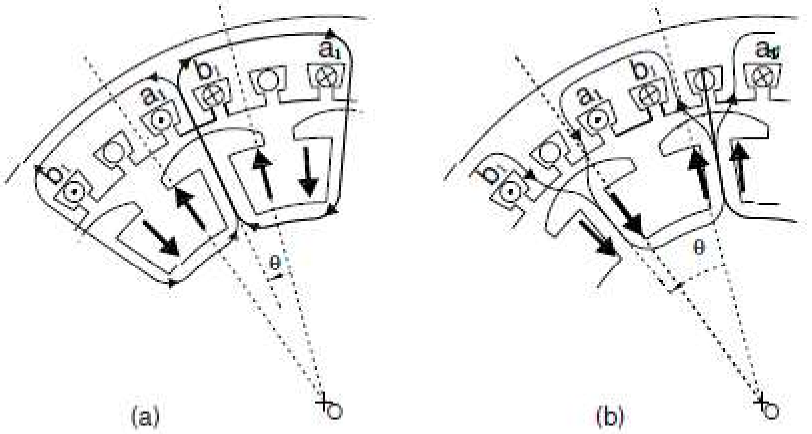

Figure 3-9Flux lines generated merely by the current in "a" windings for different rotor positions

- L_{aF} and L_{aD}: for various rotor angles the flux path is induced by positive dc current in “a” windings (Shaheri, 2011), Figure 3-10:

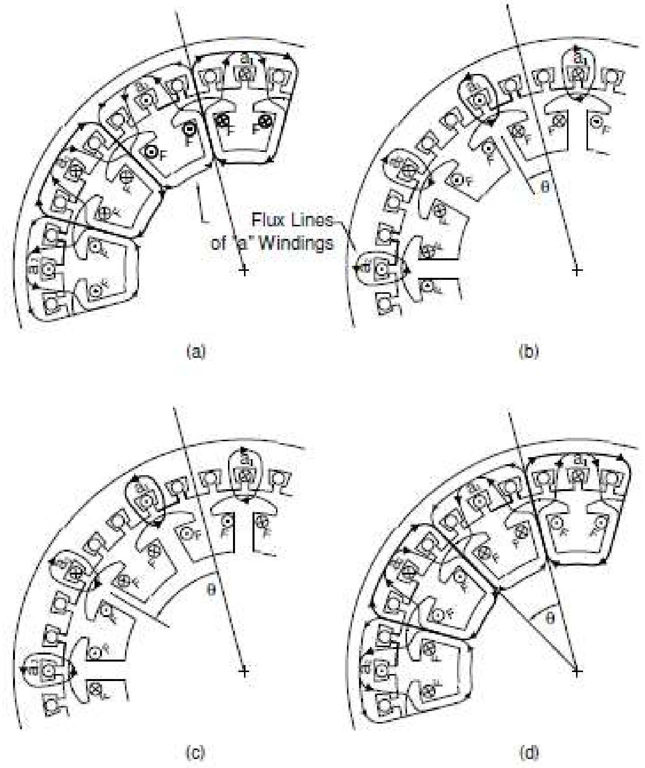

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Figure 3-10 Flux lines generated by the current in "F" windings and linking the "a" windings for different rotor positions

1) $\Theta_m = 4k\pi/n_p$ $(k = 0,1,\dots)$: the maximum magnitude are given by the flux linkages in the “F” windings. The direction of flux that passes the “F” windings and the flux that was induced by “F” windings are the same, that leads to positive mutual inductance.

2) $\Theta_m = (4k\pi + \pi)/n_p\ (k = 0,1,\dots)$: no flux linkage in the “F” windings because of the symmetry.

3) $\Theta_m = (4k\pi + 3\pi)/n_p\ (k = 0,1,\dots)$: no flux linkage in the “F” windings because of the symmetry.

4) $\Theta_m = (4k\pi + 2\pi)/n_p\ (k = 0,1,\dots)$: the maximum magnitude are given by the flux linkages in the “F” windings. The direction of flux that passes the “F” windings and the flux that was induced by “F” windings are opposite, that leads to negative mutual inductance.

- L_{aQ} , L_{FQ} and L_{DQ}: for various rotor angles the flux path is induced by positive dc current in “Q” windings (Shaheri, 2011), Figure 3-11 :

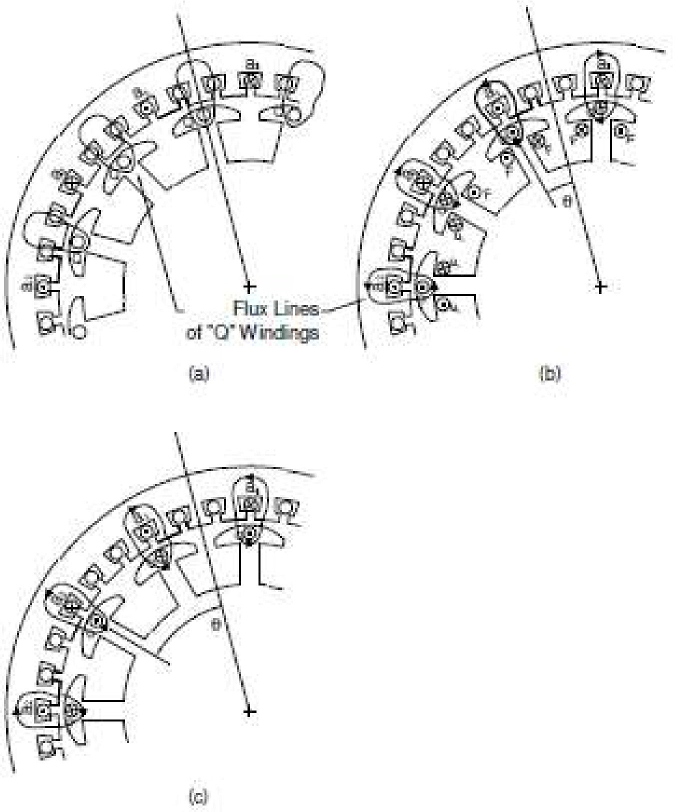


Figure 3-11 Flux lines generated by the current in "Q" windings and linking the "a" windings for different rotor positions

1) $\Theta_m = 2k\pi/n_p\ (k = 0,1,\dots)$: no flux linkage between “Q” and “F”, “Q” and “D” windings, hence $\mathrm{L}_{FQ} = \mathrm{L}_{DQ} = 0$. No flux linkage between “Q” and “a” windings due to symmetry.

2) $\Theta_m = (4k\pi + \pi)/n_p\ (k = 0,1,\dots)$: no flux linkage between “Q” and “F”, “Q” and “D” windings, hence $\mathrm{L}_{FQ} = \mathrm{L}_{DQ} = 0$. The direction of the induced flux by a current flowing into “a” winding is the same as the flux induced by current in “Q” windings. That is explaining the positive sign of mutual inductance L_{aQ} .

3) $\Theta_m = (4k\pi + 3\pi)/n_p\ (k = 0,1,\dots)$: no flux linkage between “Q” and “F”, “Q” and “D” windings, hence $\mathrm{L}_{FQ} = \mathrm{L}_{DQ} = 0$. The direction of the induced flux by a current flowing into “a” winding is the same as the flux induced by current in “Q” windings. That is explaining the negative sign of mutual inductance L_{aQ} for this case.

The Park's Transformation

For description of steady state operation of synchronous generator Park's transformation are going to be used. The idea of such type of transformation is that "to formulate the rotating magnetic field produced by 3-phase sinusoidal current flowing in the armature windings in a coordinate system fixed to the rotor" (Shaheri, 2011). It is possible to model the stator magnetic field as the result of fractious windings fixed to the rotor and carrying dc currents because of stator magnetic self- inductances of the fictitious windings and their mutual inductance with rotor windings will no longer depend on the rotor angle. To obtain the projection of the stator magnetomotive force onto the rotor frame the following expression is going to be used:

$$\boldsymbol{H} = H_a\boldsymbol{u_a} + H_b\boldsymbol{u_b} + H_c\boldsymbol{u_c} \tag{3-14}$$

where:

$\boldsymbol{H}$ – resultant magnetomotive force vector at the rotor axis measured in the stator reference frame [A.turns]

$H_a(H_b, H_c)$ – resultant magnetomotive force of the current flowing in the "a"("b", "c") winding measured in the stator reference frame [A.turns]

$\boldsymbol{u_a}(\boldsymbol{u_b}, \boldsymbol{u_c})$ – unit vector along the "a"("b", "c") axis

For rotor reference frame $\boldsymbol{H}$ can be shown as:

$$\boldsymbol{H} = \widetilde{H_d}\boldsymbol{u_d} + \widetilde{H_q}\boldsymbol{u_q} \tag{3-15}$$

where:

$\widetilde{H_d}(\widetilde{H_q})$ – magnitude of projection of H onto the "d"("q") axis, m^2

$\boldsymbol{u_d}(\boldsymbol{u_q})$ – unit vector along the "d"("q")

Terms of $\boldsymbol{u_d}$ and $\boldsymbol{u_q}$ will be used to rewrite unit vectors $\boldsymbol{u_a}, \boldsymbol{u_b}, \boldsymbol{u_c}$ to calculate $\widetilde{H_d}$ and $\widetilde{H_q}$:

$$\begin{aligned}
\boldsymbol{u_a} &= \cos\left(\frac{n_p\theta_m}{2}\right)\boldsymbol{u_d} + \sin\left(\frac{n_p\theta_m}{2}\right)\boldsymbol{u_q} \\
\boldsymbol{u_b} &= \cos\left(\frac{n_p\theta_m}{2} - \frac{2\pi}{3}\right)\boldsymbol{u_d} + \sin\left(\frac{n_p\theta_m}{2} - \frac{2\pi}{3}\right)\boldsymbol{u_q} \\
\boldsymbol{u_c} &= \cos\left(\frac{n_p\theta_m}{2} + \frac{2\pi}{3}\right)\boldsymbol{u_d} + \sin\left(\frac{n_p\theta_m}{2} + \frac{2\pi}{3}\right)\boldsymbol{u_q}
\end{aligned} \tag{3-16}$$

Putting (3-16) to (3-17) the following expression is received:

$$\begin{aligned}
\boldsymbol{H} = &\left[H_a\cos\left(\frac{n_p\theta_m}{2}\right) + H_b\cos\left(\frac{n_p\theta_m}{2} - \frac{2\pi}{3}\right) + H_c\cos\left(\frac{n_p\theta_m}{2} + \frac{2\pi}{3}\right)\right]\boldsymbol{u_d} \\
&+ \left[H_a\sin\left(\frac{n_p\theta_m}{2}\right) + H_b\sin\left(\frac{n_p\theta_m}{2} - \frac{2\pi}{3}\right) + H_c\sin\left(\frac{n_p\theta_m}{2} + \frac{2\pi}{3}\right)\right]\boldsymbol{u_q}
\end{aligned} \tag{3-17}$$

(3-16) can be rewritten as:

$$\begin{bmatrix}\widetilde{H_d}\\ \widetilde{H_q}\end{bmatrix} = \begin{bmatrix}\cos\left(\frac{n_p\theta_m}{2}\right) & \cos\left(\frac{n_p\theta_m}{2}-\frac{2\pi}{3}\right) & \cos\left(\frac{n_p\theta_m}{2}+\frac{2\pi}{3}\right)\\ \sin\left(\frac{n_p\theta_m}{2}\right) & \sin\left(\frac{n_p\theta_m}{2}-\frac{2\pi}{3}\right) & \sin\left(\frac{n_p\theta_m}{2}+\frac{2\pi}{3}\right)\end{bmatrix}\begin{bmatrix}H_a\\ H_b\\ H_c\end{bmatrix} = \tilde{P}\begin{bmatrix}H_a\\ H_b\\ H_c\end{bmatrix} \tag{3-18}$$

It is known, that:

$$\tilde{P}\tilde{P}^T = \begin{bmatrix}\frac{3}{2} & 0\\ 0 & \frac{3}{2}\end{bmatrix} \textbf{ and } \tilde{P}\begin{bmatrix}1\\ 1\\ 1\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix} \tag{3-19}$$

That is why orthonormal transformations can be applied:

$$\boldsymbol{P} = \sqrt{\frac{2}{3}}\begin{bmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\ \cos\left(\frac{n_p\theta_m}{2}\right) & \cos\left(\frac{n_p\theta_m}{2}-\frac{2\pi}{3}\right) & \cos\left(\frac{n_p\theta_m}{2}+\frac{2\pi}{3}\right)\\ \sin\left(\frac{n_p\theta_m}{2}\right) & \sin\left(\frac{n_p\theta_m}{2}-\frac{2\pi}{3}\right) & \sin\left(\frac{n_p\theta_m}{2}+\frac{2\pi}{3}\right)\end{bmatrix} \tag{3-20}$$

$$PP^T = P^TP = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

Applying the transform $\boldsymbol{P}$ to $\begin{bmatrix}H_a\\ H_b\\ H_c\end{bmatrix}$**,** the following can be obtained:

$$\begin{bmatrix}H_0\\ H_d\\ H_q\end{bmatrix} = \begin{bmatrix}\sqrt{\frac{1}{3}}(H_a+H_b+H_c)\\ \sqrt{\frac{2}{3}}\widetilde{H_d}\\ \sqrt{\frac{2}{3}}\widetilde{H_q}\end{bmatrix} = \boldsymbol{P}\begin{bmatrix}H_a\\ H_b\\ H_c\end{bmatrix} \tag{3-21}$$

In the current operation case the sum $(H_a + H_b + H_c)$ is equal to zero because of (Shaheri, 2011):

1) The armature windings are star connected, no ground connection
2) No saturation effect or no imbalance armature currents.

If the values of $(\widetilde{H_d}, \widetilde{H_q})$ are known, the values of set (H_a, H_b, H_c) can be determined any time then, and vice versa.

Application of correction factor $\sqrt{\frac{3}{2}}$, H_d and H_q allows preserving the direction of stator magnetic field measured in rotor frame and preserving its strength. Moreover $\begin{bmatrix}H_a\\ H_b\\ H_c\end{bmatrix}$ is proportional to the armature current vector $\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$. That allows usage of transform $\boldsymbol{P}$ “to study the effects of a three phase sinusoidal current flowing in the armature windings on the magnetic field seen by the rotor” (Shaheri, 2011). Transform $\boldsymbol{P}$ can also be applied to transform voltages across the armature into the rotor frame.

The Park`s transform matrix can be rewritten in the following way, inserting (3-21):

$$\boldsymbol{P} = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \cos(\theta_e) & \cos(\theta_e') & \cos(\theta_e'') \\ \sin(\theta_e) & \sin(\theta_e') & \sin(\theta_e'') \end{bmatrix} \tag{3-22}$$

Flux Linkage Equations

Park’s transformation matrix can be written as:

$$\boldsymbol{P_{ext}} = \begin{bmatrix} \boldsymbol{P} & \boldsymbol{O}^{3\times3} \\ \boldsymbol{O}^{3\times3} & \boldsymbol{I}^{3\times3} \end{bmatrix}, \boldsymbol{P_{ext}^T P_{ext}} = \boldsymbol{P_{ext} P_{ext}^T} = \boldsymbol{I} \tag{3-23}$$

Combining (3-22) with (3-23) allows receiving the following:

$$\begin{bmatrix} \psi_0 \\ \psi_d \\ \psi_q \\ \psi_F \\ \psi_D \\ \psi_Q \end{bmatrix} = \boldsymbol{P_{ext}} \begin{bmatrix} \psi_a \\ \psi_b \\ \psi_c \\ \psi_F \\ \psi_D \\ \psi_Q \end{bmatrix} = \boldsymbol{P_{ext} L} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_F \\ i_D \\ i_Q \end{bmatrix} = \boldsymbol{P_{ext} L P_{ext}^T} \begin{bmatrix} i_0 \\ i_d \\ i_q \\ i_F \\ i_D \\ i_Q \end{bmatrix} \tag{3-24}$$

where $\boldsymbol{L}$ is new inductance matrix in the rotor frame (Shaheri, 2011).

L can be independent from rotor angle, that makes possible to rewrite flux equations:

$$\begin{bmatrix} \psi_0 \\ \psi_d \\ \psi_q \\ \psi_F \\ \psi_D \\ \psi_Q \end{bmatrix} = \begin{bmatrix} L_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_d & 0 & kM_F & kM_D & 0 \\ 0 & 0 & L_q & 0 & 0 & kM_Q \\ 0 & kM_F & 0 & L_F & M_R & 0 \\ 0 & kM_D & 0 & M_R & L_D & 0 \\ 0 & 0 & kM_Q & 0 & 0 & L_Q \end{bmatrix} \begin{bmatrix} i_0 \\ i_d \\ i_q \\ i_F \\ i_D \\ i_Q \end{bmatrix} \tag{3-25}$$

where:

$$L_0 = L_s - 2M_s, L_d = L_s + M_s + k^2 L_m, L_q = L_s + M_s + k^2 L_m, k = \sqrt{\frac{3}{2}}$$

Voltage equations

The electrical connections can be explained using the conventional directions for currents and voltages. Taking into account dependence of mutual inductances on the rotor angle, the equation for voltage directions can be written:

$$\begin{bmatrix} \mathrm{v_a} \\ \mathrm{v_b} \\ \mathrm{v_c} \\ -\mathrm{v_F} \\ 0 \\ 0 \end{bmatrix} = \underbrace{\begin{bmatrix} \mathrm{r} & & & & & \\ & \mathrm{r} & & & & \\ & & \mathrm{r} & & & \\ & & & \mathrm{r_F} & & \\ & & & & \mathrm{r_D} & \\ & & & & & \mathrm{r_Q} \end{bmatrix}}_{\boldsymbol{R}} \begin{bmatrix} \mathrm{i_a} \\ \mathrm{i_b} \\ \mathrm{i_c} \\ \mathrm{i_F} \\ \mathrm{i_D} \\ \mathrm{i_Q} \end{bmatrix} - \frac{\mathrm{d}}{\mathrm{dt}} \underbrace{\begin{bmatrix} \psi_\mathrm{a} \\ \psi_\mathrm{b} \\ \psi_\mathrm{c} \\ \psi_\mathrm{F} \\ \psi_\mathrm{D} \\ \psi_\mathrm{Q} \end{bmatrix}}_{\boldsymbol{\psi}} \qquad (3\text{-}26)$$

Using extended Park's transform to (3-25) , the following will be result:

$$\begin{bmatrix} \mathrm{v_a} \\ \mathrm{v_b} \\ \mathrm{v_c} \\ -\mathrm{v_F} \\ 0 \\ 0 \end{bmatrix} = -(\boldsymbol{P_{ext} L P_{ext}^T}) \begin{bmatrix} \mathrm{i_a} \\ \mathrm{i_b} \\ \mathrm{i_c} \\ \mathrm{i_F} \\ \mathrm{i_D} \\ \mathrm{i_Q} \end{bmatrix} - \boldsymbol{P_{ext}} \frac{\mathrm{d}}{\mathrm{dt}} \left(\boldsymbol{P_{ext}^T} \begin{bmatrix} \psi_\mathrm{a} \\ \psi_\mathrm{b} \\ \psi_\mathrm{c} \\ \psi_\mathrm{F} \\ \psi_\mathrm{D} \\ \psi_\mathrm{Q} \end{bmatrix} \right)$$

$$= -(\boldsymbol{P_{ext} L P_{ext}^T}) \begin{bmatrix} \mathrm{i_0} \\ \mathrm{i_d} \\ \mathrm{i_q} \\ \mathrm{i_F} \\ \mathrm{i_D} \\ \mathrm{i_Q} \end{bmatrix} - \boldsymbol{P_{ext}} \left(\boldsymbol{P_{ext}^T} \begin{bmatrix} \dot{\psi}_0 \\ \dot{\psi}_\mathrm{d} \\ \dot{\psi}_\mathrm{q} \\ \dot{\psi}_\mathrm{F} \\ \dot{\psi}_\mathrm{D} \\ \dot{\psi}_\mathrm{Q} \end{bmatrix} + \boldsymbol{\dot{P}_{ext}^T} \begin{bmatrix} \psi_0 \\ \psi_\mathrm{d} \\ \psi_\mathrm{q} \\ \psi_\mathrm{F} \\ \psi_\mathrm{D} \\ \psi_\mathrm{Q} \end{bmatrix} \right) = -R \begin{bmatrix} \mathrm{i_0} \\ \mathrm{i_d} \\ \mathrm{i_q} \\ \mathrm{i_F} \\ \mathrm{i_D} \\ \mathrm{i_Q} \end{bmatrix} - \begin{bmatrix} \dot{\psi}_0 \\ \dot{\psi}_\mathrm{d} \\ \dot{\psi}_\mathrm{q} \\ \dot{\psi}_\mathrm{F} \\ \dot{\psi}_\mathrm{D} \\ \dot{\psi}_\mathrm{Q} \end{bmatrix} - \boldsymbol{P_{ext} \dot{P}_{ext}^T} \begin{bmatrix} \psi_0 \\ \psi_\mathrm{d} \\ \psi_\mathrm{q} \\ \psi_\mathrm{F} \\ \psi_\mathrm{D} \\ \psi_\mathrm{Q} \end{bmatrix}$$

Shortly it can be expressed as:

$$\boldsymbol{P_{ext} \dot{P}_{ext}^T} = \begin{bmatrix} \boldsymbol{P\dot{P}^T} & \boldsymbol{O}^{3\times3} \\ \boldsymbol{O}^{3\times3} & \boldsymbol{O}^{3\times3} \end{bmatrix} = \begin{bmatrix} \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{matrix} & \boldsymbol{O}^{3\times3} \\ \boldsymbol{O}^{3\times3} & \boldsymbol{O}^{3\times3} \end{bmatrix} \times \underbrace{\frac{\boldsymbol{d\theta_e}}{\boldsymbol{dt}}}_{\omega_e}$$

Consequently the voltage equations are:

$$\begin{bmatrix} \mathrm{v_0} \\ \mathrm{v_d} \\ \mathrm{v_q} \\ -\mathrm{v_F} \\ 0 \\ 0 \end{bmatrix} = \underbrace{\begin{bmatrix} \mathrm{r} & & & & & \\ & \mathrm{r} & & & & \\ & & \mathrm{r} & & & \\ & & & \mathrm{r_F} & & \\ & & & & \mathrm{r_D} & \\ & & & & & \mathrm{r_Q} \end{bmatrix}}_{\mathbf{R}} \begin{bmatrix} \mathrm{i_0} \\ \mathrm{i_d} \\ \mathrm{i_q} \\ \mathrm{i_F} \\ \mathrm{i_D} \\ \mathrm{i_Q} \end{bmatrix} - \begin{bmatrix} \dot{\psi}_0 \\ \dot{\psi}_\mathrm{d} \\ \dot{\psi}_\mathrm{q} \\ \dot{\psi}_\mathrm{F} \\ \dot{\psi}_\mathrm{D} \\ \dot{\psi}_\mathrm{Q} \end{bmatrix} - \underbrace{\begin{bmatrix} 0 \\ \omega_e \psi_\mathrm{d} \\ -\omega_e \psi_\mathrm{q} \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{\mathrm{P_{ext}\dot{P}_{ext}^T \psi}} \qquad (3\text{-}27)$$

The Swing Equation

For description of amount of energy available in rotating parts of hydropower plant, rate of its change can be given as (Shaheri, 2011):

$$J\omega_m \frac{d\omega_m}{dt} = P_{in} - P_{loss} - P_{out} = \eta_g \eta_t \rho g H_t Q_t - P_{out}$$
$$\omega_m = \frac{d\theta_m}{dt} \tag{3-28}$$

where

P_{in} – hydraulic power transferred to turbine, W

P_{out} – active electric power output at terminals of generator, W

P_{loss} – power losses through turbine and generator, W

H_t - turbine head, m

Q_t – turbine volumetric discharge, m^3/sec

η_g – overall efficiency of generator

η_t – overall efficiency of turbine (hydraulic and mechanical).

Active electric power output is:

$$P_{out} = v_a i_a + v_b i_b + v_c i_c = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}^T \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}^T \mathbf{P^T P} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$
$$= \begin{bmatrix} v_0 \\ v_d \\ v_q \end{bmatrix}^T \begin{bmatrix} i_0 \\ i_d \\ i_q \end{bmatrix} = v_0 i_a + v_d i_b + v_q i_c \tag{3-29}$$

Generator Model Simplification

The following equations are used to form a model:

- the flux linkage equation (3.16)
- the voltage equation (3.18)
- the Park's transformation (3.13)
- the swing equation (3.19)

To decrease order of the equations the following assumption can be used:

$$|\dot{\psi}_d| \ll |\omega_e \psi_q|, |\dot{\psi}_q| \ll |\omega_e \psi_d| \tag{3-30}$$

According to (Machowski, 2008) it can be assumed that:

- in *(3-30)* terms $\dot{\psi}_d$, $\dot{\psi}_q$ are ignored
- currents and voltages are balanced, consequently v_0 and i_0 , and hence ψ_0 and $\dot{\psi}_0$ will be equal to zero. That leads to removal of the flux linkage and the voltage equation along the "0" axis to reduce model order.

- Existence of one or both damping of the damper windings can be ignored
- Ignoring of $\dot{\psi}_F$ allows ignoring of dynamics of changes in the field current in (3-30). As the result the steady state of generator voltage-current relationship can be received.

Applying all mentioned above towards (3.27) the following can be obtained:

$$\begin{aligned} & \begin{bmatrix} v_d \\ v_q \end{bmatrix} = -\begin{bmatrix} r & 0 \\ 0 & r \end{bmatrix}\begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} -\omega_e \psi_q \\ \omega_e \psi_d \end{bmatrix} \\ & = -\begin{bmatrix} r & 0 \\ 0 & r \end{bmatrix}\begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} -\omega_e (L_q i_q + kM_Q i_Q \\ \omega_e (L_d i_d + kM_F i_F + kM_D i_D) \end{bmatrix} \end{aligned} \tag{3-31}$$

For stable operating network the value of ω_e, generator frequency, is not constant, and is characterized by zero-mean oscillations around the system center of inertia frequency $f_s(t)$. As it is mention in (Andersson, 2012): “Normally $f_s(t)$ has very slow variations and can be regarded constant if the time duration for studying the generator dynamics is not too long.”

The following formula can be used for calculation of the rotor electrical angle of generator:

$$\begin{gathered} \theta_e(t) = 2\pi f_s t + \delta_s(t) \\ \delta_e(t) = \delta_e(0) + \int_0^t [\omega_e(\tau) - 2\pi f_s] d\tau = \delta_e(0) + \int_0^t \Delta\omega_e(\tau) d\tau \end{gathered} \tag{3-32}$$

It can be assumed:

$$|\Delta\omega_e(t)| \ll \omega_s = 2\pi f_s \tag{3-33}$$

where f_s almost constant system frequency.

Applying above mentioned assumptions, the following is obtained:

$$\begin{gathered} \begin{bmatrix} v_d \\ v_q \end{bmatrix} = -\begin{bmatrix} r & x_q \\ -x_d & r \end{bmatrix}\begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} \omega_s kM_Q i_Q \\ \omega_s kM_F i_F + \omega_s kM_D i_D) \end{bmatrix} \\ x_d = \omega_s L_d, \; x_q = \omega_s L_q \end{gathered} \tag{3-34}$$

It is worth to mention that time accumulation of $\Delta\omega_e(t)$ influences on the system model’s voltage-current relationship due to changes in the Park’s transform. If generator is in parallel with ideal three-phase voltage source, the terminal voltages can be calculated as:

$v_a(t) = V\sin\omega_s t$, $v_b(t) = V\sin(\omega_s t - \frac{2\pi}{3})$, $v_c(t) = V\sin(\omega_s t + \frac{2\pi}{3})$.

So transformed voltages can be rewritten as $\begin{bmatrix} v_d \\ v_q \end{bmatrix} = \begin{bmatrix} V\sin\delta \\ V\cos\delta \end{bmatrix}$, that is a confirmation of the fact that $\Delta\omega_e(t)$ influences on $\begin{bmatrix} v_d \\ v_q \end{bmatrix}$ by changing rotor angle.

Generator Steady-State

It is known that steady state field current value i_f is calculated as v_f/r_f, transient term in the field current is Δi_f, that is why it can be written $i_f = \frac{v_f}{r_f} + \Delta i_f$. That allows the following transformation:

$$\begin{bmatrix} \mathrm{v_d} \\ \mathrm{v_q} \end{bmatrix} = -\begin{bmatrix} \mathrm{r} & \mathrm{x_q} & 0 \\ -\mathrm{x_d} & \mathrm{r} & -\omega_s \mathrm{kM_F/r_F} \end{bmatrix} \begin{bmatrix} \mathrm{i_d} \\ \mathrm{i_q} \\ \mathrm{v_F} \end{bmatrix} + \begin{bmatrix} -\omega_s \mathrm{kM_Q i_Q} \\ \omega_s \mathrm{kM_F} \Delta \mathrm{i_f} + \omega_s \mathrm{kM_D i_D}) \end{bmatrix} \tag{3-35}$$

That is an output equation for system with three inputs, two outputs and three states ($\mathrm{i_d}$, $\mathrm{i_q}$ and $\Delta\mathrm{i_f}$ which tend to zero as the system reaches a steady–state condition) (Shaheri, 2011). The following combinations can be chosen as independent variables: ($\mathrm{i_d}, \mathrm{i_q}, \mathrm{v_F}$), ($\mathrm{v_d}, \mathrm{i_q}, \mathrm{v_F}$), ($\mathrm{i_d}, \mathrm{v_q}, \mathrm{v_F}$), ($\mathrm{v_d}, \mathrm{v_q}, \mathrm{v_F}$). The other variables are chosen as outputs for each case. Some correlations are possible, such as ($\mathrm{i_d}$, $\mathrm{i_q}$) and ($\mathrm{v_d}$, $\mathrm{v_q}$) because of the load dynamics, or $\mathrm{v_F}$ and ($\mathrm{v_d}$, $\mathrm{v_q}$) because of the system excitation. Obviously it is expected that any disturbance of the system will lead to change of variables' values. The changes, for example in the load, can be in form of step, that allows the system getting a new steady state conditions, while terms $\mathrm{i_Q}$, $\mathrm{i_D}$ and $\Delta\mathrm{i_f}$ vanish. Then the following steady state values ($\mathrm{i_d}, \mathrm{i_q}, \mathrm{v_d}, \mathrm{v_q}, \mathrm{v_F}$) should meet the following equation:

$$\begin{bmatrix} \mathrm{v_d} \\ \mathrm{v_q} \end{bmatrix} = -\begin{bmatrix} \mathrm{r} & \mathrm{x_q} & 0 \\ -\mathrm{x_d} & \mathrm{r} & -\omega_s \mathrm{kM_F/r_F} \end{bmatrix} \begin{bmatrix} \mathrm{i_d} \\ \mathrm{i_q} \\ \mathrm{v_F} \end{bmatrix} \tag{3-36}$$

which can be shown as:

$$\begin{bmatrix} \mathrm{v_d} \\ \mathrm{v_q} \end{bmatrix} == -\begin{bmatrix} \mathrm{r} & \mathrm{x_q} \\ -\mathrm{x_d} & \mathrm{r} \end{bmatrix} \begin{bmatrix} \mathrm{i_d} \\ \mathrm{i_q} \end{bmatrix} + \begin{bmatrix} 0 \\ e_q \end{bmatrix}$$

$$e_d = 0, e_q = e_f = \omega_s \mathrm{kM_F/r_F} \tag{3-37}$$

To explain the simplest generator model the swing equations, the Park's transformation and equation (3-37) can be used. In this case damper windings are neglected and transient currents induced.

Operations' Assumptions

As it was mentioned above, system will experience different operational states. That is why it is crucial to set operations' assumptions with a view to the fact that "after any disturbance system will reach a new steady-state condition as the induced currents $\mathrm{i_Q}$, $\mathrm{i_D}$ and $\Delta\mathrm{i_f}$ vanish" (Shaheri, 2011):

- The most fast going phenomena in generator is reduction of induced current in the damper (the order of time constant is from 0,01 to 0,1seconds). That is why all duration variables, except from $\mathrm{i_Q}$ and $\mathrm{i_D}$, can be taken as constant. Consequently the generator operation within this time interval can be named as "subtransient".

- The generator operation after damper currents disappeared till the moment when generator reaches the new steady state (time interval for this period within field windings has an order from 1 to 10) is named as "transient".

These two assumptions lead to further simplifications that will be explained further.

In case of "generator dynamics with transient phenomenon" model above stated assumptions can be more detailed with taking into account transient field current effect on the generator voltage-current characteristics. Neglecting the damper currents, it can be got:

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} == -\begin{bmatrix} r & x_q \\ -x_d & r \end{bmatrix}\begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} 0 \\ \omega_s k M_F i_F \end{bmatrix} \qquad (3\text{-}38)$$

Neglecting damper current i_D the following is received:

$$\psi_F = kM_F i_F + L_F i_F \quad \Rightarrow \; i_F = \frac{1}{L_F}\psi_F - \frac{kM_F}{L_F} i_d$$

$$v_F = r_F i_F + \dot{\psi}_F \Rightarrow \; i_F = \frac{V_F}{r_F} i_d - \frac{1}{r_F}\dot{\psi}_F \qquad (3\text{-}39)$$

Combining (3-38) and (3-39) the following voltage equations can be obtained:

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} == -\begin{bmatrix} r & x'_q \\ -x'_d & r \end{bmatrix}\begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} e'_d \\ e'_q \end{bmatrix}$$

$$x'_d = \omega_s L'_d \;\; , \; x'_q = x_q$$

$$L'_d = L_d - \frac{k^2 M_F^2}{L_F} \qquad (3\text{-}40)$$

$$e'_d = 0, e'_q = \omega_s \frac{kM_F}{L_F}\psi_F$$

Eliminating i_F from both of *(3-39)* equations, it can be received:

$$\frac{v_F}{r_F} i_d - \frac{1}{r_F}\dot{\psi}_F = \frac{1}{L_F}\psi_F - \frac{kM_F}{L_F} i_d$$

Multiplying both sides by $\omega_s k M_F$, allows obtaining of the following:

$$\frac{L_F}{r_F}\omega_s \frac{kM_F}{L_F}\dot{\psi}_F + \omega_s \frac{kM_F}{L_F}\psi_F - \omega_s \frac{k^2 M_F^2}{L_F} i_d - \omega_s k M_F \frac{v_F}{r_F}$$

It can be written:

$$T'_{do}\frac{de'_q}{dt} = -e'_q + e_q + (x_d - x'_d)\, i_d$$

$$T'_{do} = \frac{L_F}{r_F} \qquad (3\text{-}41)$$

$$\text{Assumption: } \dot{\omega}_s \approx 0$$

According to (3-41) it can be said that system frequency ω_s is almost constant, that is why the following statement will work: $\frac{de'_q}{dt} = \frac{d}{dt}\left(\omega_s \frac{kM_F}{L_F}\psi_F\right) = \omega_s \frac{kM_F}{L_F}\dot{\psi}_F$.

Consequently, (3-41) swing equations, Park's transform and definitions x_d , x_q , e_d and e_q form the transient model of generator in which variations of the field current are used.

To formulate "generator dynamics with both transient and subtransient phenomena" equation *(3-38)* will be analyzed without neglecting of any terms. Using terms of $i_d, i_q,$ $\psi_F,$ ψ_D and ψ_Q, i_F, i_D and i_Q can be evaluated:

$$\psi_Q = kM_Q i_q + L_Q i_Q \Rightarrow\ i_Q = \frac{1}{L_Q}\psi_Q - \frac{kM_Q}{L_Q} i_q$$

$$\left.\begin{matrix} \psi_F = kM_F i_d + L_F i_F + M_R i_D \\ \psi_D = kM_D i_d + M_R i_F + L_D i_D \end{matrix}\right\} \Rightarrow \begin{cases} i_F = \dfrac{L_D(\psi_F - kM_F i_d) - M_R(\psi_D - kM_D i_d)}{L_F L_D - M_R^2} \\ i_D = \dfrac{L_D(\psi_D - kM_D i_d) - M_R(\psi_F - kM_F i_d)}{L_F L_D - M_R^2} \end{cases} \tag{3-42}$$

Applying (3-42) towards (3-38), the following can be received:

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = -\begin{bmatrix} r & x_q'' \\ -x_d'' & r \end{bmatrix}\begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} e_d'' \\ e_q'' \end{bmatrix}$$

$$x_q'' = \omega_s L_q'',\ \ L_q'' = L_q - \frac{kM_Q}{L_Q}$$

$$x_d'' = \omega_s L_d'',\quad L_d'' = L_{dq} - k^2\left(\frac{L_D M_F^2 + L_F M_D^2 - 2M_R M_D M_F}{L_F L_D - M_R^2}\right) \tag{3-43}$$

$$e_q'' = \omega_s k\left(\frac{L_D M_F - M_R M_D}{L_F L_D - M_R^2}\right)\psi_F + \omega_s k\left(\frac{L_D M_F - M_R M_F}{L_F L_D - M_R^2}\right)\psi_D$$

Using relations from (3-43) towards (3-25) and relations (3-43)allows getting of the following expressions for e_q'' and e_d'':

$$T_{do}' \frac{de_q''}{dt} = e_q' - e_q'' + (x_d' - x_d'')\, i_d$$

$$T_{qo}' \frac{de_q''}{dt} = e_d' - e_d'' + (x_q' - x_q'')\, i_q$$

$$T_{do}'' = \left(L_D - \frac{M_R^2}{L_F}\right)\frac{1}{r_D} \tag{3-44}$$

$$T_{do}'' = \frac{L_Q}{r_Q}$$

$$\text{Assumption: } \dot{\omega}_s \approx 0$$

Consequently, (3-43), (3-44) ,swing equations, Park's transform and definitions x_d', x_q', e_d' and e_q' form the complete model of generator.

Generator Models

As it is seen there are three models of generator work for which it can be done one general assumption (Shaheri, 2011) that generator efficiency η_g involves resistive losses in armature windings:

$$P_{out} = 3\left[V_d I_d + V_q I_q + r(I_d^2 + I_q^2)\right] \tag{3-45}$$

Formula (3-45) is going to be used further for Voltage-Current relations for generator models.

Based on theoretical explanations that were shown above the following generator models can be formulated:

1. **Steady state generator model** is the simplest model, in which transient effect is neglected. The model is shown in Table 3-5.

Table 3-5 Steady state generator model

$$\begin{bmatrix} \mathrm{V_d} \\ \mathrm{V_q} \end{bmatrix} = -\begin{bmatrix} \mathrm{r} & \mathrm{x_q} \\ -\mathrm{x_d} & \mathrm{r} \end{bmatrix} \begin{bmatrix} \mathrm{I_d} \\ \mathrm{I_q} \end{bmatrix} + \begin{bmatrix} E_d \\ E_q \end{bmatrix},$$

where:

$$E_d = 0,\ E_q = E_f = \frac{\mathrm{e_q}}{\sqrt{3}} = \frac{\omega_s \mathrm{kM_F\ v_f}}{\mathrm{r_f}\sqrt{3}},\quad \mathrm{x_d} = \omega_s \mathrm{L_D},\ \mathrm{x_q} = \omega_s \mathrm{L_q}$$

$\underline{\mathrm{V_t}} = \mathrm{e}^{j\delta_e}(\mathrm{V_q} + j\mathrm{V_d})$ –phasor of terminal voltage in the system reference frame

$\underline{\mathrm{I_t}} = \mathrm{e}^{j\delta_e}(\mathrm{I_q} + j\mathrm{I_d})$ - phasor of output current in the system reference frame

With assumption , that $\mathrm{v_o} = \mathrm{i_o} = 0$ and taking into account that $\mathrm{V_d} = \frac{\mathrm{v_d}}{\sqrt{3}}$ and the same can be applied for other voltages and currents, $\mathrm{P_{out}} = \left[\mathrm{E_d I_d} + \mathrm{E_q I_q} + (\mathrm{x_d} - \mathrm{x_q})\mathrm{I_q I_d}\right]$

The Swing Equation (steady state):

$\mathrm{P_{out}} = \eta_g \eta_t \rho g \mathrm{H_t Q_t},\ \delta_e = const$

2. **Transient state generator model**
   The model is shown in Table 3-6.

Table 3-6 Transient state generator model

$$\mathrm{T}'_{\mathrm{do}}\frac{\mathrm{dE}'_{\mathrm{q}}}{\mathrm{dt}} = -\mathrm{E}'_{\mathrm{q}} + \mathrm{E}_{\mathrm{q}} + (\mathrm{x}_{\mathrm{d}} - \mathrm{x}'_{\mathrm{d}})\,\mathrm{I}_{\mathrm{d}}$$

$$\mathrm{T}'_{\mathrm{do}} = \frac{\mathrm{L}_{\mathrm{F}}}{\mathrm{r}_{\mathrm{F}}}, \mathrm{E}'_{\mathrm{q}} = \mathrm{E}_{\mathrm{f}} = \frac{\omega_s \mathrm{kM}_{\mathrm{F}}\,\mathrm{v}_{\mathrm{f}}}{\mathrm{rL}_{\mathrm{f}}\sqrt{3}}, \mathrm{E}'_{\mathrm{q}} = \frac{\omega_s \mathrm{kM}_{\mathrm{F}}\,\mathrm{v}_{\mathrm{f}}}{\mathrm{L}_{\mathrm{f}}\sqrt{3}}\psi_{\mathrm{F}},$$

$$\mathrm{x}_{\mathrm{d}} = \omega_s \mathrm{L}_{\mathrm{d}}, \mathrm{x}'_{\mathrm{d}} = \omega_s \mathrm{L}'_{\mathrm{d}}$$

$$\mathrm{L}'_{\mathrm{d}} = \mathrm{L}_{\mathrm{d}} - \frac{\mathrm{k}^2\mathrm{M}_{\mathrm{F}}^2}{\mathrm{L}_{\mathrm{F}}}$$

$$\begin{bmatrix}\mathrm{V}_{\mathrm{d}}\\ \mathrm{V}_{\mathrm{q}}\end{bmatrix} == -\begin{bmatrix}\mathrm{r} & \mathrm{x}'_{\mathrm{q}}\\ -\mathrm{x}'_{\mathrm{d}} & \mathrm{r}\end{bmatrix}\begin{bmatrix}\mathrm{I}_{\mathrm{d}}\\ \mathrm{I}_{\mathrm{q}}\end{bmatrix}+\begin{bmatrix}E'_d\\ E'_q\end{bmatrix}$$

$$\mathrm{x}'_{\mathrm{q}} = \mathrm{x}_{\mathrm{q}} = \omega_s \mathrm{L}_{\mathrm{q}}, \ \mathrm{E}'_{\mathrm{d}} = 0$$

$\underline{\mathrm{V}_{\mathrm{t}}} = \mathrm{e}^{j\delta_e}(\mathrm{V}_{\mathrm{q}} + j\mathrm{V}_{\mathrm{d}})$ –phasor of terminal voltage in the system reference frame

$\underline{\mathrm{I}_{\mathrm{t}}} = \mathrm{e}^{j\delta_e}(\mathrm{I}_{\mathrm{q}} + j\mathrm{I}_{\mathrm{d}})$ - phasor of output current in the system reference frame

With assumption , that $\mathrm{v}_{\mathrm{o}} = \mathrm{i}_{\mathrm{o}} = 0$ and taking into account that $\mathrm{V}_{\mathrm{d}} = \frac{\mathrm{v}_{\mathrm{d}}}{\sqrt{3}}$ and the same can be applied for other voltages and currents, $\mathrm{P}_{\mathrm{out}} = \left[E'_d\mathrm{I}_{\mathrm{d}} + E'_d\mathrm{I}_{\mathrm{q}} + (\mathrm{x}'_{\mathrm{d}} - \mathrm{x}'_{\mathrm{q}})\mathrm{I}_{\mathrm{q}}\mathrm{I}_{\mathrm{d}}\right]$

The Swing Equation (steady state):

$$\mathrm{J}\omega_{\mathrm{m}}\frac{\mathrm{d}\omega_{\mathrm{m}}}{\mathrm{dt}} = \eta_{\mathrm{g}}\eta_{\mathrm{t}}\rho \mathrm{g}\mathrm{H}_{\mathrm{t}}\mathrm{Q}_{\mathrm{t}} - \mathrm{P}_{\mathrm{out}}$$

$$\omega_{\mathrm{e}} = \frac{\mathrm{n}_{\mathrm{p}}}{2}\omega_{\mathrm{m}}, \qquad \Delta\omega_{\mathrm{e}} = \omega_{\mathrm{e}} - \omega_{\mathrm{s}} = \frac{\mathrm{d}\delta_e}{\mathrm{dt}}$$

It is worth to mention that from experience of (Shaheri, 2011) for transient state explanation “any current or voltage signal within the system” can be shown as:

$$\mathbf{s(t)} = \sqrt{2}S(t)\begin{bmatrix}\cos(2\pi f_s + \alpha(t))\\ \cos(2\pi f_s - \frac{2\pi}{3} + \alpha(t))\\ \cos(2\pi f_s + \frac{2\pi}{3} + \alpha(t))\end{bmatrix}$$

where f_s is agreed reference frequency in the power system which is fixed or varies slowly (Andersson, 2012), $S(t)$ is he “root mean square (rms) value of each of the entries of the vector $\mathbf{s(t)}$" (Shaheri, 2011).

That is why for steady state operation or operating point (Shaheri, 2011):

$$\mathrm{I}_{\mathrm{to}} = \frac{\sqrt{P_o^2 + Q_o^2}}{3V_s}, \varphi_o = arctan\frac{Q_o}{P_o}, \underline{I_{to}} = I_{to}e^{-j\varphi_o} \tag{3-46}$$

where

P_o - an active power at steady state condition, [W]

Q_o - a reactive power at steady state condition, [rVA]

φ_o- a power angle at steady state, [radian]

I_{to} is RMS value of terminal phase current at steady state, [A]

$\underline{I_{to}}$ is phasor of out put current of generator at steady state, [A]

Putting $\underline{I_{to}}$ to Table 3-5 Steady state generator model will give the values for currents:

$$\begin{gathered} \mathrm{I_{do}} = -I_{to}\sin(\varphi_o + \delta_{eo}) \\ \mathrm{I_{qo}} = I_{to}\cos(\varphi_o + \delta_{eo}) \end{gathered} \tag{3-47}$$

where δ_{eo} is electrical rotor angle, [radian]

δ_{eo} can be calculated taking into in consideration Table 3-6 and be written as:

$$\delta_{eo} = arctan\frac{I_{to}(\mathrm{x}_q + \mathrm{x}_e)cos\varphi_o - I_{to}(\mathrm{r} + \mathrm{r}_e)sin\varphi_o}{\mathrm{V}_s + I_{to}(\mathrm{r} + \mathrm{r}_e)cos\varphi_o + I_{to}(\mathrm{x}_q + \mathrm{x}_e)cos\varphi_o} \tag{3-48}$$

where V_s is phase bus voltage, [V]

The voltage across the field winding using transfer function can be expressed as:

$$\mathrm{E_{fo}} = \mathrm{V}_s cos\delta_{eo} + I_{qo}(\mathrm{r} + \mathrm{r}_e) - I_{do}(\mathrm{x}_d + \mathrm{x}_e) \tag{3-49}$$

Terminal voltage is voltage reference set point for the exciter, which controls voltage across the field winding with transfer function[V], can be calculated as:

$$\mathrm{V_{tr}} = \mathrm{V}_{to} + \frac{\mathrm{E_{fo}}}{\mathrm{K}_E} \tag{3-50}$$

where

K_E is excitation system gain

V_{to} is phasor of terminal voltage of generator at steady state, [V]

3. Subtransient state generator model

This model is the most complete and takes into account subtransien calcult phenomenon in damper windings. The model details are shown in Table 3-7:

Table 3-7 Subtransient state generator model

$$\mathrm{T''_{do}}\frac{\mathrm{dE''_q}}{\mathrm{dt}} = \mathrm{E'_q} - \mathrm{E''_q} + (\mathrm{x'_d} - \mathrm{x''_d})\ \mathrm{I_d}$$

$$\mathrm{T''_{qo}}\frac{\mathrm{dE''_d}}{\mathrm{dt}} = \mathrm{E'_d} - \mathrm{E''_d} - (\mathrm{x'_q} - \mathrm{x''_q})\ \mathrm{I_q}$$

$$\mathrm{T''_{do}} = \left(\mathrm{L_d} - \frac{\mathrm{M_R^2}}{\mathrm{L_F}}\right)\frac{1}{\mathrm{r_D}},\ \mathrm{T''_{qo}} = \frac{\mathrm{L_Q}}{\mathrm{r_Q}},\ \mathrm{x''_q} = \omega_\mathrm{s}\mathrm{L''_q},\ \ \mathrm{L''_q} = \mathrm{L_q} - \frac{\mathrm{kM_Q}}{\mathrm{L_Q}}$$

$$\mathrm{x''_d} = \omega_\mathrm{s}\mathrm{L''_d},\ \ \ \mathrm{L''_d} = \mathrm{L_{dq}} - \mathrm{k}^2(\frac{\mathrm{L_D M_F^2 + L_F M_D^2 - 2M_R M_D M_F}}{\mathrm{L_F L_D - M_R^2}})$$

$$\mathrm{x'_q} = \mathrm{x_q} = \omega_s\mathrm{L_q}, \mathrm{x'_d} = \omega_s\mathrm{L'_d}\ \ , \mathrm{L'_d} = \mathrm{L_d} - \frac{\mathrm{k^2 M_F^2}}{\mathrm{L_F}},\ \mathrm{E'_d} = 0$$

$$\mathrm{T'_{do}}\frac{\mathrm{dE'_q}}{\mathrm{dt}} = -\mathrm{E'_q} + \mathrm{E_q}\ + (\mathrm{x_d}\ - \mathrm{x'_d})\ \mathrm{I_d}$$

$$\mathrm{T'_{do}} = \frac{\mathrm{L_F}}{\mathrm{r_F}}, \mathrm{E_q}\ = \mathrm{E_f} = \frac{\omega_s \mathrm{kM_F\ v_F}}{\mathrm{r_F}\sqrt{3}}, \mathrm{x_d} = \omega_s\mathrm{L_d}$$

$$\begin{bmatrix}\mathrm{V_d}\\ \mathrm{V_q}\end{bmatrix} == -\begin{bmatrix}\mathrm{r} & \mathrm{x''_q}\\ -\mathrm{x''_d} & \mathrm{r}\end{bmatrix}\begin{bmatrix}\mathrm{I_d}\\ \mathrm{I_q}\end{bmatrix} + \begin{bmatrix}E''_d\\ E''_q\end{bmatrix}$$

$\underline{\mathrm{V_t}} = \mathrm{e}^{j\delta_e}(\mathrm{V_q} + j\mathrm{V_d})$ –phasor of terminal voltage in the system reference frame

$\underline{\mathrm{I_t}} = \mathrm{e}^{j\delta_e}(\mathrm{I_q} + j\mathrm{I_d})$ - phasor of output current in the system reference frame

With assumption , that $\mathrm{v_o} = \mathrm{i_o} = 0$ and taking into account that $\mathrm{V_d} = \frac{\mathrm{v_d}}{\sqrt{3}}$ and the same can be applied for other voltages and currents, $\mathrm{P_{out}} = \left[E''_d\mathrm{I_d} + E''_d\mathrm{I_q} + (\mathrm{x''_d} - \mathrm{x''_q})\mathrm{I_q I_d}\right]$

The Swing Equation (steady state):

$$\mathrm{J}\omega_\mathrm{m}\frac{\mathrm{d}\omega_\mathrm{m}}{\mathrm{dt}} = \eta_\mathrm{g}\eta_\mathrm{t}\rho\mathrm{gH_tQ_t}\ \ - \mathrm{P_{out}}$$

$$\omega_\mathrm{e} = \frac{\mathrm{n_p}}{2}\omega_\mathrm{m},\qquad \Delta\omega_\mathrm{e} = \omega_\mathrm{e} - \omega_\mathrm{s} = \frac{\mathrm{d}\delta_e}{\mathrm{dt}}$$

4 Simulation of Hydro Power Plant library: structure

4.1 Modelica Basic Definitions

As it was stated earlier, before the following master thesis some attempts to build model library were done, for example (Shaheri, 2011), which was implemented using MATLAB as programming environment. In the following case OpenModelica will be used, which is "an open-source environment for modeling, simulation, and development of Modelica® applications" (Fritzson, 2005). Its properties are (Fritzson P. , 2006):

- Declarative and object-oriented, what means usage of acausal classes
- Equation-based
- Parallel process modeling of real-time applications according to synchronous data flow principle
- Functions with algorithms
- Class structure: models, functions, packages, parameterized classes etc.

For further work some definitions that are widely used regarding OpenModelica should be sated (Fritzson P. , 2006):

- Model is a class that cannot be used as connector class.
- Block is a class with fixed input-output causality.
- Function is a special kind of restricted class with some extensions. It can be called with arguments.
- Connector is an example of connector classes.
- Package is a container or name space for names of classes, functions, constants and other allowed definitions. While package designing some principles, such as the name of package, structuring of the package into subpackages, reusability and encapsulation of the package, dependencies on other packages should be taken into account.

Modelica® packages and subpackages are used to structure hierarchical model libraries, the example of which is Modelica library (Fritzson P. , 2006). It consists from sublibraries from different application fields, for example constants, electrical, icons, maths, mechanics, SI units, thermal etc.

The translation process in Modelica® environment is shown on Figure 4-1.

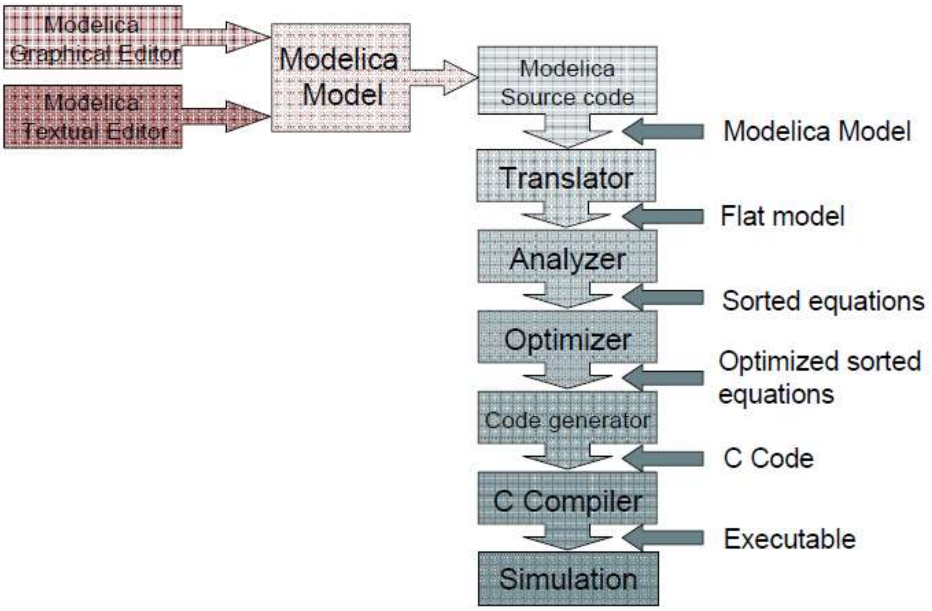


Figure 4-1 The translation process in Modelica environment (Fritzson P. , 2006)

Nowadays Modelica has a great range of environments. Modelon and Dymola were mentioned in section 1.2, but of current interest OpenModelica is, because it is free software for modelling and simulation of industrial processes (as hydrology is). However OpenModelica is open source implementation and is under constant development, that is why some sort of mistakes are expected to be faced.

4.2 Structure of Library

Considering example and previous studies for a future library it is worth to divide it into sub-libraries: hydraulic and electrical. The hydraulic sub-library will focus on the waterway and the turbine. Electrical sub-library will focus on the synchronous generator, the grid and the load. Taking in account possible difficulties that can be caused by software limitations, it is possible that the turbine and the aggregate can be put into separate sub-libraries.

Taking into account particularities of hydro power plant system that were described in Chapter 2, plus literature overview on this topic and previous works , and discussion with thesis supervisor, primary the following model library structure was offered (Figure 4-2).

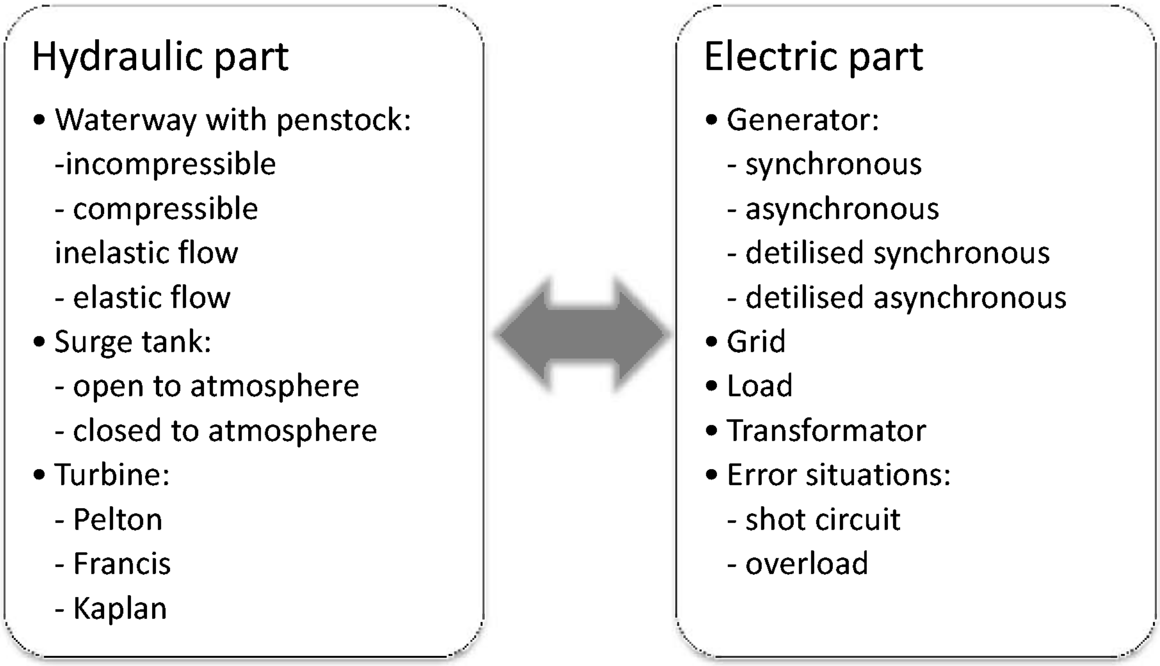


Figure 4-2 Primary model library structure for hydro power plant

Obviously, that this system will be connected with control system that will allow switching between its parts, its regimes and allows implementation of work synchronisation.

The presented model can be defiantly specified further, but taking into account example shown in Chapter 2 and the mathematical models of each hydro power plant part, presented in Chapter 3, the simplified model structure shown on Figure 3-2 will be implemented.

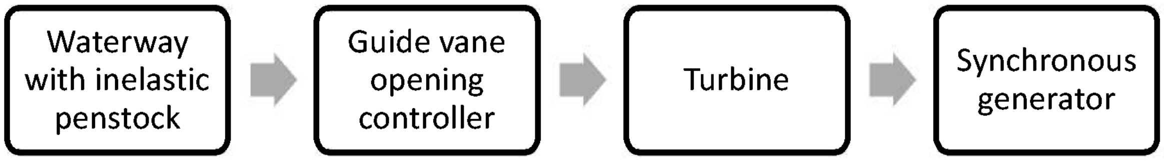


Figure 4-3 Simplified model library structure

After revision of models that has already been implemented (for example the one presented in (Shaheri, 2011) and again taking into consideration theoretical modeling it had been expected that the model library could have been presented as it is shown on Figure 4-4.

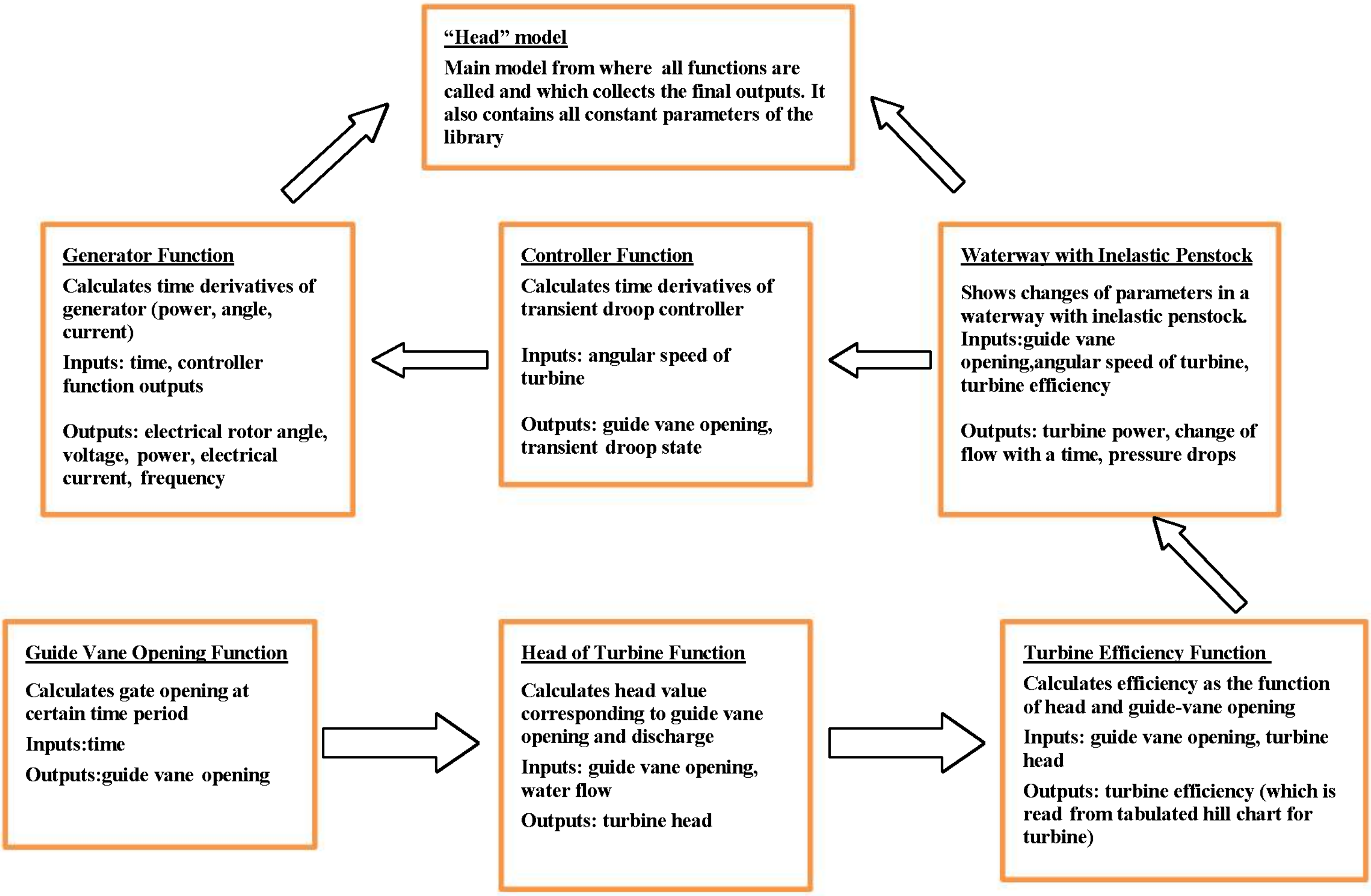


Figure 4-4 Schematic model library for hydro power plant

4.3 Model Library Realization in OpenModelica

Primary there was an attempt to use OpenModelica package definition towards Figure 4-4. Obviously mathematical models for each part of the system, presented in Chapter 3, have been followed. As it has already been mentioned in section 4.1, package is by itself a frame for potential library. That is why it was used here for model and functions combination in library.

As it was highly recommended by experienced Modelica people and moreover to implement correct interpolation, parts related to turbine efficiency and head of turbine calculation (hill charts)

Figure 4-4) were put as blocks from standard Modelica library (Figure 4-6). But due to some difficulties, which will be explained further in Chapter 5, the implementation of those tables has not been realized yet. It was assumed that angle of position that needs to be verified will be taken as constant value that equals 1.5. That also caused that efficiency was also taken constant and equals 0.88.

One other issue that has not allowed full library implementation was related to the great amount of variables and equations that appeared while library design, and OpenModelica mistake related to mismatch of number of variables and equations was shown several times while package modification and adjustments.

That is why it was made a decision to use “one model” structure (simply saying write the code in one file) and to avoid usage of “package structure”. It came out that in this case the example shown in Chapter 2 needed to be extended with primal controller and generator.

Anyway the structure, shown on

Figure 4-3, has been followed and simulation were run.

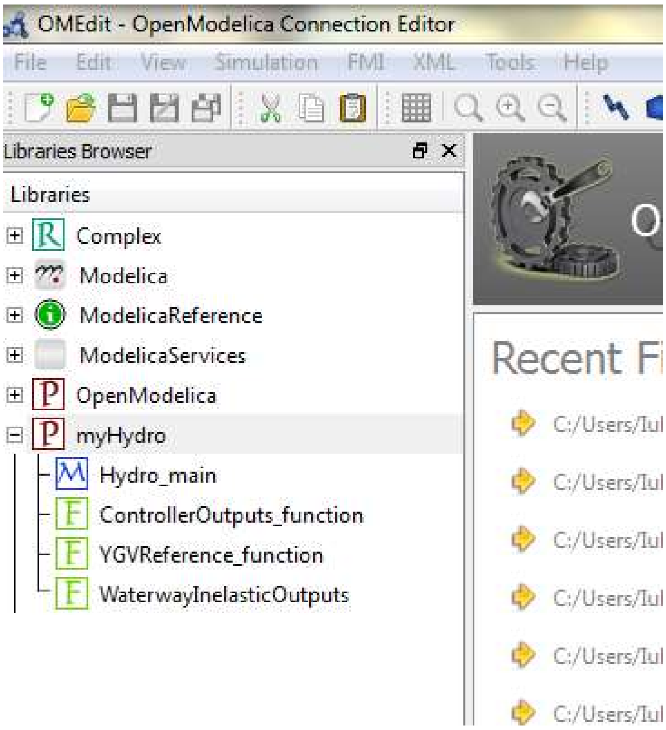
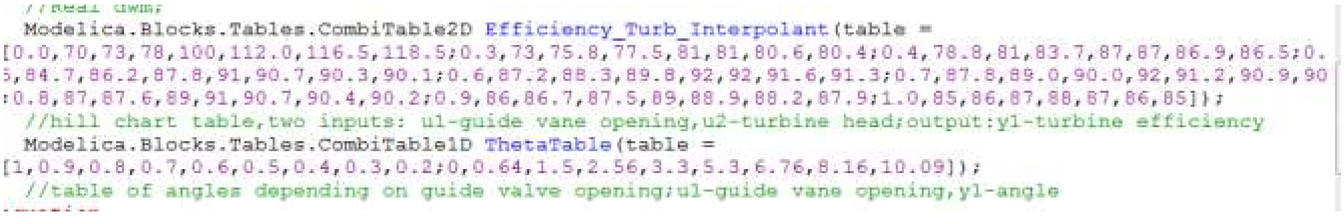


Figure 4-5 Package structure in OpenModelica: Hydro_main .mo– main model from which functions ControllerOutputs_function.mo,YGVRefererence_function.mo, WaterwayInelasticOutputs.mo are called



```
  Modelica.Blocks.Tables.CombiTable2D Efficiency_Turb_Interpolant(table =
[0.0,70,73,78,100,112.0,116.5,118.5;0.3,73,75.8,77.5,81,81,80.6,80.4;0.4,78.8,81,83.7,87,87,86.9,86.5;0.
5,84.7,86.2,87.8,91,90.7,90.3,90.1;0.6,87.2,88.3,89.8,92,92,91.6,91.3;0.7,87.8,89.0,90.0,92,91.2,90.9,90
;0.8,87,87.6,89,91,90.7,90.4,90.2;0.9,86,86.7,87.5,89,88.9,88.2,87.9;1.0,85,86,87,88,87,86,85]);
  //hill chart table,two inputs: u1-guide vane opening,u2-turbine head;output:y1-turbine efficiency
  Modelica.Blocks.Tables.CombiTable1D ThetaTable(table =
[1,0.9,0.8,0.7,0.6,0.5,0.4,0.3,0.2;0,0.64,1.5,2.56,3.3,5.3,6.76,8.16,10.09]);
  //table of angles depending on guide valve opening;u1-guide vane opening,y1-angle
```

Figure 4-6 Tables realisation in OpenModelica

4.4 Simulation results

The results of the simulation are graphs that demonstrate the working parameters of the system during the selected period of operation (1000seconds): volumetric flow rate (Figure 4-7), gate signal (*Figure 4-8*), hydro turbine kinetic flow (Figure 4-9), aggregate angular velocity (Figure 4-10). It was also assumed that guide vane opening change happens at t=200sec.

The similar graphs were built in Chapter 2 for simplified model. The first thing that can be observed that model does not crash, and is run for all simulation intervals. That is of great benefit of this model in comparison with previous one. Moreover most of the received values seem to be quite relevant to realistic (for example volumetric flow rate, guide vane opening).

However absence of some tools required for such types of systems in Open Modelica (hill chart for turbine) makes this model incomplete and requires further development.

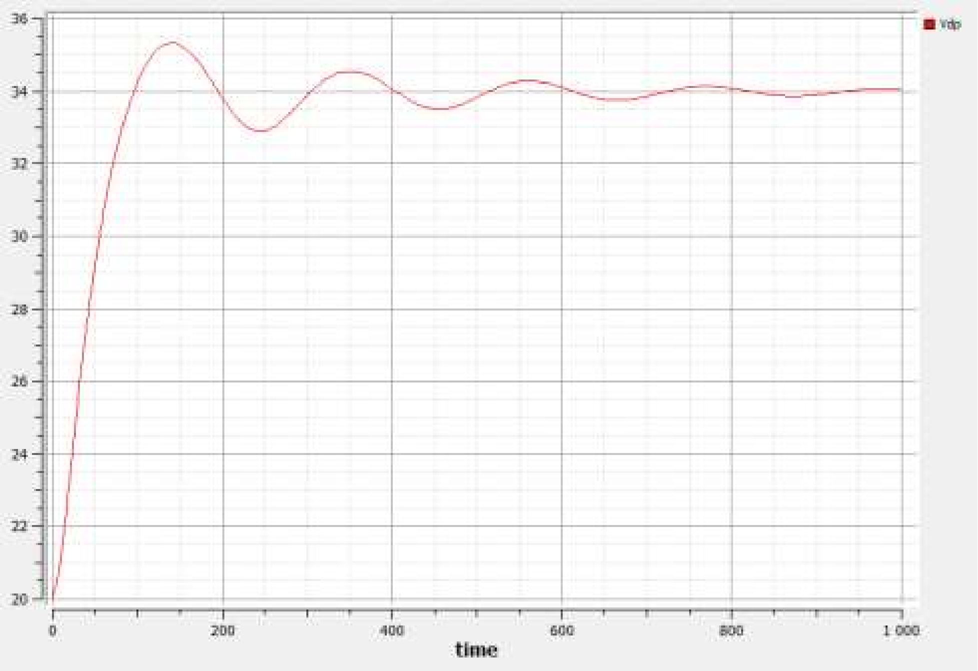


Figure 4-7 Change of penstock volumetric flow rate for model with generator, m^3/sec

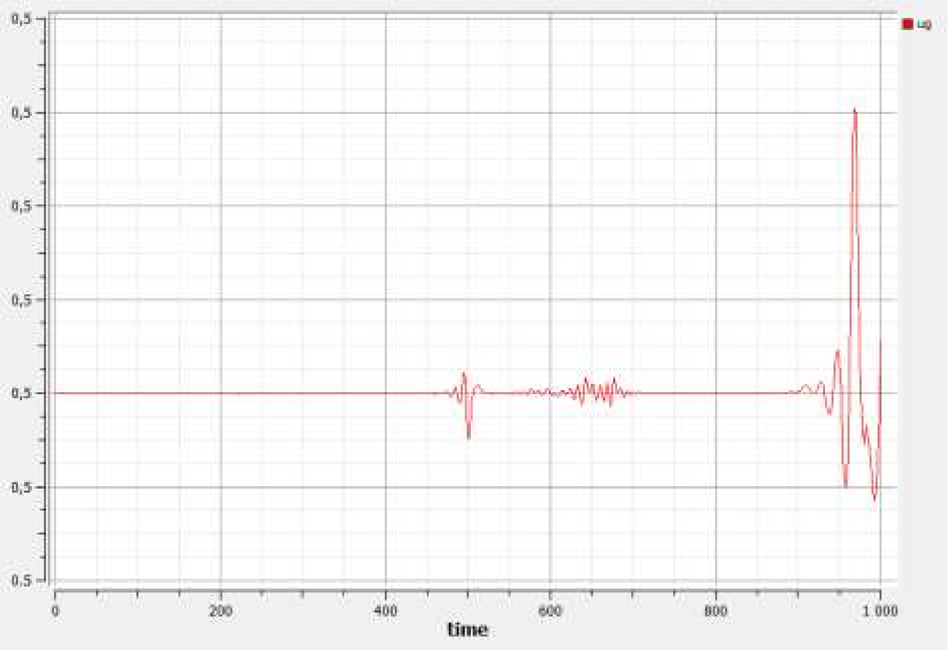


Figure 4-8 Gate signal for model with generator, p.u.

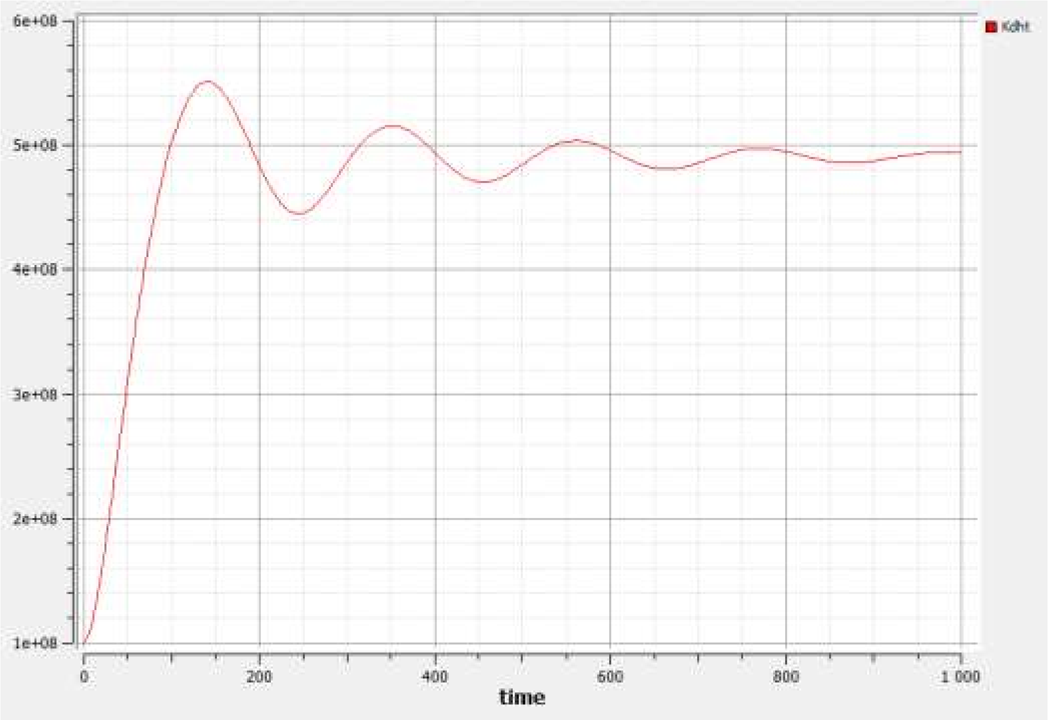


Figure 4-9 Change of hydro turbine kinetic flow, MW

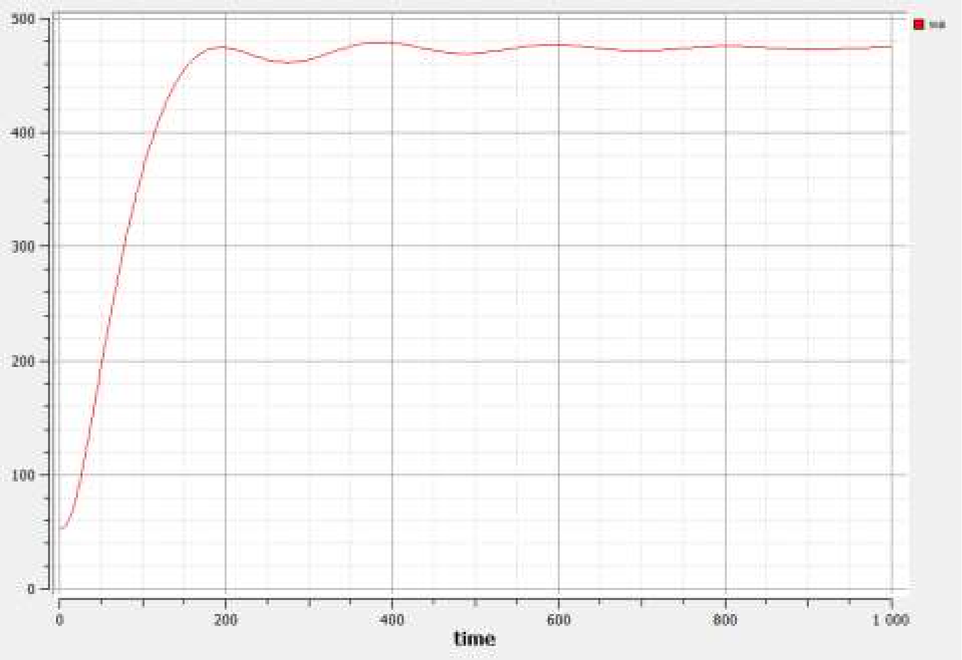


Figure 4-10 Aggregate angular velocity, rad/sec

For the extended case it is also worth to observe the reference guide vane changing (Figure 4-11) and variation of turbine active power, electrical active power, and losses (Figure 4-12). The shape of Figure 4-11 is very realistic and probably can be smoother at t=200sec. But at Figure 4-12 received values of active electric power do not look relevant to expected realistic. It can be caused firstly by imperfection of the model for electrical generator , and also values of resistances, voltage parameters need to be checked and verified.

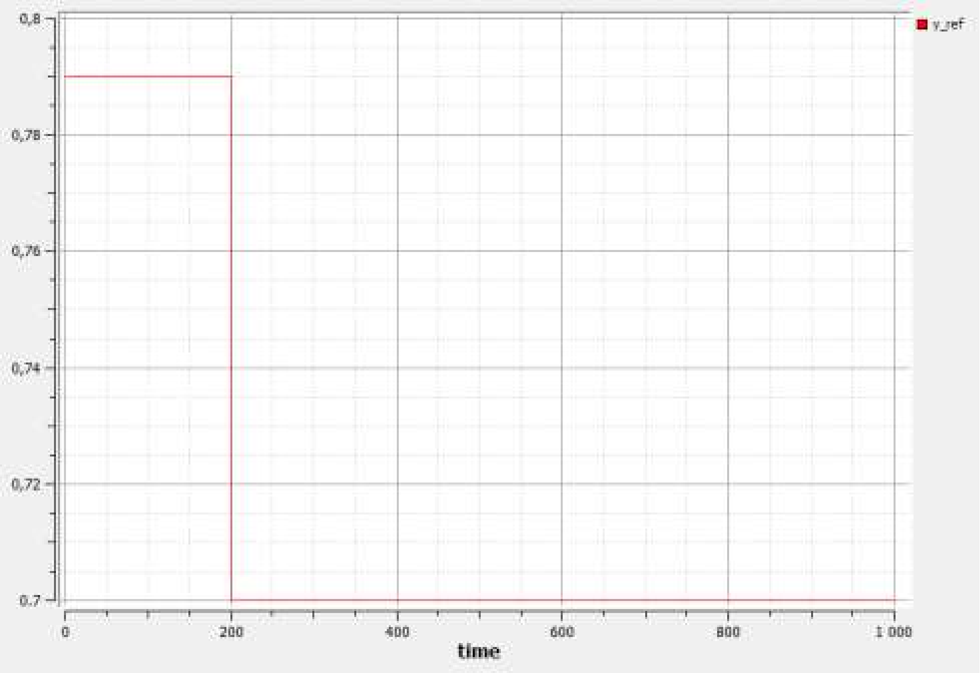


Figure 4-11 Reference guide vane opening, p.u.

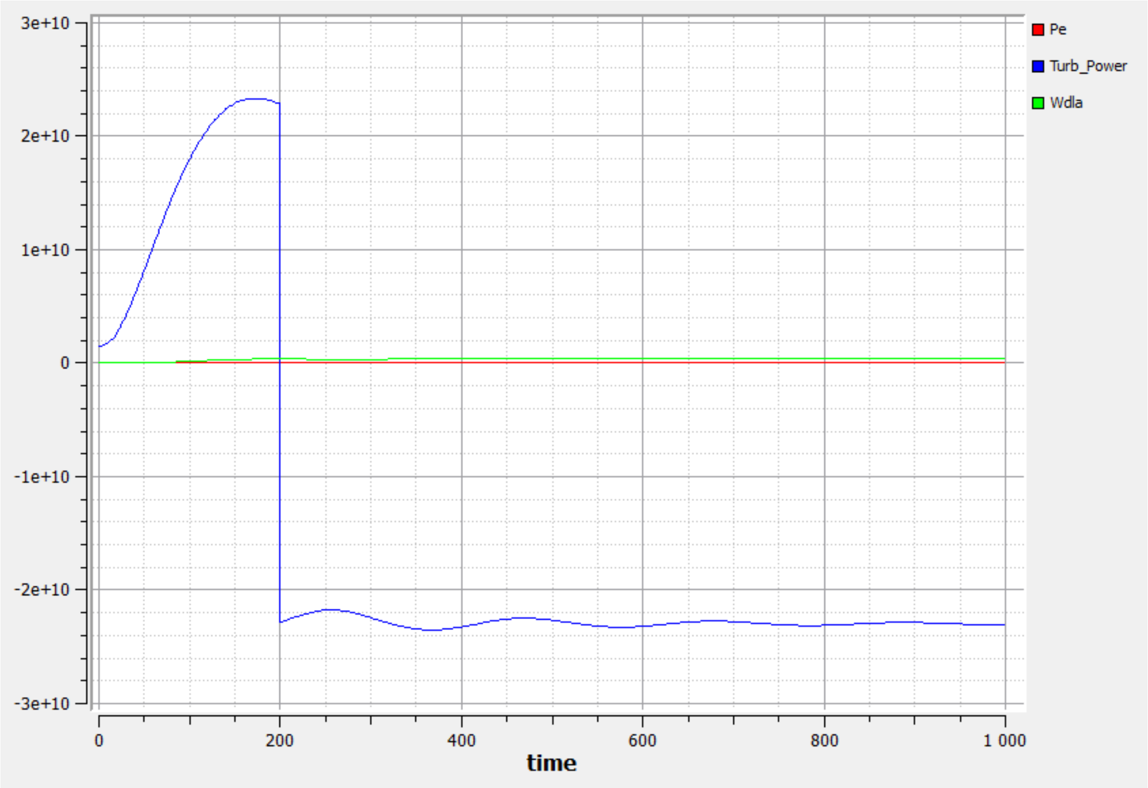
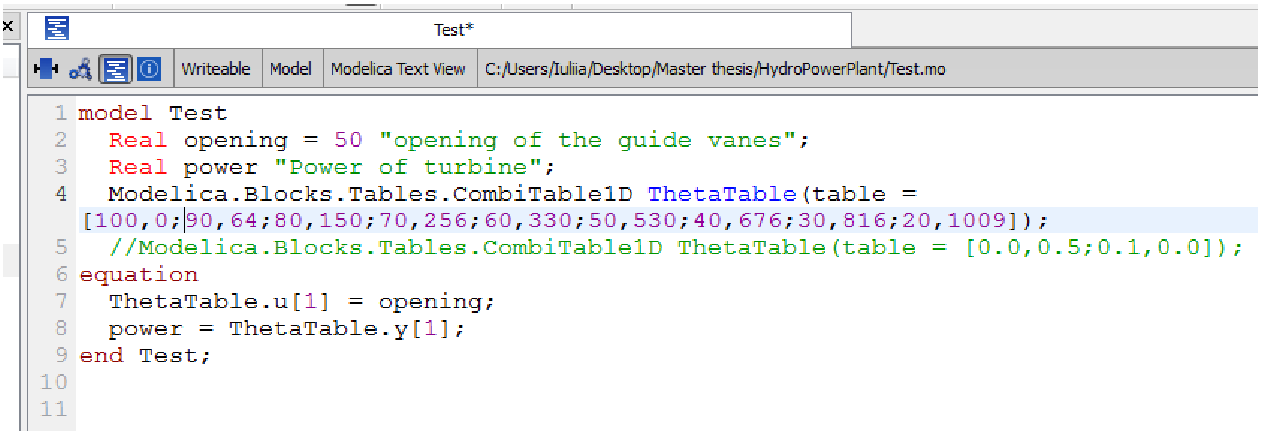


Figure 4-12 Comparison of turbine power(Turb_power), active electrical power(Pe), losses(Wdla), W

5 Discussion

If modeling part of report has demonstrated the different possibilities of model library specification, simulation part of modeling implementation has shown range of issues that can appear on this way.

First of all, while turbine modeling it was presented in Chapter 3 of this report, a lot was told about hill chart theory and their importance of application for turbine design. And it was expected that for turbine simulation one of the most important thing is to input “hill charts” into OpenModelica environment. Moreover the charts should be specific for turbine (or sets of them). Originally data for charts was tabulated in previous works related to similar topic (Shaheri, 2011). OpenModelica by itself has the block «Tables.CombiTable2D» between available Standard Library tools. It was supposed that its usage allows this block implementation for gate vane opening and head values as inputs and efficiency value as an output, using interpolation if it is required. There is also another block, called «Tables.CombiTable1D», that was expected to be used for angle evaluation based on theory. But unfortunately primary testing of «Tables.CombiTable1D» block has shown unexpected interpolation results. The example (not relevant to hydro power problem) was tested and is shown on Figure 5-1.



```
model Test
  Real opening = 50 "opening of the guide vanes";
  Real power "Power of turbine";
  Modelica.Blocks.Tables.CombiTable1D ThetaTable(table =
[100,0;90,64;80,150;70,256;60,330;50,530;40,676;30,816;20,1009]);
  //Modelica.Blocks.Tables.CombiTable1D ThetaTable(table = [0.0,0.5;0.1,0.0]);
equation
  ThetaTable.u[1] = opening;
  power = ThetaTable.y[1];
end Test;
```

Figure 5-1Test model for block «Tables.CombiTable1D» in OpenModelica

As it is seen from the Figure 5-1 the input value is 50, and it was expected to get 530 as the output from the table, that was put in model. However after running the simulation the following was got (Figure 5-2). It is clearly seen that output result equals zero. That is totally irrelevant to expected block output. After numerous manipulations with table and block it was decided to use stated values and omit interpolation. Surely it has simplified the model library and greatly influenced on library simulation results. However it is expected that further improvement of these interpolation blocks will come and it will become possible to implement them for model library.

Another issue that has influenced on received results was absence of validate data (for example hill chart data) to estimate working accuracy of model.

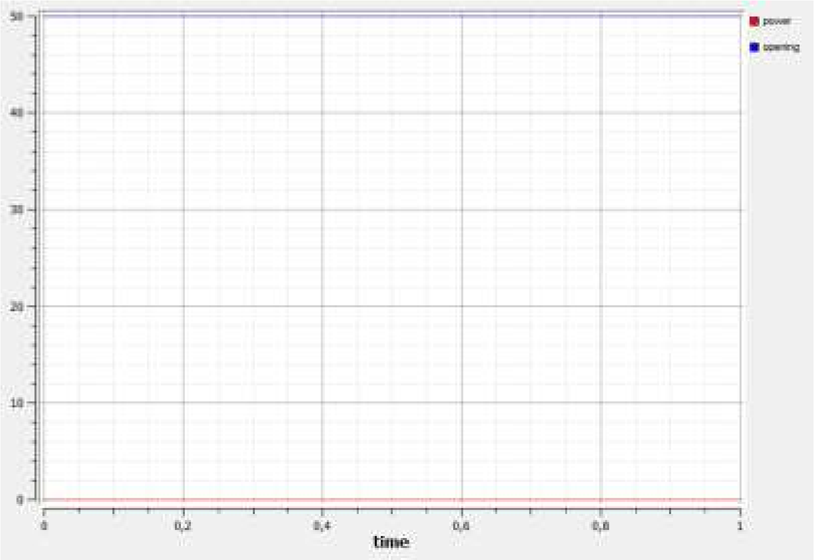


Figure 5-2 Simulation results from running of test model for block «Tables.CombiTable1D» in OpenModelica

Last, but not least, difficulty that was faced while working in OpenModelica environment, is it nightly updating. OpenModelica needs to be updated regularly (and it is not done automatically), otherwise some standard library tools cannot function properly.

6 Conclusion and Recommendations for Future Work

Nowadays, at times of high energy demands, it is of great interest to develop software tools for process observation and control and finding the ways of its optimization. The range of tools are already available at the marked, but developing of free available alternative, a model library for instance, is a way to find the best solution for specific case. For this free OpenModelica environment can be used.

Building a model library for hydro power plant is a complex problem that requires deep understanding of both modeling and simulation sides of the investigated case. That is why firstly theoretical overview of hydro power plant parts was run, and base on this overview their mathematical models were suggested.

After that the possibility of their implementation and simulation in OpenModelica was studied. For this previous experience attempts were tested. After this the simulations models of different level of specification were developed. OpenModelica has almost all tools available for this.

Model library further implementation was followed by the range of technical issues , part of which was successfully overcame and results were shown in graphical form within the report. Discussion part of report gives detailed explanation on some of them.

Despite of this the wide ranges of future work recommendations have come out:

1) Modeling part:

- Hill chart tables for specific turbine need to be available for more accurate case study.
- Electrical load of the existing electrical grid is one of the main factors that will influence on the work of the whole system. For the case of hydropower plant load is all the customers that are connected to the grid that is supplied by this plant. At previous works this part of the system has not been taken into account. That is why for developing library this part can be used as extension by separate sublibrary which will sufficiently influence on work of total system and will increase the level of its detailing.
- The point of the model library is that the system can choose the level of accuracy automatically. That is why this opportunity needs to be revised also.
- The question of turbine involvement to hydraulic part: probably the turbine unit needs to stand separately from hydraulic and electric parts, because by itself it is dependent on functioning of both hydraulic and electrical part, and its efficiency depends on governor functioning.

2) Simulation part:

- Creating separate block for input constants for the functions and main model of the library. After struggling with great amount of variables within the library it is recommended to make separate block with constant parameters and variables, which can be connected to all models and function within the package.
- The possibility of connectors’ usage within the package needs to be studied and applied. It is expected that it will optimize model library structure.
- After reviewing of most of already created Modelica libraries it was observed that they have user friendly interface. That can be also developed for library model.

- It would be also interesting to make a model in OpenModelica using its Standard Library tools and its qualities of object oriented programing environment.

All mentioned above can be the base for further library development and its implementation within educational process.

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Appendices

Appendix A: Task Description

Appendix B: The Operational Values for Simplified Model of Sundsbarm Hydro Power Plant

Appendix C: Modelica Code for Simplified Model of Sundsbarm Hydro Power Plant

Appendix D: Modelica Code for Extended Model of Sundsbarm Hydro Power Plant

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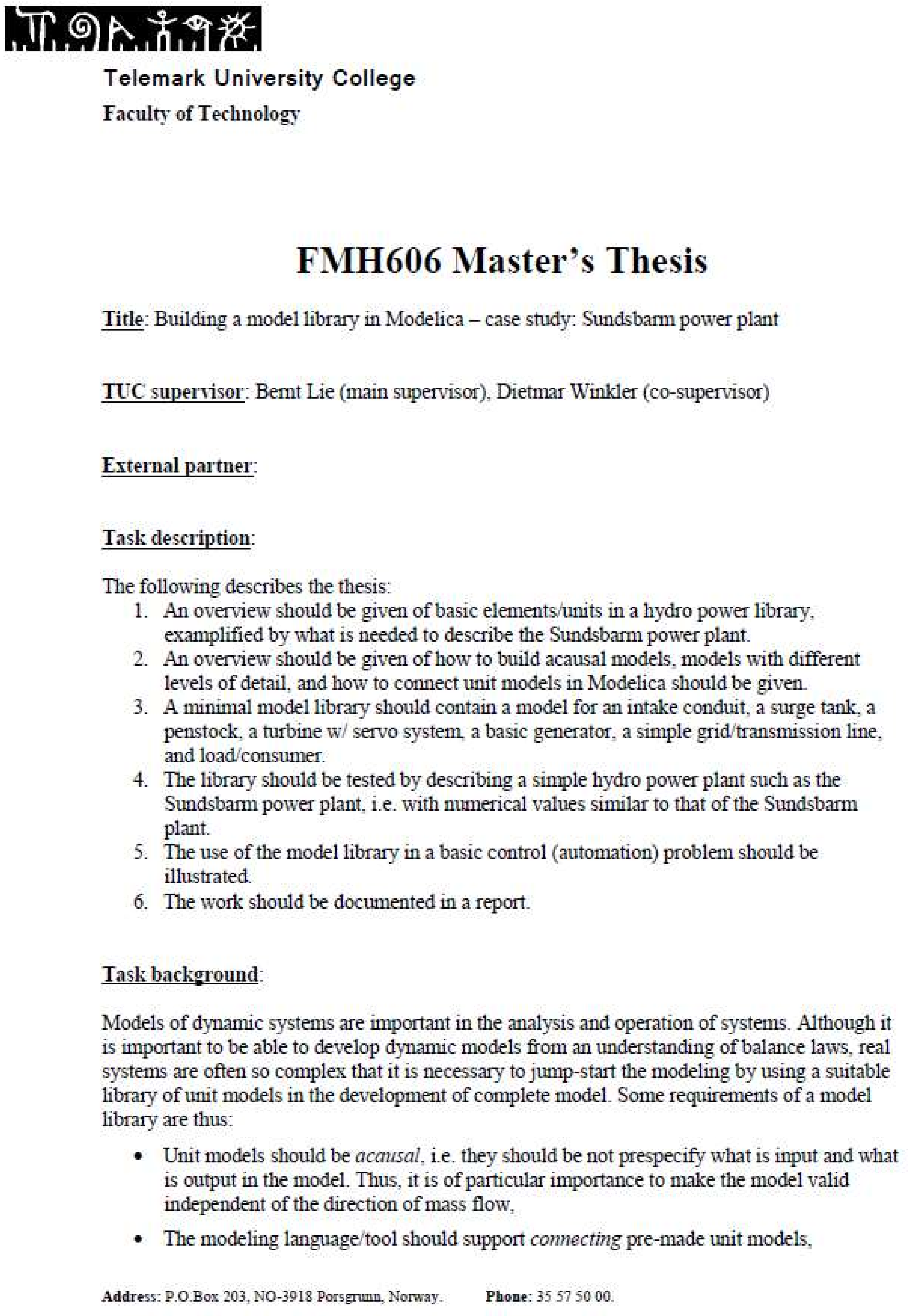
Appendix A: Task Description



Telemark University College
Faculty of Technology

FMH606 Master's Thesis

Title: Building a model library in Modelica – case study: Sundsbarm power plant

TUC supervisor: Bernt Lie (main supervisor), Dietmar Winkler (co-supervisor)

External partner:

Task description:

The following describes the thesis:

1. An overview should be given of basic elements/units in a hydro power library, examplified by what is needed to describe the Sundsbarm power plant.
2. An overview should be given of how to build acausal models, models with different levels of detail, and how to connect unit models in Modelica should be given.
3. A minimal model library should contain a model for an intake conduit, a surge tank, a penstock, a turbine w/ servo system, a basic generator, a simple grid/transmission line, and load/consumer.
4. The library should be tested by describing a simple hydro power plant such as the Sundsbarm power plant, i.e. with numerical values similar to that of the Sundsbarm plant.
5. The use of the model library in a basic control (automation) problem should be illustrated.
6. The work should be documented in a report.

Task background:

Models of dynamic systems are important in the analysis and operation of systems. Although it is important to be able to develop dynamic models from an understanding of balance laws, real systems are often so complex that it is necessary to jump-start the modeling by using a suitable library of unit models in the development of complete model. Some requirements of a model library are thus:

- Unit models should be *acausal*, i.e. they should be not prespecify what is input and what is output in the model. Thus, it is of particular importance to make the model valid independent of the direction of mass flow,
- The modeling language/tool should support *connecting* pre-made unit models,

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- The modeling tool should allow for specifying *the level of detail* in a unit model, e.g. by letting a parameter value switch between different model detailing levels. This is important if one wants to use the same unit for different applications, e.g. a simple model for optimization, a more detailed model for control synthesis, and a complex model for studying error situations,
- The modeling language should have multi physics qualities, and allow for mixing models from different physical regimes such as fluid mechanics, electrical engineering, etc.

Modelica is a language that supports all of the above requirements, and more. Telemark University College has an on-going cooperation with Skagerak Energi regarding automation and optimization of hydro power systems. To advance the study, it is of interest to develop a basic library for studying hydro power systems. A basic library could support a model for conduits of differing diameters, manifolds (for merging several conduits or for splitting a conduit), a surge tank, a penstock, a turbine, an aggregate, an electric generator, a grid/transmission line, and a consumer/load. For a realistic system, it should be possible to attach several generator to the same electric grid.

For some hydro power systems, it is necessary to consider compressible water and elastic walls in a penstock, while for other systems, an incompressible/non-elastic model suffices. Thus it is of interest to be able to switch between different levels of model detail. For a generator, it may be of interest to consider a detailed model of all three phases in a faulty situation, while for normal operation a simplified dynamic or steady state model may suffice. Again, it is of interest to be able to switch between different unit models depending on the application.

In a model library as indicated above, it is of particular importance that the models are prepared for being used in control studies and in optimization studies. Such studies may, e.g. be coordinated from MATLAB or from Python.

In educational studies, it is of particular interest that the models can be used with open source/freeware tools. OpenModelica is a free Modelica tool that supports graphical methods for connecting units. JModelica.org is a free Modelica tool with particular support for being run via Python (and MATLAB).

The outcome from the thesis should be (1) and general understanding of how to build unit models using Modelica, and (2) a basic library for use in hydro power models, (3) and understanding of how the library can be integrated in a control/optimization study.

Student category:

The main topic of the thesis is modeling of a dynamic system (a hydro power plant), where SCE, EET, and PT students have equally good background. Next, a part of the thesis deals with implementing the models in Modelica. For the Modelica part, knowledge of (basic) object oriented programming is useful. Thus, it is an advantage, but not a requirement, to have been exposed to the course Modeling and Simulation of Hydro Power Systems.

Practical arrangements:

The working place will be Telemark University College, Campus Kjølnes in Porsgrunn.

Signatures:

Student (date and signature): Iuliia Vinnik 28.01.2013

Supervisor (date and signature): Bernt Lie 28/1-2013

Appendix B: The Operational Values for Simplified Model for Sundsbarm Hydro Power Plant

Table B.1 Approximate operational values for simple penstock model of Sundsbarm hydro power plant

Steady variable	Value	Description
$\dot{V}_c^s$	$20 m^3/s$	Conduit volumetric flow rate
$m_s^s = \rho_s(h_s \; + H_p)A_s$	$-$	Mass in surge volume
h_r^s	$100 m$	Steady reservoir level above conduit entrance

Table B.2 Approximate operational values for simple penstock model of Sundsbarm hydro power plant

Parameter	Value	Description
H_p	$400 m$	Height of penstock
L_p	$700 m$	Length of penstock
A_p	$100 m^2$	Cross sectional area of penstock
ρ	$10^3 kg/m^3$	Density of water
g	$\approx 10 \quad m/s^2$	Acceleration of gravity
f_p	0.005	Fanning friction factor for penstock
T_t	$\frac{1}{10} s$	Turbine servo system time constant
C_t	$\frac{\sqrt{5}}{125}$	Turbine pressure constant (deduced)
η_t	0.95	Turbine efficiency at 85% flow of maximal water flow rate
J_a	$\frac{216 \times 10^4}{\pi^2} kg\, m^2 rad^{-2}$	Moment of inertia for aggregate (deduced)
k_{fa}	$\frac{216}{\pi^3} W\, s^3 rad^{-3}$	Friction factor for aggregate bearings (deduced)

Table B.3 Approximate parameter values for simple penstock model of Sundsbarm hydropower plant (conduit)

Parameter	Value	Description
H_c	$50m$	Height of conduit
L_c	$6600m$	Length of conduit
A_c	$20m^2$	Cross sectional area of conduit
ρ_c	$10^3 kg/m^3$	Density of (conduit) water
ρ_s	$10^3 kg/m^3$	Density of (surge volume) water
p_a	$10^5 Pa$	Atmospheric pressure
f_c	0.005	Fanning friction factor for conduit
f_c	0.005	Fanning friction factor for surge volume

Appendix C: Modelica Code for for Simplified Model of Sundsbarm Hydro Power Plant

```
model iWWModel
// Simulation of inelastic water way system
// Author: Bernt Lie
// Telemark University College, Porsgrunn, Norway
// February 4, 2013
//
///Editted by Vinnik, June 2013
// Parameter values with type and descriptive text
parameter Real hrs = 50 "Steady state reservoir level, m";
parameter Real Hc = 50 "Conduit height, m";
parameter Real Lc = 6600 "Conduit length, m";
parameter Real Ac = 20 "Conduit cross sectional area, m2";
parameter Real As = 30 "Surge tank cross sectional area, m2";
parameter Real Hp = 400 "Penstock height, m";
parameter Real Lp = 700 "Penstock length, m";
parameter Real Ap = 10 "Penstock cross sectional area, m";
parameter Real rhor = 1000.0 "Reservoir water density, kg/m3";
parameter Real rhoc = rhor "Conduit water density, kg/m3";
parameter Real rhos = rhor "Surge tank water density, kg/m3";
parameter Real rhop = rhor "Penstock water density, kg/m3";
parameter Real g = 10 "Acceleration of gravity, m/s2";
parameter Real pa = 100000.0 "Atmospheric pressure, Pa";
parameter Real fc = 0.005 "Fanning friction factor in conduit, -";
parameter Real fs = fc "Fanning friction factor in surge tank, -";
parameter Real fp = fc "Fanning friction factor in penstock, -";
parameter Real Tg = 0.1 "Time constant, gate servo system, s";
parameter Real Cg = sqrt(5) / 125 "gate valve constant, ?";
parameter Real etaht = 0.95 "Hydro turbine efficiency, -";
parameter Real Ja = 2160000.0 / 3.14 ^ 2 "Moment of inertia for aggregate, kg*m2/rad2";
parameter Real kfa = 216 / 3.14 ^ 3 "Friction factor in aggregate bearings, Ws3/rad3";
parameter Real perc = 2 * 3.14 * sqrt(Ac / 3.14) "Conduit perimeter, m";
parameter Real pers = 2 * 3.14 * sqrt(As / 3.14) "Surge tank perimeter, m";
parameter Real perp = 2 * 3.14 * sqrt(Ap / 3.14) "Penstock perimeter, m";
parameter Real cKp = 1 "Selector for Kp, -";
// Initial state parameters:
parameter Real Vdc0 = 20 "Initial conduit volumetric flow rate, m3/s";
parameter Real Vdp0 = Vdc0 "Initial penstock volumetric flow rate, m3/s";
parameter Real ms0 = rhos * (hrs + Hc) * As "Initial surge tank mass, kg";
parameter Real ug0 = 1 / 2 "Initial gate signal, -";
parameter Real wa0 = 50 * 3.14 / 3 "Initial aggregate angular velocity, rad/s";
// Setting initial values for states:
Real Vdc(start = Vdc0, fixed = true);
Real Vdp(start = Vdp0, fixed = true);
Real ms(start = ms0, fixed = true);
Real ug(start = ug0, fixed = true);
Real wa(start = wa0, fixed = true);
```

```
// Algebraic variables
Real pci "Conduit inlet pressure, Pa";
Real mc "Conduit mass, kg";
Real mp "Penstock mass, kg";
Real psx "Surge tank exit pressure, Pa";
Real Fug "gate signal function value, ?";
Real dpg "Turbine gate pressure drop, Pa";
Real ppx "Penstock exit pressure, Pa";
Real mdp "Penstock mass flow rate, kg/s";
Real Kdp "Penstock kinetic flow, J/s";
Real Kdht "Hydro turbine kinetic flow, J/s";
Real Vds "Surge tank volumetric flow rate, m3/s";
Real hs "Surge tank level, m";
Real p0 "Node inlet-surge tank-penstock pressure, Pa";
// Defining input variables:
Real us;
Real Wde;
Real hr;
// input Real us "Control input, -";
// input Real Wde "Electric power consumption, W";
// input Real hr "Reservoir level, m";
equation
// Input signals
us = 1 / 2;
Wde = 100000000.0;
hr = 10000;
// Algebraic equations
pci = pa + rhor * g * hr;
mc = rhoc * Lc * Ac;
mp = rhop * Lp * Ap;
psx = pa;
Fug = Cg * ug;
dpg = Vdp ^ 2 / Fug ^ 2;
ppx = pa + dpg;
mdp = rhop * Vdp;
Kdp = 1 / 2 * mdp * (Vdp / Ap) ^ 2;
Kdht = dpg * Vdp + cKp * Kdp;
Vds = Vdc - Vdp;
hs = ms / rhos / As;
// Differential equations
der(mc * Vdc / Ac) = (pci - p0) * Ac + mc * g * Hc / Lc - fc * perc / 2 * rhoc * Lc * abs(Vdc / Ac) *
Vdc / Ac;
der(ms) = rhos * Vds;
der(ms * Vds / As) = (p0 - psx) * As - ms * g - fs * pers / 2 * rhos * hs * abs(Vds / As) * Vds / As;
der(mp * Vdp / Ap) = (p0 - ppx) * Ap + mp * g * Hp / Lp - fp * perp / 2 * rhop * Lp * (Vdp / Ap) ^ 2;
der(ug) = (us - ug) / Tg;
der(wa) = 1 / (Ja * wa) * (etaht * Kdht - Wde - 1 / 2 * kfa * wa ^ 3);
end iWWModel;
```

Appendix D: Modelica Code for Extended Model of Sundsbarm Hydro Power Plant

```
model Hydro
// Simulation of inelastic water way system
// Author: Bernt Lie
// Telemark University College, Porsgrunn, Norway
// February 4, 2013
//
// Parameter values with type and descriptive text
///Editted by Vinnik, June 2013
// electrical generator was added; modified turbine
//for further work Modelica.Blocks.Tables.CombiTable2D should be added
parameter Real hrs = 50 "Steady state reservoir level, m";
parameter Real Hc = 50 "Conduit height, m";
parameter Real Lc = 6600 "Conduit length, m";
parameter Real Ac = 20 "Conduit cross sectional area, m2";
parameter Real As = 30 "Surge tank cross sectional area, m2";
parameter Real Hp = 400 "Penstock height, m";
parameter Real Lp = 700 "Penstock length, m";
parameter Real Ap = 10 "Penstock cross sectional area, m";
parameter Real rhor = 1000.0 "Reservoir water density, kg/m3";
parameter Real rhoc = rhor "Conduit water density, kg/m3";
parameter Real rhos = rhor "Surge tank water density, kg/m3";
parameter Real rhop = rhor "Penstock water density, kg/m3";
parameter Real g = 10 "Acceleration of gravity, m/s2";
parameter Real pa = 101325 "Atmospheric pressure, Pa";
parameter Real fc = 0.005 "Fanning friction factor in conduit, -";
parameter Real fs = fc "Fanning friction factor in surge tank, -";
parameter Real fp = fc "Fanning friction factor in penstock, -";
parameter Real Tg = 0.1 "Time constant, gate servo system, s";
parameter Real Cg = sqrt(5) / 125 "gate valve constant, ?";
parameter Real etaht = 0.95 "Hydro turbine efficiency, -";
parameter Real Ja = 2160000.0 / 3.14 ^ 2 "Moment of inertia for aggregate, kg*m2/rad2";
parameter Real kfa = 216 / 3.14 ^ 3 "Friction factor in aggregate bearings, Ws3/rad3";
parameter Real perc = 2 * 3.14 * sqrt(Ac / 3.14) "Conduit perimeter, m";
parameter Real pers = 2 * 3.14 * sqrt(As / 3.14) "Surge tank perimeter, m";
parameter Real perp = 2 * 3.14 * sqrt(Ap / 3.14) "Penstock perimeter, m";
parameter Real cKp = 1 "Selector for Kp, -";
///Editting
parameter Real wm_design = 50 * 3.14 / 180 "Angular speed designed";
//parameter Real Modelica.Blocks.Tables.CombiTable2D Efficiency_Turb_Interpolant(table =
[0.0,70,73,78,100,112.0,116.5,118.5;0.3,73,75.8,77.5,81,81,80.6,80.4;0.4,78.8,81,83.7,87,87,86.9,86.5;0
.5,84.7,86.2,87.8,91,90.7,90.3,90.1;0.6,87.2,88.3,89.8,92,92,91.6,91.3;0.7,87.8,89.0,90.0,92,91.2,90.9,9
0;0.8,87,87.6,89,91,90.7,90.4,90.2;0.9,86,86.7,87.5,89,88.9,88.2,87.9;1.0,85,86,87,88,87,86,85]) "Hill
chart table,two inputs: u1-guide vane opening,u2-turbine head; output:y1-turbine efficiency"
parameter Real Gen_xq = 12 "q_axis reactance [Ohms]";
parameter Real Gen_xd = 12 "d_axis reactance [Ohms]";
parameter Real Gen_Ra = 0.01 "Phase winding resistance [Ohms]";
```

```
parameter Real Gen_Re = 0.1 "Equivalent network resistance [Ohms]";
parameter Real Gen_xxd = 1.7 "d_axis transient reactance [Ohms]";
parameter Real Gen_xxq = 1.7 "q_axis transient reactance [Ohms]";
parameter Real Gen_xe = 1.4 "Equivalent network reactance [Ohms]";
parameter Real Vs = 15000 "Network rms voltage [V]";
parameter Real P_op = 80000000.0 "Active power drawn from generator at steady state operating
condition";
parameter Real Q_op = 50000000.0 "Reactive power drawn from generator at steady state operating
condition";
parameter Real PHI = atan(Q_op / P_op) "Power angle at steady state";
parameter Real Delta = atan((I0 * (Gen_xq + Gen_xe) * cos(PHI) - I0 * Gen_Ra + Gen_Re) * sin(PHI) /
(Vs + I0 * (Gen_Ra + Gen_Re) * cos(PHI) + I0 * (Gen_xq + Gen_xe) * sin(PHI))) "Electrical rotor
angle";
parameter Real Ef = Vs. * cos(Delta) + (Gen_Ra + Gen_Re) * Iq0 - (Gen_xd + Gen_xe) * Id0;
// Initial state parameters:
parameter Real Vdc0 = 20 "Initial conduit volumetric flow rate, m3/s";
parameter Real Vdp0 = Vdc0 "Initial penstock volumetric flow rate, m3/s";
parameter Real ms0 = rhos * (hrs + Hc) * As "Initial surge tank mass, kg";
parameter Real ug0 = 1 / 2 "Initial gate signal, -";
parameter Real wa0 = 50 * 3.14 / 3 "Initial aggregate angular velocity, rad/s";
///Generator part
parameter Real I0 = 3 * Vs / sqrt(P_op ^ 2 + Q_op ^ 2) "RMS(root mean square) current (per phase) of
generator";
parameter Real Iq0 = I0 * cos(Delta + PHI) "";
parameter Real Id0 = -I0 * sin(Delta + PHI) "Initial";
// Setting initial values for states:
Real Vdc(start = Vdc0, fixed = true);
Real Vdp(start = Vdp0, fixed = true);
Real ms(start = ms0, fixed = true);
Real ug(start = ug0, fixed = true);
Real wa(start = wa0, fixed = true);
// Algebraic variables
Real pci "Conduit inlet pressure, Pa";
Real mc "Conduit mass, kg";
Real mp "Penstock mass, kg";
Real psx "Surge tank exit pressure, Pa";
Real Fug "gate signal function value, ?";
Real dpg "Turbine gate pressure drop, Pa";
Real ppx "Penstock exit pressure, Pa";
Real mdp "Penstock mass flow rate, kg/s";
Real Kdp "Penstock kinetic flow, J/s";
Real Kdht "Hydro turbine kinetic flow, J/s";
Real Vds "Surge tank volumetric flow rate, m3/s";
Real hs "Surge tank level, m";
Real p0 "Node inlet-surge tank-penstock pressure, Pa";
///Editting
///Controller part
Real y_ref "Guide vane opening";
Real theta "Angle of position";
///Turbine part
Real Q_turb "Turbine volumetric flow rated";
```

```
Real Turb_Head "Head of turbine";
Real Turb_Head_Rated "Head of turbine rated";
Real Turb_Eff "Turbine efficiency, ideally is calculated from the table Efficiency_Turb_Interpolant";
Real Turb_Power "Power of turbine, [W]";
///Generator part
Real Iq;
Real Id;
Real Eq;
Real Ed;
Real Pe;
// Defining input variables:
Real us;
Real Wde;
Real Wdla;
Real hr;
// input Real us "Control input, -";
// input Real Wde "Electric power consumption, W";
// input Real hr "Reservoir level, m";
equation
// Input signals
us = 1 / 2;
Wde = sqrt(P_op ^ 2 + Q_op ^ 2);
//100000000.0;
hr = 1000;
// Algebraic equations
pci = pa + rhor * g * hr;
mc = rhoc * Lc * Ac;
mp = rhop * Lp * Ap;
psx = pa;
Fug = Cg * ug;
dpg = Vdp ^ 2 / Fug ^ 2;
ppx = pa + dpg;
mdp = rhop * Vdp;
Kdp = 1 / 2 * mdp * (Vdp / Ap) ^ 2;
Kdht = dpg * Vdp + cKp * Kdp;
Vds = Vdc - Vdp;
hs = ms / rhos / As;
///Editting
///Controller part
if time >= 200 then
y_ref = 0.7;
theta = 2.56;
else
y_ref = 0.79;
theta = 1.5;
end if;
///Turbine part
Q_turb = Vdp / wa * wm_design "assume turbine flow equals penstock flow";
Turb_Head = 70 / 134 * ((Q_turb + 32.2) * tan(theta) - 331.6 + 60 * 134 / 70);
Turb_Head_Rated = Turb_Head * wa ^ 2 / wm_design ^ 2;
```

```
Turb_Eff = 0.88 "because of the problems in CombiTable2D interpolation, the value was taken specifically for the current case according to y_ref=0.79 and theta=1.5";
Turb_Power = rhop * g * Turb_Head_Rated * Q_turb;
///Generator part
Ed = (Gen_xxd - Gen_xq) * Iq;
Eq = Ef + (Gen_xd - Gen_xxd) * Id;
{{Gen_Re + Gen_Ra,Gen_xe + Gen_xxd},{-Gen_xe - Gen_xxd,Gen_Re + Gen_Ra}} * {Id,Iq} = {Vs * sin(Delta) + Ed,-Vs * cos(Delta) + Eq};
Pe = 3 * (Ed * Id + Eq * Iq);
Wdla = 1 / 2 * kfa * wa ^ 3 "losses";
// Qe = sqrt(9 * Vt ^ 2 * It ^ 2 - Pe ^ 2);
///Waterway part
// Differential equations
der(mc * Vdc / Ac) = (pci - p0) * Ac + mc * g * Hc / Lc - fc * perc / 2 * rhoc * Lc * abs(Vdc / Ac) * Vdc / Ac;
der(ms) = rhos * Vds;
der(ms * Vds / As) = (p0 - psx) * As - ms * g - fs * pers / 2 * rhos * hs * abs(Vds / As) * Vds / As;
der(mp * Vdp / Ap) = (p0 - ppx) * Ap + mp * g * Hp / Lp - fp * perp / 2 * rhop * Lp * (Vdp / Ap) ^ 2;
der(ug) = (us - ug) / Tg;
///swing equation
der(wa) = 1 / (Ja * wa) * (etaht * Kdht - Wde - Wdla);
annotation(experiment(StartTime = 0.0, StopTime = 1000.0, Tolerance = 0.0001));
end Hydro;
```