Development normal pressure and frictional traction along Walls of a Steep Conical Hopper during Filling

S. Ding[a, c 1], M. Rotter[b], J. Ooi[b] and G. Enstad [c]

a. Dept. of POSTEC, Telemark Technological R&D Institute, Kjølnes ring, 3918 Porsgrunn, Norway

b. School of Engineering and Electronics, University of Edinburgh, West Mains Road, EH9 3JY, Edinburgh, Scotland

c. Dept. of Process Technology, Telemark University College, Kjølnes ring, 3914 Porsgrunn, Norway

Abstract A novel progressive filling approach was adopted in a numerical effort to represent the pilling process of particulate solids. It was implemented in a finite element analysis to investigate the development of loads along the walls of a conical steep hopper during filling. The loads were interpreted as normal pressure and frictional traction. An analysis of the conventional so-called ‘switch on’ filling was also conducted. Results form both analyses were compared with calculation based on classical theories for the loads acting on the wall of a steep hopper. A good agreement in such comparisons indicates that the progressive filling as adopted is a feasible approach as a finite element analysis to applications where analytical solutions are limited.

Keywords: a progressive filling, normal pressure, frictional traction, FEM, a steep conical hopper,

1 Introduction

The hopper is an important part of a silo, which is manifest in its effect on the mode the particulate solids is discharged [1, 2, 3, 4]. It also supports the majority of loads induced by the particulate solids, and is subject to a biaxial tension [5]. The tension as concerned is caused by the normal pressures and frictional traction developed along the walls of a hopper in various handling stages as filling, storage and discharging. It is well known that the load acting on the hopper walls varies from a stage of a filling to that of a discharging. The present investigation focuses on the development of pressures and frictional traction along the walls of a hopper with a steep inclination when the hopper is being filled with particulate solids.

The process of filling is a process in which the particulate solids are piled up from a loose surface flow [6, 7, 8]. Within the pile, a distribution of stresses, anisotropic in character, develops [9, 10]. This stress field transmits loads to the wall as filling progresses. An accurate determination of the loads

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[1] Corresponding author; telephone: +47 35574004; email: songxiong.ding@tel-tek.no.

conditions exerted along the walls of a hopper by the particulate solids is critically important.

The theories that aimed to predict the pressures exerted on hopper walls could trace back to those for the vertical walls of silos. For a vertical wall, the classical theoretical approach was the analysis by Janssen [11]. Different authors later attempted to produce extensions or modifications of Janssen's theory to the cases of conical and wedged-shaped [12, 13, 14]. Many such theories have yet gained widespread acceptance when compared with results obtained from experiments [5]. Computational modelling offers a powerful tool, and is used in the present study to develop an alternative approach to calculate the loads along the walls of a hopper during filling [15, 16, 17, 18,19].

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Admittedly, it has been a challenge with the finite element method to model a process as occurred during a silo filling process, where the particulate solid is gradually piling up and accumulated. As a result, the volume of the stored particulate solid increases, the boundary edges expands until the filling ends. In practices as usually carried out, a finite element mesh domain is defined to represent the whole stored particulate solids as concerned; loads are applied as gravity to the whole meshed region. It turns out to be a process of consolidation without initial stresses within the stored particulate solid rather than a process of progressive filling, and was conventionally referred as the so-called 'switch-on' filling. For closer representation of a filling process, various attempts have been tried, such as progressively increasing the density of the stored solids, or incremental 'progressive filling' with a small preloading in the stored solid or incrementally applying body weight over the total volume of the stored solid [17, 18, 19, 20, 21].

In this paper, efforts were made with the finite element method to investigate the development of loads along the walls of a steep hopper as occurred during filling. To simulate the filling process, the zones representing the stored particulate solid were meshed and partitioned into layers; the interaction between the stored particulate solids and the hopper wall was also modelled. They were all suspended or deactivated as an initial stage at the beginning of analysis. Steps were taken to reactivate the suspended meshes and the deactivated interactions in a designated sequence. The progressive filling process was believed to be simulated. It is regarded as a new approach; and the loads thus developed along the wall of a hopper were addressed as normal pressures and frictional tractions. In addition and for comparison, analysis in which gravity was 'switched on' for the entire mass of stored particulate solids was also conducted. These two types of analyses are referred as 'progressive filling' and 'switch-on filling'.

The finite element package Abaqus 2003 [22] was used to set up the FEM models. The models were applied to a hopper with a steep inclination angle. The results from the numerical predictions are compared with the analytical calculations used as the classical theories for the pressures acting on the wall of a steep hopper.

2 Finite Element Numerical Approach

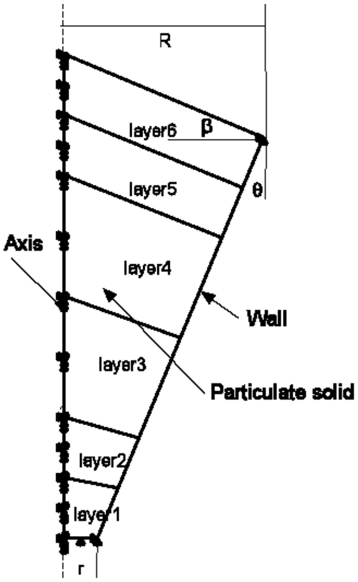
2.1 Geometry for the hopper and stored solid

A conical hopper as shown in Figure 1was considered in the present investigation. It was axi-symmetrical, 2400 mm in height with a radius R of 1200 mm for the upper inlet and a radius r of 200 mm of the outlet; the half hopper angle θ was 23°. The wall of this hopper was assumed made of stainless steel, 6mm in thickness.

A hopper is deemed to be steep if $\theta < \theta_{cri,}$holds [5]. The θ_{cri} is called a critical hopper half angle. It fits as :

$$\tan\theta_{cri} = \frac{1-\lambda}{2\tan\phi_w}$$

where: λ is the lateral pressure ratio on the vertical walls, usually has a value of 0.4 [16], ϕ_w is the friction angle between the wall and the particulate solids as stored, defined as the friction coefficient $\mu = tan\phi_w$; it was assumed to be 0.5 in this study. With such assumptions, the condition $\theta < \theta_{cri}$ held firmly; the hopper was regarded as rather steep, keeping in line with the hopper appropriated for the classic theoretical approaches.



R = 2400 mm;

r = 200mm;

$\theta = 23°$;

$\alpha = 23°$(assumed);

$\rho = 1000\ kg/m^3$ (assumed)

Figure 1. Geometry of the hopper and stored solid with same key parameters

The material filled into the hopper piled up in the due process of a surface flow and formed into a core. The shape of this cone will vary from one filling to another filling, depending on the material properties such as the angle of repose (α) and the position of feeding as well. In the present study, the material was falling in the centre, piling up into a cone with a repose angle of a α= 23°. The geometries of the particulate solid formed by the concentric filling are illustrated in Figure 1.

2.2 FE Formulation

Upon the geometries described above, finite element models were devised following the standard procedure provided by Abaqus to model the hopper wall, the stored particulate solids and the contact interaction between the stored particulate solids and the wall.

An axi-symmetric shell element was defined for the wall, and a continuum axi-symmetric element for the granular stored solid. They together formed the element domain. Within this domain, the hopper was constrained both horizontally and vertically at its top edge, and horizontally at its bottom edge. The boundary edge of the stored particulate solids was also constrained vertically at the outlet, its other edges were set free. The loading on the stored particulate solids was due to its gravity; the hopper structure was assumed to be weightless.

In a finite element model, the behaviour of each material involved is characterised by using an appropriate constitutive law. The wall of the hopper was stainless steel, was simply modelled as elastic solid. To find a model to describe the stored particulate solids is still a challenge. Particulate solids display various behaviours and the mechanical description of such assemblies is an old but still open problem. A general feature observed both in experiments and in simulation is the very heterogeneous and anisotropic character of the force network arising from the inter-granular contacts [8, 9, 10, 23, 24, 25, 26, 27]. Recently, strong interest has developed in both the engineering and physics communities to try to develop a new understanding from a description of such particulate contacts and force distribution to a macroscopic description of the stress-strain relations [26, 27, 28, 29]. In the present investigation, the classical and well established model was adopted. The stored particulate solids were assumed to be single phase, and modelled approximately as an elastic-plastic frictional material.

In similar, the interaction between the stored particulate solid and the walls of the hopper could be quite complicated, depending on the material properties of the hopper wall and the stored particulate solids. Slip-stick behaviour is observed quite often during wall friction measurements as summarised by Schwedes [30 ,31]. Modelling such a mechanical interaction can be quite complex, and is still a great challenge [32, 33, 34, 35] . In the current study, since the focus is to find an alternative approach to explore the loads as developed along the walls of a hopper during filling, the interaction between the contacting surfaces of the hopper and the stored particulate solids was modelled with a very simplified constitutive model of Coulomb friction[34, 35, 36]. A constant friction coefficient μ was assumed and implemented in the model.

To implement a progressive filling approach, the meshes representing the stored solid were defined, loads due to gravity were also defined but then suspended throughout the meshes; the interactions between the contacting surface of the particulate solid and the walls of the hopper were deactivated. A partition technique was utilized to divide the region of the stored particulate solids in layers as shown in Figure 1. The region was divided firstly into two parts of equal thickness; each of these two parts was subdivided again into two parts, giving four layers with the same thickness. The layer at the bottom and the layer at the top were again subdivided, resulting in six layers in total. By reintroducing elements, reactivating the corresponding loads and contacting interactions in a layer-by-layer upwards, the progressive filling process was simulated. This approach is not exactly equivalent to the real process of filling; but it is clear that the finer the layering, the closer the approach will be to a true filling process.

2.3 Determination of model parameters and convergence test

The wall was made of stainless steel; and was regarded as elastic, with the Young's modulus E_w = 2.0 x 10^{11} Pa and the Poisson's ratio ν_w = 0.3.

The granular material was modelled as an elastic-plastic material, with the Drucker-Prager hardening /

yield law. Under this model, the parameters involved were the Young's modulus E_p, the Poisson's ratio ν_p, the bulk density ρ, the internal friction angle φ, and the friction angle ϕ_w with the wall of the hopper. Convergence tests were carried out after setting the stored solid bulk density to $\rho = 1000$ kg/m^3, and the Poisson's ratio ν_p to 0.3, it was achieved when the Young's modulus E_p was higher than 5.5×10^4 Pa [22, 37, 38]. To avoid possible numerical problems associated with very large deformations of the mesh, the parameters, chosen to represent the stored solid, were assumed based on parameters used in Abaqus (2003) manual as summarized in Table 1.

Table 1 Constitutive model for the granular stored solid and parameters used [22]

*material, name=POW	Keyword to define a name for material
*density 1000.,	Keyword to implement the density kg/m^3
*Drucker Prager	Keyword to implement the material model
55., 1., 35.	The material angle of friction in meridional plane, ratio of the flow stress in tension to that in compression and the dilation angle in meridional plane (Abaqus, 2003).
*Drucker Prager hardening 50000., 0. 55000., 0.02 60000., 0.025 70000., 0.03 120000., 0.035 100000., 0.05	The same as above; the true stress and strain
*elastic 550000, 0.3	Keyword to implement E_p (Pa) and ν_p
*friction 0.5	Keyword to implement the interaction as a friction

3 Numerical Simulation Result and Analysis

3.1 Development of loads with a progressive filling

Under loading due to gravity, normal forces and frictional shear forces (frictional tractions) are generated across interacting surfaces between the wall and the stored solid. The normal force per unit is regarded as normal pressure on the wall, and the shear force as frictional traction distribution along the wall. The resultants of the normal pressures and frictional tractions between the contacting surfaces of stored solid and wall are interpreted as loads on the walls.

When meshes from Layer 2 to Layer 6 were suspended as shown in Figure 1, only the contact surfaces of Layer 1 and the wall were active. This interaction generated pressures and frictional tractions within this contact region. After that, Layer 2 was reactivated in the second stage of filling, and the contacts between Layer 2 and the corresponding wall surface were added back. The contact interactions were between the combined contacting surfaces of Layer 1 and Layer 2 with the wall. This new interaction brought about contact pressures and frictional tractions on the surface of a region covering Layer 1 and

Layer 2. This process was repeated until all the layers were activated. The development of the filling pressure on the hopper walls thus obtained are shown in Figure 2 for normal pressure on the wall, and in Figure 3 for frictional traction on the wall surface.

From Figure 2 one can see that the maximum normal pressure acting on the walls, from the very beginning of the filling, was not at the outlet. The maximum normal pressure increased in magnitude and moved upwards with the development of the filling process, and was located in a position around 2/5 of the length of wall from the outlet when the filling was finished. The normal pressures acting at the outlet increased in the process of filling, but tended to approach a constant value. Based on the parameters assumed in the present study, it was about half of the maximum normal pressure acting on the walls.

Figure 3 showed the development of the frictional traction on the surface of the hopper wall. It followed the same pattern as the normal pressure, with the position of the maximum frictional traction moving upwards with the filling process, and ended at a location around 2/5 of length of the wall from the outlet.

One feature for the frictional traction distribution was that the frictional traction drops to zero at the outlet. That was caused by the boundary condition adopted in the analysis. Since the node at the end was fully constrained, the stored solid cannot have any movement or movement tendency at that position, resulting in the frictional traction being zero. In other parts, there existed relative movement between the stored solid and the wall, the frictional traction thus developed.

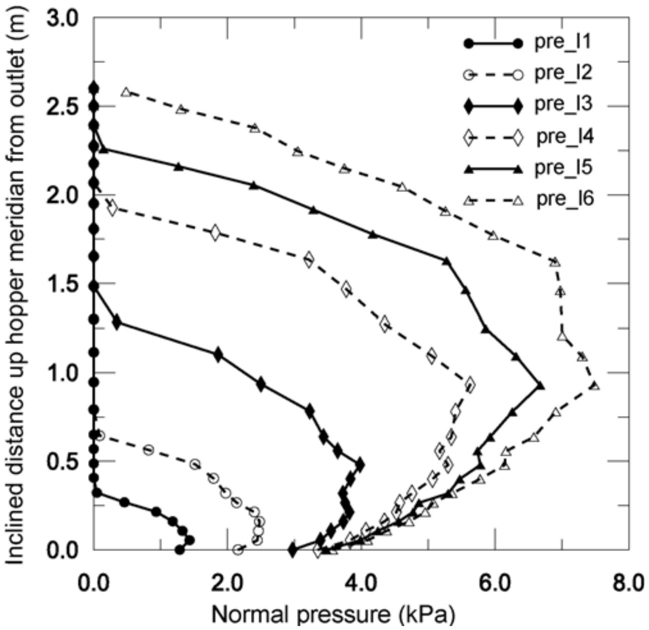


Figure 2 Development of normal pressure acting the walls of the hopper

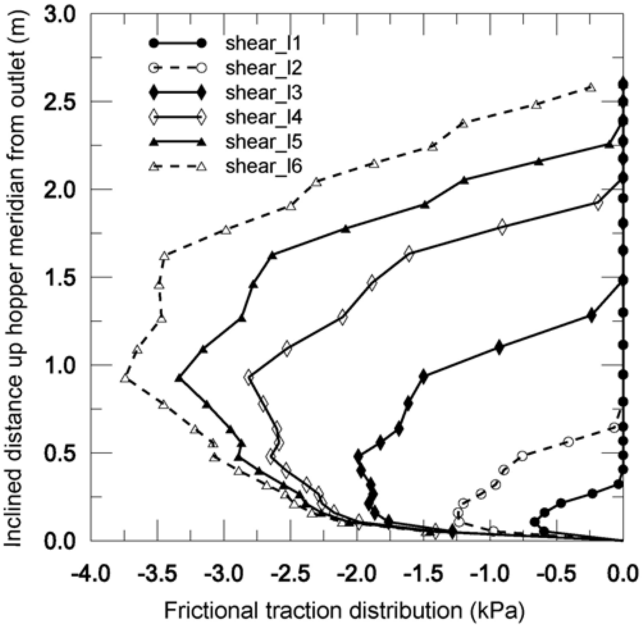


Figure 3 Development of frictional traction distribution on the walls of the hopper

3.2 Effects of filling processes on loads

To find out what affect the filling process produced, different filling processes were investigated by comparing the predictions of switch-on filling and progressive filling. In the condition of ‘switch-on’ loading, neither the mesh nor contact interactions were suspended or removed, the loading was added throughout the whole region of the granular stored solid in one step as gravity. The loads on the walls induced by a filling process of switch-on were shown in Figure 4.

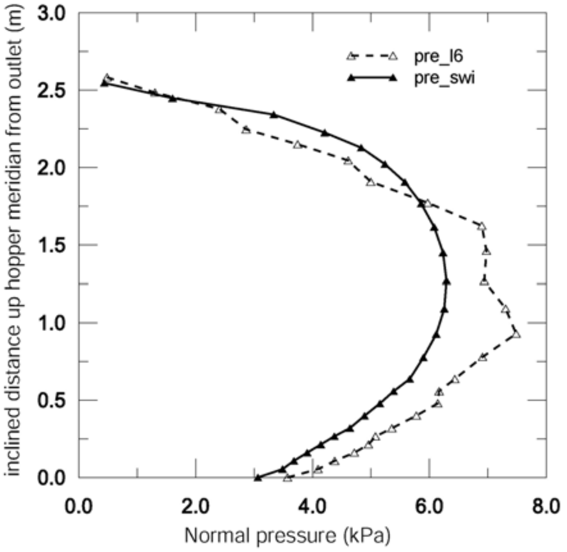


Figure 4 ‘Switch-on’ loading effects on the contact pressure on the hopper wall

Figure 4 showed the predictions of normal pressures on the hopper wall under this switch-on loading, along with the pressures for the last stage of the progressive filling (Figure 2). From Figure 4, it is clear that some differences are produced by the different filling processes. Switch-on filling decreased the maximum contact pressure and moved the location of the maximum pressure upwards. On the higher parts of the hopper wall, switch-on filling raised the normal pressure, whilst in the lower part the normal pressure was reduced.

3.3 Comparison and discussion

Many theories have been proposed for the distribution of pressures in a conical hopper. The reader is referred to [5, 39] in details. As a case study, the pressure distribution along the wall for the present model is given as a diagram as shown in Figure 5 by some typical analytical calculations (the values for the parameters required in such calculations were the same as those used in the numerical approach). For comparisons, the normal pressure at the last stage of the layer-by-layer filling as shown in Figure 2, and the normal pressure predicted by the 'switch-on' filling as in Figure 4 were extracted and plotted along the theoretical results calculated (Figure 5).

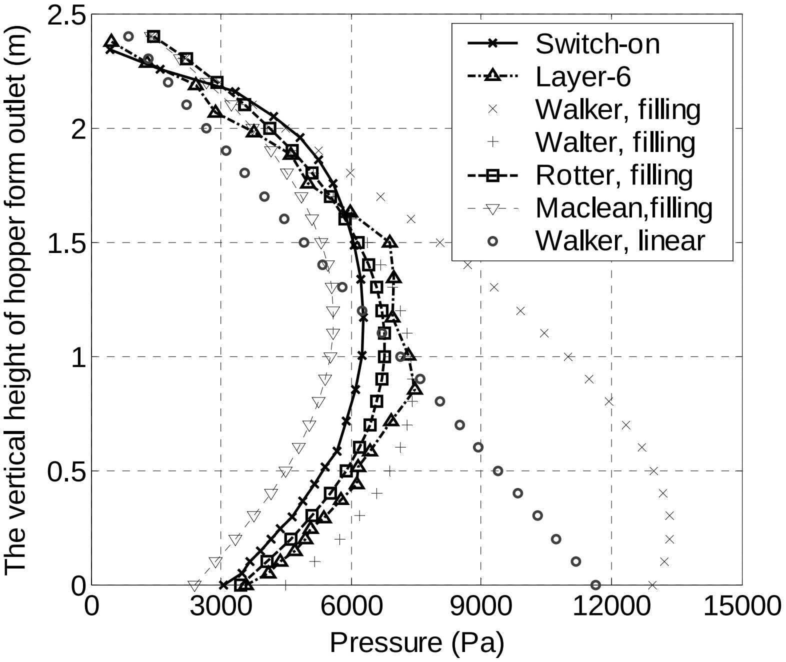


Figure 5 Comparisons of normal pressure

From Figure 5, one can see some differences existed between the theoretical results while the predictions from the finite element analyses from both the progressive and switch-on filling have the general form proposed by Walters [13], MacLean [40] and Rotter [5]; the predictions of Rotter's theory seem to be closest and lie between the two FE curves.

The frictional traction along the wall is better to be interpreted by the 'mobilised friction coefficient', which is defined as the ratio of the local frictional traction to the corresponding local normal pressure at a location [5]. The mobilised friction coefficients on the walls at the end of filling of both progressive filling and switch on filling are shown in Figure 6. One can see there that frictional traction equalled to 0.5 (i.e. the assumed value in the model). It could be assessed that the friction was fully mobilised everywhere for both progressive filling and switch-on filling in this steep hopper (except a junction region close to the outlet). This pattern of a fully developed friction mobilisation in steep hoppers matches the model of Rotter (2001), which has been adopted into the European Standard (EN 1991-4, 2004) for pressures in hoppers.

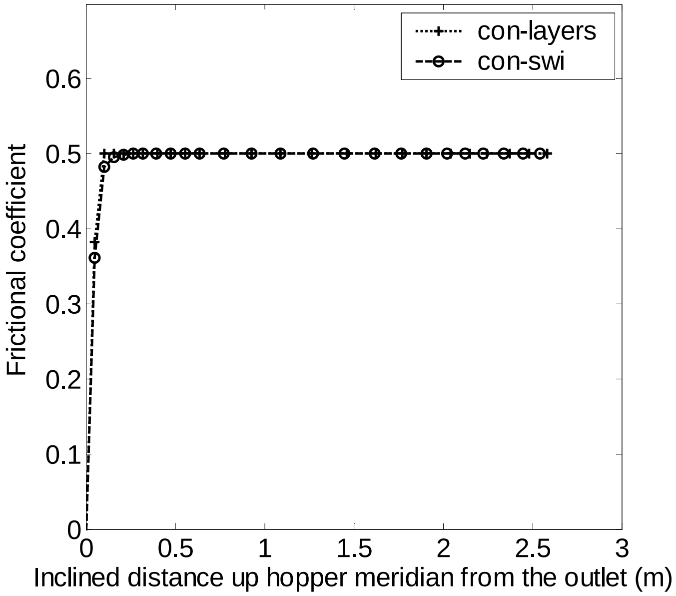


Figure 6 Mobilised friction coefficients for both progressive and switch-on filling

At the junction between the hopper and outlet, there can be no relative sliding, so the friction is unable to be mobilised. But this effect turns out to occur in only a very small region, so small that the general pattern of hopper pressures is unaffected by the small loss of mobilised friction.

4 Conclusions

It has been shown that the layer-by-layer progressive filling is a feasible approach as a finite element analysis to simulate a filling process. Such an analysis revealed that the maximum normal wall pressure in the hopper did not occur at the outlet. The maximum normal pressure increased in magnitude and moved progressively upwards during the filling process. The normal pressures acting at the outlet also increased initially, but tended to approach a constant value.

Compared with switch-on filling, progressive filling increased the maximum normal pressure and moved the location of the maximum pressure downward. In the lower part of the hopper walls, the normal pressures increased with a corresponding decrease in the higher part of the walls.

The development of the frictional traction on the surface of the hopper wall followed the same pattern as the normal pressure. The wall friction was fully mobilised everywhere in the steep hopper except very close to the outlet. This pattern of friction mobilisation matches the model of Rotter (2001) proposed for a steep hopper.

The FE predictions of normal pressure along the walls for both the progressive and switch-on filling in a steep hopper have the general forms proposed by Walters, MacLean and Rotter (2001); the predictions of Rotter’s theory seems to be closest and lie between the two FE predictions. The results obtained so far showed that the progressive filling had no convincing advantages over the conventional “switch-on” filling approach as conducted on the steep hopper, never the less, this progressive filling approach could be regarded as an alternative approach to investigate the loads developments along the walls of a hopper with a shallower inclination, where analytical solutions are few, and where its advantages might be expected to appear.

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