Investigation of the shifting-parameter as a function of particle size distribution in a fluidized bed traversing from a fixed to fluidized bed.

C. Rautenbach, M.C. Melaaen & B.M. Halvorsen
Institute for Process, Energy and Environmental Technology
Telemark University College
Norway
T: +47 3557 5222; E: christo.rautenbach@hit.no

ABSTRACT

Accurate predictions of pressure drops in fluidized beds are of great importance in the industry. Up to date no satisfactory correlation exists to predict the pressure drop in a fluidized bed as the bed is traversing from one regime to another. In the present study the powered addition correlation [1] is investigated for this particular application. It has been found that this correlation is well suited for the investigated application.

INTRODUCTION

In the present study experiments have been performed in an experimental fluidized bed reactor. The experimental tower has been equipped with a set of nine pressure sensors located at different positions along the height of the tower. The tower has a diameter of $0.072m$ and a height of $1.5m$. A procedure providing a correlation for data in the transition region between asymptotic solutions or limiting correlations have been described by Churchill and Usagi [1]. This correlation can generally be expressed as $y^s\{x\} = y_o^s\{x\} + y_\infty^s\{x\}$, where $y_o\{x\}$ and $y_\infty\{x\}$ represents the asymptotic solutions for large and small values of the independent variable x and *s* is the so called shifting parameter. Changing the value of *s* shifts the correlation given by $y\{x\}$ closer to or away from the asymptotic solutions. This procedure has been proven to give good correlations in a wide range of applications. The exact physical meaning of this shifting parameter *s*, is still unknown and the present study is part of an ongoing investigation into the physical meaning and possible mathematical expression for the shifting parameter. In the present study only one possible parameter influencing the shifting parameter is investigated, namely different particle size distributions.

A series of different powders have been used to investigate the influence of a particular parameters on the shifting parameter, *s*. Up to date no expression has been stated for this shifting parameter to govern the transition from fixed to fluidized bed. In the present study spherical glass particles have been used with different particle size distributions. By keeping all the parameters constant except the particle size distribution, the influence of the particle size distribution on the shifting parameter could be investigated. Several different drag models were used to serve as a control for investigating the shifting parameter. The results are given in the form of pressure drop data versus superficial velocity data. Experimental data are presented with the drag model correlations and the investigated values of the shifting parameter, *s*. Some of the drag models that were used were the Syamlal O'Brien drag model [2] and the extended Hill-Koch-Ladd drag correlation [3]. The results are evaluated and discussed.

PROPERTIES OF THE EXPERIMENTAL BEDS

At the point of minimum fluidization the total weight of the packed bed is supported by the upward force created by the gas moving upward through the porous structure. As the superficial velocity is increase from this point the pressure drop remains practically the same [4]. The explanation for the slight increase of pressure drop with an increase of superficial velocity may be attributed to wall effects, more specifically, slugging [5]. In the present study the pressure drop in the fluidized regime will be assumed constant. At this point of equilibrium (minimum fluidization velocity) the pressure-drop is given by

$$\Delta p = (1-\epsilon)(\rho_p - \rho_f)Lg, \tag{1}$$

with ρ_p the particle's density, ρ_f the fluid density and L the bed height. In the present study spherical glass particles were used with a density of $2485\ kg/m^3$. The three different size distribution that were used are $100-200\mu m$, $400-600\mu m$ and $750-1000\mu m$. The relevant parameters of the powders are given in Table 1.

Powder size distributions	$100-200\mu m$	$400-600\mu m$	$750-1000\mu m$
ϵ	0.39	0.37	0.36
$(1-\epsilon)(\rho_p-\rho_f)g\ [N/m^3]$	14848	15334	15578

Table 1: Relevant parameters of the powders used in the present study.

All of the data is at the point of minimum fluidization except the $750-1000\mu m$ powder. Because of a lack of experimental data in the fully fluidized regime data were used when the bed was fluidized for the first time. The only practical effect of this was that the porosity was lower than it would have been if the bed has been fluidized before. By using the correct data this should pose no problem in the accuracy of what the drag models predict.

POWERED ADDITION AND THE ASYMPTOTIC FUNCTIONS

In the work done by Chrurchill and Usagi [1] they proposed the use of a general empirical equation for correlating behavior between two asymptotic solutions or limiting correlations. In the present study the lower limiting condition will be the fixed bed regime. Different drag models will be used to model this regime. The upper limiting condition will be described by the constant pressure-drop given when the upward force created by the upward moving gas is equal to the weight of the bed. It can be shown that this constant pressure drop for the fluidized regime is given by $(1-\epsilon)(\rho_p-\rho_f)g$, as mentioned earlier. A problem arises for large values of the independent variable as a constant value is not a upper bound [1]. Through numerous graphical representation Churchill and Usagi [1] suggested equation (2) to give a linear relationship on a *log-log* plot and can be written as

$$F(q) = \frac{H(q)}{H(\infty) - H(q)}, \tag{2}$$

where $H(q)$ is the asymptotic function desired for large values of the independent variable and $H(\infty)$ is the constant value to which the asymptote will tend to. Thus using equation (2) a function can be determined for $H(q)$ that would be a asymptotic limiting condition for large values of the independent variable, q.

To determine this function $H(q)$ the data points which the function should approximate is used in equation (2). This data points are the pressure drop data in the fluidized regime. It follows that $(1-\epsilon)(\rho_p - \rho_f)g$ will be taken as the value of $H(\infty)$.

The $400 - 600\mu m$ powder will be used to serve as an example of how the function $H(q)$ is deduced. Using the data from Table 1 and the data acquired at the TUC in Norway values of the function $F(q)$ were determined. In Figure 1 (a) the positive values of $F(q)$ is given. It is clear from Figure 1 (a) that there is only three data points while the fluidized region in Figure 1 (b) has at least six data points. The missing three data points can be attributed to the prediction that $(1-\epsilon)(\rho_p - \rho_f)g$ gives. For the $400 - 600\mu\ m$ powder the theoretical prediction of equation (1) is lower than some of the data points and as the *log* of a negative value does not exist the negative values of $F(q)$ can not be plotted in Figure 1. As only two data points are required to get a linear approximation the remaining three data points are enough to produce a linear approximation. In Figure 1 (b) an example of $F(q)$ is given if the theoretical prediction of equation (1) was higher than all of the data points. The result would have been more data point and thus a more defined linear relationship.

With an approximate linear equation for $F(q)$ on a *log-log* scale, a function for the upper bound for large values of the independent variable, q, can now be determined. The general function for $H(q)$ can be expressed as

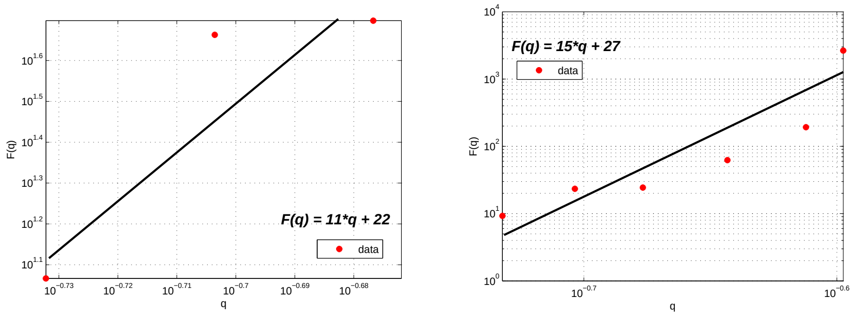
$$H(q) = \frac{H(\infty)}{\frac{1}{F_o}\left(\frac{q_o}{q}\right)^m + 1}, \tag{3}$$

with m being the gradient of the linear approximation of $F(q)$ on a *log-log* scale and F_o and q_o any point on the approximated linear curve. For the $400 - 600\mu m$ powder the upper bound for large values of q is given by

$$H(q) = \frac{15334}{\frac{1}{24.19}\left(\frac{0.199}{q}\right)^{11} + 1}, \tag{4}$$

where the point $(0.199, 24.19)$ were chosen as the arbitrary point on the linear approximation of $F(q)$. This function is not a good function for being representative of the behavior of the fully fluidized bed at high values of q. After several graphical investigations a new adequate function was formulated. It can be expressed as

$$H(q) = \frac{15334}{\frac{1}{F_o}\left(\frac{0.1q_o}{q}\right)^m + 1}, \tag{5}$$



(a) (b)

Figure 1: (a) Linear approximation to equation (3) on a *log-log* scaling. (b) Linear approximation to equation (3) on a *log-log* scaling if the calculated value of equation (1) was higher than all the data point values in the fully fluidized regime.

where the only difference to equation (3) is the addition of the factor in front of the q_o. The simple reasoning behind this factor lies in the characteristics of the investigated powders. Because two out of the three investigated powders' point of minimum fluidization were above $0.1m/s$ the function $H(q)$ was not adequate. If the minimum fluidization velocity was below $0.1m/s$ the function would produce a result that would give an accurate prediction in the fully fluidized regime (like the $100 - 200\mu m$ powder). Thus by inserting the factor of 0.1 in equation (5) the equation is assured of giving a usable function for all the powders investigated in the present study. It should be noted that this function, equation (5), is completely empirical. It is only constructed to produce a asymptotic function that would give the value of equation (1) for large values of q. This function is only created to be in accordance with the powered addition procedure described by Churchill and Usagi [1].

The general applicability of this function should also still be investigated. In other words, it should be tested for powders with different densities, different particle size distribution than the ones investigated in the present work and different particel shapes, to name but a few. It should be bore in mind that for different powders the factor in front of the q_o in equation (5) might have to be addapted. The higher the superficial velocity value at which the bed is fluidized the smaller factor is added in front of q_o. This might seem very empirical, but this is only an estimate to equation (1) and thus keeps the whole theoretical basis of the equation that it is representing. Following the procedure described by Churchill and Usagi [1] a total predictive model for fluidized beds, traversing from a fixed to fluidized regime, can be expressed as

$$\frac{\Delta p}{L} = \left(Drag\ model^{-s} + H(q)^{-s}\right)^{-\frac{1}{s}}, \tag{6}$$

were any adequate drag model can be used. The negative powers of s is because the data is a decreasing power of q.

Drag model investigation along with the shifting parameter, s.

Most drag model need some sort of definition of an average particle size. This is still a source of on going research as it is no trivial task to estimate a good representative particle size diameter. Sieving analysis was performed on the powders used in the present study. This was done to establish the particles size distribution of the powders but also to determine an effective particle size. Several definitions exist for an effective particle size in a powder with a particle size distribution. In the present study the surface-volume mean diameter will be used [4] along with the minimum and maximum particle diameter of each powder. The surface-volume mean diameter can be expressed as

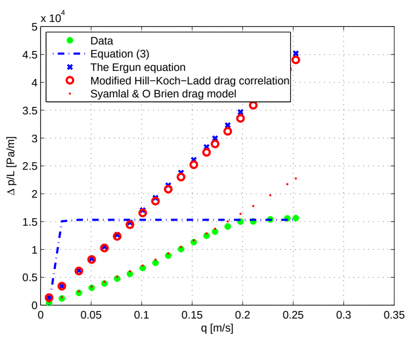
$$\bar{d}_{sv} = \frac{1}{\sum_i x_i/d_i}, \tag{7}$$

with d_i the nominal diameter and x_i the mass fraction of the total mass of the corresponding nominal size particles. Thus $\sum_i x_i$ will be equal to 1.

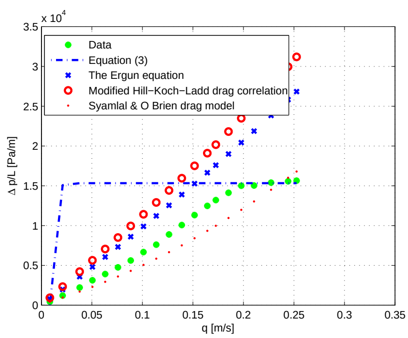
For the $400 - 600\mu m$ powder a surface-volume mean diameter ($\bar{d}_{sv}$) of 482.9 were calculated and the correlations of the different drag models using this value is given in Figure 2 (b).

In Figure 2 (a) and (b) the minimum and maximum particle size diameters were used respectfully. From these two figures it is clear to see that different models perform better with different representative particle sizes. The Ergun equation [4] and the modified Hill-Koch-Ladd drag correlation [3] performed better with large value of the representative particle diameter. The Syamlal and O'Brien drag model [2] performed very well with a low representative particle diameter.

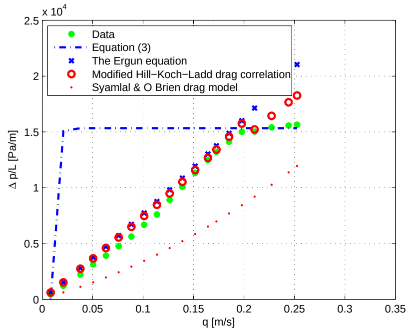
Because of the accurate prediction of the Hill-Koch-Ladd drag correlation [3] with a representative particle diameter equal to $400\mu m$, it will be used to illustrate the usefulness of the powered addition principle [1]. In Figure 3 different correlations are given with several values of the shifting parameter, *s*. It is clear to see that the higher the value of *s* the more the powered addition correlation shifts towards the asymptotes. At a value of 15 a satisfactory correlation is produced.



(a)



(b)



(c)

Figure 2: The prediction of different drag models using (a) the minimum particle size diameter ($400\mu m$), (b) the surface-volume mean diameter ($482.9\mu m$) and (c) the maximum particle size diameter ($600\mu m$). The experimental pressure drop data for the $400 - 600\mu m$ powder is given along side the predictions.

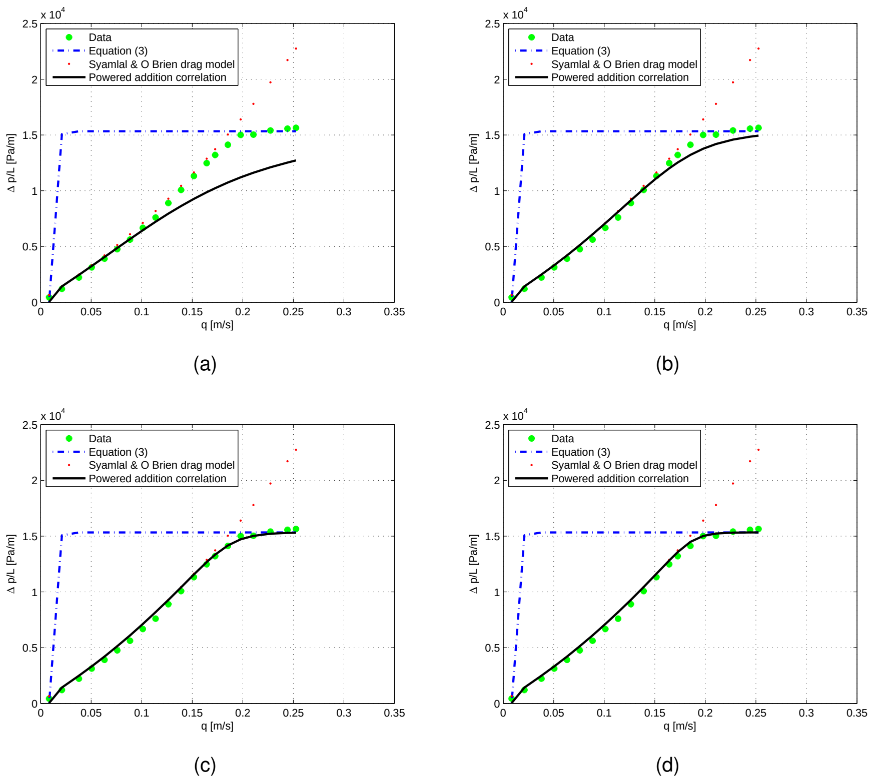
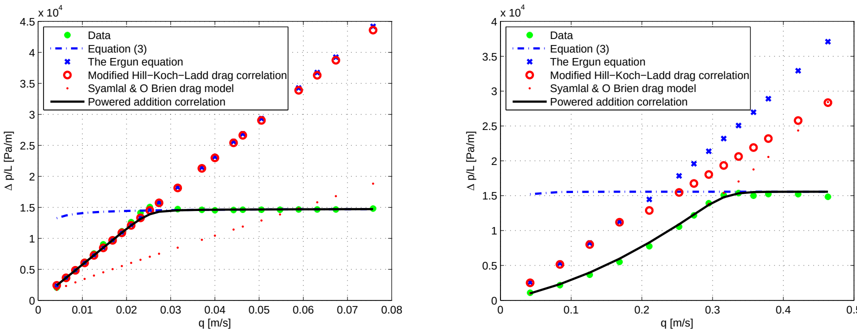


Figure 3: The powered addition correlation for a fluidized bed traversing from fixed to fluidized regime with the shifting parameter, *s*, equal to (a) 2, (b) 5, (c), 10 and (d) 15.

The accuracy of the fixed bed drag model thus plays a big role in the over all accuracy of the powered addition correlation and the drag models are very dependent on the representative particle diameter, as described earlier.

A similar analysis was done for the $100 - 200\mu m$ and $750 - 1000\mu m$ powders. Only the best results are given in Figure 4. For the $100 - 200\mu m$ powder the Syamlal and O'Brien drag model [2] was not a good representation of the data, even with a representative particle diameter of $100\mu m$. A possible explanation for this can be found in Geldarts classification of particles [4]. The $100 - 200\mu m$ powder is on the boundary between type *A* and type *B* particles whilst the $400 - 600\mu m$ powder is on the boundary between type *B* and *D* powders. The $7500 - 1000\mu m$ powder is a type *D* powder. Thus depending on the type of powder different drag models perform better. In the cases depicted in Figure 4 a particle diameter and model were chosen that best fitted the data. The reasoning was that a proper fitting in the fixed bed regime was required to produce an accurate value for the shifting parameter in each case. Using this best fitting models a value of 15 were found to give a suitable correlation in all the investigated cases.



(a) (b)

Figure 4: (a) The correlation for the $100-200\mu m$ powder using a representative particle diameter of $190\mu m$ and a s- value of 15, (b) the correlation for the $750-1000\mu m$ powder using a representative particle diameter of $750\mu m$ and a s- value of 15.

It should be mentioned that in Figure 4 (b) a very crude linear approximation was used for $F(q)$ (refer to equation (2)). The reason for this crude approximation was mainly due to the ossilations in the fluidized regime data for the $750 - 1000\mu m$ powder. Never the less, this approximation still produced an adequte asyptotic function, $H(q)$.

CONCLUSION

From the results obtained in the present work it appears that a value of 15 is adequate for the shifting parameter, s, independent of the particle size distribution. It can be concluded that the powered addition procedure [1] gives accurate correlations if the drag model used gives an accurate correlation in the fixed bed regime. Different models are suitable for different types of powders. Thus depending on the type of powder different representative particle diameter should also be used.

It is also apparent that the asymptotic function, $H(q)$, gives stable accurate result if the correct procedure is followed. Even with relatively large fluctuations in the fully fluidized data $H(q)$ still produces an accurate approximation to equation (1).

Further research into the physical meaning of the shifting parameter, *s*, is still needed and can prove very usefully in accurate prediction of different phenomena in a fluidized bed as illustrated in the present work

NOTATION

d_i	nominal diameter
$\overline{d}_{sv}$	surface-volume mean diameter
$F(q)$	function suggested to be used to produce a linear relationship on a *log-log* scaling
F_o	any point on the approximated linear curve
g	gravitational acceleration
H	asymptotic function for large values of the independent variable
m	gradient of $F(q)$ on a *log-log* scaling
p	pressure
s	shifting parameter
x	independent variable
x_i	mass fraction
y	canonical dependent variable

Greek letters

ϵ	porosity
ρ	density

Subscripts

p	particle property
f	fluid property
o	limiting condition for small values of the independent variable
∞	limiting condition for large values of the independent variable

References

[1] S. W. Churchill and R. Usagi. A standardized procedure for the production of correlations in the form of a common empirical equation. *Ind. Eng. Chem., Fundam*, 13(1):39–44, 1974.

[2] W. Rogers M. Syamlal and T. J. O'Brien. *MFIX Documentation theory guide*. U.S. Department of energy, Office of fossil energy, Morgantown energy technology center, Morgantown, West Virginia, December 1993.

[3] M. Syamlal S. Benyahia and T. J. O'Brien. Extension of the hill-koch-ladd drag correlation over all ranges of reynolds numbers and solids volume fraction. *Power Technology*, 162:166–174, 2006.

[4] D. Kunii and O. Levenspiel. *Fluidization Engineering*. Butterworth-Heinemann, 1991.

[5] G. Gibilaro Z. Chen and P. U. Foscolo. Fluid pressure loss in slugging fluidized beds. *Chemical Engineering Science*, 52(1):55–62, 1997.