Master Thesis

Heterogeneity, herding, sentiment risk and asset pricing in the Norwegian stock market

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Key words:
heterogeneity, sentiment, dispersion of beliefs, divergence of opinions, cross sectional absolute deviation of asset returns, herding, asset pricing, equity premium puzzle, correlation puzzle , stochastic discount factor

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1 Preface

This master thesis is the conclusion of the master program in finance and accounting at HiBu college in Norway. We put in good use all knowledge provided at our school be it statistics, finance, accounting or methodology. Writing a master thesis proved to be a task demanding additional competence which we had to acquire in the process. We worked our way to skills in conducting research and retrieving data using financial databases, handling econometric packages, implementing programming code, running different type of tests and interpreting the results. During this process we came across many theoretical and practical issues that arose many questions which we tried to solve as best we could. Time series became an indispensable source of information providing a wealth of data. The question of lack of stationarity and remedies to that became an important part of our daily work. While the least square regression and its assumptions are highly standardized, this is not the case for the generalized method of moments which is the method of choice for tests of consumption capital asset pricing models. We soon discovered that the symbols used for the same concepts are far from standardized in the literature. Using a multitude of sources, it proved to be a demanding task to find a way to a uniform notation in our master thesis, something we cared extra to achieve. We didn't have access to LaTex, so we had to make the best out of what is offered for mathematical symbols in word. We gave priority to keep tables in the same page in order to be easier to read rather than filling up the pages from top to the bottom. Writing this master thesis we complied with HiBu college's norms. The master thesis is a theoretical and empirical work. There are no limits when it comes to length of this thesis. Usually the master theses submitted to HiBu have a length of 100-150 pages with 1.5 line spacing excluding appendices.

Writing the master thesis together as a team meant the share of ideas and thoughts. We learned a lot from each other. Despite stumbling on far too many stones to turn around and see what is underneath we feel we laid down a fair amount of work and got the satisfaction of finding working solutions to the challenges we faced. We hope that this work is going to be helpful to others researching the same or similar issues contained in our master thesis.

Notice: In this master thesis we chose to use the terms divergence of opinions and dispersion of beliefs interchangeably. See the definitions of opinions and beliefs in appendix A - i
for an explanation.

2 Abstract

The research issues addressed in our master thesis concern heterogeneity in different forms as a red thread that connects together the tests we carried out. We examined the role of sentiment risk as a rate of change in heterogeneity of opinions in the stock exchange. Heterogeneity in the behavior of market agents can lead to departures from the predicted linear relationship between the equity return dispersions and market return. Heterogeneity in consumption can generate higher volatility in consumption growth, which is a needed feature in the stochastic discount factor used in consumption capital asset pricing models but at the same time mitigate the correlation between the stochastic discount factor and the equity premium, which is a shortcoming. The Hansen-Jagannathan bound which sets a theoretical prediction of the relation between the Sharpe ratio and the stochastic discount factor is outside the reach of conventional consumption capital asset pricing models due among other issues to low volatility in consumption of non-durables and services. The setting of our research is the Norwegian stock market.

We conducted a research on these issues using linear regression analysis, non-linear regression analysis and the generalized method of moments. In some cases we employed stochastic regression imputation and Monte Carlo simulation. Data was retrieved from the financial databases Factset and Datastream. Other data sources utilized by us were OECD, Statistics Norway, Norway's central bank and the Norwegian national institute for consumer research. We carried out also a series of interviews with financial analysts and other professionals in the financial sector of Norway as background information to characteristics of Oslo Stock Exchange.

Sentiment risk helps explain the volume of trading and volatility in the Norwegian stock market. We found a non-linear relation between the cross sectional absolute deviation of returns and the market return which can be interpreted as heterogeneity in the behavior of market agents. Heterogeneity as an added ingredient in consumption capital asset pricing increases the volatility of consumption and has the potential to contribute to a better explanation of the risk premium.

Our main conclusion is that heterogeneity in its various forms can be an important ingredient in models attempting to explain the volume of trading, the volatility of market return, the behavior of market participants and the equity risk premium.

3 Introduction

Trading volume in stock markets far exceeds the predictions of theoretical models that assume among other things rational expectations and common interpretation of information (Odean 1999, Chordia, Roll, and Subrahmanyam 2010). The capital asset pricing model predicts a linear relationship between the equity return dispersions and market return (Chang 2000). Consumption capital asset pricing models assume often homogeneity with respect to consumption in the form of a representative agent (Lucas 1978, Campbell and Cochrane 1999).

The relaxation of the assumption of homogeneity can lead to models which can better predict and explain the stylized fact of high trading volume and the excess volatility of stock returns. The relationship between the cross sectional absolute deviation of stock returns and the market return can be non-linear. Relaxing the assumption of a representative agent can yield consumption capital asset pricing models which explain more convincingly the empirical equity risk premium. Heterogeneity is an integrated feature in behavioral finance but can be also achieved in the setting of rational expectation models.

The master thesis is organized as follows. In the chapter of the classical asset pricing theory we present the basic asset pricing equation, the stochastic discount factor, the capital asset pricing model, Lucas' consumption capital asset pricing model and the fundamental theorem of asset pricing. We show explicitly how the consumption capital asset pricing model is derived from the capital asset pricing model. In the next chapter we present bubbles as examples of excessive trading and summarize the historical episodes of stock market crashes starting from tulip mania in the Netherlands in the Middle Ages up until recent times. Then we present empirical observations that depart from the predictions of the classical asset pricing theory and constitute puzzles. These include the equity premium puzzle, the risk free rate puzzle, the correlation puzzle and the volatility puzzle. We describe attempts to reconcile the facts with theory in the setting of the main stream financial theory and in the setting of behavioral finance. Our empirical tests come next. They concern heterogeneity and consist of three groups. One test group is related to sentiment risk. The second test group is related to the cross sectional absolute deviation of returns. The third test group is related to the equity risk premium and consumption capital asset pricing. We present and discuss the test results for each group including the model assumptions. We end up the master thesis with our conclusions.

4 Theoretical literature review

4.1 Financial assets and trading

In the traditional asset pricing theory there does not seem to be much scope for trading. The no trade theorem (Milgrom and Stokey 1982) for instance states that under the assumptions of rational expectations (see appendix A - xiii), complete markets, Pareto efficient allocation of resources and concordant prior beliefs that is common interpretation of information, there is no incentive to trade beyond the first period. The argument for this is that given a Pareto efficient allocation of resources any offer to trade is based on private information that is unfavorable to other traders. Due to this issue there is no universally acceptable trade. Another argument used for low levels of trading is that the stock market is efficient and adapts almost instantaneously to new information.

Tirole (1982) set ups a model with a finite number of infinitely lived agents with homogeneous beliefs. Under the assumption that the last trader knows his position in the queue, he proves by backward induction that there cannot be trade under rational expectations. In an overlapping generations' model with an infinite number of finite lived agents, Tirole (1985) shows that a bubble doesn't exist as long as the interest rate is greater than the growth rate of the economy, due to the agent's budget constraint.

Trading volume in stock markets far exceeds the predictions of theoretical models that assume among other things rational expectations and common interpretation of information (Odean 1999, Chordia, Roll, and Subrahmanyam 2010).

4.1.1 Excessive trading, Ponzi Games, Bubbles, and Crashes

Charles Ponzi was a criminal in 1920 that allured capitals from investors under the promise of extraordinary interest payments. Keeping this promise depends on a larger group of new investors contributing to meet the pay-offs which are due to earlier investors. This game leans on a continuous stream of a sufficient number of new arrivals whose investments at time t are used to compensate investors who arrived previously. The repayment of debt is done by issuing more debt. The game goes on as long as revenue at time t covers the obligations incurred previously. This set up collapses when investors withdraw their money faster than the disbursement of revenue through new arrivals. Investors entering first the game make a good return on their investment while those entering towards the last stages before it collapses have to take the loss. The catch is that one doesn't know in advance when the scheme is going to blow on one's face.

It might be hard to believe that people fall for fraud schemes whose logic is obvious. An explanation to it can be short-sightedness. Folks tend to worry about the outcome just a few steps ahead and are counting with leaving the game before it breaks down. An evidence of people being allured consequently over a longer period is Madoff's Ponzi type hedge fund scheme from 1980 to 2008 where 50 billion dollars were invested, by private persons and respected institutional investors alike.

Bubbles bear similarities to Ponzi games in that the expectation of large profits is the motive for people entering the market (Hens and Rieger 2010, p. 242). The strategy of the investor is to get in as the stock price takes off and get out before the bottom falls off. During a bubble, asset prices are inflating without a change in the assets fundamental value (see appendix A - xiv). Historical evidence exhibits striking examples of assets sold at market prices way above their fundamentals, the prices driven by expectations of reselling later at an even greater price. Bubbles can be sustained by noise traders chasing the trend (Shleifer 2000, p. 154). Bubbles are likely to appear in periods of optimism and technological innovations which are thought being able to transform the productive capabilities of society to considerably higher levels. High trading volume is associated with asset price bubbles (Scheinkman 2013, pp. 3 and 7). Crashes in the stock market are in many cases preceded by bubbles and in several cases associated with excessive trading volumes, for instance on 24th October 1929 a record 12,9 million shares were traded (encyclopedia Britannica) and in black Tuesday on 29th October 1929 investors traded 16 million shares on one day (NYSE Timeline). In Black Monday on 19th October 1987 the share volume traded in NYSE was very high (Carlson 2007 p. 9) at 604,3 million shares (USAtoday). In the flash crash of May 6th 2010 the trading volume was 5,094 million futures contracts and 1,030 million trades compared to a trading volume of 2,397 million futures contracts and 0,446 million trades in the period May 3rd – May 5th (Kirilenko, Samadi, Kyle and Tuzun 2011 p. 42). Baghestanian, Lugovskyy and Puzzello (2013) propose a model in which speculators generate the crash by massive selling to noise traders.

The occurrence of speculative bubbles and panics does not commensurate with rationality. Memory effects in the movements of stock prices shows that the rules of motion are not consistent with the random walk hypothesis (see appendix A - xi). Can bubbles be interpreted as the outcome of speculation and as evidence of market irrationality?

Consider the discounted component of an asset price P at time $t+k$, $E_t\left[\left(\frac{1}{1+R}\right)^k P_{t+k}\right]$ .In case it doesn't converge to 0 as $k \to \infty$, then there are infinite solutions to $P_t = E_t\left[\frac{P_{t+1}+D_{t+1}}{1+R}\right]$ All the solutions are of the form:

$P_t = V_t^* + B_t$	4-1

Where $V_t^*$is the asset's fundamental value and B_t is a bubble given by:

$B_t = E_t\left[\dfrac{B_{t+1}}{1+R}\right]$	4-2

The bubble B_t is consistent with rationale expectations (Campbell et al. 1997, p. 258). Blanchard and Watson (1982, pp. 295-316) come up with a bubble of the form:

$B_{t+1} = \begin{cases} \left(\dfrac{1+R}{\pi}\right) B_t + \zeta_{t+1} & \text{with probability } \pi \\ \zeta_{t+1} & with\ probability\ 1-\pi \end{cases}$	4-3

where ζ is a shock with $E_t[\zeta_{t+1}] = 0$, $1-\pi$ is the probability of bursting and $\frac{1+R}{\pi} - 1 > R$ is the bubble's growth rate.

An alternative explanation of bubbles is as the result of herding behavior. Contagion of optimism leads traders to long positions and market prices above the assets fundamental values. This could be followed by panic if a triggering event like a rumour for a bad event would create a critical mass with an avalanche effect. Contagion of opinion and behavior creates fluctuations around the assets intrinsic values (Lux 1995).

Bubbles have dominated at times the financial markets throughout history as a testimony of their power to create waves of feverish trade activity. Here is a table with indicative figures of the change in the index level on Bubbles and Crashes incidences, as a percentage drop from peak to trough, where peak is associated with excessive trading:

Table 4-1: Percentage drop of crashes

Incident	**Percentage drop**
Tulip mania	Tulip prices fell by 94,6% from 3rd Feb.1637 to May 1st 1637. (Thompson 2007)
South Sea Bubble	The South Sea's share value fell by 83,3% from June to the end of December 1720. (Source: Harvard Business School Historical Collections (2012).
British railway boom	The railway equity securities listed on the London Stock Exchange fell by 64,1 % from October 1845 to April 1850. (source: Campbell 2009)
Florida land boom	From 1926 to 1930 real estate prices fell by 68,1%. (Source: Grebler et al. 1956).
U.S Stock Market boom	From the midsummer of 1929 to October 29 in 1929, share prices fell by 48%. (Source: Encyclopedia Britannica, Stock Market Crash of 1929)
Black Monday	On October 19th in 1987, the Dow Jones Industrial Average fell by 22,6%. (Shleifer 2000, p.20)
Dotcom bubble	From March 11th 2000 to October 9th 2002, the Nasdaq Composite index fell by 77,9%. (Source: Beattie 2012).
Housing bubble	By September 2008, average U.S. housing prices had declined by over 20% from their mid-2006 peak. (Source: The Economist Newspaper Limited 2012).
Flash Crash	In May 6th 2010 the Dow Jones Industrial Average plunged 9,3% from the intraday high to the intraday low.(Source: Lauricella and Mckay 2012).

In the following table are summarized some of the most known bubbles and crashes in history:

Table 4-2: History of bubbles and crashes

Incident	Start	Description	End
Tulip mania	1630	Estates surrounded by beautiful gardens were a signal of prosperity and status. Bulbs infected with the "tulip breaking virus" developed a multitude of rare shades of colors. Trading of tulip bulbs got off to extraordinary heights. (Source: Shleifer 2000, p.170)	1637
South Sea Bubble	1720	UK financed its debt in the succession war against Spain through conversion of debt to shares of a company called South Sea. The company's share value inflated from £128 per share in January to £1050 by June 1720. This was done by circulated false tales of South Sea riches. The price plummeted down to £175by the end of the year. (Source: Harvard Business School Historical Collections 2010)	1720
British railway boom	1845	The rising of the importance of railway building projects in UK brought about a frenzy of investing on new railway lines. This trend was exacerbated by Ponzi schemes using investment capital on new railways to pay out dividends to investors of earlier projects. (Sources: Shleifer 2000, p.170 and Campbell 2009)	1850
Florida land boom	1920	Florida real estate became a popular investment object in 1920's aided by advertisement campaigns as a destination of sunshine and leisure. At its peak, city lots were bought and sold a tenfold of times in a single day. (Source: Harvard Business School Historical Collections 2012)	1926
U.S. Stock Market boom	1920	The stock market in US experienced an unprecedented expansion in the 1920's. Stock prices rallied to extraordinary heights. Consumers took mortgages on their houses to finance investments on the stock market. Shortly after the stock prices stopped rising. People rushed to sell their shares and the stock market crashed. (Source: Encyclopedia Britannica, Stock Market Crash of 1929)	1929
Black Monday	1987	In 19th of October 1987 the Dow Jones Industrial Average (DJIA) crashed. $ 500 million were stripped off the market. Value of US securities. Within a month DJIA dropped to 1/3 of its previous high. One explanation is excessive trading due to differentials of stock index futures and stock prices. This behavior was exacerbated by the use of data algorithms which were triggering along additional trading. (Source: Solberg 1992, p.211)	1987
Dotcom bubble	1994	Investing on internet companies with due haste became a powerful trend in the 90's reaching its peak on march 2000. Most of the stock gains dissolved into thin air shortly afterwards. (Source: Galbraith and Hale 2004)	2000
Housing bubble	1997	Real estate prices increased by 124% between 1997 and 2006. Subprime lending and speculative purchases were contributing factors to the bubble and its subsequent burst. (Source: The Economist, CSI: credit crunch, Oct 18th 2007)	2008
Flash Crash	2010	The flash crash occurred in May 6th 2010. It was the biggest point decline in just one day. The index took a harrowing plunge by 998,5 points, the biggest one in the history of the Dow Jones Industrial Average. An explanation given is that this was a liquidity event caused by high frequency trading as a result of an accumulation of losses in the recent past. (Source: The Wall Street Journal, Dow Takes a Harrowing Point Trip, May 7th 2010)	2010

Whether certain crashes have been preceded by bubbles is an issue that has been debated. There have been divergent opinions on this issue concerning Black Monday 1987 (Malkiel 1989) Tulip mania (Thompson 2007) and the British railway boom (Campbell 2009).

4.1.1.1 *The Tulip Mania*

"Many individuals grew suddenly rich. A golden-bait hung temptingly out before the people, and one after another, they rushed to the tulip marts, like flies around a honey- pot. At last, however, the more prudent began to see that this folly could not last forever. Rich people no longer bought the flowers to keep them in their gardens, but to sell them again at cent per cent profit. It was seen that somebody must lose fearfully in the end. As this conviction spread, prices fell, and never rose again." (Mackay 1841, cited in Library of Economics and Liberty)

The citing is on the Tulip mania which is one of the most spectacular get-rich-quick rushes in history (Malkiel 1991, p.35).In between November 1636 to May 1637 people were driven to madness investing all their money to tulip bulbs in anticipation of even higher return to their investment. Expectations were driving bulb prices up into a self-fulfilling prophecy. There was a mass hysteria as market prices for tulip bulbs roared towards extraordinary heights. But then the scheme collapsed:

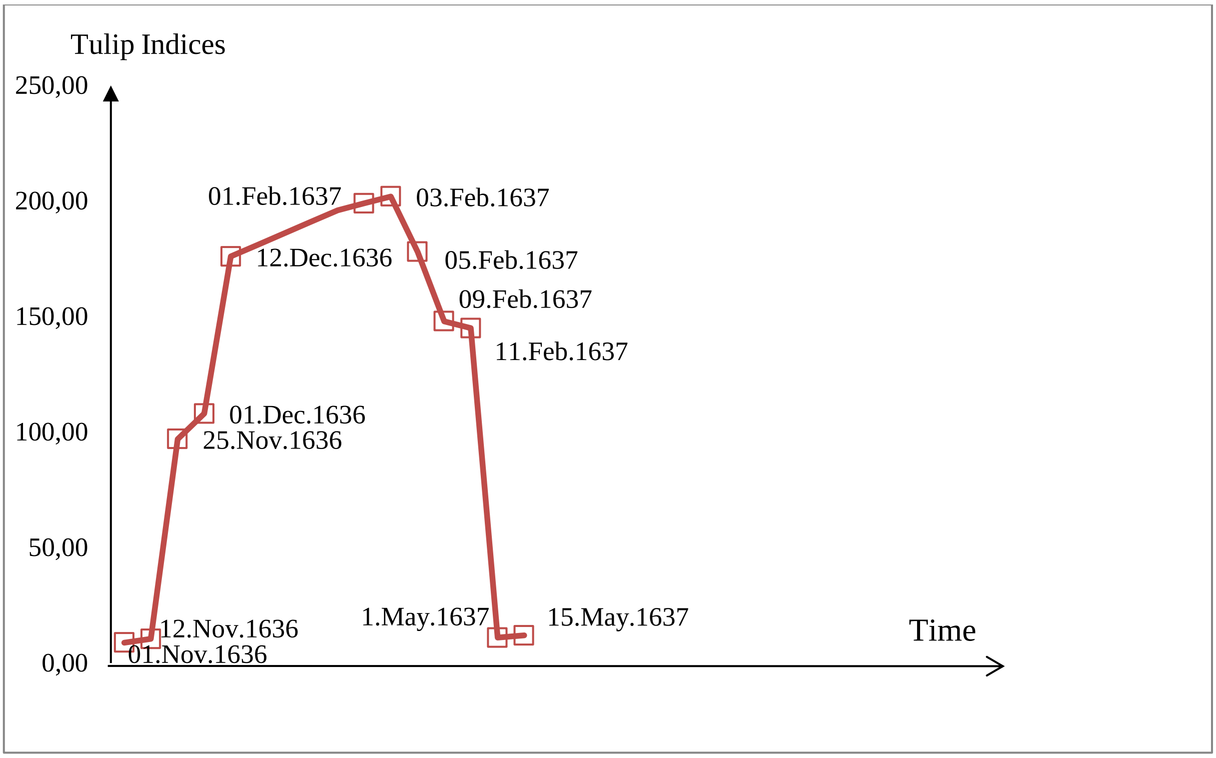


Figure 4-1: Tulipmania: Development of tulip prices between 12th November and 3rd February 1636-1637 during the tulip mania period.
(Adjusted from Thompson 2007)

4.1.1.2 *Black Monday*

In what is referred to as "Black Monday", share prices around the world fell by 22,6% in October 19th 1987:

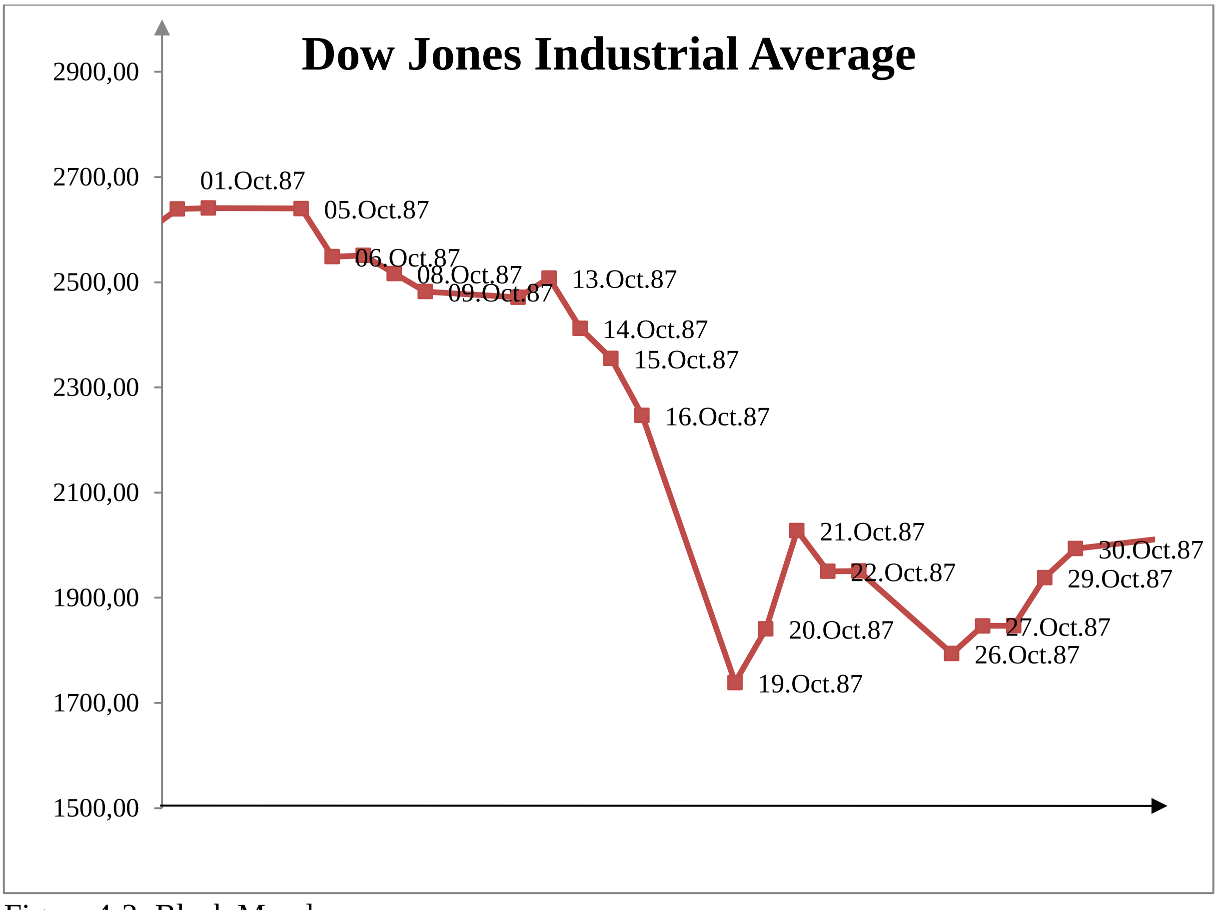


Figure 4-2: Black Monday
(Adjusted from Kiyono et al 2006)

A survey conducted in the aftermath showed that (Shiller 1987):

- no news story or rumour was responsible for investor behavior
- many investors thought that they could predict the market
- the general perception was that the market was overvalued
- investors blamed the crash on the psychology of other investors
- investors were influenced by technical analysis considerations

4.1.1.3 Flash crash and market microstructure

Changes in the stock market microstructure can also explain the high volume of trading and flash crashes with sudden trading collapse. Although high frequency firms represent just 2% of the trading firms they account for over 70% of the US equity trading. Market makers make a profit by earning a spread between sellers and buyers. The crash on May 6th 2010 featured the biggest drop ever of the Dow Jones Industrial Average which declined by 998,5 points. The backdrop to that event is the overtaking of market making by high frequency intermediaries. Market makers are taking the opposite side of a transaction providing liquidity in the stock market. Because of asymmetric information and adverse selection, the market makers run the risk to inflict losses. This happens when market makers trade with agents who have inside information. These kinds of order flows are called toxic. When toxicity is high, the market makers liquidate their inventory and stop participating in trading. This has the effect of liquidity drying. Easley, de Prado and O'Hara (2011) investigated the flash crash event and concluded that it was caused by market microstructure features such as the computerization of market making and the high frequency trading rules.

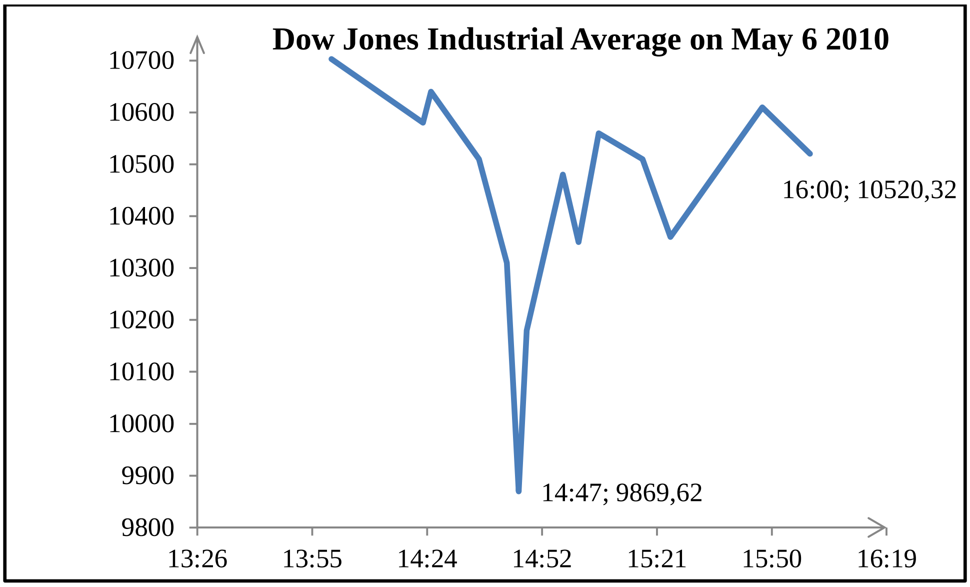


Figure 4-3: Flash Crash
(Adjusted from Wall Street Journal, Phillips 2011)

4.1.2 Attempts to explain trading

Trading can be explained by two schools of thought. The first one is attributes trading to noise traders who introduce noise risk and speculative trading in the market place. The other school of thought explains trading with diversity in interpretation of available information.

Trading can be the result of heterogeneity in beliefs. Heterogeneity in beliefs can arise because people interpret the same information differently (Iori 2002, Hommes 2006, p. 56). Heterogeneity in beliefs due to asymmetric information is informative in the sense that agents assume that persons willing to trade are better informed whereas heterogeneity of beliefs based on opinion differences is uninformative. When heterogeneity is uninformative, trade occurs. If prior beliefs are different, agents would also trade. Another reason for trading is relative changes in wealth (Xiouros 2009, p.111). Behavioral finance offers as possible explanations noise and psychological bias like overconfidence (Barberis 2003, p. 1102). Lux (1995) explains bubbles as a self-organizing process based on contagion of behavior across heterogeneous agent groups. Small deviations of the asset prices from their fundamental values create a powerful amplifying effect due to the interaction of noise traders with fundamentalists.

A large part of significant stock market moves are difficult to explain on the basis of information on fundamental values (Cutler, Poterba and Summers 1989).The tulip mania in 17th century and the stock crash in 1987 are examples of phenomena that cast doubts on the assumption of rational behavior. For that matter also that share prices follow a random walk at all points of time.

Campbell, Lo and MacKinlay (1997, p. 259) maintain that bubbles can be ruled out in a rational expectations world on theoretical grounds using the following three arguments

● The first argument is that there cannot be a negative bubble for an asset with limited liability since an asset cannot have a negative price.

● The second argument is that a bubble cannot arise in the course of an asset pricing process unless it was present at time 0. This is because if the bubble has ever a zero value, its expected future value is zero too.

● The third argument is that the price of an asset has an upper limit due to firms issuing stock when there are large price increases.

Abreu and Brunnermeier (2003) make an effort to explain bubbles and crashes in the presence of rational arbitrageurs. They show that in their setting a bubble can originate and persist over a longer period of time. In their model the price process is assumed to be exogenous. At time t = 0 the price is $p_0 = 1$ without loss of generality. Until the time t_0 the price is justified by fundamentals. From that time onwards, price grows at rate g, so $p_t = e^{gt}$. For any $t > t_0$, only a fraction $1 - \beta\ (t - t_0)$ of the price is justified by fundamentals with t_0 being a random variable with exponential distribution, $\Phi(t_0) = 1 - e^{-\lambda t_0}$. The fraction $\beta(t - t_0)$ represents the bubble component.

The origins of the excessive growth rate are assumed being the result of a series of positive shocks. The price is assumed to be kept above its fundamental value by behavioral traders. The bubble will burst exogenously at $t_0 + \bar{\tau}$. When the arbitrageurs wanting to sell reaches a fraction κ which exceeds the absorption level of behavioral traders, the price drops by $\beta(t - t_0)$. The bubble can also burst for exogenous reasons at $t_0 + \bar{\tau}$ when it reaches its maximum size $\bar{\beta}$. In this case the price drops by $\bar{\beta}(t_0 + \bar{\tau}\ )$. Rational agents (arbitrageurs) are assumed being risk-neutral and having mass 1. They become sequentially aware of the bubble. At each $t < t_0 < t_0 + \eta$ a mass $\frac{1}{\eta}$ of arbitrageurs becomes aware of deviation between the asset's market price and its fundamental value, where $[t_0, t_0 + \eta]$ is the awareness window. Arbitrageurs do not observe t_0 and don't know how many of the others arbitrageurs are aware of the mispricing. Abreu and Brunnermeier (2003) then go on to show that under the common knowledge of bubble there is unique equilibrium in which bubble bursts immediately. However, because of the sequential information line constraint and due to the arbitrageurs' agnosticism on their position in the line, the common knowledge requirement breaks. The larger κ or η the longer the bubble will persist because either the absorption capacity of the behavioral traders gets larger or the time it takes the arbitrageurs to get aware of the mispricing increases. In this setting there is no equilibrium in which all arbitrageurs sell their stocks simultaneously. Lack of synchronization and coordination between the arbitrageurs is in the heart of bubble persistence. A public piece of information may act as a coordination signal and have a large impact on the strategy choice. In this sense even news which have little informative content, may lead to a market crash. In conclusion, the model by Abreu and Brunnermeier predicts that the presence of rational arbitrageurs doesn't preclude the existence of bubbles. A bubble may last for a long time even if agents are aware of it, due to lack of common knowledge. News can have a large impact on the behavior of agents by acting like a signal that triggers coordination.

4.2 Sentiment

There are two approaches to sentiment (Baker and Wurgler 2007). One approach is based on psychological bias such as overconfidence, representativeness or conservatism. The other approach revolves around the divergence of opinions. Shleifer (2000, p.112-153) relates sentiment to overreaction and underreaction to asset prices. In an efficient market, stock prices quickly and rationally reflects all public information. Nevertheless, in the view of market anomalies, there are gaps between prices and fundamentals. Traders overreact or underreact to new information. Subsequent price corrections lead eventually to mean reversion. When a large earnings increase is forecasted, actual earnings turn out on average to be lower than expected and vice versa. Stock prices don't behave as a random walk (De Bondt 1991). An explanation could be that the interaction of informed and uninformed influences the development of share prices in the stock market.

Overreaction and underreaction depend on the fundamental value compared to market price, on the asset prices being on an increasing or decreasing trend and on the type of information classified as god news (GN) or bad news (BN). Market efficiency depends on how quickly new information is reflected on asset prices and on the degree market price reflects an asset's fundamental value. We show this in Table 4-3.

Table 4-3: Fundamental value and market price

Relation Value/Price Trends	Fundamental value > Market price	Fundamental value < Market price
Increasing	Underreaction given GN	Overreaction given GN
Decreasing	Overreaction given bad news BN	Underreaction given BN

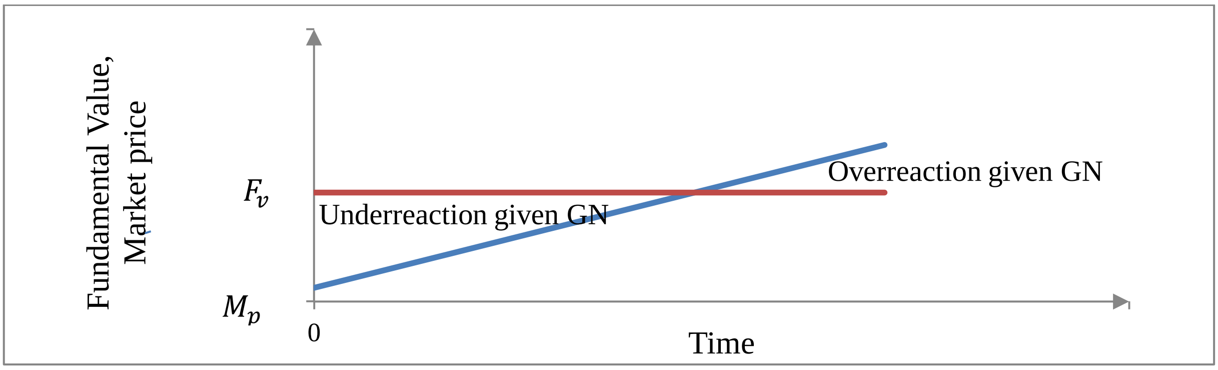


Figure 4-4: Underreaction /Overreaction given GN (god news)

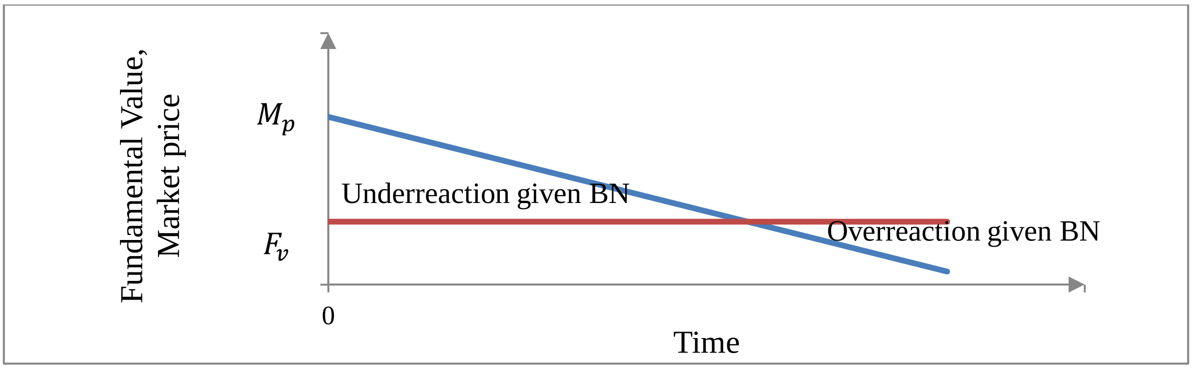


Figure 4-5: Underreaction and Overreaction given BN (bad news)

where

F_v is the fundamental value

M_p is the market price

The figures above show that judgment on overreaction or underreaction of asset prices t news should not be passed without considering the position of the asset's market price against the fundamental value. An asset prices moving upwards in an event of good news can be an underreaction if the fundamental asset value lies above the asset's market price. Likewise, an asset price moving downwards can be an underreaction in an event of bad news if the fundamental asset value lies below the asset's market price.

Shleifer (2000, pp.134-153) attributes underreaction to conservatism. That boils down to insufficient reevaluation of the agents' expectations given new information. Overreaction is pinned down to the representativeness bias which leads to the overestimation of the probabilities assigned to events. Shleifer presents a model of agents with adaptive expectations (see appendix A - xiii) where the asset price process depends on:

a) earnings shocks

b) the deviation between the market price and the asset's fundamental value

Shleifer lets asset prices satisfying the following equation:

$P_t = \frac{N_t}{\delta} + y_t(p_1 - p_2 q_t)$	4-4

where N is a random walk, y is a shock to earnings q is the probability that y was generated by a specific Markov process; p is a parameter that depends on the probability for high and low states in the economy and the transition probabilities from one Markov process to another; δ is the discount rate. The first term, $\frac{N_t}{\delta}$ shows the asset price process obtained using the random walk N to forecast earnings. The second term, $y_t(p_1 - p_2 q_t)$ shows the asset price's deviations from its fundamental value.

Whether the regime will be one of overreaction or underreaction depends on the nature of the sequence of previous events. The value of regime switching parameters is determined exogenously. The model by Shleifer predicts that the return difference between two portfolios, one following a positive and the other a negative realization of earnings, decline over time. More specifically, when the return of the portfolio with a positive realization of earnings is greater than the return of the portfolio of the firms with a negative realization of earnings, then the market underreacts. When the return of the portfolio with a positive realization of earnings is lower than the return of the portfolio of the firms with a negative realization of earnings, then the market overreacts.

Xiouros (2009, p.106) describes sentiment risk as the degree of diversity in interpretation of new information. Xiouros provides an asset pricing model with time varying sentiment risk which plays a significant role in explaining stylized facts in finance like trading volume and volatility. The sentiment risk is determined exogenously. The model predicts that high sentiment risk is positively correlated with the volatility of beliefs and trading volume and negatively correlated with the asset price level. Xiouros uses the deviation of individual forecast of professional forecasters as a proxy for belief dispersion.

We note that the model by Shleifer (2000) describes switches between two regimes, overreaction and underreaction while the model by Xiouros (2009) assumes a continuum of dispersion of beliefs.

An example of sentiment can be found in closed-end funds. These funds are issued a fixed number of shares. After the issuing they are traded on the stock exchange market. Closed-end funds trade at a premium at issuing time, then at a discount and at termination at net asset value, which is the value of the fund assets minus the value of the liabilities. The funds swing in the period between issuance and termination substantially over and under their net asset value but usually sell at 10% discount (Barberis and Thaler 2003, pp.1051-1121). Explanations consistent with rational expectations have been proposed like management fees but they can't explain all aspects of the puzzle. Another explanation proposed by Lee, Shleifer and Thaler (1991) is that the financial institutions selling the closed-end fund choose issuing at a time of optimism in the capital market. After issuing, noise traders swing from optimist to pessimist mood affecting the market price. Because of the noise risk, rational traders demand a discount to net asset value. At termination the noise risk ceases and so fund shares raise to their fundamental value. Lee, Shleifer and Thaler (ibid) predict that closed-end funds co-move because a change in the sentiment of noise traders is having an encompassing effect. They find empirical evidence that the movement of the prices of closed end funds is strongly correlated. Furthermore they attain a co-movement between asset funds and small stocks typically owned by individuals.

Baker and Wurgler (2007) study the effect of investor sentiment to cross sectional stock returns. They interpret the bubbles and crashes in the US stock market as a consequence of investor sentiment. They argue that a mispricing is the result of sentiment based demands and difficulties in exploiting arbitrage opportunities. Based on this intuition they predict that when sentiment is high, the stocks get overvalued for certain types of stocks which are difficult to arbitrage. Examples of such securities are small cap stocks, high volatility stocks, non-dividend paying stocks, and issuing of public offerings (IPO) stocks. As a proxy for sentiment they use a composite index consisting of the closed – end fund discount, the NYSE turnover, the first-day returns on IPOs, the equity share in new issues and the dividend premium.

Hong and Stein (1999) develop a theory that differences of opinion can generate market crashes under short sales constraints. Investors are of three types, bearish and pessimistic, bullish and optimistic and rational arbitrageurs. In bull markets bearish investors bail out and asset prices reflect the optimistic investors. In bear markets bullish investors bail out and asset prices reflect the pessimistic investors. Arbitrageurs would like to average out the signals of bullish and bearish investors. Because of short sale constraints asset prices don't fully reflect the information held by pessimistic investors. When bad news is diffused, the pessimistic agent's enter the market exacerbating the asset prices spiral downwards. One of the predictions of their theory is that the higher the dispersion of beliefs, the higher the trading volume and the more negatively skewed become the asset prices.

In another article, Hong and Stein (1999) attempt to formulate a theory which unifies underreaction and overreaction in asset markets. Momentum strategies are used as the link which leads from underreaction to overreaction. In their model there are two types of agents with bounded rationality, newswatchers and momentum traders. Newswatchers have a lower bound of rationality and cannot extract private information form prices. Momentum traders exploit underreaction chasing the trend. Their action leads eventually to overreaction if they employ univariate strategies. With univariate strategies is meant only looking at the last price change. The assumptions their model is based upon are bounded rationality and the slowly diffusion of news on asset fundamentals. The prediction of their theory is in the short term can momentum strategies be profitable especially for small cap stocks with few analysts following them. Overreaction in the long run is more likely for private information than public news. The investment horizon of momentum traders is related to the pattern of return autocorrelations.

Daniel, Hirshleifer and Subrahmanyam (1998) ascribe overreaction to private information signals and underreaction to public information signals to the psychological bias of overconfidence and self-attribution. Self-attribution denotes the individuals' propensity to assign success to own abilities and failures to external noise. Self-attribution enhances overconfidence in case of success but doesn't weaken overconfidence in case of failure.

Miller (1977) proposes a theory of investor behavior which relaxes the assumption of homogeneity in estimates of return and risk from every security. Under the assumption that the number of shares are limited, the investors with the highest evaluation of asset's fundamental value will end up owning the shares if the magnitude of their demand equals the number of available shares. So asset prices reflect the most optimistic expectations.

Behavioural finance explains sentiment risk by means of overreaction and underreaction. These types of behavior are pinned down to the psychological bias of representativeness and conservatism. Overreaction given good news drives the price of an asset above its fundamental value while overreaction given bad news drives the asset price below its fundamental value. Underreaction given good news drives the price of an asset below its fundamental value while underreaction given bad news drives the asset price above its fundamental value. Dispersion of beliefs gives room for the preservation of rational expectations while allowing for sentiment risk. Dispersion of beliefs arises when individuals interpret commonly known information differently. This can happen by assigning different probabilities for the same state of the market. If probabilities cannot be estimated objectively then probabilities become a subjective matter. Divergence of opinions (meaning the same as dispersion of beliefs) would in many cases be the most plausible situation to entertain in research since it seems a reasonable assumption that individuals assign subjective probabilities to the same events.

4.3 Herding

Herding is defined by Brunnermeier and Kim (2010) as the tendency of different agents to take similar actions at the same time. The psychological need for eliminating dissonance and achieve an alignment of beliefs is documented in Sherif's experiment on the autokinetic effect (1936, cited in Sherif 2009, p.138). Sherif showed that people's perception of the movement of a fixed light beam in a dark room is influenced by the group norms. Herding can be intentional or unintentional (Gebka and Wohar 2013). Unintentional herding occurs when all agents share correlated information and interpret it in the same way. An example could be an announcement of positive earnings with the result that all analysts revise their forecasts upwards. Intentional herding arises when individuals suppress their own beliefs and follow signals of other investors. In informational cascades it is assumed that some investors possess superior information. Following these agents can be rational for agents who think they possess inferior information. Expectations based on a feeling of euphoria can create market bubbles. In situations of panic individual investors are driven by psychological factors like fear. Panic can lead to market crashes or bank runs. Intentional herding can also find place when the incentive compatibility constrain in contracts motivates mimicking the actions of other investors (Scharfstein and Stein. 1990). Herding behaviour is not restricted to uninformed traders. Analysts can also imitate each other in their stock recommendations in order to conform to a market consensus. It is more likely for analysts of reputable brokerages or analysts that make infrequent recommendation revisions to not deviate from the general judgment (Jegadeesh and Kim 2010). In a model by Trueman (1994), analysts with low abilities issue forecasts imitating the predictions of high ability analysts. Herding can also be attributed to information costs (Lin, Tsai and Sun 2009). Herding can create excess overvaluation or undervaluation of asset prices through self-amplifying reactions (Lux 1995). De Long, Shleifer, Summers and Waldmann (1990) suggest a model which explains the survival of noise traders in competition with rational investors. Friedman conjectured in 1953 that investors with incorrect expectations will lose money to rational investors and be wiped out. De Long et al. argue that noise traders introduce a new type of risk that has not previously been accounted for, namely noise risk. De Long et al. add a distortion term in the asset return equation due to the influence of noise traders. If noise traders' expectations are bullish enough the distortion term exceeds the fraction of noise traders in the market and noise traders earn greater expected returns than rational investors (see appendix A - ii).

Lux (1995) explains bubbles as a self-organizing process where herding arises due to contagious behavior. Optimism and expectations about higher asset prices reinforce each other and create a willingness to adopt other's anticipation of a bullish market. When there are not any additional optimistic buyers entering the market and asset prices start relapsing, the mood can shift to pessimism leading to herding in the opposite direction. Cont and Bouchaud (2000) present a model which establishes a link between imitating behavior with random communication and the stylized fact of heavy tails in asset prices distributions. In their set up the kurtosis of the asset returns is equal to the kurtosis of excess demand. Large changes in excess demand create large price fluctuations and excess kurtosis. Alfarano, Lux and Wagner (2010) introduce a model with two types of traders, fundamentalists and noise traders, in order to derive analytical expressions which can help explain the stylized facts of heteroscedasticity, fat tails and long range volatility dependence in asset returns.

Herding behavior can be stimulated by the interaction of heterogeneous trader types. Notwithstanding the lack of consistency of terminology concerning trader types, the main archetypes are informed and uninformed. Terms like noise traders and chartists are used interchangeably with the term uninformed whereas terms like fundamentalists is used interchangeably with informed. Insiders possessing privileged information are a special case of informed agents while contrarians and liquidity traders are special cases of uninformed.

4.3.1 The Beja and Goldman herding model

Beja and Goldman present in their seminal article (1980) a model of herding in the context of heterogeneity of agent types. The excess demand of the fundamentalists is:

$$ED_F = a\,(p_f - p_t), \tag{4-5}$$

where a is the sensitivity of fundamentalists' excess demand, p_f is the fundamental value and p_t is the market price at time t

The excess demand of the chartists is:

$$ED_C = b\rho$$

$$\rho = E\left[\frac{dp}{dt}\right] = E[\dot{p}(t)] \tag{4-6}$$

where b is the sensitivity of chartists' excess demand and ρ is the expected price change (the expected capital and dividend gain or loss).

The chartists' adaptive expectations to observed price change $\dot{p}(\mathrm{t})$ is:

$$\dot{p}(t) = \lambda(ED_F + ED_C) \rightarrow \dot{p}(t) = \frac{dp}{dt} = \lambda\big[a(p_f - p_t) + b\rho\big] \tag{4-7}$$

where λ is the adjustment speed on excess demand and $ED_F + ED_C$ is the overall excess demand.

Rational expectations predict that price changes by a random shock, when new information arrives:

$$p = p^* + \varepsilon \tag{4-8}$$

where $p^*$is the equilibrium price and ε is the stochastic error term.

The chartist's adaptive adjustment of expectations $\dot{\rho}(t)$ is:

$\dot{\rho}(t) = \frac{d\rho}{dt} = c\ [\dot{p}(t) - \rho]$	4-9

where c is the adjustment speed of adaptive expectations .

From 4-7 and 4-9 we have:

$\dot{\rho}(t) = \frac{d\rho}{dt} = c\ [\lambda[a(p_f - p_t)] + b\ \rho] - \rho]$	4-10

At equilibrium the expectation of price change is 0 and the market price is equal to the fundamental value:

$\left.\begin{matrix} \rho = 0 \\ p_t = p_f \end{matrix}\right\} \rightarrow \dot{p}(t) = \dot{\rho}(t) = 0$	4-11

The equilibrium analysis gives the following conditions:

$\frac{dp}{dt} = 0 \rightarrow a\left(p_f - p_t\right) + b\rho = 0 \rightarrow -ap_t = -b\rho - ap_f \rightarrow ap_t = b\rho + ap_f \rightarrow$ $p_t^* = \frac{b}{a}\rho + p_f$	4-12
$\frac{d\rho}{dt} = 0 \rightarrow c\left[[\lambda[a\left(p_f - p_t\right) + b\ \rho] - \rho\right] = 0 \rightarrow c\ \left[(\lambda\ a\ p_f - \lambda\ a\ p_t + \lambda b\ \rho) - \rho\right] = 0$ $\xrightarrow{division\ by\ c} \lambda\ a\ p_f + \lambda b\ \rho - \rho = \lambda\ a\ p_t \rightarrow p_t = \frac{\lambda\ a\ p_f}{\lambda\ a} + \frac{\lambda b\ \rho}{\lambda\ a} - \frac{\rho}{\lambda\ a} \rightarrow$ $p_t^* = p_f + \rho\ (\frac{\lambda b - 1}{\lambda\ a})$	4-13

- When $ED_C = 0$ then $p_t^* = p_f$, the market price is equal to the fundamental value.
- When $\rho < 0 \leftrightarrow ED_C < 0$ then $p_t < p_f$, the market price is below the fundamental value
- When $\rho > 0 \leftrightarrow ED_C > 0$ then $p_t > p_f$, the market price is above the fundamental value

From 4-7 and 4-10 we have:

$$\frac{dp}{dt} = -\lambda\, a\,(p_t - p_f) + \lambda b\, \rho$$

$$\frac{d\rho}{dt} = -c\,\lambda\, a\,(p_t - p_f) + c\;(\lambda b - 1)\rho \qquad \text{4-14}$$

The determinant of the above equation system is.

$$a_{11} = -\lambda\, a \qquad a_{12} = \lambda b \qquad a_{21} = -c\,\lambda\, a \qquad a_{22} = c\,\lambda b - c = c\,(\lambda b - 1) \qquad \text{4-15}$$

$$A = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = \begin{vmatrix} -\lambda a & \lambda b \\ -c\lambda a & c(\lambda b - 1) \end{vmatrix}$$

The equilibrium is asymptotically stable iff:

$$\det(A) = -\lambda a c(\lambda b - 1) - (-c\lambda a)\lambda b > 0 \qquad \text{4-16}$$

$$tr\,(A) = -\lambda a + c(\lambda b - 1) < 0$$

The above conditions are called the Lyapunov conditions for asymptotic stability (Sydsæter bind II 1990 p.266).

Assume that the fundamental asset price is 10. At this point the excess demand is 0. When new information arrives at the market with positive news the fundamental asset value jumps to 11. The fundamentalists start buying. The chartists observe the action of the fundamentalists and they start buying as well. When the asset's market price exceeds its fundamental value, the fundamentalists start selling. The chartists mimic this behavior and they start selling too. The result of this interaction will in the end lead to equilibrium where the asset market price is equal to its fundamental value. The price adjustment is not instantaneous. This is shown in Figure 4-6.

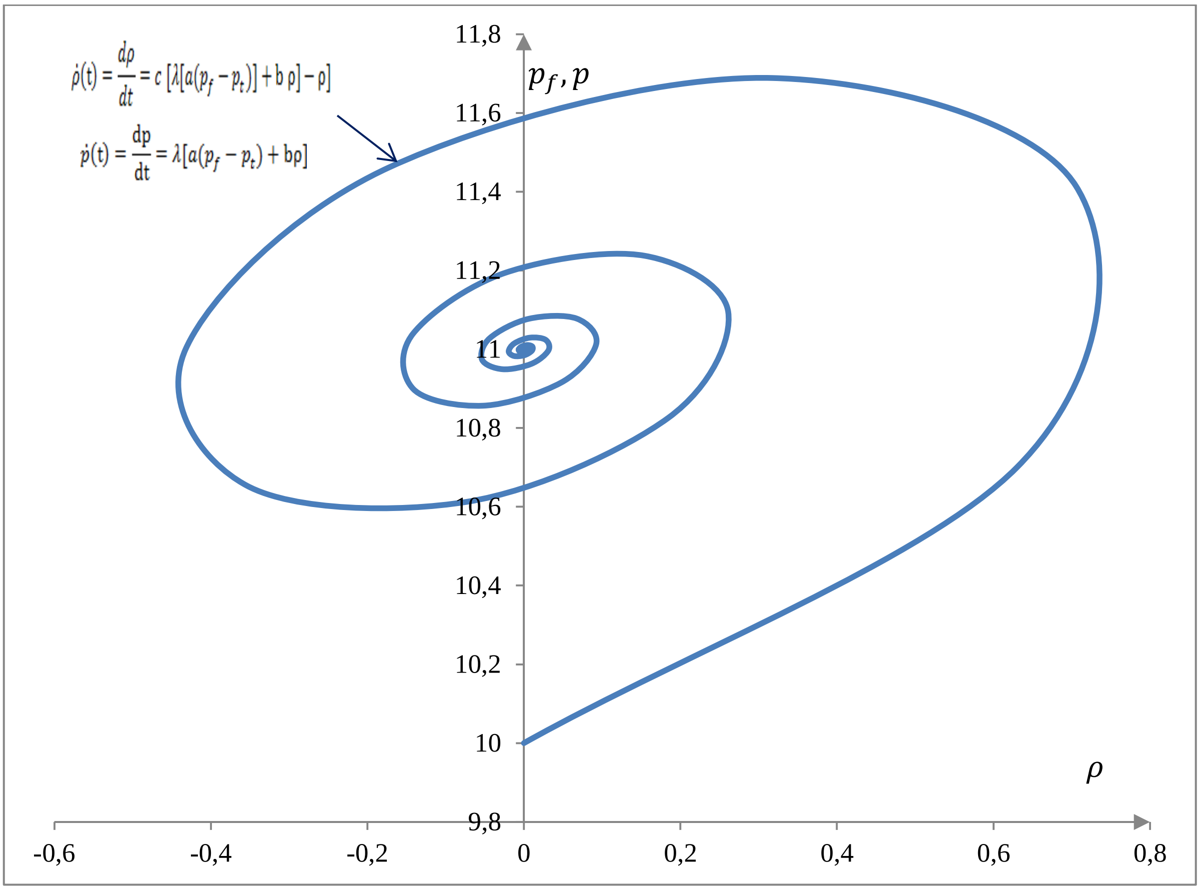
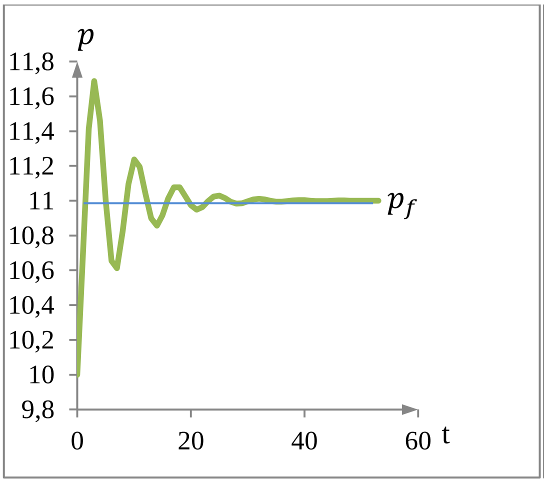


Figure 4-6: The asset price converging to equilibrium

The vertical price shows the fundamental asset value and the asset price. The horizontal axis shows the price changes. The price adjustment is characterized by the interaction between the chartists and the fundamentalists. The above diagram is based on a simulation we did in Excel with $a = 0{,}7$, $b = 0{,}8$, $c = 0{,}9$, $\lambda = 1$, $p_{f,1} = 10$ and $p_{f,2}$ =11.



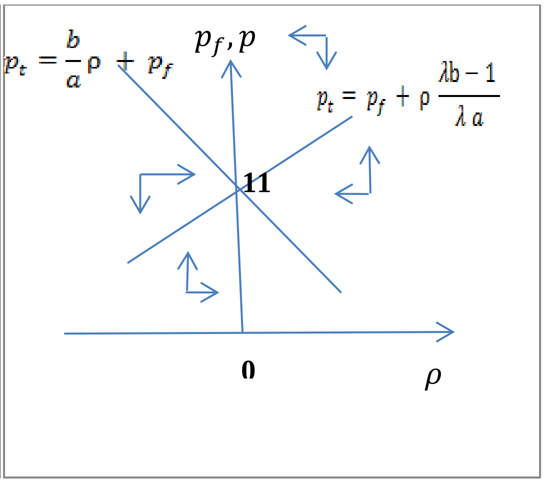


Figure 4-7: Damped fluctuation

Figure 4-8: Phase diagram of the Beja and Goldman model

Convergence to equilibrium assumes that the determinant of the equation system 4-14is positive and its trace negative.

The efficient market hypothesis (see appendix A - x) assumes that information is reflected on prices which adjust instantaneously to new information. Instantaneous adjustment occurs when the information shared among the market participants, the information disseminated by the companies and the information transmitted by the media is symmetric (Marisetty 2003). There is empirical evidence that this is not the case (Sinnakkannu and Nassir 2006). Information costs create a band of agnosticism (De Grauwe 1995, pp. 181-185). Lux (1995) asserts that herd behaviour of speculative traders can be explained as irrational behaviour, as an attempt to draw information from what the others do or as a reputation consideration.

4.4 Asset Pricing

4.4.1 The Fundamental Theorem of Asset Pricing

The fundamental theory of asset pricing is at heart of the classical financial theory. According to the fundamental theorem of asset pricing in discrete time, there are no arbitrage opportunities if and only if there is an equivalent martingale measure (Duffie 2001, p. 30). Additional technical conditions have to be fulfilled for if and only if statement to apply in continuous time (Duffie 2001, ch. 6). An equivalent martingale measure to another probability measure is a measure that assigns a positive probability and a zero probability for the same states of the world as the other probability measure (Pennacchi p. 207). The equivalent martingale measure in this setting is the probability measure in a risk neutral world, i.e. the probability under which the price of an asset is the expected cash flows discounted with the risk free rate (Duffie 2001, p. 28 and p. 108). It is customary to denote the equivalent martingale measure with $\mathcal{Q}$. The transformation from a probability measure $\mathcal{P}$ in a risk world to a probability measure $\mathcal{Q}$ in a risk neutral world is done by means of the Girsanov theorem.

Let a stochastic process:

$$dY = \beta(t,\omega)dt + \sigma(t,\omega)dB \tag{4-17}$$

Under the assumption that there exists a process $u(t,\omega)$so that:

$$\sigma(t,\omega)u(t,\omega) = \beta(t,\omega) - \alpha(t,\omega) \tag{4-18}$$

Putting:

$$M = e^{-\int_0^t u dB_s - \frac{1}{2}\int_0^t u^2 ds} \tag{4-19}$$

And

$$d\mathcal{Q} = M_T d\mathcal{P} \tag{4-20}$$

Then:

$$dY = \alpha(t,\omega)dt + \sigma(t,\omega)d\hat{B} \tag{4-21}$$

by using the transformation:

$\hat{B} = \int_0^t u(t, \omega) ds + B$ The process u should satisfy the Novikov condition: $E_P\left(e^{\frac{1}{2}\int_0^T u^2 ds}\right) < \infty$, which means that u is a square integrable function	4-22

Where

$\hat{B}$ is a Brownian motion with respect to the probability measure Q (Øksendal 2000, p.155) and ω is an event in the probability space Ω (Øksendal 2000, p.156).

Y and B depend on t; μ, σ, α and u depend on t and ω.

For an example of using Girsanov's theorem for transforming a probability measure $\mathcal{P}$ to Q so that the risk free rate r_f can be used for finding the asset price (see appendix A - ix).

Assuming that an asset price follows a fractional Brownian motion, i.e. a Brownian motion which increments are correlated, one can construct arbitrage investment portfolios (Sottinen 2003). Hu and Øksendal (2003) expanded the fundamental theorem of asset pricing to include the fractional Brownian motion in a non-arbitrage fashion, using a mathematical operator called Wick product. This is the product of two square integrable random variables. Øksendal (2004) interprets the Wick product as a value process that becomes an asset price when observed by an economic agent like observations in quantum mechanics. The expansion of the fundamental asset pricing theorem to fractional Brownian motions has been met with counterarguments by Björk and Hult (2005) and Bender, Sottinen and Valkeila (2007) for either lacking a sound economic interpretation or producing arbitrage under some observations.

The fundamental theorem of asset pricing supports the notion of efficient capital markets.

4.4.2 The Stochastic Discount Factor

Let a price functional that maps payoffs into prices of the form $\boldsymbol{q} = \pi(\boldsymbol{x})$. It can be shown by Riesz Representation Theorem (Ødegaard 2013) that under certain conditions there exist a stochastic variable $\mathfrak{m}$ so that:

$$\pi(\boldsymbol{x}) = E[\mathfrak{m}\boldsymbol{x}]$$

Ødegaard lists up the following conditions:

- The set of payoffs is a linear space H.
- The conditional expectation defines an inner product on this linear space. If $\boldsymbol{x}, \mathfrak{m}$ are in the space H, the conditional expectation $E[\mathfrak{m}\boldsymbol{x}]$ is an inner product.
- The set of payoffs with the inner product of conditional expectation is a Hilbert Space.

In a Hilbert space every Cauchy sequence has a limit to converge (Bierens 2007, Borowski and Borwein 1989).). A Cauchy sequence is a sequence which values can be brought arbitrarily closed together.

Let a bounded linear functional πon a Hilbert space. Then, according to Riesz Representation Theorem, there exists a unique element x_0 in H such that

$$\pi(\boldsymbol{x}) = \langle \boldsymbol{x}, x_0 \rangle$$

Substituting the conditional expectation for the inner product we have:

$$\pi(\boldsymbol{x}) = E[\mathfrak{m}\boldsymbol{x}]$$

The stochastic variable $\mathfrak{m}$ used in finance is called:

- the stochastic discount factor (SDF)
- the pricing kernel
- the intertemporal marginal rate of substitution of consumption

SDF can be equal to the equivalent martingale measure (the Radon-Nikodym derivative) under certain conditions. Let Q be the equivalent probability measure in a risk neutral world and $\mathcal{P}$ the true probability measure. Then the equivalent martingale measure is $\frac{dQ}{dP}$ which is also called the Radon-Nikodym derivative. Given a strictly positive stochastic process $\mathfrak{m}$ which satisfies the equation $\pi(\boldsymbol{x}) = E[\mathfrak{m}\boldsymbol{x}]$ for all assets we can write $\pi(\boldsymbol{x}) = R_f E_Q[\boldsymbol{x}]$ where R_f is the risk free gross return (Duffee 2012, pp. 21-22)

Another version of $\mathfrak{m}$ is the behavioral stochastic discount factor:

$$\mathfrak{m}_b = \Lambda\mathfrak{m}$$

$$\Lambda = \frac{d\mathrm{P}}{d\mathcal{P}}$$

Where P is the representative agent's probability measure and Λ is a sentiment risk (Shefrin 2007). If $\mathrm{P} = \mathcal{P}$ the sentiment risk disappears and $\mathfrak{m}_b$ collapses back to $\mathfrak{m}$.

An example of a stochastic discount factor (SDF) is (Campbell et al. 1997, p. 294):

$$\mathfrak{m}_{t+1} = \delta\frac{U'(C_{t+1})}{U'(C_t)} \tag{4-23}$$

where δ is the subjective discount factor and $U'(C_t)$is the derivative of the utility of consumption.

According to Campbell et al. (1997, p.296), a stochastic discount factor can be constructed for every pair of utility functions $U'(C_{t+1})$ and $U'(C_t)$. Given complete markets the stochastic discount factor is going to be unique. The unique stochastic discount factor is related to the equivalent martingale measure. The equivalent martingale measure transforms the investment from a world with risk to a risk neutral world. In incomplete markets there can be many stochastic discount factors due to idiosyncratic marginal utilities.

Cvitanic and Malamud (2010) ascertain that homogeneity in consumption preferences and beliefs is relevant for defining a unique stochastic discount factor when there are more than two types of agents. Given complete markets, homogeneous preferences and homogeneous beliefs, the stochastic discount factor and assets prices are uniquely defined. Bhamra and Uppal (2010) come up with a closed-form solution for the stochastic discount factor in an economy with two heterogeneous types of agents without assuming specific utility function values.

4.4.3 The Basic Asset Pricing Equation

Here we follow the presentation by Cochrane (2005, pp. 4-5).

Consider the following consumption utility maximization problem over two periods:

$V = \underbrace{max}_{wrt.\ \xi} U(e_t - P_t\xi) + E_t\delta U(e_{t+1} + R_{i,t+1}\xi)$ Subject to: $C_t = e_t - P_t\xi$ $C_{t+1} = e_{t+1} + R_{i,t+1}\xi$	4-24

The notation is as follows:

ξ is the amount of the asset the agent chooses to buy

$R_{i,t+1}$is the payoff on an asset

e_t is the endowment

δ is the subjective discount factor.

The first order condition (F.O.C.) is:

$\frac{\partial V}{\partial \xi} = U'(e_t - P_t\xi) \times (-P_t) + E_t\delta U'\left(e_{t+1} + R_{i,t+1}\xi\right) \times \left(R_{i,t+1}\right) = 0 \rightarrow$ $U'(C_t) \times (-P_t) + E_t\delta U'(C_{t+1}) \times \left(R_{i,t+1}\right) = 0 \rightarrow$ $E_t\delta U'(C_{t+1}) \times \left(R_{i,t+1}\right) = U'(C_t)(P_t) \rightarrow P_t = E_t\delta \frac{U'(C_{t+1})}{U'(C_t)} R_{i,t+1}$	4-25

Equation 4-25 is called the basic asset pricing equation.

4.4.4 The Capital Asset Pricing Model

The Capital Asset Pricing Model was developed independently by Sharpe (1964), Treynor (1962), Lintner (1965a, b) and Mossin (1966).

Let the expected return and variance of a portfolio be:

$$E(R_p) = aE(R_i) + (1-a)E(R_m) \tag{4-26}$$

$$\sigma(R_p) = [a^2\sigma_i^2 + (1-a)^2\sigma_m^2 + 2a(1-a)\sigma_{im}]^{\frac{1}{2}} \tag{4-27}$$

where

R_p is the rate of return of a portfolio, $\sigma(R_p)$ is the variance of the portfolio

R_i is the rate of return for the ith asset, σ_i^2 is the variance of the risky asset

R_m is the rate of the market return, σ_m^2 is the variance of the market portfolio

σ_{im} is the covariance of the ith asset and the market portfolio

The F.O.C. with respect to a are:

$$\frac{\partial E(R_p)}{\partial a} = E(R_i) - E(R_m) \tag{4-28}$$

$$\frac{\partial \sigma(R_p)}{\partial a} = \frac{1}{2}[a^2\sigma_i^2 + (1-a)^2\sigma_m^2 + 2a(1-a)\sigma_{im}]^{-\frac{1}{2}} \times (2a\sigma_i^2 - 2\sigma_m^2 + 2a\sigma_m^2 + 2\sigma_{im} - 4a\sigma_{im}) \tag{4-29}$$

Setting the excess return $a = 0$:

$$\frac{\partial \sigma(R_p)}{\partial a} = \frac{1}{2}(\sigma_m^2)^{-\frac{1}{2}}(-2\sigma_m^2 + 2\sigma_{im}) = \frac{\sigma_{im} - \sigma_m^2}{\sigma_m} \tag{4-30}$$

Then

$$\frac{\frac{\partial E(R_p)}{\partial a}}{\frac{\partial \sigma(R_p)}{\partial a}} = \frac{E(R_i) - E(R_m)}{\frac{\sigma_{im} - \sigma_m^2}{\sigma_m}} \quad (4\text{-}31)$$

This has to be equal to the slope of the Capital Market Line which shows a linear relationship between expected return and the risk of a portfolio:

$$\frac{E(R_i) - E(R_m)}{\frac{\sigma_{im} - \sigma_m^2}{\sigma_m}} = \frac{E(R_m) - R_f}{\sigma_m} \rightarrow E(R_i) = R_f + \left[E(R_m) - R_f\right]\frac{\sigma_{im}}{\sigma_m^2} \quad (4\text{-}32)$$

The above equation is the CAPM. The main prediction of this model is that the return of the risky asset depends on the risk premium $\left[E(\tilde{R}_m) - R_f\right]$ and asset's risk $\beta_i = \frac{\sigma_{im}}{\sigma_m^2}$.

The sample equation is $R_i = a_i + b_i R_m + \varepsilon_i$ with $var(R_i) = b_i^2\sigma_m^2 + \sigma_\varepsilon^2$. The systematic risk is $b_i^2\sigma_m^2$ and the unsystematic risk is σ_ε^2.

4.4.5 The Lucas Asset Pricing Model and the Consumption Capital asset Pricing Model

Lucas derived the equilibrium price of a risky asset (a tree) with dividends (fruits) in an endowment economy, i.e. an economy where the production is given exogenously. It is assumed that the fruit cannot be stored for later consume so that all production has to be consumed immediately. That implies that consumption equals production at each time t. The asset return is given by (Pennacchi 2008, pp. 125-126):

$$R_{i,t} = \frac{d_{i,t+1} + P_{i,t+1}}{P_{i,t}} \quad (4\text{-}33)$$

An investor's intertemporal utility maximization problem can be formulated in the following manner (Campbell et al. 1997, p.293):

$$\text{Max } E_t\left[\sum_{j=0}^{\infty} \delta^j U(C_{t+j})\right] \quad (4\text{-}34)$$

where $\delta = \frac{1}{1+\vartheta}$ is the subjective discount factor, ϑ is the subjective discount rate which shows time preferences, C is the consumption and U is the utility of consumption.

An optimality condition of consumption between two periods is:

$U'(C_t) = \delta E_t\left[\left(R_{i,t+1}\right)U'(C_{t+1})\right]$	4-35

where $R_{i,t+1} = 1 + r_{i,t+1}$, $r_{i,t+1}$ is the rate of return for asset i at time $t+1$.

The above equation is a first order condition for optimal consumption between two periods. It's a so called Euler equation. This holds for every investor.

Dividing both sides with $U'(C_t)$ we get the equation for the Consumption Capital Asset Pricing Model (CCAPM):

$1 = E_t\left[\left(R_{i,t+1}\right)\mathfrak{m}_{t,t+1}\right]$	4-36

where $\mathfrak{m}_{t,t+1} = \delta\,\frac{U'(C^*_{t+1})}{U'(C^*_t)}$ is the stochastic discount factor.

CCAPM says that the expected rate of return on a risky asset depends on the marginal rate of substitution of utility between two periods, adjusted with the time discount rate. CAPM says in comparison that the expected rate of return on a risky asset depends on the risk premium and its sensitivity to changes in market returns.

Plugging

4-33 into the consumption asset pricing model

4-36 and solving for $P_{i,t}$ we get:

$P_{i,t} = E_t\left[\sum_{j=1}^{T}\delta^j\,\frac{U'\left(C^*_{t+j}\right)}{U'(C^*_t)}\,d_{i,t+j} + \delta^T\,\frac{U'(C^*_{t+T})}{U'(C^*_t)}\,P_{i,t+T}\right]$	4-37

where C^*_t shows the optimal level of consumption at period t.

Assuming a representative agent with infinite horizon we can let the last term to go to zero:

$\lim_{T\to\infty} E_t\left[\delta^T\,\frac{U'(C^*_{t+T})}{U'(C^*_t)}\,P_{i,t+T}\right] = 0$	4-38

which is equivalent to the assumption of no bubble.

Then the equilibrium price of the asset becomes:

$$P_{i,t} = E_t\left[\sum_{j=1}^{\infty} \delta^j \frac{U'\left(C^*_{t+j}\right)}{U'(C^*_t)} d_{i,t+j}\right] = E_t\left[\sum_{j=1}^{\infty} \mathfrak{m}_{t,t+j}\, d_{i,t+j}\right] \qquad \text{4-39}$$

where $\mathfrak{m}_{t,t+j} = \delta^j \frac{U'\left(C^*_{t+j}\right)}{U'(C^*_t)}$.

4.4.6 Puzzles

Anomalies are discrepancies from predictions of the market efficiency hypotheses at the level of individual stocks. Puzzles are aggregate anomalies at the level of the stock market as a whole (Ackert and Deaves 2010, p. 237).

Lucas' asset pricing model made it possible to examine the issue of equity premium which was beyond the reach of the capital asset pricing model. Mehra (2008) contends that the magnitude of the average US historical equity premium at 6,36 % in the period 1889 to 2005 cannot be justified in the context of classical finance. This is called the equity premium puzzle. Empirical data for other countries shows that the question of the magnitude of equity premium poses itself at a global scale. The equity premium is not justified because it requires either very high risk aversion or very high volatility in consumption growth. Even if one was willing to accept high risk aversion it would lead to another puzzle since the risk free rate would have to be extraordinary high. Mehra calculates a maximum equity risk premium of 1,4 % which cannot be reconciled with empirical data. The Hansen-Jagannathan bound which implies a lower bound on the standard deviation of the stochastic discount factor is also far off the empirical volatility. Mehra points out that models which employ recursive utility functions disentangle the intertemporal rate of substitution with the risk aversion coefficient. These models might have the potential to generate high equity premium without having to assume unrealistically high magnitude of risk aversion. Testing them requires data for the agents' wealth portfolio which is unobservable. Another challenge is that the elasticity of the intertemporal substitution of consumption is small, which makes it problematic to explain the risk free rate puzzle. Habit formation models introduced by Constantinides are taking advantage of a fundamental feature of human psychology. They are classified as internal and external. Habit models can increase the coefficient of relative risk aversion without altering the risk aversion parameter. In an external habit model utility of consumption is influenced by the lagged average consumption per capita. This class of models are called catching up or keeping up with the Joneses (the neighbours). In an internal habit model the utility of consumption is affected by the past private consumption. Heterogeneity representing idiosyncratic risk is yet another class of models trying to provide plausible explanations to the equity premium puzzle. Models employing the prospect theory from behavioural finance introduce utility functions with the characteristics of loss aversion. Mehra concludes that although the research efforts to explain the equity premium puzzle has deepened our understanding of the factors surrounding this issue, no single explanation has satisfactory explained the puzzle.

4.4.6.1 The Equity Premium Puzzle

The Sharpe ratio $\frac{E(R_i)-R_f}{\sigma(R_i)}$ shows the risk premium return per volatility unit. Hansen and Jagannathan (1991) derived an upper bound for the Sharpe ratio called the HJ-bound. It is given by $\frac{\sigma(m)}{E(m)}$. This relation is derived as follows.

Let the intertemporal budget constraint be:

$$C_{t+1} = y_{t+1} + (W_t + y_t - C_t)\sum_{i=1}^{n} \omega_{t+1,i} R_{t+1,i} \qquad \text{4-40}$$

where y is the income, W is the wealth, C is the consumption and $(W_t + y_t - C_t)$ is the individual's savings at time t.

The individual's maximization problem can then be stated as:

$$\max U(C_t) + \delta E_t[U(C_{t+1})] \text{ with respect to } C_t, \omega_{t+1,i} \qquad \text{4-41}$$

$$\text{when } C_{t+1} = y_{t+1} + (W_t + y_t - C_t)\sum_{i=1}^{n} \omega_{t+1,i} R_{t+1,i}$$

Subject to

$$\sum_{i=1}^{n} \omega_{t+1,i} = 1$$

The Lagrange function is:

$$L = U(C_t) + \delta E_t\left[U\left(y_{t+1} + (W_t + y_t + C_t)\sum_{i=1}^{n}\omega_{t+1,i}R_{t+1,i}\right)\right] - \gimel\left(\sum_{i=1}^{n}\omega_{t+1,i} - 1\right)$$

$$\rightarrow L = U(C_t) + \delta E_t[U(C_{t+1})] - \gimel\left(\sum_{i=1}^{n}\omega_{t+1,i} - 1\right) \tag{4-42}$$

The first-order conditions with respect to C_0 is:

$$\frac{\partial L}{\partial C_t} = U'(C_t) - \delta E_t\left[U'(C_{t+1})\sum_{i=1}^{n}\omega_{t+1,i}R_{t+1,i}\right] = 0 \rightarrow$$

$$U'(C_t) = \delta E_t\left[U'(C_{t+1})\sum_{i=1}^{n}\omega_{t+1,i}R_{t+1,i}\right] \rightarrow$$

$$U'(C_t) = \sum_{i=1}^{n}\omega_{t+1,i}\,\delta E_t\left[U'(C_{t+1})R_{t+1,i}\right] \tag{4-43}$$

The first-order conditions with respect to $\omega_{t,i}$ is:

$$\left.\begin{array}{r}\frac{\partial L}{\partial \omega} = \delta E_t[U'(C_{t+1})]R_{t+1,i} - \frac{\gimel}{W_t + y_t + C_t} = 0 \\ \lambda = \frac{\gimel}{W_t + y_t + C_t}\end{array}\right\} \rightarrow$$

$$\frac{\partial L}{\partial \omega} = \delta E_t[U'(C_{t+1})]R_{t+1,i} - \lambda = 0 \tag{4-44}$$

Plugging 4-44 into 4-43 we get:

$$U'(C_t) = \sum_{i=1}^{n}\omega_{t+1,i}\,\lambda \rightarrow U'(C_t) = \lambda \tag{4-45}$$

Substituting λ for $U'(C_t)$ into 4-44 gives:

$\delta E_t[U'(C_{t+1})R_{t+1,i}] - U'(C_t) = 0 \rightarrow \delta E_t[U'(C_{t+1})R_{t+1,i}] = U'(C_t)$	4-46

Let:

$r_{t+1,i} = \left(\frac{P_{t+1,i} + d_{t+1,i} - P_{t,i}}{P_{t,i}}\right) \rightarrow r_{t+1,i} = \left(\frac{P_{t+1,i} + d_{t+1,i}}{P_{t,i}} - \frac{P_{t,i}}{P_{t,i}}\right)$ $\rightarrow r_{t+1,i} + 1 = \left(\frac{P_{t+1,i}+d_{t+1,i}}{P_{t,i}}\right) \rightarrow R_{t+1,i} = \left(\frac{P_{t+1,i}+d_{t+1,i}}{P_{t,i}}\right)$	4-47

Then:

$U'(C_t) = \delta E_t\left[U'(C_{t+1})\left(\frac{P_{t+1,i} + d_{t+1,i}}{P_{t,i}}\right)\right] \rightarrow$ $P_{t,i}U'(C_t) = \delta E_t[U'(C_{t+1})(P_{t+1,i} + d_{t+1,i})]$	4-48

So:

$P_{t,i} = E_t\left[\delta \frac{U'(C_{t+1})(P_{t+1,i} + d_{t+1,i})}{U'(C_t)}\right]$	4-49

Normalizing the price of the asset to be equal to 1 and setting $\eta_{t,t+1} = \delta \frac{U'(C_{t+1})}{U'(C_t)}$ we have:

$\frac{P_{t,i}}{P_{t,i}} = E_t\left[\eta_{t,t+1}\left(\frac{P_{t+1,i} + d_{t+1,i}}{P_{t,i}}\right)\right] \rightarrow$ $1 = E_t[\eta_{t,t+1}R_{t+1,i}] \rightarrow 1 = E_1[\eta_{t,t+1}]E_t[R_{t+1,i}] + \mathrm{cov}[\eta_{t,t+1}, R_{t+1,i}] \rightarrow$ $1 = E_t[\eta_{t,t+1}]\left(E_t[R_{t+1,i}] + \frac{\mathrm{cov}[\eta_{t,t+1}, R_{t+1,i}]}{E_t[\eta_{t,t+1}]}\right)$	4-50

The relation between the risk free rate and the stochastic discount factor is as follows:

$E_t\left[\frac{\delta U'(C_{t+1})}{U'(C_t)}\right] = E_t[\eta_{t,t+1}] = \frac{1}{R_f}$	4-51

From 4-50 and 4-51 we have:

$R_f = E_t[R_{t+1,i}] + \dfrac{\operatorname{cov}[\mathfrak{m}_{t,t+1}, R_{t+1,i}]}{E_t[\mathfrak{m}_{t,t+1}]}$	4-52

$E_t(R_{t+1,i}) = R_f - \dfrac{\operatorname{cov}[\mathfrak{m}_{t,t+1}, R_{t+1,i}]}{E_t[\mathfrak{m}_{t,t+1}]} = R_f - \dfrac{\operatorname{cov}[U'(C_{t+1}), R_{t+1,i}]}{E_t[U'(C_{t+1})]}$	4-53

Equation 4-53 (Pennacchi 2008 p. 86) shows that the expected return of asset i is equal to the risk free rate minus the ratio of covariance between the marginal utility of consumption at time $t+1$ divided by the expected marginal utility of consumption at time $t+1$. Assets that pay off relatively higher when consumption is low are more attractive than assets that pay off relatively lower when consumption is low. This is based on the assumption that consumers prefer to smooth out consumption. Mehra (2006, pp. 10-11) gives insurance policies as an example of such assets.

Exchanging covariance with correlation we have:

$E_t(R_{t+1,i}) = R_f - \rho_{\mathfrak{m}_{t,t+1},R_{t+1,i}} \dfrac{\sigma_{\mathfrak{m}_{t,t+1}} \sigma_{R_{t+1,i}}}{E_t[\mathfrak{m}_{t,t+1}]} \rightarrow$ $\dfrac{E_t(R_{t+1,i}) - R_f}{\sigma_{R_{t+1,i}}} = -\rho_{\mathfrak{m}_{t,t+1},R_{t+1,i}} \dfrac{\sigma_{\mathfrak{m}_{t,t+1}}}{E_t[\mathfrak{m}_{t,t+1}]} \xrightarrow{\rho_{\mathfrak{m}_{t,t+1},R_{t+1,i}}=1}$ $\dfrac{E_t(R_{t+1,i}) - R_f}{\sigma_{R_{t+1,i}}} = -\dfrac{\sigma_{\mathfrak{m}_{t,t+1}}}{E_t[\mathfrak{m}_{t,t+1}]} \rightarrow -\dfrac{E_t(R_{t+1,i}) - R_f}{\sigma_{R_{t+1,i}}} = \dfrac{\sigma_{\mathfrak{m}_{t,t+1}}}{E_t[\mathfrak{m}_{t,t+1}]} \rightarrow$ $\left\lvert\dfrac{E_t(R_{t+1,i}) - R_f}{\sigma_{R_{t+1,i}}}\right\rvert \leq \dfrac{\sigma_{\mathfrak{m}_{t,t+1}}}{E_t[\mathfrak{m}_{t,t+1}]} = \sigma_{\mathfrak{m}_{t,t+1}} R_f$	4-54

Consider the basic asset pricing equation:

$$P_{t,i} = E_t\left(\text{ɱ}_{t,t+1}X_{t+1,i}\right) \tag{4-55}$$

where ɱ is the stochastic discount factor and $X_{t+1,i}$ is the asset's pay off.

Dividing both sides by $P_{t,i}$ we get:

$$1 = E_t\left(\text{ɱ}_{t,t+1}\frac{X_{t+1,i}}{P_{t,i}}\right) \rightarrow 1 = E_t\left(\text{ɱ}_{t,t+1}R_{t+1,i}\right) \rightarrow$$
$$1 = E_t\left(\text{ɱ}_{t,t+1}\right)E_t\left(R_{t+1,i}\right) + cov\left(\text{ɱ}_{t,t+1}, R_{t+1,i}\right) \tag{4-56}$$

where $R_{t+1,i}$ is the return on asset i.

Since

$$cov\left(\text{ɱ}_{t,t+1}, R_{t+1,i}\right) = \rho_{\text{ɱ}_{t,t+1},R_{t,i}}\sigma(R_{t+1.i})\sigma(\text{ɱ}_{t,t+1}) \tag{4-57}$$

where ρ is the correlation coefficient.

From 4-56 and 4-57 we have:

$$1 = E_t\left(\text{ɱ}_{t,t+1}\right)E_t\left(R_{t+1,i}\right) + \rho_{\text{ɱ}_{t,t+1},R_{t+1.i}}\sigma(R_{t+1,i})\sigma(\text{ɱ}_{t,t+1}) \tag{4-58}$$

hence

$$E_t\left(\text{ɱ}_{t,t+1}\right)E_t\left(R_{t+1,i}\right) = 1 - \rho_{\text{ɱ}_{t,t+1},R_{t,i}}\sigma\left(R_{t+1,i}\right)\sigma\left(\text{ɱ}_{t,t+1}\right)$$
$$\rightarrow E_t\left(R_{t+1,i}\right) = \frac{1}{E_t\left(\text{ɱ}_{t,t+1}\right)} - \rho_{\text{ɱ}_{t,t+1},R_{t+1,i}}\frac{\sigma(\text{ɱ}_{t,t+1})}{E_t\left(\text{ɱ}_{t,t+1}\right)}\sigma(R_{t+1,i})$$
$$\rightarrow E_t\left(R_{t+1,i}\right) = R_f - \rho_{\text{ɱ}_{t,t+1},R_{t+1,i}}\frac{\sigma(\text{ɱ}_{t,t+1})}{E_t\left(\text{ɱ}_{t,t+1}\right)}\sigma(R_{t+1,i})$$
$$\rightarrow \frac{E_t\left(R_{t+1,i}\right) - R_f}{\sigma(R_{t+1,i})} = -\rho_{\text{ɱ}_{t,t+1},R_{t+1,i}}\frac{\sigma(\text{ɱ}_{t,t+1})}{E_t\left(\text{ɱ}_{t,t+1}\right)} \tag{4-59}$$

Because $-1 \leq \rho_{\mathfrak{m}_{t,t+1},R_i} \leq 1$ we conclude that

$$-\rho_{\mathfrak{m}_{t,t+1},R_{t+1,i}} \frac{\sigma(\mathfrak{m}_{t,t+1})}{E_t(\mathfrak{m}_{t,t+1})} \leq \frac{\sigma(\mathfrak{m}_{t,t+1})}{E_t(\mathfrak{m}_{t,t+1})}$$

$$\rho = -1 \rightarrow \frac{\sigma(\mathfrak{m}_{t,t+1})}{E_t(\mathfrak{m}_{t,t+1})} = \frac{\sigma(\mathfrak{m}_{t,t+1})}{E_t(\mathfrak{m}_{t,t+1})}$$

$$\rho = 1 \rightarrow \left|-\rho_{\mathfrak{m}_{t,t+1},R_{t+1,i}} \frac{\sigma(\mathfrak{m}_{t,t+1})}{E_t(\mathfrak{m}_{t,t+1})}\right| = \frac{\sigma(\mathfrak{m}_{t,t+1})}{E_t(\mathfrak{m}_{t,t+1})}$$

$$\rho \in (-1,1) \rightarrow -\rho_{\mathfrak{m}_{t,t+1},R_{t+1,i}} \frac{\sigma(\mathfrak{m}_{t,t+1})}{E_t(\mathfrak{m}_{t,t+1})} < \frac{\sigma(\mathfrak{m}_{t,t+1})}{E_t(\mathfrak{m}_{t,t+1})}$$

$$\left|\frac{E_t(R_{t+1,i}) - R_f}{\sigma(R_{t+1,i})}\right| \leq \frac{\sigma(\mathfrak{m}_{t,t+1})}{E_t(\mathfrak{m}_{t,t+1})} \quad (4\text{-}60)$$

The above inequality is called the Hansen-Jagannathan (HJ) bound.

Here is a numerical example. Assume that:

$E_t(R_{t+1,i}) = 0{,}07$

$R_f = 0{,}02$

$\sigma(R_{t+1,i}) = 0{,}3$

$\frac{E_t(R_{t+1,i}) - R_f}{\sigma(R_{t+1,i})} = 0{,}1667$

$\sigma(\mathfrak{m}_{t,t+1}) = 0{,}05$

$E_t(\mathfrak{m}_{t,t+1}) = 0{,}3$

$\frac{\sigma(\mathfrak{m}_{t,t+1})}{E_t(\mathfrak{m}_{t,t+1})} = \frac{0{,}05}{0{,}3} = 0{,}1667$

Table 4-4: Hansen- Jagannathan bound, numerical example

$\rho_{\mathfrak{m}_{t,t+1},R_{t+1,i}}$	$-\rho_{\mathfrak{m}_{t,t+1},R_{t+1,i}}\dfrac{\sigma(\mathfrak{m}_{t,t+1})}{E_t(\mathfrak{m}_{t,t+1})}$	$\left\lvert\dfrac{E_t(R_{t+1,i})-R_f}{\sigma(R_{t+1,i})}\right\rvert$	$\dfrac{\sigma(\mathfrak{m}_{t,t+1})}{E_t(\mathfrak{m}_{t,t+1})}$
-1	0,16667	0,16667	0,1667
-0,7	0,11667	0,11667	0,1667
-0,4	0,06667	0,06667	0,1667
-0,1	0,01667	0,01667	0,1667
0	0	0	0,1667
0,2	-0,03333	0,03333	0,1667
0,5	-0,08333	0,08333	0,1667
0,8	-0,13333	0,13333	0,1667
1	-0,16667	0,16667	0,1667

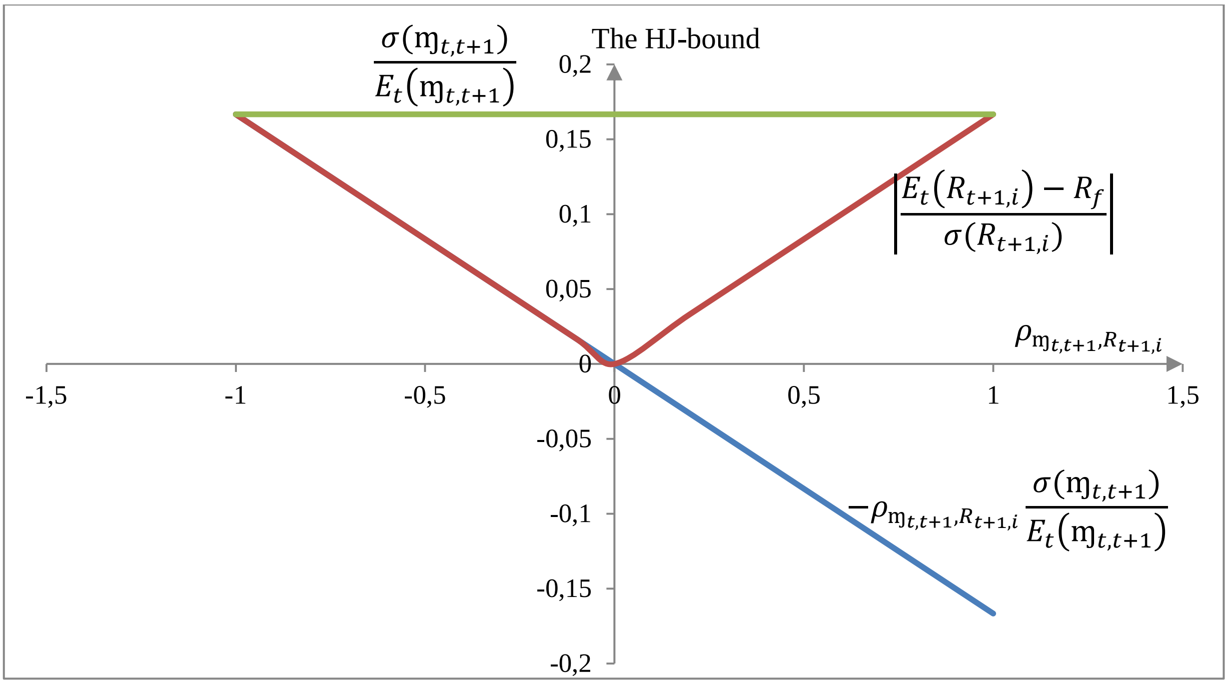


Figure 4-4: The Hansen – Jagannathan bound

An important property of this bound is that it is independent of the form of the utility function and can be used to test whether a pair of a discount factor and a utility function may serve as reasonable model for analyzing a market. The HJ-bound can be used to evaluate if a specific asset pricing model is a reasonable approximation to market by testing it's predictions against market data (Cochrane 2005, pp. 455-484).

Observe that:

$$R_{t+1,e} = R_{t+1,i} - R_f \rightarrow E_t\left(R_{t+1,e}\right) = E_t\left(R_{t+1,i} - R_f\right) \rightarrow$$
$$E_t\left(R_{t+1,e}\right) = E_t\left(R_{t+1,i}\right) - R_f \tag{4-61}$$

Then:

$$\sigma^2\left(R_{t+1,e}\right) = \sigma^2\left(R_{t+1,i} - R_f\right) \rightarrow \sigma^2\left(R_{t+1,e}\right) =$$
$$\sigma^2\left(R_{t+1,i}\right) - \sigma^2\left(R_f\right) - 2cov\left(R_{t+1,i}, R_f\right) \rightarrow$$
$$\sigma^2\left(R_{t+1,e}\right) = \sigma^2\left(R_{t+1,i}\right) \rightarrow \sigma\left(R_{t+1,e}\right) = \sigma\left(R_{t+1,i}\right) \tag{4-62}$$

and

$$\frac{E_t\left(R_{t+1,i}\right) - R_f}{\sigma\left(R_{t+1,i}\right)} = \frac{E_t(R_{t+1,e})}{\sigma(R_{t+1,e})} \tag{4-63}$$

The HJ bound can be written as:

$$\frac{\sigma(\mathfrak{m}_{t,t+1})}{E_t\left(\mathfrak{m}_{t,t+1}\right)} \geq \frac{E_t(R_{t+1,e})}{\sigma(R_{t+1,e})} \tag{4-64}$$

The following notation is used:

$E_t\left(\mathfrak{m}_{t,t+1}\right)$ is the expectation of the stochastic discount factor

$\sigma(\mathfrak{m}_{t,t+1})$ is the standard deviation of the stochastic discount factor

$E_t(R_{t+1,e})$ is the expectation of excess return

Mehra and Prescott (1985) researched the return of the US stock market over the period 1889-1994. They found that the equity return is so high compared to the alternative risk free investment (bonds) that in order to explain it by standard asset pricing models one has to assume extremely high risk aversion or too high stochastic discount rate. The excess returns required by investors cannot be explained by models like CCAPM (Consumption CAPM). This is called the equity premium puzzle.

The equity premium puzzle can be illustrated by the following example (Pennacchi 2008, pp. 89-90): Let R_i be the return on a well-diversified portfolio of say S & P 500 U.S. stocks. Using historical data over the past 75 years one finds that a reasonable estimate of this portfolio's annual excess return over the risk free interest rate is 8,3 %:

$$E_t\left[R_{t+1,i}\right] - R_f = 0{,}083 \tag{4-65}$$

The portfolio's annual standard deviation for the same period is estimated to 17 %. Then the Sharpe ratio is:

$$\frac{E_t\left[R_{t+1,i}\right] - R_f}{\sigma_{R_{t+1,i}}} = 0{,}49 \tag{4-66}$$

Assuming a power utility function of the form $\frac{C^{\gamma}}{\gamma}$ can we calculate the HJ-bound from 4-64 as follows:

$$\frac{\sigma(\mathfrak{m}_{t,t+1})}{E_t[\mathfrak{m}_{t,t+1}]} = \frac{\sqrt{Var\left[e^{(\gamma-1)ln\left(\frac{C_{t+1}}{C_t}\right)}\right]}}{E_t\left[e^{(\gamma-1)ln\left(\frac{C_{t+1}}{C_t}\right)}\right]} = \frac{\sqrt{E_t\left[e^{2(\gamma-1)ln\left(\frac{C_{t+1}}{C_t}\right)}\right] - E_t\left[e^{(\gamma-1)ln\left(\frac{C_{t+1}}{C_t}\right)}\right]^2}}{E_t\left[e^{(\gamma-1)ln\left(\frac{C_{t+1}}{C_t}\right)}\right]}$$

$$= \sqrt{\frac{E_t\left[e^{2(\gamma-1)ln\left(\frac{C_{t+1}}{C_t}\right)}\right]}{E_t\left[e^{(\gamma-1)ln\left(\frac{C_{t+1}}{C_t}\right)}\right]^2} - 1} = \sqrt{\frac{e^{2(\gamma-1)\mu_c + 2(\gamma-1)^2\sigma_c^2}}{e^{2(\gamma-1)\mu_c + (\gamma-1)^2\sigma_c^2}} - 1} = \sqrt{e^{(\gamma-1)^2\sigma_c^2} - 1}$$

$$\overset{e^x = 1 + x + \frac{x^2}{2!}\dots}{\approx} \sqrt{1 + (\gamma-1)^2\sigma_c^2 - 1} = \sqrt{(\gamma-1)^2\sigma_c^2} = (1-\gamma)\sigma_c \tag{4-67}$$

where $e^x = 1 + x$ is an approximation of e for small x.

For a power utility function of the form $\frac{c^{\gamma}}{\gamma}$ is

$\left\lvert \frac{E_t[R_{t+1,i}] - R_f}{\sigma_{R_{t+1,i}}} \right\rvert \leq (1-\gamma)\sigma_c$	4-68

Assuming a broadly diversified portfolio, for example the stock index, we can write:

$\frac{E_t[R_{t+1,i}] - R_f}{\sigma_{R_{t+1,i}}} = (1-\gamma)\sigma_c \rightarrow E_t[R_{t+1,i}] - R_f = (1-\gamma)\sigma_c\sigma_{R_{t+1,i}}$	4-69

So for a broadly diversified portfolio the equity premium is proportional to $(1-\gamma)$, σ_c and $\sigma_{R_{t+1,i}}$.

Equation

4-69 can be rewritten as:

$\frac{\frac{E_t[R_{t+1,i}] - R_f}{\sigma_{R_{t+1,i}}}}{\sigma_c} = (1-\gamma) \rightarrow \frac{E_t[R_{t+1,i}] - R_f}{\sigma_{R_{t+1,i}}\sigma_c} = (1-\gamma)$	4-70

The annual standard deviation of consumption growth in USA for 1933-2008 (Pennacchi 2008, p. 89) has been estimated to be between 0,01 and 0,0386 and the equity premium $E_t[R_{t+1,i}] - R_f = 0{,}083$, where R_f is the S&P 500. Then the risk aversion γ is between:

$-48 < \gamma = 1 - \frac{E_t[R_{t+1,i}] - R_f}{\sigma_{R_{t+1,i}}\sigma_c} < -11{,}7$	4-71

This result doesn't harmonize with the expected range for risk aversion calculated from other sources being between -5 to -1. The volatility of consumption growth is too low compared to the premium demanded by investors for holding stocks.

4.4.6.1.1 *Deriving an equation for the volatility of the stochastic discount factor*

Let the regression equation for estimating ɱ be (Cochrane 2005, p.94):

$$\text{ɱ}_{t-1,t} = E(\text{ɱ}) + (\boldsymbol{x_t} - E(x))'\boldsymbol{\beta} + \varepsilon_t \tag{4-72}$$

where $\boldsymbol{x}_t$ and $\boldsymbol{\beta}$ are $N \times 1$vectors

$$\begin{aligned}
&P_{t-1} = E\left(\text{ɱ}_{t-1,t}, \boldsymbol{x_t}\right) \xrightarrow{\text{suppressing the indices}} \\
&P = E(\text{ɱ})E(x) + cov(\text{ɱ}, \boldsymbol{x}) \rightarrow \\
&P = E(\text{ɱ})E(x) + cov[E(\text{ɱ}) + (\boldsymbol{x} - E(x))'\beta + \varepsilon, \boldsymbol{x}\,] \rightarrow \\
&P = E(\text{ɱ})E(x) + cov(\boldsymbol{x}, \boldsymbol{x}')\beta \xrightarrow{\Sigma = var(x)} \\
&P = E(\text{ɱ})E(x) + \sum \beta \;\; \rightarrow \\
&\beta = \Sigma'[p - E(\text{ɱ})E(x)] \text{ or } \beta = \Delta^{-1}[1 - E(\text{ɱ})E(x)]
\end{aligned} \tag{4-73}$$

Taking variance from both sides of 4-72:

$$\begin{aligned}
&var(\text{ɱ}) = \sigma^2(\text{ɱ}) \\
&var[E(\text{ɱ}) + (\boldsymbol{x_t} - E(x))'\boldsymbol{\beta} + \varepsilon_t] = var[(\boldsymbol{x} - E(x))'\beta] + \sigma^2(\varepsilon) \\
&\sigma^2(\varepsilon_t) \geq 0 \rightarrow \sigma^2(m) \geq var\left[\left(\boldsymbol{x} - E(x)\right)'\beta\right] \\
&var(\text{ɱ}) \geq [p - (E(\text{ɱ})E(x))]^T var(x)^{-1}[p - E(\text{ɱ})E(x)] \xrightarrow{p=1, x=R_m} \\
&var(\text{ɱ}) \geq [1 - (E(\text{ɱ})E(R_m))]^T var(R_m)^{-1}[1 - E(\text{ɱ})E(R_m)] \rightarrow \\
&\left.\begin{array}{l} var(\text{ɱ}) \geq \dfrac{\left[1 - \left(E(\text{ɱ})E(R_m)\right)\right]^2}{var(R_m)} \\ \sqrt{var(\text{ɱ})} = \sigma(\text{ɱ}) \end{array}\right\} \rightarrow \sigma(\text{ɱ}) \geq \pm\sqrt{\dfrac{\left[1 - \left(E(\text{ɱ})E(R_m)\right)\right]^2}{var(R_m)}} \rightarrow \\
&\sigma(\text{ɱ}) \geq \left|\frac{1 - \left(E(\text{ɱ})E(R_m)\right)}{\sigma(R_m)}\right|
\end{aligned} \tag{4-74}$$

Equation 4-74 is useful for constructing the HJ-bound.

4.4.6.2 The Risk-Free Rate Puzzle

From Lucas model we derived in 4-67 that:

$\dfrac{\sigma(\mathfrak{m}_{t,t+1})}{E_t[\mathfrak{m}_{t,t+1}]} = (1-\gamma)\sigma_c$	4-75

Here we follow the presentation by Pennacchi (2008, pp.80-90). Starting with the Lucas tree asset pricing model (

4-36) we get the following relation:

$\left.\begin{matrix} 1 = E_t\left[\mathfrak{m}_{t,t+1}\dfrac{X_{t+1,i}}{P_{t,i}}\right] \\ \dfrac{X_{t+1,i}}{P_{t,i}} = R_{t+1,i} \end{matrix}\right\} \rightarrow 1 = E_t[\mathfrak{m}_{t,t+1}R_{t+1,i}]$	4-76

Equation 4-76 shows the relationship between the stochastic discount factor m and the risk free discount factor R_f.

For a risk free asset equation 4-76 becomes:

$1 = E_t[\mathfrak{m}_{t,t+1}R_f] \rightarrow \dfrac{1}{R_f} = E_t[\mathfrak{m}_{t,t+1}]$	4-77

where $\mathfrak{m}_{t,t+1} = \delta\frac{U'(C_{t+1})}{U'(C_t)}$ is the stochastic discount factor, δ is the subjective discount factor and C is the consumption.

Let the utility function be (Pennacchi 2008, p. 83):

$U(C) = \frac{C^\gamma}{\gamma}, \gamma < 1$	4-78

where γ is the risk aversion equal to $1-\alpha$ (Mehra 2006, p. 13).

Table 4-5: The relation between γ and α

Relation between γ and α / $\alpha = 1-\gamma$	$\gamma = 1-\alpha$	$\frac{C^{\gamma}}{\gamma} = \frac{C^{1-\alpha}}{1-\alpha}$
$\alpha = 0$	$\gamma = 1-0 = 1$	$\frac{C^{1}}{1} = \frac{C^{1-0}}{1-0} = \frac{C^{1}}{1}$
$\alpha = \frac{1}{2}$	$\gamma = 1-\frac{1}{2} = \frac{1}{2}$	$\frac{C^{\frac{1}{2}}}{\frac{1}{2}} = \frac{C^{1-\frac{1}{2}}}{1-\frac{1}{2}} = \frac{C^{\frac{1}{2}}}{\frac{1}{2}}$
$\alpha = 100$	$\gamma = 1-100 = -99$	$\frac{C^{-99}}{-99} = \frac{C^{1-100}}{1-100} = \frac{C^{-99}}{-99}$

So $\gamma \in (-\infty, 1)$, $\alpha \in (0, +\infty)$

Then:

$$\mathfrak{m}_{t,t+1} = \delta \frac{\frac{1}{\gamma}\gamma c_{t+1}^{\gamma-1}}{\frac{1}{\gamma}\gamma c_{t}^{\gamma-1}} \rightarrow \mathfrak{m}_{t,t+1} = \delta \left(\frac{c_{t+1}}{c_t}\right)^{\gamma-1} \quad (4\text{-}79)$$

This can be rewritten as:

$$\mathfrak{m}_{t,t+1} = \delta e^{(\gamma-1)ln\left(\frac{C_{t+1}}{C_t}\right)} \quad (4\text{-}80)$$

where $ln\left(\frac{C_{t+1}}{C_t}\right)$ is the logarithmic growth in consumption.

Let $\frac{C_{t+1}}{C_t}$ be lognormally distributed, then:

$$ln\left(\frac{C_{t+1}}{C_t}\right) \sim N(\mu_c, {\sigma_c}^2) \text{ and} \quad (4\text{-}81)$$

The probability density function PDF of $\frac{C_{t+1}}{C_t}$ is:

$$PDF\left(\frac{c_{t+1}}{c_t}\right) = \frac{1}{\sigma_c\sqrt{2\pi}} \frac{1}{\left(\frac{C_{t+1}}{C_t}\right)} e^{-\frac{\left[ln\left(\frac{C_{t+1}}{C_t}\right)-\mu_c\right]^2}{2{\sigma_c}^2}} \quad (4\text{-}82)$$

So:

$$E_t\left[ln\left(\frac{c_{t+1}}{c_t}\right)\right] = \mu_c + \frac{1}{2}{\sigma_c}^2 \quad (4\text{-}83)$$

By the same token:

$$ln\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1} \sim N((\gamma-1)\mu_c, (\gamma-1)^2{\sigma_c}^2) \quad (4\text{-}84)$$

and:

$$E_t\left[ln\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1}\right] = (\gamma-1)\mu_c + \frac{1}{2}(\gamma-1)^2\sigma_c^2 \quad (4\text{-}85)$$

Plugging the above equation to $\frac{1}{R_f} = E[\mathrm{m}_{t,t+1}]$ we get:

$$\frac{1}{R_f} = E_t[\mathrm{m}_{t,t+1}] \to \frac{1}{R_f} = \delta E_t\left[e^{ln\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1}}\right] \to \frac{1}{R_f} = \delta\left[e^{E_t\left[ln\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1}\right]}\right] \to$$

$$\frac{1}{R_f} = \delta e^{(\gamma-1)\mu_c + \frac{1}{2}(\gamma-1)^2\sigma_c^2} \to$$

$$ln(R_f)^{-1} = \ln(\delta) + (\gamma-1)\mu_c + \frac{1}{2}(\gamma-1)^2\sigma_c^2 \to$$

$$-ln(R_f) = \ln(\delta) + (\gamma-1)\mu_c + \frac{1}{2}(\gamma-1)^2\sigma_c^2 \to$$

$$ln(R_f) = -\ln(\delta) + (1-\gamma)\mu_c - \frac{1}{2}(1-\gamma)^2\sigma_c^2 \quad (4\text{-}86)$$

Assuming 0,01 (1%) of time preference is $\delta = 0{,}99$. The historical annual growth rate of consumption is $\mu_c = 0{,}018$. Plugging in $\gamma = -11$ and $\sigma_c = 0{,}036$ we get:

$ln(R_f) = 0{,}0105 + 0{,}216 - 0{,}093 = 0{,}133 \to e^{ln(R_f)} = e^{0{,}133} \to R_f = 1{,}142$

So r_f is calculated in this setting to be 14,2%. This is too high a number and doesn't conform to the empirical data since the risk free interest rate for the same period averaged 1% in the United States.

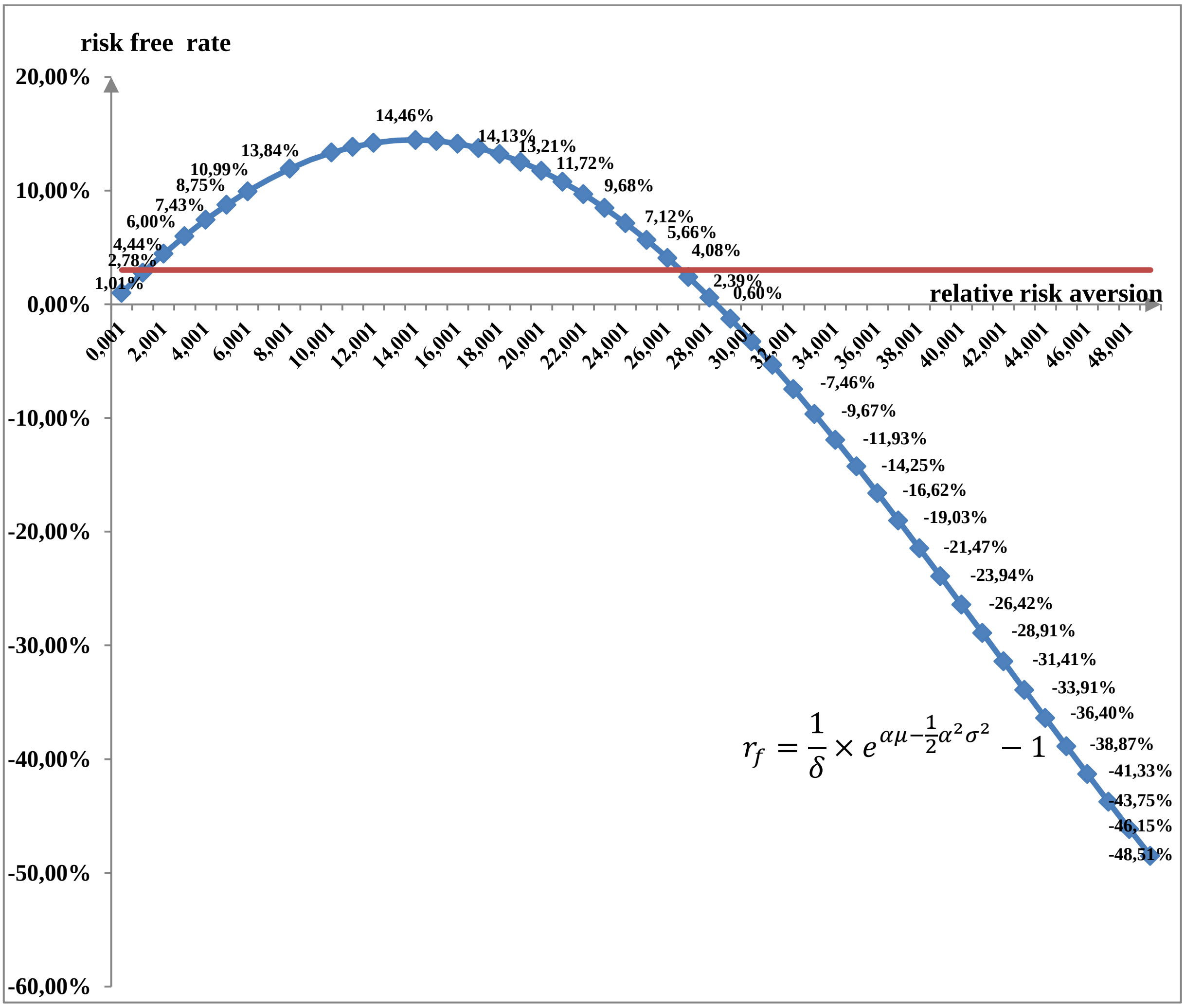


Figure 4-9: The risk free rate r_f as a function of the relative risk aversionα.

The equation used in the diagram is derived as follows:
The net risk free rate is given by:

$$r_f = e^{ln(R_f)} \tag{4-87}$$

In discrete time:

$$r_f = R_f - 1 \tag{4-88}$$

From 4-86 we get:

$$ln(R_f) = -\ln(\delta) + \alpha\mu_c - \frac{1}{2}\alpha^2\sigma_c^2 \tag{4-89}$$

Then:

$e^{ln(R_f)} = e^{-\ln(\delta)} \times e^{\alpha\mu - \frac{1}{2}\alpha^2\sigma^2} \rightarrow$ $R_f = \frac{1}{\delta} \times e^{\alpha\mu - \frac{1}{2}\alpha^2\sigma^2} \rightarrow$ $R_f - 1 = \frac{1}{\delta} \times e^{\alpha\mu - \frac{1}{2}\alpha^2\sigma^2} - 1 \rightarrow$ $r_f = \frac{1}{\delta} \times e^{\alpha\mu - \frac{1}{2}\alpha^2\sigma^2} - 1$	4-90

For high values of α, for instance $\alpha = 27$, the risk free rate is "reasonable" at 2,39%. High values of risk aversion imply high changes of the term $\alpha\mu_c$ for small changes in the expected consumption growth μ_c. This would imply high changes in the risk free rate, which doesn't agree with empirical data (Mehra 2006, p. 21).

$$ln(R_f) = -\ln(\delta) + \alpha\mu_c - \frac{1}{2}\alpha^2\sigma_c^2$$

Table 4-6: Higher values of α and changes in $\alpha\mu_c$

α	μ_c	$\alpha\mu_c$	$\Delta\alpha\mu_c$
50	0,018	0,9	
50	0,019	0,95	0,05

Table 4-7: Lower values of α and changes in $\alpha\mu_c$

α	μ_c	$\alpha\mu_c$	$\Delta\alpha\mu_c$
10	0,018	0,18	
10	0,019	0,19	0,01

We conclude that high risk aversion leads to unreasonably high risk free rate. This is called the risk-free rate puzzle.

4.4.6.3 The Correlation Puzzle

The HJ-bound is set under the assumption that the absolute value of the correlation between the consumption growth for non-durables and services and stock returns is equal to one. Tests with US data show that the empirical risk free rate is 0,01, the empirical Sharpe ratio is $\frac{E_t(R_{t+1,\xi})}{\sigma(R_{t+1,\xi})} = 0{,}5$, the empirical volatility of consumption growth 0,01 and the empirical correlation is 0,2.

That implies:

$$\frac{\sigma(\mathfrak{m}_{t,t+1})}{E_t(\mathfrak{m}_{t,t+1})} \geq \frac{E_t(R_{e,t+1})}{\sigma(R_{e,t+1})} = 0{,}5 \times 5 = 2{,}5$$

Using $E_t(\mathfrak{m}_{t,t+1}) = \frac{1}{R_f}$ we find that:

$$E_t(\mathfrak{m}_{t,t+1}) = \frac{1}{1 + 0{,}01} \approx 0{,}99$$

Approximating $\sigma(\mathfrak{m}_{t,t+1})$ with $\gamma\sigma\left(\frac{C_{t+1}}{C_t}\right)$ we have:

$$\gamma \geq \frac{0{,}99 \times 2{,}5}{0{,}01} = 247{,}5$$

which is a huge risk aversion coefficient.

When the correlation between the consumption growth for non-durables and services and stock returns is not perfect, the HJ-bound becomes much more difficult to fulfill. For empirically observed small volatilities in the consumption growth for non-durables and services, the risk aversion coefficient has to be excessively big to satisfy the HJ-bound. (Cochrane 2005, p. 457)

4.4.6.4 The Volatility Puzzle

The stock's intrinsic value at time t is:

$$V_t = \frac{d_{t+1}}{1+r} + \frac{d_{t+2}}{(1+r)^2} + \cdots = \sum_{i=1}^{\infty} \frac{d_{t+i}}{(1+r)^i} \tag{4-91}$$

where C_{t+i} is the dividend cash flow of stock i at time t, and r is the discount rate.

In finite time:

$$V_t = \sum_{i=1}^{T-t} \frac{d_{t+i}}{(1+r)^i} + \frac{V_T}{(1+r)^{T-t}} \tag{4-92}$$

where $t < T$ and V_T is the fundamental value of the stock.

Shiller (1981) investigated whether the stock prices' volatility is greater than the dividend cash flows' volatility by testing equation 4-92 in ex-post fashion; with T as the present time and t some time points in the past. Because the test was ex post the cash flows were known. Shiller used the present asset price as an approximation of V_T. Utilizing the market data, Shiller calculated an estimate for V_t and compared it with the actual asset price at time t, P_t.

The prediction of EMH is that P_t is the optimal forecast, i.e. the best predictor, at time t of V_t. A best predictor $\hat{X}$ of a variable X has the following property (Ruppert 2004, pp.443-444):

$$Var(X) = Var(\hat{X}) + E(X - \hat{X})^2 \tag{4-93}$$

which implies that $\hat{X}$ and $X - \hat{X}$ are uncorrelated.

From 4-93 follows the following inequality:

$$Var(\hat{X}) \leq Var(X) \tag{4-94}$$

Had P_t been an optimal forecast of V_t , should P_t be less volatile than V_t. Contrary to this expectation, the market data shows that the volatility of the asset prices is much greater than the volatility of the asset's dividend cash flows. This is called the excess volatility puzzle.

Shiller (1981) concludes that asset prices are not optimal forecasts of the present value of discounted future dividends.

Shiller discusses in his book “Irrational Exuberance” (Shiller 2005, pp. 74-76, 147-156 and 189-194) possible explanations of excess market volatility such as crowd psychology and naturally occurring Ponzi schemes. Excess Volatility can be interpreted as overreaction to new information (Daniel, Hirshleifer and Subrahmanyam 1998, p. 1841).

The dynamic Gordon growth model allows for a high volatility of stock prices for small changes in the expected stock return. This is under the assumption of the logarithm of the required rate of return r following a persistent process (Campbell et al. 1997, p. 265).

4.4.6.4.1 Models of stochastic volatility

The assumption of share prices having constant volatility might be true in the short run. However, is not unreasonable to assume that the volatility of share prices is not constant in the long run. So a more realistic model of share prices is formed by relaxing the assumption of constant volatility. There are periods of high volatility and periods of low volatility. The variability of volatility is called volatility drift.

In order to describe the volatility of share prices as a stochastic variable, two stochastic differential equations are needed (Hens and Rieger 2010, pp. 329-332):

$$dS(t) = \mu S(t)dt + \sigma(t)S(t)dB_1(t) \tag{4-95}$$

$$d\sigma(t) = \alpha(\sigma(t))dt + \beta(\sigma(t))dB_2(t) \tag{4-96}$$

where B_1 and B_2 are Brownian motions.

The square root stochastic volatility version is also used:

$$dS(t) = \mu S(t)dt + \sqrt{\sigma(t)}S(t)dB_1(t) \tag{4-97}$$

$$d\sigma(t) = \alpha(\sigma(t))dt + \beta(\sigma(t))dB_2(t) \tag{4-98}$$

The rational for using the square root of the standard deviation is that it is yields analytical solutions in option pricing (Ishida and Engle 2002).

Another stylized fact is that the share price volatility is mean reverting. This is modelled in the following way:

$\alpha\big(\sigma(t)\big) = \theta(\omega - \sigma(t))$	4-99

$\beta\big(\sigma(t)\big) = \xi\sigma(t)^{\gamma}$	4-100

where ω is a long term volatility mean and $\xi > 0$ is a constant.

The constant $\theta > 0$ shows the speed of adjustment to mean ω. Exponent $\gamma > 0$ is a constant.

Some popular models in this framework are the following:

● The Heston model, where $\gamma = 1/2$.

● The Generalized Autoregressive Conditional Heteroscedasticity model (GARCH), where $\gamma = 1$.

One can run empirical tests to find the value of γ.

Some other stylized facts about the stock price volatility is that it is higher in bear markets and lower in bull markets. This is called "the leverage effect" and doesn't fare well with the known assumption of risk return being proportional to volatility. Volatility and stock prices are correlated negatively in losses but not necessarily so in gains. This is called "volatility asymmetry". Stylized facts explained by models of interacting agents are summarized by Lux (2009).

4.4.7 Attempts to explain the puzzles

4.4.7.1 Behavioural Finance

The presumption of efficient markets, rational expectations and symmetric information, constitutes a neat set of premises. On these grounds is derived a framework on the formation of security prices in financial markets. The appealing rationale of efficient markets is that there are no free lunches. There is however a collection of real life phenomena that cannot be satisfactory supported within this theoretical space. Behavioral finance departs from the assumption of market efficiency and the rational expectations equilibrium. Models of behavioral finance accept the existence of traders that have expectations which depend on the past. Behavioural finance demonstrates the feasibility of co-existence of irrational with rational traders. Noise traders can have a lasting impact on asset prices. The appeal of behavioural finance is allowing for psychological bias and looking squarely into the eyes the issue of cognitive bias. Market psychology, mood and sentiment are invaluable instruments in explaining bubbles, panics, and crashes. The existence of fat tails in the probability distribution of stock prices is explained through the introduction of noise risk. The conditions for the survival of noise traders can be justified in a complex chaotic world with bounded rationality. The cornerstones behavioral finance is building on are limits to arbitrage (see appendix A - xv) and psychology (Barberis and Thaler, 2003, pp.1051-1121).

4.4.7.1.1 Beliefs and psychological bias

People are not inclined to find evidence that contradict their established conceptions. Even when such evidence arises they might choose to ignore it. This is called selective attention. It lends itself to a tendency of awareness to some constituencies of the environment excluding others.

Keynes (1936, cited in Shiller 2011) asserted that picking stocks is much the same as the majority’s voting for the most beautiful women in a beauty contest. Sherif's experiment on the autokinetic effect the same year (1936, cited in Sherif 2009, p.138) showed that people’s perception of the movement of a fixed light beam in a dark room is influenced by the group norms.

Cognitive psychology research has resulted in a pile up of evidence on systematic biases in people's formation of beliefs.

The following biases imply a departure from the assumptions of rational expectations (Berk and De Marzo 2010, pp. 417-423):

- The familiarity bias. Investors prefer to invest on companies that they are familiar with.
- The overconfidence bias. Investors tend to overestimate their knowledge like football fans second guessing coaching decisions.
- The sensation seeking bias. Investors like the excitement of handling investments as lottery tickets.
- The disposition bias. Investors tend to sell out shares that have risen in value and hold on to shares that have lost value.
- Ambiguity aversion: Because probabilities are not objectively known, individuals built their beliefs on subjective probabilities. In ambiguous circumstances will individuals make choices that render subjective probabilities that are inconsistent with each other, see Ellsberg 1961.
- The sentiment bias. Investors are influenced in their decisions by mood and the market psychology.

Barberis and Thaler (2003, pp.1051-1121) describe beliefs as the process of forming expectations. Forming of beliefs is influenced by a number of psychological traits. Experiments show that people assign too high probabilities to events that occur more often and too low probabilities to events that occur more rarely. This is pinned down to overconfidence.

When an initial estimate on an unknown subject is asked for, people pick an arbitrary value. The provision of new information leads to adjustments but not far off the initial values. People tend to cling on too much to their initial guess. This is attributed to the anchoring effect.

According to Bayes" rule is the probability of an event B given an event A as following:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Kahneman and Tversky (1974) provide experimental evidence that the prior probability $P(A)$doesn't have an effect on the probability belief outcome which is Bayes' rule prediction.

4.4.7.2 Heterogeneity

Fama (1970) contains concordant beliefs as one of three sufficient conditions for capital market efficiency, the other two being no transaction costs and the availability of information to all market participants. Rubinstein (1975) puts forward a set of increasingly stronger conditions on informational capital market efficiency: 1) non speculative beliefs 2) consensus beliefs and 3) homogeneous beliefs. Non-speculative beliefs are beliefs for which portfolio revision is not an optimal strategy. Consensus beliefs are beliefs which generate the same equilibrium prices as an heterogeneous economy. He ascertains that the existence of homogeneous beliefs is a sufficient but not necessary condition for consensus beliefs and non-speculative beliefs. Heterogeneity in Rubinstein's terms mean individuals assigning different probabilities to the occurrence of a certain state of nature. It can also mean different tastes expressed as diverse utility functions.

Heterogeneity can be related to beliefs, risk aversion and time preferences. Heterogeneity in beliefs is expressed as the assignment of different probabilities by different investors to the same event. Shefrin (2000, pp.107-109) presents a model of two investors, one optimist and the other pessimist with logarithmic utility functions with binomial beliefs, i.e. at each time t the state of the economy evolves only in two states. Then he derives in this setting an equilibrium price density function with fat tails.

Varian (1985) sets up a model with diversity of opinions but common time and state separable utility functions. In Varian's terminology is diversity of opinions equivalent to Rubinstein's heterogeneity in beliefs, that is assigning different probabilities to the same state. One of his model predictions is that given a utility function with constant relative risk aversion, the diversity of opinions is inversely proportional to asset prices if the absolute risk aversion is greater than 1.

Bhamra and Uppal (2010) assert that heterogeneous preferences and beliefs boost the ability of their model to match characteristics of asset returns. Their setting is an endowment economy with two types of agents who have different power utilities, i.e. different relative risk aversions, different subjective discount factors, i.e. different time preferences and different beliefs, i. e. different stochastic discount factors. They conclude that heterogeneity in beliefs, time preferences and risk aversion increases the market price of risk and the volatility of asset prices. The consequence is a considerably higher equity premium.

In a classical financial theory framework in the sense of Fama it is assumed that agents interpret information in the same way. So if information is publicly know they form homogenous beliefs about the future. Relaxing the assumption of homogenous interpretation of information gives rise to heterogeneity in beliefs in a rational expectations setting. Xiouros (2009) assumes that agents don’t know the true data generating process. As a consequence they use a range of models that are statistically indistinguishable in order to form their beliefs. Information costs make agents to choose randomly among these models. By this chain of arguments he arrives to dispersion of belies due to disparate interpretation of information without resorting to behavioral factors.

Xiouros and Zapatero (2010) put forward a discrete time model of heterogeneous agents with different degrees of risk aversion but with the same time preferences and beliefs. Agents have a power utility function with different degrees of risk aversion. The financial market they operate in is assumed to be complete, i.e. there is a unique security asset for every state of nature (Copeland et al. 2005, pp. 77, 78).The model includes an external consumption habit which depends on individual's previous consumption and on the aggregate consumption level in a "keep up with the Joneses" fashion. Xiouros and Zapatero derive a closed form solution for the equilibrium state price density. After a careful calibration of the distribution of agent types they conclude that it is unlikely that the heterogeneity in risk aversion alone can explain the volatility in stock prices. This is ascribed by the authors of the paper to the cross sectional redistribution of wealth in this setting being too low.

Cvitanic, Jouini, Malamud and Napp (2011) present a model in a complete financial market populated with agents of constant absolute risk aversion and heterogeneity in beliefs, risk aversion or liquidity preferences. They find that heterogeneity is constant at individual level but fluctuates at the aggregate level. In their setting, heterogeneity leads to excess volatility of asset prices and an additional risk premium in the long run.

4.4.7.3 Habit formation

Habit formation is proposed as an explanation to both the Equity Premium Puzzle and the volatility puzzle. Utility functions of which past consumption is affecting current utility are said to display habit persistence Pennacchi (2008, pp. 295-316). There are two main model types of habit persistence in the literature: The internal habit persistence models and the external habit persistence models. In the internal habit persistence models, asset prices are perfectly elastic. In the external habit persistence models, asset prices are perfectly inelastic.

In an internal habit model the consumption level is assumed to be formed at a point of time and then influence consumption levels onwards. This is because the individual gets accustomed to a certain consumption standard. With internal habit persistence the demand for the risky asset decreases, resulting in a higher return which could be an explanation to the equity premium puzzle (Constantinides 1990). This is a setting that accommodates heterogeneity amongst consumers since the individual consumption enters explicitly into the model. In an external habit model the individual consumption depends on society's aggregate consumption. Because of that, external habit models are called "Keeping up with the Joneses" in order to point out that the utility of consumption for one person depends on the utility of consumption of the other consumers (the aggregate consumption). In a sense one doesn't want to be left behind in terms of consumption habits. External habit models predict that the risk premium varies together with business cycles. The equity premium increases in recessions and decreases in booms (Campbell and Cochrane 1999).

4.4.7.4 Recursive utilities as an explanation to the Equity Premium Puzzle and the risk free rate puzzle

Time inseparable utilities are developed in discrete time by Kreps and Porteus (1978) and Epstein and Zin (1989) and in continuous time by Duffie and Epstein (1992). Recursive utility depends on a function which aggregates the future lifetime utility. A characteristic of recursive utility functions is the separation of the relative risk aversion from the elasticity of the intertemporal substitution of consumption. The relative risk aversion shows choices between portfolios of different risks while the elasticity of the intertemporal substitution of consumption shows choice of consumption at different points of time. The advantage of recursive utility functions is differentiating between two components of utility functions which are conceptually different. In this setting it is possible to combine low risk aversion with low elasticity of intertemporal substitution with the potential to explain the equity premium puzzle and the interest rate puzzle. Recursive utilities are claimed to explain the Equity Premium Puzzle and the risk free rate puzzle.

4.4.7.5 Myopic loss aversion

Benartzi and Thaler (1995) offer an explanation to the equity premium puzzle based on Kahneman's and Tversky's prospect theory. In this context, investors are assumed to be loss averse, i.e. more sensitive to losses than gains. A second assumption is that investors evaluate their portfolios frequently and react adversely to short term losses. Benartzi and Thaler set up a model where the investor maximizes the following function:

$$f(v) = E_{\pi} v\left[(1-\omega)R_{f,t+1} + \omega R_{t+1} - 1\right]$$

where

$$v\begin{cases} x^{a} & if\ x\ \geq 0 \\ -\lambda(-x)^{a} & if\ x < 0 \end{cases}$$

$$\pi_i = w(p_i) - w(p_i^{*})$$

$$w(p) = \frac{p^{\gamma}}{(p^{\gamma} + (1-p)^{\gamma})^{\frac{1}{\gamma}}}$$

where v is a variable capturing loss aversion, ω is the fraction of wealth invested on stocks, and π is the decision weight associated with the event probabilities, λ is the coefficient of loss aversion which shows the relative sensitivity to gains and losses, a is the sensitivity to loss aversion, $w(p)$is the weighting probability function, γ is the sensitivity to the probability and $w(p_i^{*})$ is the weighting probability which makes an investor indifferent to the decision of accepting or not accepting a bet. Benartzi and Thaler contain that investors are more sensitive to losses than gains and are subject to the mental accounting bias which is the tendency to segregate individual gambles from each other. Benartzi and Thaler argue that the combination of loss aversion and mental accounting would make it unattractive to accept a bet of 50% probability for a gain of $200 and 50 % probability for a $200 even for small values of λ , say $\lambda = 2$. If the bet was played only once or evaluated one at a time but it would be attractive if the bet was played more than one period and the investor didn’t have to watch. The combination of short sightedness and loss aversion causes myopic loss aversion and is proposed as a possible explanation of the equity premium puzzle. The trick of the game is that small values of loss aversion can generate a large equity premium.

5 Empirical literature review

In this section we include also articles that are based on experiments and articles that use simulation for examining the performance of theoretical models against stylized facts.

Sentiment

Xiouros (2009, pp.104-159) provides a stock pricing model which predicts that belief dispersion and sentiment risk are positively correlated with the volatility of beliefs, the trading volume and the stock return. Xiouros uses the deviation of individual forecast of professional forecasters as a proxy for belief dispersion and sentiment risk. In this setting, the higher the level of disagreement, the bigger the speculative trade becomes and the higher the price changes. This leads to a prediction of a proportional relation between volume, stock returns, stock returns' volatility and belief heterogeneity which he confirms with empirical tests.

Shleifer (2000, pp.134-153) introduces a model which presents sentiment risk as the result of overreaction and underreaction. Overreaction is attributed to the representativeness bias which leads to the overestimation of the probabilities assigned to events. Underreaction is the result of conservatism and implies inadequate revision of beliefs. Whether the regime will be one of overreaction or underreaction depends on the nature of the sequence of previous events. The value of regime switching parameters is determined exogenously. Shleifer parameterizes the model and carries through simulation experiments which produce results consistent with a broad empirical evidence such as that of De Bondt and Thaler (1985), Bernard and Thomas (1989) and Jegadeesh and Titman (1993). De Bondt and Thaler find that portfolios of prior losers outperform prior winners over 3-5 years, consistent with the predictions of the overreaction hypothesis. Bernard and Thomas study the post earnings announcement drift and conclude that it cannot be easily reconciled with rational pricing. Jegadeesh and Titman document that strategies of buying stocks that have performed well in the past and selling stocks that have performed poorly generate positive returns in holding periods up to 7 months. This situation reverses in the period 8 to 20 months after the portfolios were formed. They propose as possible explanations either positive feedback traders who buy past winners and sell past losers so that prices move away from their fundamental values or as an underreaction to the short term prospects and overreaction to the long term prospects of firms. Their results suggest investor expectations being systematically biased. Schwert (2003) reviews anomalies and market efficiency and stresses that well-known anomalies in the finance literature don't hold in sample periods after the papers which highlighted them got published. This is an indication of market participants taking advantage of them to the point they disappear. Fama and French (1996) test momentum strategies using a three factor model which extends CAPM with two risk factors. These are the risk factors to size and overvaluation. Fama and French explain the abnormal returns on the long-term reversal strategy of DeBondt and Thaler to small distressed firms but find no explanation to the short term momentum effects pointed out by Jegadeesh and Titman.

Iori (2002) proposes a model with heterogeneous agents. The heterogeneity is manifested through a threshold value. An action is triggered whenever a common external signal is interpreted in such a way that it overtakes or undertakes a threshold value that varies from individual to individual. The model by Iori doesn’t require specific utility functions. The model by Iori produces through simulation stylized facts like high trading volume and volatility clustering.

Diether, Malloy and Scherbina (2002 p. 2113) test the hypothesis that prices will reflect the optimistic view. This hypothesis is based on the view that in a market of agents with heterogeneous expectations, the investors with the highest evaluation of the return are dominating the price setting (Miller 1977 p. 1152). The prediction of this hypothesis is that the relationship between the dispersion of beliefs and asset returns is inversely proportional. They find evidence that stocks with higher dispersion in analysts' earnings forecasts earn lower returns.

Grigaliuniene and Cibulskienė (2010) use the consumer confidence indicator (CCI) and the economic sentiment indicator (ESI) as proxies for sentiment risk. They test the relation between market return and sentiment risk in Scandinavian countries for the period 1989 to 2009. They find that both CCI and ESI are statistically significant predictors of stock returns at aggregate level and that in most of cases the relations between sentiment and stock returns are negative. The sentiment risk seems to be associated to macroeconomic indicators such as the consumer price index, the change in industrial production, the gross domestic product and the short term T-Bill rate.

Stenstad and Rabben (2012) construct portfolios of stocks in Oslo Stock Exchange based on revisions of EPS-earnings in analyst forecasts. Using portfolios with the most and least favourable EPS-revision ratios, they follow a strategy of buying the stocks with the most favourable revisions and sell the stocks with the least favourable revisions. They find that this portfolio gives a significant risk free abnormal return of 1% per month. Then they split each portfolio into two sub portfolios by their level of analysts' forecast dispersion. The results show that by buying the sub portfolio with the lowest dispersion and selling the sub portfolio with the highest dispersion, they obtain a significant risk free monthly return of 1,33 % over the sample period 2005-2011. They interpret the high dispersion of analysts' forecast is a signal of high uncertainty and large forecast errors.

Baker and Wurgler (2007) test how investor sentiment affects the cross sectional stock returns. As a proxy for sentiment they use a composite index made of the closed-end fund discount, NYSE share turnover, the number and average first-day returns on IPOs, the equity share in new issues and the dividend premium. The closed-end fund discount is the average difference between the net asset values of closed-end stock fund shares and their market prices. A closed en d fund is sold as a portfolio with a fixed number of shares in an initial public offering (IPO). After the IPO closed end funds typically trade on a secondary market. Based on earlier tests, the closed-end-fund discount is expected to be inversely proportional to sentiment. NYSE share turnover is a proxy for liquidity and equals the ratio of reported share volume to average shares listed from the NYSE Fact Book. Liquidity is proportional to optimism and shows overvaluation. First day returns on IPOs are viewed as a proxy for investor enthusiasm. The equity share in new issues is gross equity issuance divided by gross equity plus gross long-term debt issuance. The equity share is expected to capture sentiment. The proxy used for relative investor demand for dividend paying stocks is the market-to-book ratios of dividend paying stocks to no dividend paying stocks. The relative investor demand for dividend paying stocks is also expected to be associated to sentiment. Their empirical tests show that sentiment affects securities which are difficult to arbitrage. Sentiment at the beginning of a period is inversely proportional to subsequent returns for small stocks, young stocks, high volatility stocks, unprofitable stocks, non-dividend-paying stocks, extreme growth stocks, and distressed stocks.

Jubbega (2011) examine the role of Twitter as an instrument for capturing the public sentiment for the stock prices in American stock exchanges of Coca-Cola, IBM, Microsoft, Google, McDonald’s, Intel, Nokia, Disney, Toyota and Cisco. She finds a significant relation for Coca-Cola , Toyota, Microsoft, Disney and Nokia.

Herding

Institutional investors are holding a prominent share of the stock market and account for a large proportion of the trading volume. The bulk of their holding and trading consists of large cap stocks. Lakonishok, Shleifer and Vishny (1992) investigate the effect of institutional investors' trades on stock prices. As a measure for herding they use the absolute value of the difference between the proportion of institutional investors buying in any given quarter relative to the number active minus the expected proportion of money managers buying in that quarter relative to the number active. Ina sense their measure is the absolute value between the realized probability and the expected probability of buying in a given time period. Lakonishok, Shleifer and Vishny find weak evidence of herding for small cap stocks but don't find support for herding in the large cap stocks. Hagen and Joshi (2009) investigate how the trading behaviour of state agents (departments and the public pension fund administered by the national bank of Norway), individuals, financial firms, non-financial firms, and foreign investors influences returns and volatility in the Oslo Stock Exchange. They find indications of the foreign investor group acting as arbitrageurs and spurious herding where informed investors drive prices towards fundamental values. Lin, Tsai and Sun (2009) argue that the measure by Lakonishok, Shleifer and Vishny doesn't capture the sequential interactions of market agents in a higher frequency context. Lin, Tsai and Sun assert that the order flow of buy or sell orders can reveal runs and provides a more realistic characterization of herding. Building on a statistic of runs, that is sequences of buys or sells which realization exceeds the expected probability of their occurrence, they find evidence of herding on an intraday level for the highest returns stocks in bull market. The measure of herding employed by Lin, Tsai and Sun is conceived by Patterson and Sharma (2006). Information cascades arise when agents suppress their own beliefs and align with the observed market consensus. Cascades are more likely to form among small traders with higher information costs. The information cascade hypothesis is that herding should be more likely in small cap stocks. Lin, Tsai and Sun find that order flow herding is inversely proportional to time it takes to fill an order. Herding is more likely for trading of high cap liquid stocks, at market open. The information cascade effect is more likely at market close. Their findings support the theory of search costs by Vayanos and Wang (2007). Search costs are the costs for finding counterparts. Vayanos and Wang assume that investors are heterogeneous with respect to their investment horizon. Investors seek counterparts when their evaluation of the asset switches to a lower level. Their switching rate is inversely proportional to their investment horizon. The higher the liquidity of an asset, the lower the search costs and the higher the order flow. Another finding by Lin, Tsai and Sun is that herding is more prominent in stocks with returns ranking at the highest deciles. This renders support the directional asymmetry in the autocorrelation of returns documented by McQueen, Pinegar and Thorley (1996).

In a review article on herding by Bikhchandani and Sharma (2001), the cross sectional dispersion of returns or standard deviations is described as a measure for a particular type of herding which is asset specific. It doesn't capture herding that shows up in the common component of returns which can cause returns of all assets with the same characteristics to move to the same direction. The common component of returns is the one which cannot be diversified away (Richards 1999). Absence of evidence against the asset specific type of herding doesn't preclude the existence of other types of herding.

Jegadeesh and Kim (2010) investigate herding among sell side analysts. They invent a measure of herding based on an asset's abnormal returns as a function of the difference of an analyst's recommendations minus the consensus recommendation. Recommendations are non-information driven and show the tendency of each analyst to deviate from or conform to the consensus. Jegadeesh and Kim find evidence of herding among analysts. The herding effect is more pronounced in recommendations downgrades and among analysts following stocks with a small divergence of opinions. Herding among analysts can be due to their compensation scheme which might favor analysts mimicking their star colleagues. Amundsen and Bay (2011) study the mandatory notifications of stock trade and the corresponding stock returns in Oslo Stock Exchange made by investment experts who have a status of being financial celebrity. Amundsen and Bay find evidence of abnormal returns and herding for stocks traded by investment experts with high media profiles.

Chang, Chen and Khorana (2000) propose using the relation between the cross sectional absolute deviation of asset returns (CSAD) and the market return for detecting herding behavior. Using equally weighted portfolios they examine this relation in US, Hong Kong, Japan, South Korea and Taiwan. They find evidence of herding in South Korea and Taiwan. They find also that the rate of increase in asset return dispersion as a function of the market return is higher in states of bull markets, which is consistent with the directional asymmetry theory proposed by McQueen, Pinegar and Thorley (1996). Tan, Chiang, Mason and Nelling

(2008) examine herding behavior in Chinese stock markets for A - and B -shares. The A type of shares can be traded only by domestic investors while the B type can be traded also by foreigner investors. They detect herding using the cross sectional absolute deviation test devised by Chang, Cheng and Khorana and find evidence of herding for both share types. The herding behavior for A - shares is more prominent in states of bull markets, high trading volume and high volatility. Economou, Kostakis and Philippas (2011) test the existence of herding in the stock markets of Portugal, Italy, Spain and Greece using the CSAD test. They also test for correlational effects between these countries. Evidence of herding is found for the Greek and Italian stock markets for both equally weighted and value weighted portfolios. The evidence of herding in the Greek stock market is more prominent in Bull market states, that is in days with rising asset prices. For the Portuguese stock market the results vary depending on using an equally weighted or a value weighted portfolio. When an equally weighted portfolio is employed they find no evidence of herding while the opposite happens when they use a value weighted portfolio. Economou, Kostakis and Philippas find evidence of herding in the Spanish stock market in states of bear markets, that is on days with falling asset prices. They find also cross country correlations of CSAD between all stock markets. Kallinterakis and Lodetti (2009) explore the relation between herding and illiquidity in the Montenegro stock market using the CSAD-test and correcting for thin trading. Their tests include equally weighted portfolios and volume weighted portfolios before and after correcting for thin trading. They find evidence of herding only in states of bull markets after correcting for thin trading. Besides that they find a positive non-linear relation between the cross sectional absolute deviation of asset returns and market return. Correcting for thin trading reduces the magnitude of non-linearity. Al-Shboul (2012) finds evidence of herding in the Australian equity market using the CSAD-test and equally weighted portfolios in both bull and bear markets. Araghi, Mavi and Alidoost (2011) examine and find evidence of herding behavior in the Iranian stock market employing the CSAD methodology with equally weighted portfolios. Prosad, Kapoor and Sengupta (2012) test for presence of herding in the Indian equity market applying CSAD in equally weighted portfolios. They find evidence of herding behaviour in bull markets. Gebka and Wohar (2013) investigate the presence of herding in the global equity market across sectors. Their country sample consists of Argentina, Australia, Austria, Brazil, Canada, Chile, China, Colombia, Czechia, France, Greece, Hong Kong, India, Ireland, Israel, Italy, Japan, Korea, Luxembourg, Malaysia, Netherlands, New Zealand, Norway, Pakistan, Philippine, Singapore, South Africa, Spain, Thailand, Turkey, UK and US. The sectors examined are basic materials, consumer services, consumer goods, financials, Health, Industrial and oil and gas. They use an extension of the CSAD-test which takes into account cross country sector correlations. Gebka and Wohar find that disaggregating the national indices to economic sectors they find evidence of herding in basic materials, consumer services and oil and gas stocks. Asset prices for assets in the same sectors but in different countries become more dispersed though they have common fundamentals. Gebka and Wohar explain this with what they call localized herding which occurs as a body of investors moves between countries, creating a sector bubble on their target market. Lindhe (2012) investigates the existence of herd behavior in Sweden, Denmark, Norway and Finland. Using equally weighted asset portfolios she finds evidence of herding behavior in Finland but not in Norway, Sweden and Denmark. She finds no evidence of asymmetric effects in the state of high market returns compared to the state of low market returns.

CCAPM

Kahneman and Tversky (1979) investigated if preferences vary when the same payoff is presented in a different setting. Consider the following gamble:

To an initial endowment of 1 000 add a gamble of event A of 1 000 with probability 0,5 or event B of 500 with probability 0,5.

A gamble with the same event payoff is the following:

To an initial endowment of 2 000 add a gamble of event C of -1 000 with probability 0,5 or event D of -500 with probability 0,5.

Although A has the same payoff as C and B the same payoff as D, experiments show that people prefer B in the first gamble and C in the second gamble. This result is not compatible with the expected utility framework, and particularly the axiom of transitivity. This is because since B is as good as D and A is as good as C, given a preference for B over A one should also prefer D over C. But this is not the case in real life.

Kahneman and Tversky propose a utility function that is concave for gains and convex for losses. Instead of probabilities they use decision weights that are a function of the event probabilities:

$$V(p, \Delta w_1, \Delta w_2) = \pi(p)v(\Delta w_1) + \pi(1-p)v(\Delta w_2)$$

where V is the value function, $\pi(p)$ and $\pi(1-p)$ are the decision weights and $v(\Delta w_1)$, $v(\Delta w_2)$ are the values of wealth changes for two mutually exclusive events.

This type of utility function is the cornerstone of the Prospect theory.

Mehra and Prescott (1985) address the question whether the equity premium can be accounted for by general equilibrium models. Mehra and Prescott present earlier estimates of the magnitude of the relative risk aversion coefficient to be from close to 0 to 2. Mehra and Prescott (1985 p. 155) place an upper bound of 10 to the value of relative risk aversion with the argument that their results are robust below this value for a variety of consumption processes. They estimate the equity premium for US using the Standard and Poor's 500 composite stock index in the period 1889 to 1978 to be 6,18 %. Using a variation of Lucas' consumption capital asset pricing model they predict that the equity premium shouldn't be higher than 0,35 %. The equity premium puzzle is the difference between the observed empirically equity premium and the equity premium forecasted by their model.

Empirical tests show only a modest success of habit models in explaining the equity premium puzzle (Ferson and Constantinides 1991, Tallarini and Zhang 2005).

Benartzi and Thaler (1995) parameterize loss aversion with the value of 2,77 which is not very far from values for loss aversion estimated in experimental settings. Kahneman and Tversky (1992) for instance have estimated the loss aversion to be 2,25 for a similar value function. In a classical asset theory setting investors should have coefficients of relative risk aversion (RRA) in excess of 30 to explain the historical equity premium whereas previous estimates of a plausible parameter value for RRA had been close to 1,0 (Mehra and Prescott 1985). Benartzi and Thaler find that their model produces results consistent with the empirical data for equity premium of 6,5% if the investors' evaluation period for which bonds have the same prospective utility as stocks is 12 months. Investors seem to evaluate their portfolios as if their investment horizon is just one year even if they are investment plan period is much longer. Benartzi and Thaler call this effect myopic loss aversion which is a combination of loss aversion and short term reevaluation horizon.

Christensen (2011) finds support for the existence of an equity premium puzzle in Denmark, Norway, Sweden, Belgium, France, Germany, Netherlands, Switzerland and the UK. He tests if myopic loss aversion can explain the equity premium. With myopic loss aversion is meant that the investors have a short investment horizon which increases the probability of experiencing a loss on equities (Benartzi & Thaler 1995). This leads to a high premium requirement. He finds that myopic loss aversion, explains the observed equity premium in Sweden, and to some extent in Norway and Denmark, but not in the other countries. The results are sensitive to the choice of the magnitude of the loss aversion parameter which is not estimated endogenously

Barberis, Huang and Santos (2001) put forward a general equilibrium model which explores dynamical aspects of prospect theory. More specifically, they try to capture the idea that the degree of loss aversion depends on prior gains and losses. In this setting, prior gains make investors less scared of stocks, increasing the demand for stocks, pushing asset prices higher relative to dividends, increasing the return volatility (Barberis and Thaler 2003, pp.1051-1121). This is called the House Money Effect due to its resemblance to the willingness of casino gamblers to risk money recently won. The degree of success of this model in explaining the volatility puzzle depends highly on the parameter values used. Ackert, Charupat, Church and Deaves (2006), conduct an experiment where market participants compete via a sealed-bid auction to acquire an asset under different endowment settings. They find that traders' bids and price predictions are influenced by the amount of money they are endowed with but not by increases in wealth.

Hansen and Singleton (1982) introduce a method for estimating and testing nonlinear rational expectation models using Euler equations as orthogonality conditions. The method is called the generalized method of moments. By this method they estimate the parameters of risk aversion and the subjective discount factor in Lucas consumption capital asset pricing for the period 1959 to 1978 in US. Their data is monthly. The estimated values of risk aversion range between -0,95 to -0,68. The estimated values of the subjective discount factor range between 0,9925 to 0,9981.

Dimson, Marsh and Staunton (2008) construct a database of stocks, bonds, bills, inflation and currency returns for 17 countries. They calculate a world index for the period 1900 to 2005. Using the US bond as the risk free rate they find that the world equity premium is 3-3,5 % on a geometric mean basis and 4,5 - 5 % on an arithmetic mean basis. The equity return for Norwegian stocks in this period is 7,08 % and the equity premium relative to bills 5,7% expressed as arithmetic means. The index returns from 1900 to 1969 are derived using the statistics yearbook and the Oslo Stock Exchange indices since 1970. In the period 1970 to 1982 the industrial index, in the period 1983 to 1995 the general index and in the period 1996 to 2005 the all share index. The risk free rate is Norway's central bank discount rate for the period 1900 to1971, the money market rates for the period 1972 to 1983 and the Norwegian T – bills rates for the period 1984 to 2005.

Ødegaard (2012) calculates the monthly arithmetic equity premium for Norwegian stocks in the period 1980 to 2011 using four different indices to be 1,05 % for an equally weighted index, 1,35 % for a value weighted index, 0,38 % for an index consisting of the most liquid assets and 0,62% for the total index (all share index). The Norwegian monthly intra bank offered rate (NIBOR) is used as the risk free rate. Is worth noting that monthly intra bank offered rates are more volatile than t - bills rates. The annualized equity premium is 18,14 % for an equally weighted index, 22,17 % for a value weighted index, 5,84 % for an index consisting of the most liquid assets and 10,08% for the total index (all share index). Donadelli and Prosperi (2011 pp. 7 - 9) estimate the Norwegian monthly equity premium to be 0,82 % for the period 1969 - 2010, 1,18 % for the period 2000 to 2010 and 0,89% for the period 1988 to 2010 using the one month t - bills rates. The annual equity premium is calculated to 10,63% for the period between 1988 to 2010.The standard deviation range for developed countries is from 0,0432 in US to 0,074 for the Norwegian market. Båtvik (2008) calculates the arithmetic equity premium to be 5,9 %. for the period 1900 to 2008 and 11,36 % for the period 1970 – 2008. He finds that the implied parameter on relative risk aversion predicated from the basic Hansen - Jagannathan bound for the period 1900 to 2005 to be 6,07. In the basic Hansen Jagannathan bound it is assumed that the correlation between consumption growth and stock returns is 1. He then concludes that there is not an equity premium puzzle in Norway since Mehra and Prescott (1985) ascertain that the theoretical upper bound of the relative risk aversion coefficient can be as high as 10. The data set used in Båtvik's study is the same one used in the study of Dimson, Marsh and Staunton (2008). Mellingen and Kleiven (2012) using the realized average model and the dividend growth model calculate the arithmetic equity premium to be 8, 24 % and 7,1 % respectively in the period 1970 to 2011. As risk free rate they use NIBOR rates.

6 Our empirical tests based on stock returns in Oslo Stock Exchange

6.1 Characteristics of OSE

In order to develop our understanding of Oslo Stock Exchange we conducted 5 interviews of people working in OSE and broker houses situated in Oslo. This took place in summer 2012. The interviews were based on open ended questions regarding factors influencing stock prices in OSE, twin shares (see appendix A - xv) and arbitrage opportunities, the market microstructure, the market sentiment and the role of analysts. The description of OSE below is based mainly on these interviews.

6.1.1 Factors influencing the stock prices in OSE

The most important factor influencing stock prices in Oslo Stock Exchange (OSE) is oil price. OSE is more volatile than the big stock exchanges because is little, is based on cyclical shares and is dominated by foreign investors. At bad times OSE experiences massive exits. OSE is also less liquid.

6.1.2 Twin shares and arbitrage opportunities

Brent oil correlates with West Texas Intermediate (WTI) oil which is a crude oil used as a benchmark. If the prices of Brent oil and WTI deviate too much from each other then there is an arbitrage opportunity. In general, stocks which are tightly correlated, for instance in the drilling sector, can be used to test for arbitrage opportunities. A twin share example is Seadrill which is sold both in Norway and USA.

6.1.3 Market microstructure

Market makers

There are two types of market makers in OSE.

i) A market maker obligated to carry out market making in the shares/primary capital certificates for at least 85% of the opening hours of Oslo Stock Exchange with binding bid and offer prices applying to a minimum value of NOK 40 000 on both the offer and the bid side and the largest spread not representing more than 4% (Fondsfinans).

ii) The free market maker with no such obligations.

Institutional investors

OSE is characterized by big institutional investors like DnB and Storebrand which sell or buy stocks through broker houses. Broker houses become market makers by making contracts with a company. The companies which make such agreements get promoted from OB standard to OB match companies, i.e. companies of better liquidity.

The order book

The order book is public information about supply and demand. The quality of order book is proportional to liquidity and inverse proportional to spread. Seldom does supply and demand match each other. Usually is one side in the order book greater than the other. There is the visible order book and the non-visible order book. The visible order book doesn't contain complete information of the volumes asked or demanded before orders are executed. This is because market makers are not supposed to have an impact on market prices through volumes. However, market makers can use the non-visible order information to their own advantage. The European Commission's documentation (2003) on legislation in financial transactions (2003) in EEA countries shows that in some countries is exploitation of information in order books by brokers, also called "Front Running", prohibited explicitly, unless this information is publicly known. In Norway is order book information public information but the supply and demand clients are not.

Inside trading

OSE follows with equity market volume to uncover inside trading. Uncovered short selling and inside trading on unpublished information are not allowed. However, foreigners who are equity holders and live abroad can trade on inside information and conceal their identity using nominees. Nominees are representatives into whose name transactions are registered.

The financial newspaper's ("Finansavisen") inside portfolio can indicate that insiders have better feeling with information, which although is public, can be subject to different interpretations. Transaction costs don't eliminate the abnormal returns of the inside portfolio (Tønnesen 2010 p. 34). However, the stocks in this portfolio are not as easy to trade. Lack of liquidity makes it more difficult to exploit this anomaly.

Robot trading

Robots belong to members of OSE. Robots are playing the role of market makers and stand for a big part of trading volume. An example is smart order routers which are sending orders to several different exchanges trading Norwegian stocks (NASDAQ among others) and try to get the best deal for the investor. Natural flow daily traders are overtaken by robot trading which are much faster in trading and exploiting short term trends. However, in the Timber Hill robot case in OSE, two daily traders managed to exploit repetitive robot algorithms for making a profit.

There are market maker robots and statistical arbitrage robots. Market maker robots are executing orders from the order book. Statistical arbitrage robots are trying to test the market vulnerability by creating and exploiting short term trends. This is done by placing big volume orders which are withdrawn before they are executed. OSE is trying to mitigate this behaviour by imposing a fee whenever the number of non-executed orders transcends the executed orders by a certain per cent. OSE has also introduced circuit breakers in order to reduce excess intraday volatility and avoid flash crashes.

Opening and closing prices

Opening prices reflect information since the evening before, so their volatility is bigger. Closing prices are influenced by the closing of NYSE. There is an opening price auction and a closing price auction. These auctions are similar. OSE has reduced the opening time by 1 hour the 7th august and this is expected to have an effect at the opening prices. The opening time since 7th august is similar with the pre 2008-era.

6.1.4 Market sentiment

Market sentiment influences stock prices. The VIX index in Chicago is an index of market sentiment. Index-options at the money, close to the exercise date, is a better indicator of market sentiment than an individual stock. The analysts' estimates can also be an indicator of market sentiment notwithstanding that analysts are biased towards buy suggestions. This bias is due to broker houses getting easier stock issuing orders from companies. Another indicator of sentiment is P/E ratios. They are higher at good times and lower at bad times even if the fundamentals of a company haven't changed. This is called multiples contraction. Long term estimates (3 or 6 months ahead) are exposed to changes in risk premium over time, so that the analysts' estimates don't come true. Underreaction is likely for liquid shares. Overreaction is possible for shares that are not liquid since trading occurs less frequently (Chopra, Lakonishok and Ritter 1992).

6.1.5 The role of analysts

A type of analysis frequently used is called consensus analysis. Investors in OSE are interested in consensus estimates, i.e. the mean or median of estimates. They are also attentive to the highest and lowest estimates so much so that they contact the respective analysts for background information. Investors focus mainly on operating cash flow (OCF), net cash flow (NCF), earnings before interest, tax, depreciation and amortization (EBITDA), earnings per share (EPS) and enterprise value (EV). Models of multiples based on ratios like price/earnings (P/E),, earnings before interest, tax, depreciation and amortization/enterprise value (EBITDA/EV) and book to market (B/M) are frequently used to come up with a fair value. These models are calibrated for the business sector a company operates in. An analyst compares these figures with the prevailing yardstick and investigates potential underlying factors to significant deviations from the norm for the line of business at question. For the real estate or the shipping sector are market prices most relevant for estimating a company's assets. For companies producing commodities like aluminum, fertilizers are costs of capacity replacement a relevant factor. The same applies for sea-farming.

The submission of accounting reports leads to examination of reported results against their corresponding estimates. Deviations between these two sets produce changes in stock prices at the announcement day. Analysts are solicitous over the influence of accounting choices on the reporting figures. Accounting practices can have a significant impact on the accounting statements. Managers are known to exert efforts to sway reported figures towards a desired direction (Healy 1985). Bonus contracts can provide a motive to this behavior. The analysts' job is to eliminate these effects by making suitable adjustments.

6.2 Testing sentiment risk

6.2.1 Hypotheses

With the theory of heterogeneity based on the models by Xiouros (2009) and Iori (2002) as a starting point we formulated a set of hypotheses. Because of non-stationarity issues which are discussed in another section in our thesis, we had to use the differencing of the dispersion of beliefs. This can fit nicely in our tests since it is related in this setting to the sentiment risk. We tested the following hypotheses for data on Oslo Stock Exchange:

H - 1: The absolute values of stock returns are proportional to the sentiment risk expressed as the absolute value of differencing of analysts' stock price targets.

H - 2: The absolute values of stock returns are proportional to the sentiment risk expressed as the absolute value of differencing of analysts' EBITDA estimates.

H - 3: The absolute values of stock returns are proportional to the changes of sentiment risk expressed as the absolute value of the second differencing of analysts' EPS estimates.

H - 4: Stock returns' volatility is proportional to the sentiment risk expressed as the absolute value of differencing of analysts' stock price targets.

H - 5: Trade volume of stock is proportional to the sentiment risk expressed as the absolute value of differencing of analysts' stock price targets.

H - 6: Trade volume of stock is proportional to stock returns' volatility.

Model specification

Using data from the financial databases Datastream and Factset (see appendix A - xvii) we run in EViews least square regressions of the following equations:

H - 1

The hypothesis was tested in two ways, using data for market value MV from the financial database Datastream and data for enterprise value EV from the financial database Factset:

$$\left|\sum_{i=0}^{N}\left[\left(\frac{MV_{i,t}}{\sum_{j=0}^{N} MV_{j,t}}\right)(r_{i,t})\right]\right| = \alpha + \mathbf{1}\gamma \left| d^{1} \sum_{i=0}^{N}\left[\left(\frac{MV_{i,t}}{\sum_{j=0}^{N} MV_{j,t}}\right)\sigma(P_{\tau,i,t})\right]\right| + \hat{\varepsilon}_{t,t-1}$$

M - 1

$$\left|\sum_{i=0}^{N}\left[\left(\frac{EV_{i,t}}{\sum_{j=0}^{N} EV_{j,t}}\right)(r_{i,t})\right]\right| = \alpha + \mathbf{1}\gamma \left| d^{1} \sum_{i=0}^{N}\left[\left(\frac{EV_{i,t}}{\sum_{j=0}^{N} EV_{j,t}}\right)\sigma(P_{\tau,i,t})\right]\right| + \hat{\varepsilon}_{t,t-1}$$

M - 2

where d^1 denotes the first differencing.

H - 2

$$\left|\sum_{i=0}^{N}\left[\left(\frac{EV_{i,t}}{\sum_{j=0}^{N} EV_{j,t}}\right)(r_{i,t})\right]\right| = \alpha + \mathbf{1}\gamma \left| d^{1} \sum_{i=0}^{N}\left[\left(\frac{EV_{i,t}}{\sum_{j=0}^{N} EV_{j,t}}\right)\sigma(EBITDA_{\tau,i,t})\right]\right| + \hat{\varepsilon}_{t,t-1}$$

M - 3

H - 3

$$\left|\sum_{i=0}^{N}\left[\left(\frac{EV_{i,t}}{\sum_{j=0}^{N} EV_{j,t}}\right)(r_{i,t})\right]\right| = \alpha + \mathbf{1}\gamma \left| d^{2} \sum_{i=0}^{N}\left[\left(\frac{EV_{i,t}}{\sum_{j=0}^{N} EV_{j,t}}\right)\sigma(EPS_{\tau,i,t})\right]\right| + \hat{\varepsilon}_{t,t-1,t-2}$$

M - 4

where d^2 denotes the second differencing.

H - 4

The hypothesis was tested using $\sqrt{\left(r_{i,t}-r_{i,t-1}\right)^2}$ as measure of volatility. We tested the hypothesis in two ways, using data for market value MV from the financial database Datastream and data for enterprise value EV from the financial database Factset:

$$\sum_{i=0}^{N}\left[\left(\frac{MV_{i,t}}{\sum_{j=0}^{N} MV_{j,t}}\right)\sqrt{\left(r_{i,t}-r_{i,t-1}\right)^2}\,\right] = \alpha + \mathbf{1}\gamma\left|d^1\sum_{i=0}^{N}\left[\left(\frac{MV_{i,t}}{\sum_{j=0}^{N} MV_{j,t}}\right)\sigma\left(P_{\tau,i,t}\right)\right]\right| + \hat{\varepsilon}_{t,t-1} \tag{M - 5}$$

$$\sum_{i=0}^{N}\left[\left(\frac{EV_{i,t}}{\sum_{j=0}^{N} EV_{j,t}}\right)\sqrt{\left(r_{i,t}-r_{i,t-1}\right)^2}\,\right] = \alpha + \mathbf{1}\gamma\left|d^1\sum_{i=0}^{N}\left[\left(\frac{EV_{i,t}}{\sum_{j=0}^{N} EV_{j,t}}\right)\sigma\left(P_{\tau,i,t}\right)\right]\right| + \hat{\varepsilon}_{t,t-1} \tag{M - 6}$$

H - 5

$$\sum_{i=0}^{N}\left[\left(\frac{MV_{i,t}}{\sum_{j=0}^{N} MV_{j,t}}\right)\left(k_{i,t}\right)\right] = \alpha + \mathbf{1}\gamma\left|d^1\sum_{i=0}^{N}\left[\left(\frac{MV_{i,t}}{\sum_{j=0}^{N} MV_{j,t}}\right)\sigma\left(P_{\tau,i,t}\right)\right]\right| + \hat{\varepsilon}_{t,t-1} \tag{M - 7}$$

H - 6

The hypothesis was tested using $\sqrt{\left(r_{i,t}-r_{i,t-1}\right)^2}$ as measure of volatility:

$$\sum_{i=0}^{N}\left[\left(\frac{MV_{i,t}}{\sum_{j=0}^{N} MV_{j,t}}\right)\left(k_{i,t}\right)\right] = \alpha + \mathbf{1}\gamma\sum_{i=0}^{N}\left[\left(\frac{MV_{i,t}}{\sum_{j=0}^{N} MV_{j,t}}\right)\sqrt{\left(r_{i,t}-r_{i,t-1}\right)^2}\,\right] + \hat{\varepsilon}_{t} \tag{M - 8}$$

The symbols in the above equations are as follows:

1 stands for an indicator function which takes the value of 0 if an observation is not available and 1 otherwise.

$\hat{\varepsilon}_t$ stands for the residual, where $\varepsilon_t \sim N(0,\sigma_t^2)$

$MV_{i,t}$ stands for market value of stock i at time t

$r_{i,t}$ stands for netto return of stock i at time t

$\sigma\left(P_{\tau,i,t}\right)$ stands for standard deviation of analysts' price target τ of stock i at time t

$k_{i,t}$ stands for the trade volume measured in thousands of stocks for company i at time t.

$EV_{i,t}$ stands for enterprise value of stock i at time t.
$EBITDA$ stands for earnings before interests and taxes, depreciation and amortization and $\sigma(EBITDA_{\tau,i,t})$ stands for standard deviation of analysts' $EBITDA$ estimates of stock i at time t.

EPS stands for earnings per stock and $\sigma(EPS_{\tau,i,t})$ stands for standard deviation of analysts' EPS estimates τ of stock i at time t.

6.2.2 Methodology

Data sources

Our data sources were the financial databases Factset and Datastream. Factset includes a greater number of analysts' estimates than Datastream. Using two databases contributed in improving the robustness of our test results. Our data consists of daily, weekly and monthly observations for 163 Norwegian companies for the period 1.1.2007-12.7.2012 with some slight variations depending on the data source and the choice of frequency.

Choice of method

For testing the hypotheses H - 1 to H - 6 we considered the following alternatives:
i) Run time series regressions for each company in our data set. That would have created 163 time series regressions.
ii) Run cross sectional regressions for each point of time. That would have created roughly 1400 regressions.
iii) Doing first cross sectional weighted averages across the companies for each point of time we created a time series that resulted in one regression. Compared to alternatives (i) and (ii) this is a solution that is much less tedious and summarizes the results in a parsimonious way. The cross sectional time series means that we are testing if the hypotheses hold on average over a time period rather than holding for each individual company or each point of time. We used LS (least squares regression) in the econometric program EViews.

Estimation procedure

The regressions are run as follows:

For data which source is the financial database Datastream we set up a matrix consisting of data series for individual companies for the same variables and the same period of time. The variables used were returns, volatility of returns, volume and the standard deviation of analysts' price targets. In that way we created a data pool in form of a 1391×163 matrix as a starting point, where rows show daily observations and columns show companies. For weekly observations we get a 289×163 matrix.

For data which source is the financial database Factset we set up a matrix consisting of data series for individual companies for the same variables and the same period of time. The variables used were returns, EBITDA, EPS and the standard deviation of analysts' price targets. In that way we created a data pool in form of a 1365×270 matrix as a starting point, where rows show daily observations and columns show companies. For weekly observations we got a 290×270 matrix. The data sets from "Datastream" are for the period 01/01/2007-12/07/2012. The datasets from "Factset" are for the time period 29/12/2006-19/07/2012.

The daily observations in each row were summed up to create 1391×1 and 1365×1 vectors for data from Datastream and Factset correspondingly. In the same way the weekly observations in each row were summed up to create 289×1 and 290×1 vectors for data from Datastream and Factset respectively. For the variables of returns, volume, and analysts' price target and analysts' estimates we used the cross sectional averages weighted with their market value for data from Datastream and with their enterprise value for data from Factset. The data was also transformed to weekly observations because practically speaking we don't expect analysts to daily evaluate price targets, estimates for EBITDA or estimates for EPS. Then we run the regressions using least squares (LS) in EViews. As a measure for volatility we used the standard deviation of two subsequent observations of returns. The observations were weighted by the stocks' market value. The dimensions of the matrices and vectors varied depending on the test at hand.

6.2.3 Empirical test results

All variables used in the regression equations below are tested for unit roots and are stationary. The regression results were corrected both for heteroscedasticity and autocorrelation using the Newey–West estimators and were as follows (the numbers in parentheses show t-values and the numbers in brackets p-values for a two sided test, EViews 7 Users Guide II, pp. 12-13):

H - 1

Table 6-1: Regression results market value weighted stock returns and sentiment risk based on price targets

$$\left|\sum_{i=0}^{N}\left[\left(\frac{MV_{i,t}}{\sum_{j=0}^{N} MV_{j,t}}\right)(r_{i,t})\right]\right| = \alpha + \mathbf{1}\gamma \left|d^1 \sum_{i=0}^{N}\left[\left(\frac{MV_{i,t}}{\sum_{j=0}^{N} MV_{j,t}}\right)\sigma(P_{\tau,i,t})\right]\right| + \varepsilon_{t,t-1}$$

α	γ	Adjusted R^2	Period	N
0,012906 (22,16399) [0,0000]	0,005204 (2,258280) [0,0241]	0,011005	Daily data 02/01/2007- 12/07/2012	1391
0,005230 (11,95498) [0,0000]	0,008499 (1,876033) [0,0617]	0,023212	Weekly data 01/01/2007- 09/07/2012	289
0,002259 (8,915494) [0,0000]	0,007322 (1,363931) [0,1773]	0,015215	Monthly data 2007M01- 2012M07	67

Data source: Datastream.

Table 6-2: Regression results enterprise value weighted stock returns and sentiment risk based on price targets

$$\left|\sum_{i=0}^{N}\left[\left(\frac{EV_{i,t}}{\sum_{j=0}^{N} EV_{j,t}}\right)(r_{i,t})\right]\right| = \alpha + \mathbf{1}\gamma \left|d^1 \sum_{i=0}^{N}\left[\left(\frac{EV_{i,t}}{\sum_{j=0}^{N} EV_{j,t}}\right)\sigma(P_{\tau,i,t})\right]\right| + \varepsilon_{t,t-1}$$

α	γ	Adjusted R^2	Period	N
0,012118 (22,41232) [0,0000]	0,001872 (1,881861) [0,0601]	0,015747	Daily data 03/01/2007- 17/07/2012	1365
0,005208 (12,09431) [0,0000]	0,002083 (1,423917) [0,1556]	0,014260	Weekly data 29/12/2006- 13/07/2012	290
0,002553 (7,875944) [0,0000]	0,005070 (1,449750) [0,1519]	0,065330	Monthly data 2007M01- 2012M07	67

Data source: Factset.

H - 2

Table 6-3: Regression results enterprise value weighted stock returns and sentiment risk based on EBITDA

$$\left\lvert\sum_{i=0}^{N}\left[\left(\frac{EV_{i,t}}{\sum_{j=0}^{N} EV_{j,t}}\right)(r_{i,t})\right]\right\rvert = \alpha + \mathbf{1}\gamma \left\lvert d^1 \sum_{i=0}^{N}\left[\left(\frac{EV_{i,t}}{\sum_{j=0}^{N} EV_{j,t}}\right)\sigma(EBITDA_{\tau,i,t})\right]\right\rvert + \varepsilon_{t,t-1}$$				
α	γ	Adjusted R^2	Period	N
0,012579 (21,12711) [0,0000]	9,21E-07 (0,993861 [0,3205]	-0,000393	Daily data 03/01/2007- 17/07/2012	1365
0,005461 (9,961501) [0,0000]	1,85E-06 (0,847227) [0,3976]	-0,002398	Weekly data 29/12/2006- 13/07/2012	290
0,002728 (5,713245) [0,0000]	7,65E-06 (1,219625) [0,2270]	0,001763	Monthly data 2007M01- 2012M07	67

Data source: Factset.

H - 3

Table 6-4: Regression results enterprise value weighted stock returns and changes in sentiment risk based on EPS

$$\left\lvert\sum_{i=0}^{N}\left[\left(\frac{EV_{i,t}}{\sum_{j=0}^{N} EV_{j,t}}\right)(r_{i,t})\right]\right\rvert = \alpha + \mathbf{1}\gamma \left\lvert d^2 \sum_{i=0}^{N}\left[\left(\frac{EV_{i,t}}{\sum_{j=0}^{N} EV_{j,t}}\right)\sigma(EPS_{\tau,i,t})\right]\right\rvert + \varepsilon_{t,t-1,t-2}$$				
α	γ	Adjusted R^2	Period	N
0,012470 (20,15322) [0,0000]	1,97E-06 (2,406842) [0,0162]	0,000833	Daily data 04/01/2007- 17/07/2012	1335
0,005259 (10,56345) [0,0000]	2,99E-06 (1,440322) [0,1509]	-0,000439	Weekly data 29/12/2006- 13/07/2012	288
0,002721 (5,791382) [0,0000]	8,03E-06 (1,427766) [0,1581]	0,001963	Monthly data 2007M01- 2012M07	67

Data source: Factset.

H - 4

Table 6-5: Regression results market value weighted volatility of stock returns and sentiment risk based on price targets

$\sum_{i=0}^{N}\left[\left(\frac{MV_{i,t}}{\sum_{j=0}^{N} MV_{j,t}}\right)\sqrt{\left(r_{i,t}-r_{i,t-1}\right)^{2}}\right]=\alpha+\mathbf{1}\gamma\left\|d^{1}\sum_{i=0}^{N}\left[\left(\frac{MV_{i,t}}{\sum_{j=0}^{N} MV_{j,t}}\right)\sigma\left(P_{\tau,i,t}\right)\right]\right\|+\varepsilon_{t,t-1}$				
α	γ	Adjusted R^2	Period	N
0,026456 (29,02417) [0,0000]	0,006946 (3,068367) [0,0022]	0,011283	Daily data 03/01/2007- 12/07/2012	1442
0,025539 (19,05279) [0,0000]	0,026743 (2,288743) [0,0228]	0,049080	Weekly data 01/01/2007- 09/07/2012	289
0,022253 (12,05895) [0,0000]	0,116365 (2,106942) [0,0390]	0,228040	Monthly data 2007M01- 2012M07	67

Data source: Datastream

Table 6-6: Regression results enterprise value weighted volatility of stock returns and sentiment risk based on price targets

$\sum_{i=0}^{N}\left[\left(\frac{EV_{i,t}}{\sum_{j=0}^{N} EV_{j,t}}\right)\sqrt{\left(r_{i,t}-r_{i,t-1}\right)^{2}}\right]=\alpha+\mathbf{1}\gamma\left\|d^{1}\sum_{i=0}^{N}\left[\left(\frac{EV_{i,t}}{\sum_{j=0}^{N} EV_{j,t}}\right)\sigma\left(P_{\tau,i,t}\right)\right]\right\|+\varepsilon_{t,t-1}$				
α	γ	Adjusted R^2	Period	N
0,027126 (32,10215) [0,0000]	0,004008 (2,664111) [0,0078]	0,044463	Daily data 03/01/2007- 17/07/2012	1365
0,026249 (19,96628) [0,0000]	0,010931 (2,098367) [0,0367]	0,093944	Weekly data 29/12/2006- 13/07/2012	290
0,025520 (14,65532) [0,0000]	0,030707 (1,526325) [0,1318]	0,108755	Monthly data 2007M01- 2012M07	67

Data source: Factset

H - 5

Table 6-7: Regression results market value weighted trading volume of stocks and sentiment risk based on price targets

$$\sum_{i=0}^{N}\left[\left(\frac{MV_{i,t}}{\sum_{j=0}^{N} MV_{j,t}}\right)(k_{i,t})\right] = \alpha + \mathbf{1}\gamma\left|d^{1}\sum_{i=0}^{N}\left[\left(\frac{MV_{i,t}}{\sum_{j=0}^{N} MV_{j,t}}\right)\sigma(P_{\tau,i,t})\right]\right| + \varepsilon_{t,t-1}$$

α	γ	Adjusted R^2	Period	N
6153,135 (34,13104) [0,0000]	1292,972 (4,788616) [0,0000]	0,014919	Daily data 02/01/2007- 12/07/2012	1391
6032,934 (18,63726) [0,0000]	3211,286 (2,707216) [0,0072]	0,018811	Weekly data 01/01/2007- 09/07/2012	289
5749,539 (9,978904) [0,0000]	12064,07 (2,334196) [0,0227]	0,054848	Monthly data 2007M01- 2012M07	67

Data source: Datastream

H - 6

Table 6-8: Regression results market value weighted trading volume of stocks and volatility of stock returns

$$\sum_{i=0}^{N}\left[\left(\frac{MV_{i,t}}{\sum_{j=0}^{N} MV_{j,t}}\right)(k_{i,t})\right] = \alpha + \mathbf{1}\gamma\sum_{i=0}^{N}\left[\left(\frac{MV_{i,t}}{\sum_{j=0}^{N} MV_{j,t}}\right)\sqrt{(r_{i,t} - r_{i,t-1})^{2}}\right] + \varepsilon_{t}$$

α	γ	Adjusted R^2	Period	N
5205,927 (19,32685) [0,0000]	38339,57 (6,610332) [0,0000]	0,052421	Daily data 03/01/2007- 12/07/2012.	1390
4644,540 (8,708447) [0,0000]	57977,02 (5,100360) [0,0000]	0,095739	Weekly data. 01/01/2007- 09/07/2012.	289
4732,927 (5,081665) [0,0000]	55770,15 (2,858593) [0,0057]	0,069390	Monthly data. 2007M01- 2012M07	67

Data source: Datastream.

H - 1: We find support for the hypothesis that the absolute values of stock returns are proportional to the sentiment risk expressed as the absolute value of differencing of analysts' stock price targets. The confidence intervals are 90 % (Factset) to 95 % (Datastream) for daily observations and 90 % (Datastream) for weekly observations. Adjusted R^2 for the models with significant results show that sentiment risk explains roughly 1,1 % (daily) to 2,3 % (weekly) of the variations of the absolute values of stock returns.

H - 2: We don't find support for the hypothesis that the absolute values of stock returns are proportional to the sentiment risk expressed as the absolute value of differencing of analysts' EBITDA estimates. Negative adjusted R^2 denotes poor fitting of this model with the daily and weekly data.

H - 3: We find support in daily data for the hypothesis that the absolute values of stock returns are proportional to the changes in sentiment risk expressed as the absolute value of the second differencing of analysts' EPS estimates. The confidence interval is 95 % for daily observations. Adjusted R^2 for the model with significant results show that sentiment risk explains roughly 0,1 % of the variations of the absolute values of stock returns. The model fits poorly with weekly data since the adjusted R^2 is negative.

H - 4: We find support for the hypothesis that stock returns' volatility is proportional to the sentiment risk expressed as the absolute value of differencing of analysts' stock price targets except for the case of monthly observations with Factset data . The confidence intervals are 99 % (Factset and Datastream) for daily observations, 95 % (Factset and Datastream) for weekly observations and 95 % (Datastream) for monthly observations. Adjusted R^2 for the models with significant results show that the sentiment risk explains roughly 1,1 % (daily) to 23 % (monthly) of the variations of stock returns volatility.

H - 5: We find support for the hypothesis that the trading volume of stocks is proportional to the sentiment risk expressed as the absolute value of differencing of analysts' stock price targets. The confidence intervals are 99 % for daily and weekly observations, and 95% for monthly observations. Adjusted R^2 for the models with significant results show that sentiment risk explains roughly 1,5 % (daily) to 6,9 % (monthly) of the variations of the absolute values of stock returns.

H - 6: We find support for the hypothesis that the trading volume of stocks is proportional to stock returns' volatility. The confidence intervals are 99 % for daily, weekly and monthly observations. Adjusted R^2 show that volatility of stock returns explains roughly 5,2 % (daily) to 6,9 % (monthly) of the variations of the trading volume.

The relations between stock returns, volatility, trading volume and sentiment risk are summarized in the table underneath.

Table 6-9: The relations between stock returns, volatility, trading volume and sentiment risk

	Absolute value of stock returns	Volatility of Stock returns	Volume of trading	Absolute value of sentiment risk
Absolute value of stock returns	proportional	not tested	not tested	proportional
Volatility of Stock returns	not tested	proportional	proportional	proportional
Volume of trading	not tested	proportional	proportional	proportional
Absolute value of sentiment risk	proportional	proportional	proportional	proportional

6.2.4 Discussion of the results

Regarding hypotheses H - 1, H - 2 and H - 3 we see that the adjusted R-square values are low. These hypotheses concern the relation between asset returns and sentiment risk based on analysts' targets for asset prices, EBITDA and EPS. The low R-square values indicate that sentiment risk accounts little for variations in asset returns. We observe that the regressions with monthly data provide the highest R^2 scores for hypotheses H - 4 and H - 5 regarding the relation between volatility, trading volume and sentiment risk. It seems reasonable that analysts don't update their target estimates on a day to day or week to week basis but rather on a month to month basis. Sentiment risk seems to be much more successful in explaining variations in trading volume and asset volatility.

Despite sentiment risk explaining too little of the stock returns stock it does have a significant statistical relation to it when the sentiment risk based on analysts' price targets or analysts' EPS targets. We didn't find any relation between the stock returns and sentiment risk concerning EBITDA. It appears that sentiment risk related to price targets or EPS targets matters more to investors than sentiment risk related to EBITDA targets.

The international empirical evidence on the relation between volatility and volume is inconclusive. Chuang, Liu and Susmel (2011) find a positive contemporaneous relation between trading volume and return volatility in Hong Kong, Korea, Singapore, China, Indonesia, and Thailand, but a negative one in Japan and Taiwan. Collado, Galiay and Ureche-Rangau, (2011) conclude that the sign of the relationship cannot be clearly set after investigating the relation between stock market trading volume and volatility in 23 developed and 15 emerging markets. Bredi, Hyde and Muckley (2013) find that the contemporaneous coefficient between volume and volatility is negative in the carbon finance market which they interpret as liquidity traders dominating informed traders. They employ a VAR model which takes into account the elapsed time between trades. Our test results indicate a positive contemporaneous relation between trading volume and volatility in the Norwegian stock market. An implicit assumption in our test is that the trade is equally spaced.

6.2.5 Contribution to research

Our contribution to research is testing and finding significant relations between:

i) Sentiment risk and market volume

ii) Sentiment risk and stock market volatility

Sentiment risk depends on the dispersion of analyst's price targets and earnings targets. Our literature research indicates that similar tests have not been run for the Norwegian stock market previously.

6.2.6 Conclusion

The sentiment risk battery of tests has investigated the relation between sentiment risk, asset returns, trading volume and volatility of asset returns.

The examination of these relations was undertaken in order to determine the role of sentiment risk as explanatory variable of important dimensions of the Norwegian stock market.

Sentiment risk in our setting is a function of the dispersion of analysts' beliefs which is a measure of heterogeneity at individual level.

Using data from two different financial databases (Datastream and Factset), we find that sentiment risk measured by the absolute value of differencing analysts' dispersion of beliefs, is an important factor in examining and explain the variations in the volatility of stock returns and the trading volume. Our tests are run in a heterogeneous perspective with dispersion of beliefs as the starting point.

The relations between stock returns, volatility, trading volume and sentiment risk are in the majority of our tests proportional and render support to the predictions of the model of heterogeneous investors put forward by Xiouros (2009) and Iori (2002).

Our study was contained to using only one sentiment risk measure.

An extension of the research presented here is using other sentiment risk indices like the consumer confidence indicator (CCI), the economic sentiment indicator (ESI) and the Put Call parity.

6.2.7 Testing the regression assumptions

Here we follow the presentation by Berry (1993, p. 12)

Regression assumption 1: Quantitativeness

All independent variables (x_1, x_2,…,x_i) are quantitative and the dependent variable, Y, is quantitative, and continuous and measured without error.

Our variables are quantitative and continuous and our data come from reliable sources.

The descriptive statistics for the variables used in our tests of difference of opinions were as following:

Table 6-10: Descriptive statistics for Datastream variables

	R_MVW_A	SDR_MVW_A	DIFF_PTSD_MVW_A	VOLUME_MVW_A
Mean	0,000396	0,027035	-0,001165	6264,432184
Median	0,001163	0,022383	-0,000148	5796,621326
Maximum	0,105620	0,178043	3,684254	29523,420484
Minimum	-0,092178	0,000000	-4,126600	682,498398
Std. Dev.	0,018886	0,017359	0,285769	2873,520474
Skewness	-0,234330	2,844832	1,673945	1,599481
Kurtosis	7,190	15,975	114,790	9,001
Jarque-Bera	1030,222	12060,690	752053,783	2680,380
Probability	0,000000	0,000000	0,000000	0,000000
Sum	0,550495	38,984644	-1,681540	8713825,168
Sum Sq. Dev.	0,495775	0,434226	117,759056	11477396683
N	1391	1442	1443	1391

Table 6-11: Descriptive statistics for Factset variables

	R_EVW_A	SDR_EVW_A	DIFF_PTSD_EVW_A	DIFF_EBITDA_SD_EVW_A	S_DIFF_EPS_SD_EVW_A
Mean	0,000088	0,028291	-0,009120	1,512971	-2,143803
Median	0,001041	0,023439	0,002295	-1,907773	1,264122
Maximum	0,092755	0,159019	8,967589	5835,356	2067,948
Minimum	-0,088092	0,008883	-16,776480	-2067,957000	-5835,349000
Std. Dev.	0,017848	0,016373	0,915570	269,3762	270,7689
Skewness	-0,382937	2,759392	-7,502870	8,744599	-8,771658
Kurtosis	6,923029	14,22287	161,6347	193,1418	193,2769
Jarque-Bera	929,9782	8895,806	1444064	2073651	2031040
Probability	0,000000	0,000000	0,000000	0,000000	0,000000
Sum	0,122823	38,61726	-12,448130	2065,206	-2861,977000
Sum Sq. Dev.	0,444710	0,365662	1143,397	98976664	97803293
N	1397	1365	1365	1365	1335

where

R_MVW_A is a time series created using the market value weighted cross sectional stock returns.

SDR_MVW_A is a time series created using the market value weighted cross sectional stock returns' volatility $\sqrt{\left(r_{i,t} - r_{i,t-1}\right)^2}$.

DIFF_PTSD_MVW_A is a time series created using the market value weighted cross sectional differencing of analysts' stock price targets.

VOLUME_MVW_A is a time series created using the market value weighted cross sectional trade volume of stocks.

R_EVW_A is a time series created using the enterprise value weighted cross sectional stock returns.

SDR_EVW_A is a time series created using the enterprise value weighted cross sectional stock returns' volatility.

DIFF_PTSD_EVW_A is a time series created using the enterprise value weighted cross sectional differencing of analysts' stock price targets.

DIFF_EBITDA_SD_EVW_A is a time series created using the enterprise value weighted cross sectional differencing of analysts' EBITDA estimates.

S_DIFF_EPS_SD_EVW_A is a time series created using the enterprise value weighted cross sectional the second differencing of analysts' EPS estimates.

The Jarque-Bera test rejects the null hypothesis of normal distribution for all variables with 99% confidence interval. Kurtosis is a descriptive statistic for fat tails which shows the probability for extreme events (in finance called "black swans"). When kurtosis is greater than 3 the variable does not follow a normal distribution. From the above tables we see that none of the variables used in our regression equations are normally distributed. It is a stylized fact that many financial time series do not follow a normal distribution (Cont 2001, Andersen, Davis, Kreiss and Mikosch 2009 p. 120).

Regression assumption 2 Variance

All variables have some variance.

The descriptive statistics show that our variables have nonzero variance.

Regression assumption 3: Multicollinearity

This assumption says that there is not an exact linear relationship between two or more of the independent variables, i.e. there is not perfect multicollinearity.

Multicollinearity is not relevant for the PTSD-tests because in each model tested there is only one independent variable.

Regression assumption 4: Mean of the error term

At each set of values for the i independent variables, $x_i, x_{i-1}, \dots, x_1$, $E(\varepsilon_i | \varepsilon_{i-1}, \dots, \varepsilon_1, x_i, x_{i-1}, \dots, x_1) = 0$ (i.e. the conditional expected mean value of the error term is zero).

At this point we assume that the conditional and the unconditional expected mean of the error term are equal. The conditional and the unconditional expectations are equal when the error term is independent from the regressors by the law of iterated expectations (Bailey 2005 p. 59). This is tested under assumptions 5 and 6.

We test the null hypothesis that the unconditional expected mean of the residual is 0 by means of the Jarque-Bera test in EViews.

Table 6-12: Jarque-Bera test of the expected mean of the residual

Test Equation	Jarque-Bera [Probability] Daily	Jarque-Bera [Probability] Weekly	Jarque-Bera [Probability] Monthly
M - 1	3,06E-14 [1,0000] N=1391	-7,70E-15 [1,0000] N=289	4,24E-15 [1,0000] N=67
M - 2	-8,72E-15 [1,0000] N=1365	9,04E-15 [1,0000] N=290	1,74E-15 [1,0000] N=67
M - 3	-1,18E-14 [1,0000] N=1365	8,07E-15 [1,0000] N=290	1,20E-15 [1,0000] N=67
M - 4	-1,69E-14 [1,0000] N=1335	6,38E-15 [1,0000] N=290	-1,22E-15 [1,0000] N=67
M - 5	1,59E-13 [1,0000] N=1442	1,32E-14 [1,0000] N=289	8,83E-15 [1,0000] N=67
M - 6	5,36E-14 [1,0000] N=1365	1,16E-14 [1,0000] N=290	2,22E-15 [1,0000] N=67
M - 7	-9,58E-14 [1,0000] N=1391	-3,43E-14 [1,0000] N=289	9,57E-15 [1,0000] N=67
M - 8	-1,48E-13 [1,0000] N=1390	-3,82E-14 [1,0000] N=289	4,62E-15 [1,0000] N=67

The Jarque-Bera tests show that we can't reject the null hypothesis that the unconditional expected mean of the residual is 0.

Regression assumption 5: Correlation of regressors with the error term

For each x_i, $cov(x_i, \varepsilon_i) = 0$ (i.e., each independent variable is uncorrelated with the error term).

See under assumption 6.

Regression assumption 6: Variance of the error term

At each set of values for the i independent variables, $x_i, x_{i-1}, \dots, x_1$, $var(\varepsilon_i|\varepsilon_{i-1}, \dots, \varepsilon_1, x_i, x_{i-1}, \dots, x_1) = \sigma^2$ where σ^2 is a constant (i.e., the conditional variance of the error term is constant); this is known as the assumption of homoscedasticity.

To test assumptions 5 and 6 we used the White-test statistic which is a test of the null hypothesisof no correlation of the explanatory variables with the residual and no-heteroskedasticity (EViews 7 2009 User Guide II, pp. 163-165).

Table 6-13: Homoscedasticity test

Test Equation	F-statistic	Obs*R-squared	Scaled explained SS	N
M - 1	10,34587 [0,0000]	20,43188 [0,0000]	110,5849 [0,0000]	1391
M - 2	43,72448 [0,0000]	82,35394 [0,0000]	392,3557 [0,0000]	1365
M - 3	0.331321 [0,7180]	0.663778 [0,7176]	3.360115 [0,1864]	1365
M - 4	0,205740 [0,8141]	0,412279 [0,8137]	2,106356 [0,3488]	1335
M - 5	13,66379 [0,0000]	26,87419 [0,0000]	199,7091 [0,0000]	1442
M - 6	43,84502 [0,0000]	82,56724 [0,0000]	526,5849 [0,0000]	1365
M - 7	0,036136 [0,9645]	0,072425 [0,9644]	0,301065 [0,8602]	1391
M - 8	0,044456 [0,9565]	0,089099 [0,9564]	0,382580 [0,8259]	1390

“Obs” in the table above stands for observations.

The test results show that the null hypothesis is rejected for models M - 1, M - 2, M - 5 and M - 6.

For the models which didn’t pass White’s test we run a test of the relation between the regressors and the residuals. The null hypothesis is that the covariance between the regressors and the residuals is zero.

Table 6-14: Covariance test of regressors with the residuals

Model	Regressor	Covariance regressor with residuals	t-statistics	probability	N
M - 1	@ABS(DIFF_PTSD_MVW_A)	-3,53E-18	-3,57E-14	1,0000	1391
M - 2	@ABS(DIFF_P_TGT_SD_EVW_A)	-5,92E-18	-2,01E-14	1,0000	1365
M - 5	@ABS(DIFF_PTSD_MVW_A)	-3,70E-18	-2,98E-14	1,0000	1442
M - 6	@ABS(DIFF_P_TGT_SD_EVW_A)	-1,99E-17	-5,29E-14	1,0000	1365

All the models in the table above pass this test.

The regression results were corrected using the Newey–West estimators. That means that the standard errors and as a consequence the t-values were adjusted to account for heteroscedasticity (EViews 7 2009 User Guide II, pp. 32-33).

Regression assumption 7: Autocorrelation.

For any two observations, $(x_{1j}\ x_{2j},\ldots,x_{ij})$ and $(x_{1h}, x_{2h},\ldots,x_{ih})$, $cov(\varepsilon_j, \varepsilon_h\ ) = 0$ (i.e., error terms for different observations are uncorrelated); this assumption is known as lack of autocorrelation.

The null hypothesis is that there is not serial correlation. This is tested by means of the Breusch-Godfrey serial correlation Lagrange multiplier (LM) test.

Table 6-15: Breusch-Godfrey serial correlation LM test

Test Equation	Lag	F-statistic Prob.	Obs*R-squared Prob. Chi-Square	N
M - 1	1	89,37005 [0,0000]	84,14529 [0,0000]	1391
M - 2	1	57,25459 [0,0000]	55,06589 [0,0000]	1365
M - 3	1	84,32814 [0,0000]	79,58631 [0,0000]	1365
M - 4	1	81,52033 [0,0000]	76,99191 [0,0000]	1335
M - 5	1	598,7237 [0,0000]	423,6883 [0,0000]	1442
M - 6	1	673,7529 [0,0000]	451,7605 [0,0000]	1365
M - 7	1	1421,635 [0,0000]	703,8260 [0,0000]	1391
M - 8	1	1273,525 [0,0000]	665,3572 [0,0000]	1390

“Obs” in the table above stands for observations.

The test results show that we reject the null hypothesis of no autocorrelation for all models.

The regression results were corrected using the Newey–West estimators. That means that the standard errors and as a consequence the t-values were adjusted to account for autocorrelation (EViews 7 2009 User Guide II, pp. 32-33).

Regression assumption 8: Distribution of the error term

At each set of values for the i independent variables, the error term ε_i is normally distributed.

The null hypothesis is that the standardized residuals are normally distributed. This is assessed by means of the Jarque-Bera normality test.

Table 6-16: Jarque-Bera normality test

Test Equation	Jarque-Bera [Probability]	N
M - 1	5880,095 [0,0000]	1391
M - 2	4395,477 [0,0000]	1365
M - 3	5077,112 [0,0000]	1365
M - 4	5065,054 [0,0000]	1335
M - 5	11921,96 [0,0000]	1442
M - 6	8127,621 [0,0000]	1365
M - 7	2960,300 [0,0000]	1391
M - 8	3243,009 [0,0000]	1390

The test results show no-normal distribution of the residuals for all models tested. A violation of this assumption is not as serious as heteroscedasticity and autocorrelation. A moderate departure from normality does not impair the conclusion when the data set is large (Bhattacharyya and Johnson 1977 p. 359). Greene (2012 pp. 64-67) states that a normal distribution of the error term is not necessary for establishing results that allow statistical inference. This is because statistical inference can be based on the law of large numbers which concerns consistency and the central limit theorem which concerns the asymptotic distribution of the estimator.

Regression assumption 9: Stationarity.

The independent variables are stationary processes.

The null Hypothesis is that the time series have a unit root. This is assessed by means of the augmented Dickey-Fuller unit root test.

Table 6-17: Augmented Dickey-Fuller unit root test

Time Series	N	t-Statistic	Prob.	Data source
PTSD_MVW_A	1443	-1,042702	0,7397	Datastream
Diff_PTSD_MVW_A	1442	-38,59308	0,0000	Datastream
R_MVW_A	1358	-37,49063	0,0000	Datastream
SDR_MVW_A	1437	-6,965002	0,0000	Datastream
Volume_MVW_A	1240	-6,324258	0,0000	Datastream
Diff_PTGT_SD_EVW_A	1188	-16,09674	0,0000	Factset
R_EVW_A	1365	-35,10707	0,0000	Factset
EBITDA_SD_EVW_A	1365	-3,118235	0,1024	Factset
Diff_EBITDA_SD_EVW_A	1335	-37,65884	0,0000	Factset
EPS_SD_EVW_A	1365	1,850249	0,9998	Factset
Diff_EPS_SD_EVW_A	1335	-2,530979	0,1083	Factset
S_Diff_EPS_SD_EVW_A	1307	-37,38227	0,0000	Factset
SDR_EVW_A	1230	-5,154813	0,0000	Factset

The tests showed that we can’t reject the null hypothesis of a unit root for the data series of analysts’ standard deviation of stock price targets, EBITDA estimates and EPS estimates. The data series for analysts’ standard deviation of stock price targets was integrated of order one. The same was true for the data series with EBITDA estimates. We dealt with non-stationarity by differencing the series once. We checked that the differenced series were stationary with unit root tests. The data series of EPS estimates was integrated of order two and was differenced twice.

6.3 Testing for herding and nonlinearity of the Cross Sectional Absolute Deviation of Asset Returns and Stock Market Returns

Let the cross sectional absolute deviation of returns ($CSAD$) be

$CSAD_{t,EW} = \frac{1}{N_t} \sum_{i=1}^{N_t} \lvert R_{i,t} - R_{m,t} \rvert$	6-1

Chang, Cheng and Khorana (2000) propose $CSAD$ as a mean for detecting herding behavior:

$CSAD_t = \alpha + \gamma_1 \lvert R_{m,t} \rvert + \gamma_2 R_{m,t}^2 + \varepsilon_t,\ \varepsilon_t \sim N(0, \sigma_t^2)$

where

$R_{i,t}$ stands for the i stock return at time t and $R_{m,t}$ for the market return at time t, N_t for a time varying number of stocks due to enlisting and delisting and EW stands for equally weighted.

Chang et al.'s argument is that given rational expectations, should the relation between $CSAD$ and the market return R_m be linear.

Starting with the following variant of CAPM:

$E_t[R_i] = \gamma_0 + \beta_i E_t(R_m - \gamma_0)$

where $\gamma_0$is the return of the beta-zero portfolio.

The CAPM applied to market return gives:

$E_t[R_m] = \gamma_0 + \beta_m E_t(R_m - \gamma_0)$

Observe that the absolute deviation of returns can be written as:

$AD_{i,t} = \ \lvert E_t[R_m] - E_t[R_i] \rvert = \lvert [\gamma_0 + \beta_m E_t(R_m - \gamma_0)] - [\gamma_0 + \beta_i E_t(R_m - \gamma_0)] \rvert$

Then the expected cross sectional absolute deviation of returns is:

$ECSAD_t = \frac{1}{N_t} \sum_{i=1}^{N_t} \lvert \beta_{i,t} - \beta_{m,t} \rvert \, E_t(R_m - \gamma_0)$	6-2

Taking the first and second partial derivative of R_m we get:

$$\frac{\partial ECSAD_t}{\partial E_t[R_m]} = \frac{1}{N_t}\sum_{i=1}^{N_t}\left|\beta_{i,t} - \beta_{m,t}\right| > 0$$

$$\frac{\partial^2 ECSAD_t}{\partial E_t[R_m]^2} = 0$$

Chang et al. (2000) use $CSAD_t$ as a proxy for $ECSAD_t$. Henker, Henker and Mitsios (2006) have shown that 6-1 and 6-2 are equivalent.

Since $\frac{\partial^2 ECSAD_t}{\partial E_t[R_m]^2} = 0$ the prediction of the rational expectations in this setting is that the relation between $CSAD_t$ and the stock market return R_m is linear. $CSAD_t$ is calculated as the absolute value of the average deviation of the individual stock return relative to the equally weighted market return.

The hypothesis that the investor behavior exhibited in upward movements of markets is different than the behavior exhibited in downward movements is examined by Chang et al. (2000),Tan, Chiang, Mason and Nelling (2008), Araghi, Mavi and Alidoost (2011), Prosad, Kapoor and Sengupta (2012), Al-Shboul (2012). Since the market return, volume and volatility are important aspects of the stock market, upward and downward movements are investigated with respect to these variables. In periods of high financial stress, denoted for instance by high volatility, it is hypothesized that individuals suppress their individual beliefs and align with the crowd (Economou, Kostakis and Philippas 2011). We look into the asymmetric effects in the relation between $CSAD$ and R_m (McQueen et al. 1996, Chang et al. 2000) in samples based on the lower and upper percentiles of returns, volume and volatility. Positive γ_2 signifies the opposite of herding. Since herding shows a convergence of opinions and beliefs, we interpret positive γ_2 as a signal for divergence of opinions and beliefs which shows heterogeneity among investors.

In our tests we use the market value weighted $CSAD$:

$$CSAD_{t,MVW} = \sum_{i=1}^{N_t} MVW_{i,t}\left|R_{i,t} - R_{m,t,MVW}\right|$$

where

$$MVW_{i,t} = \frac{MV_{i,t}}{\sum_{i=1}^{n} MV_{i,t}}$$

$$R_{m,t,MVW} = \sum_{i=1}^{N_t} MVW_{i,t} R_{i,t} = \sum_{i=1}^{N_t} \frac{MV_{i,t}}{\sum_{i=1}^{n} MV_{i,t}} R_{i,t}$$

6.3.1 Hypotheses

We formulate the following hypotheses:

H - 7: The cross sectional absolute deviation of returns has a quadratic relation to the market return in times of higher market return.

H - 8: The cross sectional absolute deviation of returns has a quadratic relation to the market return in times of lower market return.

H - 9: The cross sectional absolute deviation of returns has a quadratic relation to the market return in times of higher trade volume of stocks.

H - 10: The cross sectional absolute deviation of returns has a quadratic relation to the market return in times of lower trade volume of stocks.

H - 11: The cross sectional absolute deviation of returns has a quadratic relation to the market return in times of higher volatility of the stock market.

H - 12: The cross sectional absolute deviation of returns has a quadratic relation to the market return in times of lower volatility of the stock market.

Model specification

Using data from Datastream (see appendix A - xvii) we run in EViews least square regressions of the following equations:

H - 7:

$CSAD_{t,\mathrm{MVW},hr} = \alpha + \gamma_{1,high\ return}\left|R_{m,t,\mathrm{MVW},high\ return}\right| + \gamma_{2,high\ return}\left(R_{m,t,\mathrm{MVW},high\ return}\right)^2 + \varepsilon_t$	M - 9

H - 8:

$CSAD_{t,\mathrm{MVW},lr} = \alpha + \gamma_{1,low\ return}\left|R_{m,t,\mathrm{MVW},low\ return}\right| + \gamma_{2,low\ price}\left(R_{m,t,\mathrm{MVW},low\ return}\right)^2 + \varepsilon_t$	M - 10

H - 9:

$CSAD_{t,\mathrm{MVW},hv} = \alpha + \gamma_{1,high\ volume}\left|R_{m,t,\mathrm{MVW},high\ volume}\right| + \gamma_{2,high\ volume}\left(R_{m,t,\mathrm{MVW},high\ volume}\right)^2 + \varepsilon_t$	M - 11

H - 10:

$CSAD_{t,\mathrm{MVW},lv} = \alpha + \gamma_{1,low\ volume}\left|R_{m,t,\mathrm{MVW},low\ volume}\right| + \gamma_{2,low\ volume}\left(R_{m,t,\mathrm{MVW},low\ volume}\right)^2 + \varepsilon_t$	M - 12

H - 11:

$CSAD_{t,\mathrm{MVW},h\sigma^2} = \alpha + \gamma_{1,high\ \sigma^2}\left|R_{m,t,\mathrm{MVW},high\ \sigma^2}\right| + \gamma_{2,high\ \sigma^2}\left(R_{m,t,\mathrm{MVW},high\ \sigma^2}\right)^2 + \varepsilon_t$	M - 13

H - 12:

$CSAD_{t,\ MVW,l\sigma^2} = \alpha + \gamma_{1,low\ \sigma^2}\left|R_{m,t,\mathrm{MVW},low\ \sigma^2}\right| + \gamma_{2,low\ \sigma^2}\left(R_{m,t,\mathrm{MVW},\ \ low\ \sigma^2}\right)^2 + \varepsilon_t$	M - 14

Where

$R_{m,t,MVW,high\ \sigma^2}$ is the market return when market volatility is high and $R_{m,t,MVW,low\ \sigma^2}$ is the market return when market volatility is low. $CSAD_{t,MVW,h\sigma^2}$ corresponds to $R_{m,t,MVW,high\ \sigma^2}$ and $CSAD_{t,MVW,l\sigma^2}$ corresponds to $R_{m,t,MVW,low\ \sigma^2}$.

$R_{m,t,high\ volume}$ is the market return when the trading volume is high and $R_{m,t,MVW,low\ volume}$is the market return when trading volume is low. $CSAD_{t,MVW,hv}$ corresponds to $R_{m,t,MVW,high\ volume}$ and $CSAD_{t,MVW,lv}$ corresponds to $R_{m,t,MVW,low\ volume}$.

$R_{m,t,MVW,high\ return}$ is the market return when the market return is high and $R_{m,t,MVW,low\ return}$ is the market return when the market return is low. $CSAD_{t,MVW,hr}$ corresponds to $R_{m,t,MVW,high\ return}$. and $CSAD_{t,MVW,lr}$ corresponds to $R_{m,t,MVW,low\ return}$.

Hypotheses H - 7 and H - 8 concern high and low market returns.

Hypotheses H - 9 and H - 10 concern high and low trading volume.

Hypotheses H - 11 and H - 12 concern high and low stock market return volatility.

The reason for using the absolute value of R_m is for being easier to compare the coefficient of the linear term in the up vs. down-market (Chang et al. 2000, p. 1656).

6.3.2 Methodology

Data sources

The data source was the financial database Datastream. Our data consists of daily, weekly and monthly observations for 163 Norwegian companies for the period 1.1.2007-12.7.2012.

Choice of method

We used LS (least squares regression) in the econometric program EViews. The data in the regressions is time series based on cross-sectional averages. This is in line with the method used by Chang, Cheng and Khorana (2000).

6.3.3 Empirical test results

All variables used in the regression equations below are tested for unit roots and are stationary. The regression results were corrected both for heteroscedasticity and autocorrelation using the Newey–West estimators and were as follows (the numbers in parentheses shows t-values and the numbers in brackets p-values; a negative t-value shows that the expected value of the population mean is greater than the sample mean since $t = \frac{\bar{X}-\mu}{s/\sqrt{n}}$):

H - 7

Table 6-18: Regression results $CSAD$ and R_m when the market return is high

$CSAD_{t,\text{MVW},hr} = \alpha + \gamma_{1,high\ return}\left\|R_{m,t,\text{MVW},high\ return}\right\| + \gamma_{2,high\ return}\left(R_{m,t,\text{MVW},high\ return}\right)^2 + \varepsilon_t$						
α	γ_1	γ_2	Adjusted R^2	Period	N	Percentile
0,012451 (6,023045) [0,0000]	0,064802 (0,596856) [0,5516]	2,498544 (2,332990) [0,0211]	0,405374	01/01/2007-12/07/2012	140	highest decile
0,010024 (26,98282) [0,0000]	0,191794 (4,540843) [0,0000]	1,463772 (2,294653) [0,0221]	0,342609	01/01/2007-12/07/2012	696	upper half

H - 8

Table 6-19: Regression results $CSAD$ and R_m when the market return is low

$CCSAD_{t,\text{MVW},lr} = \alpha + \gamma_{1,low\ return}\left\|R_{m,t,\text{MVW},low\ return}\right\| + \gamma_{2,low\ price}\left(R_{m,t,\text{MVW},low\ return}\right)^2 + \varepsilon_t$						
α	γ_1	γ_2	Adjusted R^2	Period	N	Percentile
0,016181 (9,114252) [0,0000]	-0,169204 (-1,579843) [0,1164]	4,097348 (3,016089) [0,0031]	0,318296	01/01/2007-12/07/2012	140	lowest decile
0,010871 (28,72175) [0,0000]	0,092201 (2,195369) [0,0285]	1,598635 (2,346123) [0,0193]	0,244576	01/01/2007-12/07/2012	695	lower half

H - 9

Table 6-20: Regression results $CSAD$ and R_m when the market volume is high

$CCSAD_{t,\text{MVW},hv} = \alpha + \gamma_{1,high\ volume}\lvert R_{m,t,\text{MVW},high\ volume}\rvert + \gamma_{2,high\ volume}\left(R_{m,t,\text{MVW},high\ volume}\right)^2 + \varepsilon_t$						
α	γ_1	γ_2	Adjusted R^2	Period	N	Percentile
0,012473 (12,82409) [0,0000]	0,047805 (0.521300) [0.6030]	2.583153 (1.774638) [0.0782]	0.276793	01/01/2007- 12/07/2012	140	highest decile
0,010909 (23,82759) [0,0000]	0,164642 (3,153031) [0,0017]	1,146235 (1,494746) [0,1354]	0,328297	01/01/2007- 12/07/2012	722	upper half

H - 10

Table 6-21: Regression results $CSAD$ and R_m when the market volume is low

$CSAD_{t,\text{MVW},lv} = \alpha + \gamma_{1,low\ volume}\lvert R_{m,t,\text{MVW},low\ volume}\rvert + \gamma_{2,low\ volume}\left(R_{m,t,\text{MVW},low\ volume}\right)^2 + \varepsilon_t$						
α	γ_1	γ_2	Adjusted R^2	Period	N	Percentile
0,008407 (17,52708) [0,0000]	0,106784 (1,565415) [0,1198]	2,570017 (2,228804) [0,0275]	0,251196	01/01/2007- 12/07/2012	140	lowest decile
0,010252 (27,91523) [0,0000]	0,089227 (2,744859) [0,0062]	1,968053 (5,484649) [0,0000]	0,184578	01/01/2007- 12/07/2012	669	lower half

H - 11

Table 6-22: Regression results $CSAD$ and R_m when the market volatility is high

$CSAD_{t,\text{MVW},h\sigma^2} = \alpha + \gamma_{1,high\ \sigma^2}\lvert R_{m,t,\text{MVW},high\ \sigma^2}\rvert + \gamma_{2,high\ \sigma^2}\left(R_{m,t,\text{MVW},high\ \sigma^2}\right)^2 + \varepsilon_t$						
α	γ_1	γ_2	Adjusted R^2	Period	N	Percentile
0,012352 (7,111478) [0,0000]	0,275151 (2,074484) [0,0399]	-0,475254 (-0,304343) [0,7613]	0,334012	01/01/2007- 12/07/2012	140	highest decile
0,011504 (20,90555) [0,0000]	0,138027 (2,671848) [0,0077]	1,305529 (1,848264) [0,0650]	0,294511	01/01/2007- 12/07/2012	696	upper half

H - 12

Table 6-23: Regression results $CSAD$ and R_m when the market volatility is low

$CSAD_{t,\ MVW,l\sigma^2} = \alpha + \gamma_{1,low\ \sigma^2}\lvert R_{m,t,\text{MVW},low\ \sigma^2}\rvert + \gamma_{2,low\ \sigma^2}\left(R_{m,t,\text{MVW},\ low\ \sigma^2}\right)^2 + \varepsilon_t$						
α	γ_1	γ_2	Adjusted R^2	Period	N	Percentile
0,009952 (14,65733) [0,0000]	-0,115180 (-0,829859) [0,4081]	9,768722 (1,685786) [0,0941]	0,036532	01/01/2007- 12/07/2012	140	lowest decile
0,010201 (39,35436) [0,0000]	0,046228 (0,889414) [0,3741]	3,600147 (1,815613) [0,0699]	0,093146	01/01/2007- 12/07/2012	695	lower half

H - 7: We have support for the hypothesis that the relation between $CSAD_{t,MVW,hr}$ and $R_{m,t,MVW,high\ return}$ is quadratic at both the upper half and the highest decile. The nonlinear coefficient is positive and significant different than zero at the 5% significance level.

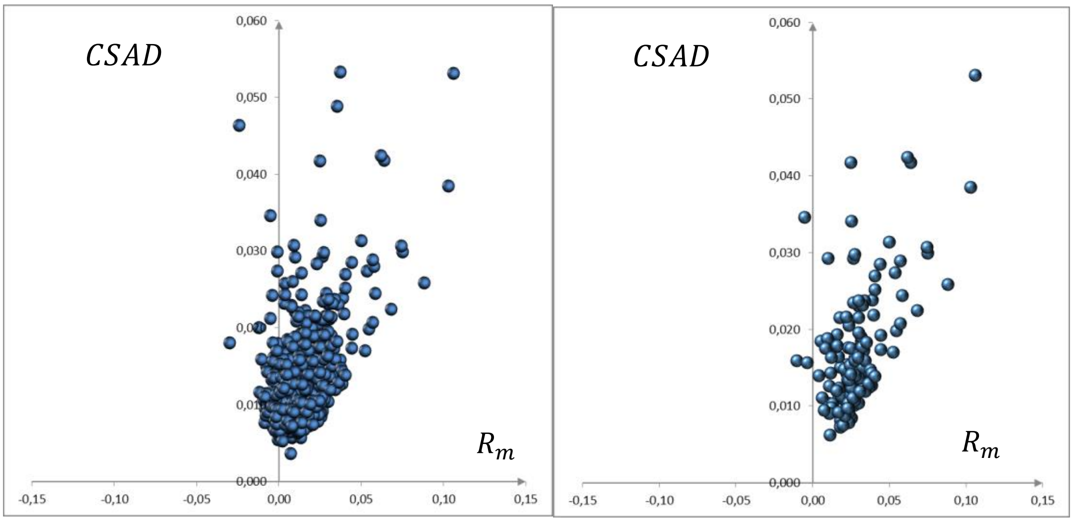


Figure 6-1: $CSAD$ and R_m when the stocks' return is high for the upper half in the left diagram and the highest decile in the right diagram

H - 8: We have support for the hypothesis that the relation between $CSAD_{t,MVW,lr}$ and $R_{m,t,MVW;low\ return}$ is quadratic at both the lower half and the lowest decile. The nonlinear coefficient is positive and significant different than zero at the 5% and 1% significance level respectively.

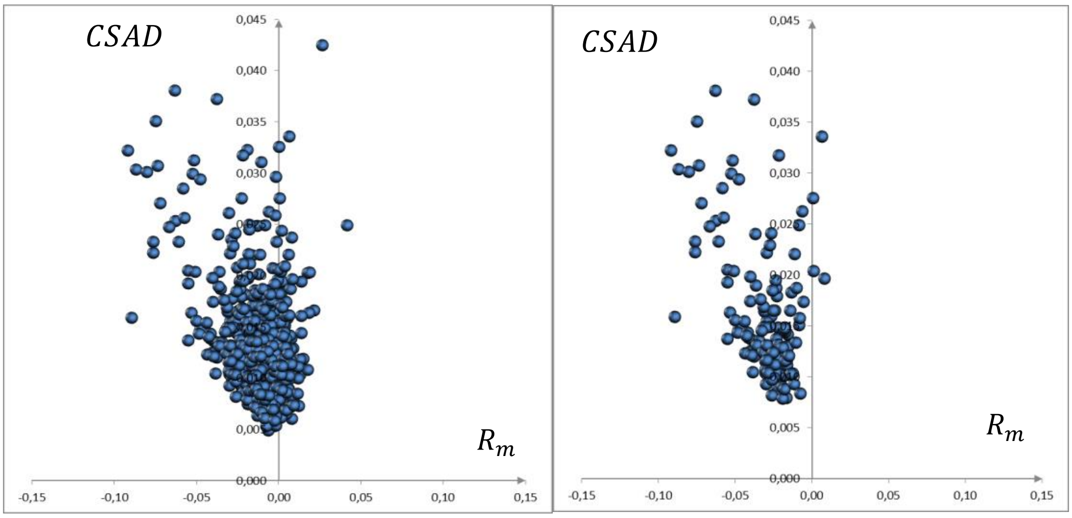


Figure 6-2: $CSAD$ and R_m when the stocks' return is low for the lower half in the left diagram and the lowest decile in the right diagram

The Wald coefficient test shows that the probability of $\gamma_{1,high\ return} = \gamma_{1,low\ return}$ and $\gamma_{2,high\ return} = \gamma_{2,low\ return}$ at the upper and lower half and the highest and lowest decile is 0 % and 3,16 % respectively.

H - 9: We have support in the highest decile of trading volume for the hypothesis that the relation between $CSAD_{t,MVW,hv}$ and $R_{m,t,MVW,high\ volume}$ is quadratic. The nonlinear coefficient in the highest decile is positive and significantly different than zero at the 10% significance level. There isn't any significant nonlinear relation between $CSAD_{t,MVW,hv}$ and $R_{m,t,MVW,high\ volume}$ in the upper half of trading volume.

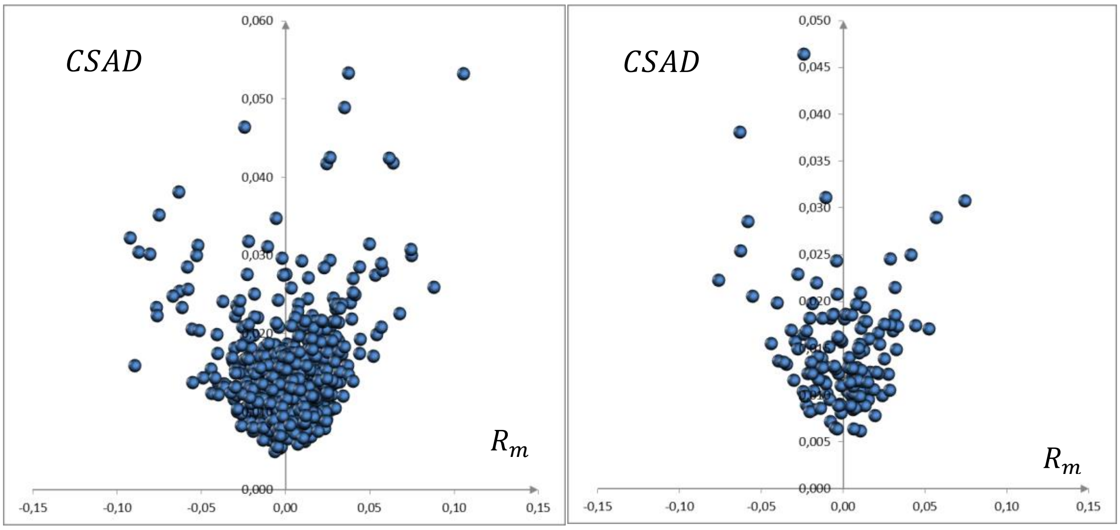


Figure 6-3: $CSAD$ and R_m when the stocks' trading volume is high for the upper half in the left diagram and the highest decile in the right diagram

H - 10: We have support for the hypothesis that the relation between $CSAD_{t,MVW,lv}$ and $R_{m,t,MVW,low\ volume}$ is quadratic at both the lower half and the lowest decile. The nonlinear coefficient is positive and significant different than zero at the 1 % and 5 % significance level respectively.

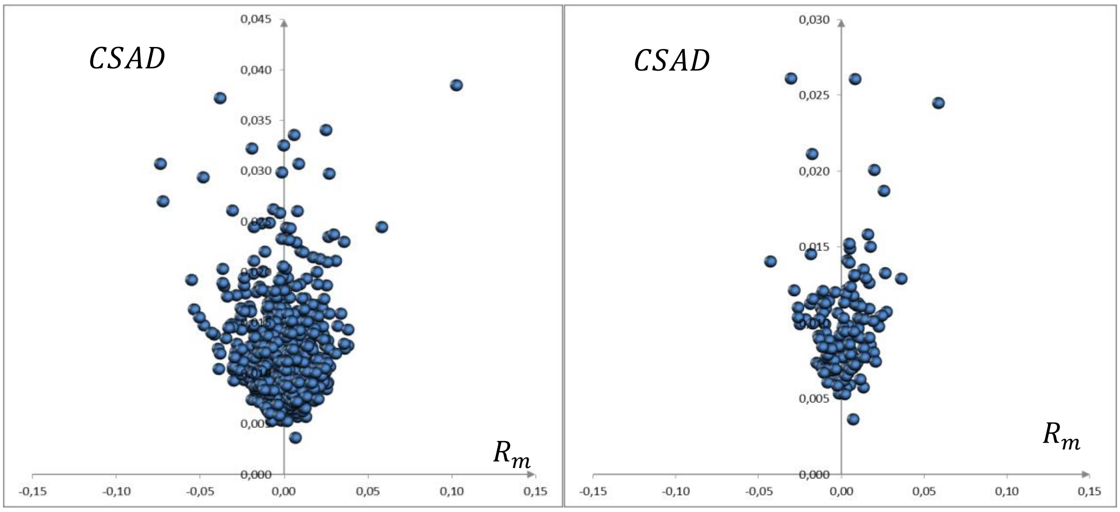


Figure 6-4: $CSAD$ and R_m when the stocks' trading volume is low for the lower half in the left diagram and the lowest decile in the right diagram

The Wald coefficient test shows that the probability of $\gamma_{1,high\ volume} = \gamma_{1,low\ \ volume}$ and $\gamma_{2,\ \ high\ volume} = \gamma_{2,low\ volume}$ at the upper and lower half and the highest and lowest decile is 17,72 % and 46,49 % respectively.

H - 11: We have support in the upper half of stock market volatility for the hypothesis that the relation between $CSAD_{t,MVW,h\sigma^2}$ and $R_{m,t,MVW,high\ \sigma^2}$ is quadratic. The $\gamma_2$estimator is positive and significantly different from zero at the 5% significance level. We don't have support in the highest decile of stock market volatility for the hypothesis that the relation between $CSAD_{t,MVW,h\sigma^2}$ and $R_{m,t,MVW,high\ \sigma^2}$ is quadratic.

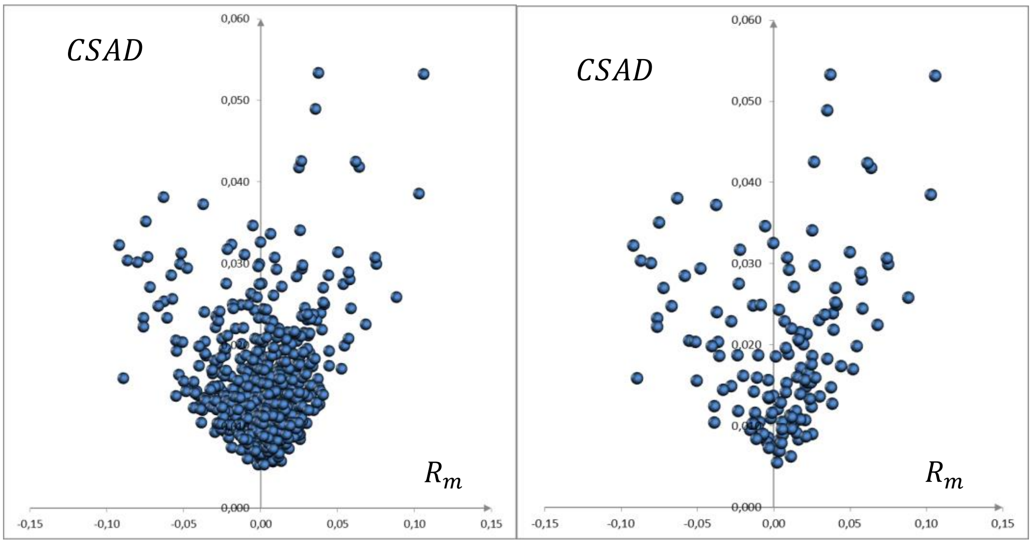


Figure 6-5: $CSAD$ and R_m when the stocks' return volatility is high for the upper half in the left diagram and the highest decile in the right diagram

H - 12: We have support for the hypothesis that the relation between $CSAD_{t,MVW,l\ \sigma^2}$ and $R_{m,t,MVW,low\ \sigma^2}$ is quadratic. The $\gamma_2$estimator is positive and significantly different than zero at 10 % significance level in both the lower half and the lowest decile of stock market volatility.

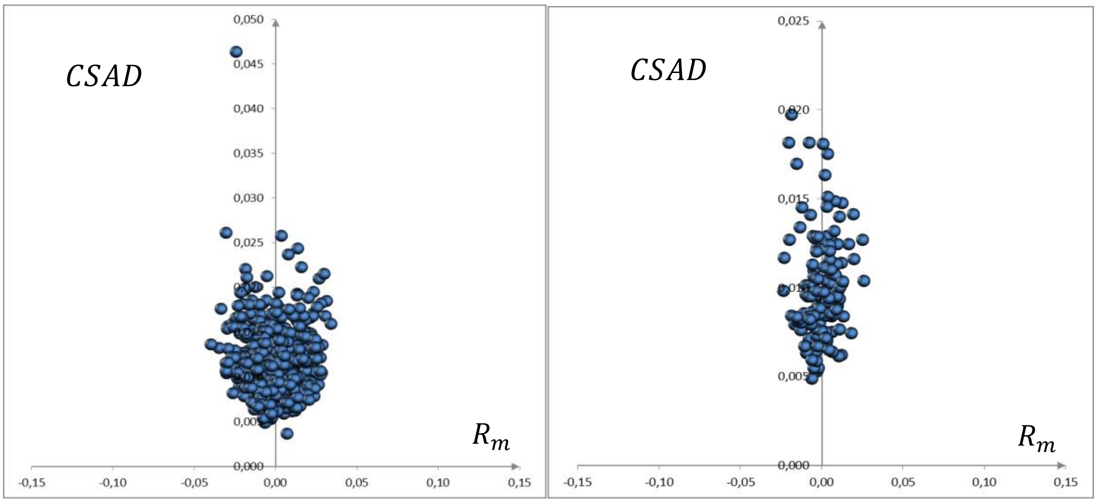


Figure 6-6: $CSAD$ and R_m when the stocks' return volatility is low for the lower half in the left diagram and the lowest decile in the right diagram

The Wald coefficient test shows that the probability of $\gamma_{1,high\ \sigma^2} = \gamma_{1,low\ \sigma^2}$ and $\gamma_{2,high\ \sigma^2} = \gamma_{2,low\ \sigma^2}$ at the upper and lower half and the highest and lowest decile is 32,87 % and 4,43 % respectively.

6.3.4 Discussion of the results

When the coefficient $\gamma_2$of the non-linear term is significantly different from 0 we cannot reject the hypothesis that the relation between $CSAD_t$and $R_{m,t}$ is non-linear. A non-linear relation with a negative coefficient for the non-linear term is interpreted as herding. In herding behavior the agents suppress their individual beliefs and follow the crowd. In this sense herding becomes a convergence of beliefs. A non-linear relation with a positive coefficient for the non-linear term is interpreted as antiherding. Since herding shows a convergence of beliefs, we interpret antiherding as a divergence of opinions.

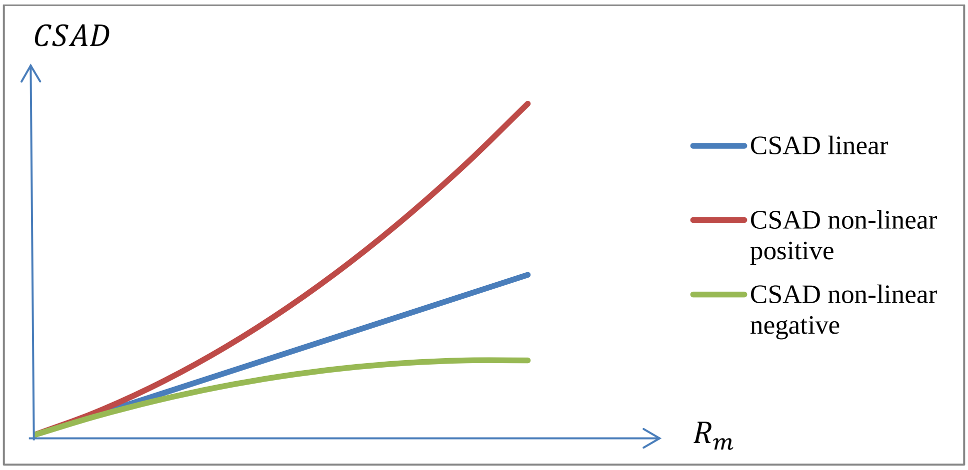


Figure 6-7: Relationships between the $CSAD$ and R_m

The test results don't supply evidence of herding behavior. The relation between CSAD and R_m is non-linear positive both in high and low states of market return. In states of high and low market volume and high and low market volatility we observe non-linearity with a positive sign. In the states of upper half trading volume and highest decile stock market volatility the relation between CSAD and R_m is linear. The Wald coefficient test shows directional asymmetry between the upper half and lower half for stock market returns and the highest and lowest decile for stock market returns and stock market volatility. The same goes for the highest decile and lowest decile.. The upper half yields higher adjusted R^2 values compared to the lower half. This implies asymmetric effects in the explanation power of the regressors. In other words, the explanation power of the regressors depends on the market state.

We see that divergence of opinions occurs in both low and high market states. The positive non-linear term suggests that the dispersion of returns is proportional to the divergence of opinions in the context of trading volume in three out of four market states (highest decile, lower half and lowest decile). This seems to be in line with the predictions of the theory of heterogeneity in beliefs (Varian 1985, Xiouros 2009) that trading volume is proportional to divergence of opinions. It appears to be a contradiction concerning the market return since the CSAD test shows a proportional relation between the dispersion of returns and the divergence of opinions in all states of stock market returns while the theory of heterogeneity in beliefs predicts an inverse proportional relation between asset returns and divergence of opinions. However, the test results can be considered to be in line with the auction theory of asset prices which predicts that the relationship between the dispersion of beliefs and asset returns is inversely proportional. This is due to agents with the highest evaluation of returns dominating the price setting (Miller 1977 p. 1152, Diether, Malloy and Scherbina 2002 p. 2113).

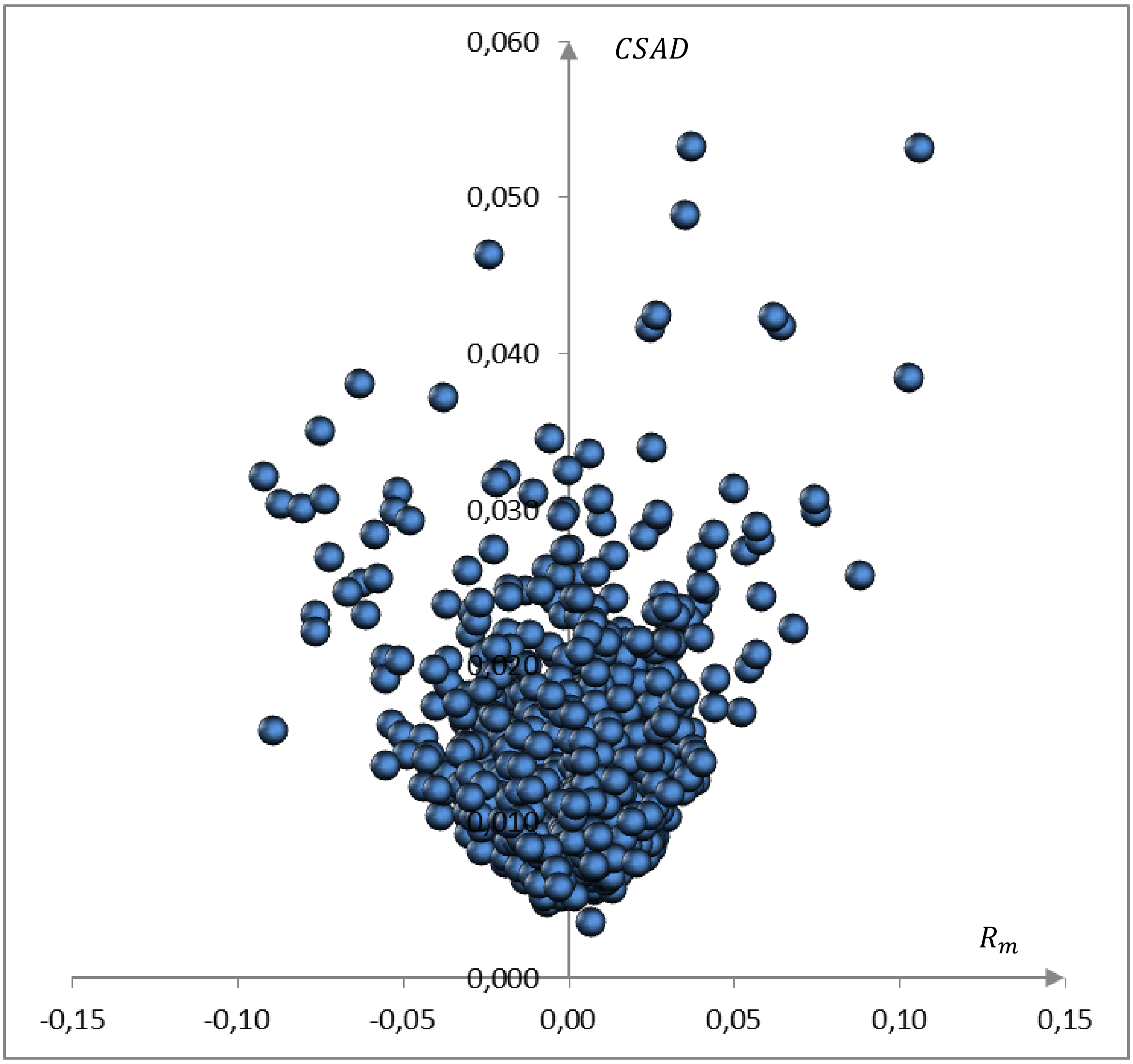


Figure 6-8: $CSAD$ and R_m for the stock market as a whole

The Norwegian stock market exhibits as a whole a non-linear relation between the cross sectional dispersion of asset returns and the stock market return for the period primo 2007 to middle 2012 with the nonlinear coefficient being positive and significantly different than zero at the 1 % significance level (appendix A - xxii).

6.3.5 Contribution to research

Our $CSAD$ tests contribute to previous research in the following ways:

i) Test for the presence of herding behavior in the Norwegian stock market using a market value weighted stock market return. In our research of literature we didn't find this test to have been carried out previously for the Norwegian stock market.

ii) Test for directional asymmetry in different market states of the Norwegian stock market. The empirical literature we have been through didn't indicate that directional asymmetry has been tested earlier for Norway.

ii) A novel interpretation of a non-linear positive relation between $CSAD$ and the stock market return as divergence of opinions and heterogeneity. In our research of literature we didn't find this interpretation to have been precedently used in $CSAD$ tests.

6.3.6 Conclusion

The cross sectional absolute deviation test series have delved into the dispersion of asset returns as a means for examining the suppression of own beliefs in favour of the market consensus in the backdrop of high and low market states with respect to trading volume and volatility.

The study set out to explore the case of nonlinearity in the relation between the cross sectional absolute deviation of asset returns and the stock market return.

We run our tests using the market value weighted cross sectional absolute deviation since we wanted big cap stocks to weight more than small cap stocks to mitigate the effects of illiquidity and thin trading. We found convincing evidence of nonlinearity in the upper and lower half of stock returns including the highest and lowest deciles, in the highest and lowest decile of trading volume, in the lower half of trading volume and in the upper half, lower half and lowest decile of stock market volatility. Nonlinearity was consistently positive. Directional asymmetry was discovered in the upper and lower states of stock market returns including the highest and lowest decile and in the highest and lowest deciles of stock market volatility.

The implication of our findings is threefold. To begin with we didn't find evidence of herding. Secondly, we found evidence of dispersion of beliefs and heterogeneity. Thirdly, we found support for asymmetric effects in bull and bear markets.

Although $CSAD$ is not a measure of herding or antiherding, the relation of the dispersion of returns and the market return can be used as an instrument for spotting convergence or divergence of opinions. The findings are illuminated in the context of other relevant information such as the market states of trading volume, market volatility and market return.

Our research was contained to low frequency data and to CSAD as an indicator for spotting herding behavior. A subject for future exploration is herding behavior in the Norwegian stock market with the Patterson Sharma measure which captures the intraday herding behavior using higher frequency data. Future research could investigate in greater depth the relation between the dispersion of asset returns and stock returns with respect to dispersion of beliefs, sentiment risk and the optimistic hypothesis that the agents with the highest valuation of asset returns set the stock prices.

6.3.7 Testing the regression assumptions

Regression assumptions

Here we follow the presentation by Berry (1993, p. 12)

Regression assumption 1: Quantitativeness

All independent variables (x_1, x_2,…,x_i) are quantitative and the dependent variable, Y, is quantitative, and continuous and measured without error.

Our variables are quantitative and continuous and our data come from reliable sources.

Table 6-24: Descriptive statistics for the market value weighted stock return series

	RM_MVW
Mean	0.000396
Median	0.001163
Maximum	0.105620
Minimum	-0.092178
Std. Dev.	0.018886
Skewness	-0.234330
Kurtosis	7.189936
Jarque-Bera	1030.222
Probability	0.000000
Sum	0.550495
Sum Sq. Dev.	0.495775
N	1391

Table 6-25: Descriptive statistics for cross sectional absolute deviation of returns in upper half, highest decile, lower half and lowest decile market return states

	CSAD_RI_U_MVW	CSAD_RI_U_D_MVW	CSAD_RI_L_MVW	CSAD_RI_L_D_MVW
Mean	0,013059	0,016993	0,012713	0,016369
Median	0,011389	0,014821	0,011421	0,014367
Maximum	0,053347	0,053202	0,042543	0,038110
Minimum	0,003662	0,006318	0,004930	0,007897
Std. Dev.	0,006123	0,008078	0,005404	0,006801
Skewness	2,559084	1,648819	1,782950	1,260313
Kurtosis	13,11376	6,302285	7,283961	3,913028
Jarque-Bera	3726,032	127,0471	899,676600	41,925200
Probability	0,000000	0,000000	0,000000	0,000000
Sum	9,088844	2,379073	8,835315	2,291699
Sum Sq. Dev.	0,026056	0,009071	0,020265	0,006429
N	696	140	695	140

Table 6-26: Descriptive statistics for cross sectional absolute deviation of returns in upper half, highest decile, lower half and lowest decile trading volume states

	CSAD_RI_V_U_MVW	CSAD_RI_V_U_D_MVW	CSAD_RI_V_L_MVW	CSAD_RI_V_L_D_EW
Mean	0,013936	0,014692	0,011752	0,009978
Median	0,012274	0,012989	0,010531	0,009195
Maximum	0,053347	0,046411	0,038557	0,026140
Minimum	0,004930	0,006168	0,003662	0,003662
Std. Dev.	0,006324	0,005990	0,004875	0,003661
Skewness	2,295717	1,962841	1,953402	2,067982
Kurtosis	11,050890	9,258677	8,333432	9,054990
Jarque-Bera	2584,103000	318,395200	1218,378	313,653100
Probability	0,000000	0,000000	0,000000	0,000000
Sum	10,062080	2,056817	7,862084	1,396868
Sum Sq. Dev.	0,028834	0,004988	0,015872	0,001863
N	722	140	669	140

Table 6-27: Descriptive statistics for cross sectional absolute deviation of returns in upper half, highest decile, lower half and lowest decile stocks' return cross sectional volatility states

	CSAD_RI_SD_U_MVW	CSAD_RI_SD_U_D_MVW	CSAD_RI_SD_L_MVW	CSAD_RI_SD_L_D_MVW
Mean	0,014548	0,019511	0,011221	0,009945
Median	0,012777	0,017318	0,010631	0,00949
Maximum	0,053347	0,053347	0,046411	0,019782
Minimum	0,005320	0,005578	0,003662	0,00493
Std. Dev.	0,006938	0,009901	0,003613	0,002951
Skewness	1,789009	1,066971	2,126123	0,967316
Kurtosis	7,657291	4,065808	16,307750	3,866539
Jarque-Bera	1000,285000	33,189660	5652,024000	26,21318
Probability	0,000000	0,000000	0,000000	0,000002
Sum	10,125380	2,731484	7,798781	1,392297
Sum Sq. Dev.	0,033456	0,013627	0,009059	0,001211
N	696	140	695	140

Where

RM_MVW is a time series of market value weighted market returns based on163 Norwegian stocks.

CSAD_RI_U_MVW is a time series of the cross sectional absolute deviation of stock returns in the upper half of daily observations of market return.

CSAD_RI_U_D_MVW is a time series of the cross sectional absolute deviation of stock returns in the highest decile of daily observations of market return.

CSAD_RI_L_MVW is a time series of the cross sectional absolute deviation of stock returns in the lower half of daily observations of market return.

CSAD_RI_L_D_MVW is a time series of the cross sectional absolute deviation of stock returns in the lowest decile of daily observations of market return.

CSAD_RI_V_U_MVW is a time series of the cross sectional absolute deviation of stock returns in the upper half of daily observations for market trading volume.

CSAD_RI_V_U_D_MVW is a time series of the cross sectional absolute deviation of stock returns in the highest decile of daily observations for market trading volume.

CSAD_RI_V_L_MVW is a time series of the cross sectional absolute deviation of stock returns in the lower half of daily observations for market trading volume.

CSAD_RI_V_L_D_MVW is a time series of the cross sectional absolute deviation of stock returns in the lowest decile of daily observations for market trading volume

CSAD_RI_SD_U_MVW is a time series of the cross sectional absolute deviation of stock returns in the upper half of daily observations of cross sectional market volatility.

CSAD_RI_SD_U_D_MVW is a time series of the cross sectional absolute deviation of stock returns in the highest decile of daily observations of cross sectional market volatility.

CSAD_RI_SD_L_MVW is a time series of the cross sectional absolute deviation of stock returns in the lower half of daily observations of cross sectional market volatility.

CSAD_RI_SD_L_D_MVW is a time series of the cross sectional absolute deviation of stock returns in the lowest decile of daily observations of cross sectional market volatility.

Kurtosis is a descriptive statistic for fat tails which shows the probability for extreme events (in finance called “black swans”). When kurtosis is greater than 3 the variable does not follow a normal distribution. The Jarque-Bera test is based on the ratio of kurtosis and skewness. From the above tables we see that none of the variables used in our regression equations is normally distributed. This is in line with the stylized fact that asset returns do not follow a normal distribution (Cont 2001).

Regression assumption 2 Variance

All variables have some variance.

The descriptive statistics show that our variables have nonzero variance.

Regression assumption 3 Multicollinearity

There is not an exact linear relationship between two or more of the independent variables, i.e. there is not perfect multicollinearity.

Our model is set up as a quadratic regression equation. In a strict sense the relation between R_m and $(R_m)^2$ doesn't violate the assumption of no multicollinearity (Gujarati 2003 pp. 343-344). However, since R_m and $(R_m)^2$ are functionally related, the coefficients of R_m and $(R_m)^2$ are highly correlated, which makes the standard errors of the estimators bigger and increases the risk of type II errors.

According to Gujarati (2003 p. 359) multicollinearity cannot be tested but can be measured. One commonly used measure is the variance inflating factor (VIF).

Table 6-28: VIF measurement

Model	Percentile	Uncentered VIF α $\lvert R_{m,t,MVW}\rvert$ $(R_{m,t,MVW})^2$	Centered VIF α $\lvert R_{m,t,MVW}\rvert$ $(R_{m,t,MVW})^2$	N
M - 9	Highest decile	13,04378 32,52658 10,12983	NA 6,917561 6,917561	140
M - 9	Upper half	2,571147 5,706751 3,246268	NA 2,928659 2,928659	696
M - 10	Lowest decile	14,05064 79,03947 37,76759	NA 26,73884 26,73884	140
M - 10	Lower half	2,467063 14,30410 10,64299	NA 9,361468 9,361468	695
M - 11	Highest decile	3,531628 19,29526 12,57701	NA 9,757477 9,757477	140
M - 11	Upper half	3,430109 13,26604 8,648176	NA 7,970507 7,970507	722
M - 12	Lowest decile	2,852398 7,335045 4,219880	NA 3,783613 3,783613	140
M - 12	Lower half	2,083737 7,812404 5,649179	NA 5,129652 5,129652	669
M - 13	Highest decile	4,296300 41,76416 31,22117	NA 26,38250 26,38250	140
M - 13	Upper half	2,511783 14,82716 11,06294	NA 9,791363 9,791363	696
M - 14	Lowest decile	7,884136 26,14356 11,63461	NA 8,643712 8,643712	140
M - 14	Lower half	2,554243 10,85610 7,471866	NA 6,534112 6,534112	695

Testing the CSAD regressions in EViews with VIF we get centered and uncentered values. Centered VIF has no ability to discover collinearity involving the intercept (Gross 2003 p. 304). Uncentered VIF values are above 10. According to Gross this shows collinearity involving the intercept. Gross suggests in cases with high uncentered VIF removing the intercept. This would lead to the advantage of lower standard errors and higher t-values for the coefficients of R_m and R_m^2. The disadvantage of this method is that one forces the regression line to go through the origo instead of going through the line that minimizes least squares. Moreover there is a risk that econometric packages are not calibrated for calculating regression equations without intercepts (Eisenhower 2003). For instance, preliminary tests showed that after removing the intercept, 10 out of 12 models we used in our tests, receive negative R-square values (EViews 7 users Guide II 2009 p. 13 and Hayashi 2000 p. 21). Another issue is that one needs a theoretical justification for assuming the intercept equal to zero.

Because of the above issues we let the intercept remain in the model.

Regression assumption 4: Mean of the error term

At each set of values for the i independent variables, $x_i, x_{i-1}, \dots, x_1$, $E(\varepsilon_i | \varepsilon_{i-1}, \dots, \varepsilon_1, x_i, x_{i-1}, \dots, x_1) = 0$ (i.e. the conditional expected mean value of the error term is zero). The conditional and the unconditional expectations are equal when the error term is independent from the regressors by the law of iterated expectations (Bailey 2005 p. 59).

At this point we assume that the conditional and the unconditional expected mean of the error term are equal. This postulation is tested under assumptions 5 and 6.

We test the null hypothesis that the unconditional expected mean of the residual is zero with the Jarque-Bera test in EViews.

Table 6-29: Jarque-Bera test of the expected mean of the residual

Model	Percentile	t-statistic [Probability]
M - 9	Highest decile	-7,72E-15 [1,0000] N=140
M - 9	Upper half	-6,13E-15 [1,0000] N=696
M - 10	Lowest decile	1,86E-14 [1,0000] N=140
M - 10	Lower half	-3,05E-17 [1,0000] N=695
M - 11	Highest decile	1,24E-14 [1,0000] N=140
M - 11	Upper half	-2,90E-14 [1,0000] N=722
M - 12	Lowest decile	-1,10E-14 [1,0000] N=140
M - 12	Lower half	2,54E-14 [1,0000] N=669
M - 13	Highest decile	1,77E-15 [1,0000] N=140
M - 13	Upper half	8,46E-15 [1,0000] N=696
M - 14	Lowest decile	5,10E-16 [1,0000] N=140
M - 14	Lower half	2,70E-14 [1,0000] N=695

The Jarque-Bera tests show that we can't reject the null hypothesis that the unconditional expected mean of the residual is 0.

Regression assumption 5: Correlation with the error term

For each x_i, $cov(x_i, \varepsilon_i) = 0$ (i.e., each independent variable is uncorrelated with the error term).

This assumption is tested together with the regression assumption 6.

Regression assumption 6: Variance of the error term

At each set of values for the i independent variables, $x_i, x_{i-1}, \dots, x_1$, $var(\varepsilon_i | \varepsilon_{i-1}, \dots, \varepsilon_1, x_i, x_{i-1}, \dots, x_1\ ) = \sigma^2$ where σ^2 is a constant (i.e., the conditional variance of the error term is constant); this is known as the assumption of homoscedasticity.

To test assumptions 5 and 6 we used the White-test statistic (1980) which is a test of the null hypothesis of no correlation of the explanatory variables with the residual and no heteroscedasticity (EViews 7 2009 User Guide II, pp. 163-165).

Table 6-30: Homoscedasticity test

Model	Percentile	F-statistic	Obs*R-squared	Scaled explained SS	N
M - 9	Highest decile	0,598613 [0,6643]	2,439861 [0,6554]	5,688564 [0,2236]	140
M - 9	Upper half	5,883789 [0,0001]	22,92465 [0,0001]	130,0719 [0,0000]	696
M - 10	Lowest decile	2,950556 [0,0224]	11,25536 [0,0238]	25,52989 [0,0000]	140
M - 10	Lower half	2,717225 [0,0289]	10,77789 [0,0292]	36,25201 [0,0000]	695
M - 11	Highest decile	0,289990 [0,8841]	1,192672 [0,8793]	6,768302 [0,1487]	140
M - 11	Upper half	6,641361 [0,0000]	25,79497 [0,0000]	133,8110 [0,0000]	722
M - 12	Lowest decile	0,841368 [0,5013]	3,405229 [0,4924]	13,58500 [0,0087]	140
M - 12	Lower half	1,297802 [0,2694]	5,189724 [0,2684]	18.74463 [0,0009]	669
M - 13	Highest decile	1,214087 [0,3078]	4,861338 [0,3018]	9,631074 [0,0471]	140
M - 13	Upper half	3,307534 [0,0107]	13,07552 [0,0109]	44,10448 [0,0000]	696
M - 14	Lowest decile	1,617029 [0,1735]	6,400991 [0,1711]	7,973122 [0,0926]	140
M - 14	Lower half	2,346006 [0,0533]	9,325200 [0,0535]	67,17285 [0,0000]	695

“Obs” in the table above stands for observations.

The White test statistics show that the models in Table 6-30 don’t pass White’s test.

Since the models in Table 6-30 don't pass the White test we want to examine if this is due to covariance between the regressors and the residuals. The null hypothesis is that the covariance between the regressors and the residuals is zero.

Table 6-31: Covariance test of regressors with the residuals

Model	Percentile	Regressors	Covariance regressor with residuals	t-statistics	probability	N
M - 9	Highest decile	$\lvert R_{m,t,\text{MVW},high\ return}\rvert$ $(R_{m,t,\text{MVW},high\ return})^2$	1,12E-19 6,09E-21	1,24E-14 7,06E-15	1,0000 1,0000	140
M - 9	Upper half	$\lvert R_{m,t,\text{MVW},high\ return}\rvert$ $(R_{m,t,\text{MVW},high\ return})^2$	1,51E-19 1,08E-21	6,31E-14 6,72E-15	1,0000 1,0000	696
M - 10	Lowest decile	$\lvert R_{m,t,\text{MVW},low\ return}\rvert$ $(R_{m,t,\text{MVW},low\ return})^2$	-4,34E-19 -1,57E-20	-4,88E-14 -1,99E-14	1,0000 1,0000	140
M - 10	Lower half	$\lvert R_{m,t,\text{MVW},low\ return}\rvert$ $(R_{m,t,\text{MVW},low\ return})^2$	3,28E-20 -1,00E-20	1,32E-14 -6,19E-14	1,0000 1,0000	695
M - 11	Highest decile	$\lvert R_{m,t,\text{MVW},high\ volume}\rvert$ $(R_{m,t,\text{MVW},high\ volume})^2$	5,34E-20 2,21E-21	1,24E-14 7,06E-15	1,0000 1,0000	140
M - 11	Upper half	$\lvert R_{m,t,\text{MVW},high\ volume}\rvert$ $(R_{m,t,\text{MVW},high\ volume})^2$	2,70E-19 1,29E-20	9,29E-14 6,30E-14	1,0000 1,0000	722
M - 12	Lowest decile	$\lvert R_{m,t,\text{MVW},low\ volume}\rvert$ $(R_{m,t,\ \text{MVW},low\ volume})^2$	6,23E-21 -1,50E-22	2,62E-15 -1,51E-15	1,0000 1,0000	140
M - 12	Lower half	$\lvert R_{m,t,\text{MVW},low\ volume}\rvert$ $(R_{m,t,\text{MVW},low\ volume})^2$	7,96E-21 2,79E-21	4,32E-15 2,66E-14	1,0000 1,0000	669
M - 13	Highest decile	$\lvert R_{m,t,\text{MVW},high\ \sigma^2}\rvert$ $(R_{m,t,\text{MVW},high\ \sigma^2})^2$	-3,47E-19 -8,97E-21	-2,07E-14 -5,99E-15	1,0000 1,0000	140
M - 13	Upper half	$\lvert R_{m,t,\text{MVW},high\ \sigma^2}\rvert$ $(R_{m,t,\text{MVW},high\ \sigma^2})^2$	1,21E-20 1,17E-20	3,31E-15 4,42E-14	1,0000 1,0000	696
M - 14	Lowest decile	$\lvert R_{m,t,\text{MVW},low\ \sigma^2}\rvert$ $(R_{m,t,\text{MVW},low\ \sigma^2})^2$	-1,02E-20 -5,57E-22	-7,53E-15 -1,81E-14	1,0000 1,0000	140
M - 14	Lower half	$\lvert R_{m,t,\text{MVW},low\ \sigma^2}\rvert$ $(R_{m,t,\text{MVW},low\ \sigma^2})^2$	-1,64E-20 -2,24E-21	-1,65E-14 -7,94E-14	1,0000 1,0000	695

All models in Table 6-31 pass this test. We conclude that assumption 5 of zero covariance between the regressors and the error term is fulfilled while assumption 6 of homoscedasticity is not. The remedy to the violation of assumption 6 was a correction of the regression results using the Newey–West estimators. That means that the standard errors and as a consequence the t-values were adjusted to account for heteroscedasticity (EViews 7 2009 User Guide II, pp. 32-33).

Regression assumption 7: Autocorrelation.

For any two observations, (x_{1j} x_{2j},…,x_{ij}) and (x_{1h}, x_{2h},…,x_{ih}), $cov(\varepsilon_j, \varepsilon_h) = 0$ (i.e., error terms for different observations are uncorrelated); this assumption is known as lack of autocorrelation.

The null hypothesis is that there is not serial correlation. This is tested by means of the Breusch-Godfrey serial correlation Lagrange multiplier (LM) test.

Table 6-32: Breusch-Godfrey serial correlation LM test

Test Equation	Percentile	Lag	F-statistic [Prob.]	Obs*R-squared [Prob. Chi-Square]	N
M - 9	Highest decile	1	3,195315 [0,0761]	3,213787 [0,0730]	140
M - 9	Upper half	1	39,97875 [0,0000]	38,01368 [0,0000]	696
M - 10	Lowest decile	1	0,989753 [0,3216]	1,011502 [0,3145]	140
M - 10	Lower half	1	30,36463 [0,0000]	29,25485 [0,0000]	695
M - 11	Highest decile	1	11,48948 [0,0009]	10,90605 [0,0010]	140
M - 11	Upper half	1	54,73027 [0,0000]	51,13719 [0,0000]	722
M - 12	Lowest decile	1	9,428962 [0,0026]	9,076972 [0,0026]	140
M - 12	Lower half	1	86,47990 [0,0000]	76,98816 [0,0000]	669
M - 13	Highest decile	1	11,58931 [0,0009]	10,99336 [0,0009]	140
M - 13	Upper half	1	85,52823 [0,0000]	76,56011 [0,0000]	696
M - 14	Lowest decile	1	1,417540 [0,2359]	1,444180 [0,2295]	140
M - 14	Lower half	1	22,28787 [0,0000]	21,71644 [0,0000]	695

"Obs" in the table above stands for observations.

The test results show that we reject the null hypothesis of no autocorrelation for models M - 9, M - 10 (lower half), M - 11 , M - 12 , M - 13 , M - 14 (lower half) with 90 % or higher confidence interval. That means that assumption 7 is not fulfilled for these models. The remedy to the violation of assumption 7 is the correction of the regression results using the Newey–West estimators. That means that the standard errors and as a consequence the t-values were adjusted to account for autocorrelation (EViews 7 2009 User Guide II, pp. 32-33).

Regression assumption 8: Distribution of the error term

At each set of values for the i independent variables, the error term ε_i is normally distributed.

The null hypothesis: The standardized residuals are normally distributed.

Table 6-33: Jarque-Bera normality test

Test Equation	Percentile	Jarque-Bera [Probability]	N
M - 9	Highest decile	99,73704 [0,0000]	140
M - 9	Upper half	3184,245 [0,0000]	696
M - 10	Lowest decile	78,21020 [0,0000]	140
M - 10	Lower half	994,2820 [0,0000]	695
M - 11	Highest decile	679,8193 [0,0000]	140
M - 11	Upper half	2656,006 [0,0000]	722
M - 12	Lowest decile	324,6278 [0,0000]	140
M - 12	Lower half	1207,394 [0,0000]	669
M - 13	Highest decile	69,92857 [0,0000]	140
M - 13	Upper half	1010,715 [0,0000]	696
M - 14	Lowest decile	22,43301 [0,0000]	140
M - 14	Lower half	5031,875 [0,000]	695

The test results show no-normal distribution of the residuals for all models tested in Table 6-33. A violation of assumption 8 is not as serious as the violation of assumption of homoscedasticity and autocorrelation. Departure from normality does not impair inferences when the data set is large (Bhattacharyya and Johnson 1997, p. 359). Greene (2012, pp. 64-67) states that a normal distribution of the residual is not necessary for establishing results that allow statistical inference. This is because statistical inference can be based on the law of large numbers which concerns consistency and the central limit theorem which concerns the asymptotic distribution of the estimators.

Regression assumption 9: Stationarity.

The independent variables are stationary processes.

The null Hypothesis is that the time series have a unit root. This is assessed by means of the augmented Dickey-Fuller unit root test.

Table 6-34: Augmented Dickey-Fuller unit root test

Time Series	N	t-Statistic	Prob.
$R_{m,t}$	1358	-37,49063	0,0000
$CSAD_t$	1125	-2,831729	0,0542

The unit root tests show that we can reject the null hypothesis of a unit root at 1% significance level for $R_{m,t}$ and 10 % significance level for $CSAD_t$. So we consider the processes as stationary.

6.4 Testing the equity premium puzzle

The equity premium puzzle is based on the empirical fact that the difference between return on a risky asset and a risk free asset is too great compared to what is assumed to be a normal level of risk aversion.

Let the price of an asset be (Cochrane 2005, p. 6):

$P_t = E[\mathfrak{m}(P_{t+1} + d_{t+1})]$.

Dividing both sides by P_t we get:

$$\frac{P_t}{P_t} = E\left[\mathfrak{m}\left(\frac{P_{t+1} + d_{t+1}}{P_t}\right)\right] \to 1 = E[\mathfrak{m}R]$$

where R is the gross return.

The Euler equations are:

$$\left.\begin{array}{r} E[\mathfrak{m}\, R_m] - 1 = 0 \\ E[\mathfrak{m}\, R_f] - 1 = 0 \end{array}\right\} \to E[\mathfrak{m}R_e] = 0$$

where $\mathfrak{m} = \delta \times IMRS$ is the stochastic discount factor, δ is the subjective discount factor, $IMRS$ is the intertemporal marginal rate of substitution of consumption between periods and R_p is the risk premium,

$$E[\mathfrak{m}R_e] = 0 \to E[\mathfrak{m}]E[R_e] + cov(\mathfrak{m}, R_e) = 0 \to$$

$$E[\mathfrak{m}]E[R_e] + \rho(\mathfrak{m}, R_e)\sigma(\mathfrak{m})\sigma(R_e) = 0 \to$$

$$\sigma(\mathfrak{m})\rho(\mathfrak{m}, R_e) = -E[\mathfrak{m}]\frac{E[R_e]}{\sigma(R_e)} \to \sigma(\mathfrak{m}) = -\frac{1}{\rho(\mathfrak{m}, R_e)}E[\mathfrak{m}]\frac{E[R_e]}{\sigma(R_e)} \to$$

$$-\rho(\mathfrak{m}, R_e)\frac{\sigma(\mathfrak{m})}{E[\mathfrak{m}]} = \frac{E[R_e]}{\sigma(R_e)} \to \frac{\sigma(\mathfrak{m})}{E[\mathfrak{m}]} \geq \left|\frac{E[R_e]}{\sigma(R_e)}\right|$$

The last inequality is the HJ-bound (Cochrane 2005, p. 93).

For logarithmic utility functions the HJ-bound transforms to (Pennacchi 2008, p. 307):

$$\frac{|E[R_e]|}{\sigma(R_e)} \leq RRA\sigma_c$$

where RRA stands for the coefficient of relative risk aversion

$$RRA = -C_t\frac{U''(C_t)}{U'(C_t)}$$

and σ_c for the volatility of consumption growth.

The HJ-bound can be traced in the $E[\mathfrak{m}]$, $\sigma(\mathfrak{m})$ space using the following equation:

$$\sigma^2(\mathfrak{m}) = \left[1 - E[\mathfrak{m}]\boldsymbol{E}[\boldsymbol{R}]\right]^T \boldsymbol{\Sigma}^{-1}\left[1 - E[\mathfrak{m}]\boldsymbol{E}[\boldsymbol{R}]\right]$$

where T is the transposed, $\boldsymbol{R}$ is a $n \times 1$ vector of returns and $\boldsymbol{\Sigma^{-1}}$ is a $n \times n$ variance-covariance matrix. For $n = 1$ the expression simplifies to:

$$\sigma^2(\mathfrak{m}) = \left[1 - E[\mathfrak{m}]E[R]\right]^2 \frac{1}{\sigma^2(R)} \to \sigma(\mathfrak{m}) = \left|\left[1 - E[\mathfrak{m}]E[R]\right] \frac{1}{\sigma(R)}\right| \to$$

$$\sigma(\mathfrak{m}) = \left|\frac{1}{\sigma(R)} - \frac{E[R]}{\sigma(R)} E[\mathfrak{m}]\right|$$

The HJ-bound can be used for predicting a quantified relation between the equity premium and the stochastic discount factor and can be used for testing CCAP-models against this prediction.

Consumption capital asset pricing models connect together consumption with asset returns. They have certain common features such as an intertemporal utility function, a periodic utility function, a pricing kernel alias stochastic discount factor, a subjective discount factor which shows time preferences, a risk aversion coefficient and a relative risk aversion coefficient. In addition they make certain assumptions on the time separability of utility and the homogeneity or heterogeneity of consumers.

6.4.1 Hypotheses

We tested the following hypothesis:

H - 13: The stochastic discount factor specified by the power utility preferences explains the equity premium and satisfies the HJ-bounds.

6.4.2 Model specification

We expect also that the models tested are not misspecified. This consists of testing that

$$\frac{1}{T}\sum_{t=1}^{T} E[f_i(\theta, X_t)] = 0$$

where θ is the parameter to be estimated and X a variable which denotes a data series.

The null hypothesis that the model is not misspecified cannot be rejected at a given confidence level q if $J < \chi^2_{k-l,q}$ where χ^2 is a chi-squared distribution, k is the number of variables and l is the number of estimates. We calculated the J - statistic in the econometric program EViews by first running the equation:

$scalar\ jval = s1.@regobs * s1.@jstat$

Where "regobs" is the number of observations and $s1$ is the system of equations:

$$E_t\left[\mathfrak{m}_{t,t+1} R_{t+1,i} - 1\right] = 0$$

$$E_t\left[\mathfrak{m}_{t,t+1} R_{t+1,f} - 1\right] = 0$$

Then we run the equation

$scalar\ pval = 1 - @cchisq(jval, number)$

where "number" is a scalar showing the degrees of freedom obtained as:

$df = (nr\ instrumental\ variables + nr\ constants) \times nr\ equations\ -\ nr\ estimates$

and $@cchisq$ is the χ^2 distribution.

6.4.3 Methodology

Data sources

Our datasets consisted of quarterly data for gross return of stocks, three-months Norwegian treasury bills, consumption growth for non-durables and services, inflation and population. The data for stock returns was extracted from Datastream. The data for consumption for non-durables and services was provided by Statistics Norway (in Norwegian Statistisk Sentralbyrå, for short SSB). The data source for 3 months T-bills was Bank of Norway (NorgesBank). The data for a subsistence level of consumption was extracted from the minimum consumption budgets of the Norwegian Institute for Consumption research.(in Norwegian called Statens Institutt for Forbruksforskning, for short SIFO).

Choice of method

We used the Generalized Method of Moments (GMM).

GMM makes no assumption on normality of the error terms. The method allows for heteroscedasticity of unknown form (Pynnonen 2008, p. 35 and Sørensen 2007). GMM can be used for estimating parameters for models that cannot be solved analytically form the first order conditions (Verbeek 2004, p. 161). GMM is a semiparametric estimation method. In a semiparametric estimation there are no distributional assumptions (Newey, Powell and Walker 1990, Hansen and Singleton 1982, p. 1280). The estimator is based on general relationships which hold in the population, for instance orthogonality conditions (Greene 2012, pp. 439-440 and pp. 468-507). Euler equations in CCAPM are such orthogonality conditions.

We did a literature search and found no requirement for GMM on absence of multicollinearity. Multicollinearity between the subjective discount rate which shows time preferences and the risk aversion can be used to evaluate the different models against a theoretical prediction. Research has presented evidence that the subjective discount rate and risk aversion are related (Praag and Booij 2003).

MM requires that the number of instrumental variables is at least as high as the number of parameters to be estimated. In other words, the dimension of $f(\theta)$ is at least as large as the dimension of θ, where $f(\theta)$ is the moment function and θ the parameter vector.

GMM assumes that the instrumental variables are uncorrelated to the error term:

$$\frac{1}{N}\sum_{i=1}^{N} \mathbf{z}_i' \, \varepsilon_i \overset{a}{\sim} N(0, \Sigma)$$

where $\mathbf{z}_i'$ are the transposes of the instrumental variables (Costa Dias 2008). EViews doesn't include a test of this assumption for a GMM equation system. However, using different sets of instrumental variables in a sufficient number so that the equation system was overidentified, we examined by means of the Hansen J-statistic (Hansen and Singleton 1982, pp. 1277-1278) which set was best specified.

The validity of the model restrictions is tested by Hansen J-statistic of overidentifying restrictions. The minimized distance $J \equiv n\boldsymbol{g}_n(\boldsymbol{\theta}_0)'W^{-1}\boldsymbol{g}_n(\boldsymbol{\theta}_0)$ follows a χ^2 distribution with $L - K$ degrees of freedom (Hayashi 2000, p. 481).

The null hypothesis in the J-test is that $f(\boldsymbol{\theta}_0) = 0$. The probability value in the J-test shows the probability that it exists a $\boldsymbol{\theta}_0$ so that $f(\boldsymbol{\theta}_0) = 0$. In practice the GMM method uses some numerical minimization algorithm for finding the value of $\boldsymbol{\theta}_0$ which minimizes the test restrictions. In the setting of CCAPM tests the test restrictions are the Euler equations.

GMM is based on moment conditions written as orthogonality conditions of the form $E[f_i(\boldsymbol{Z_t}, \boldsymbol{u_t})] = 0$ between the error term $\boldsymbol{u_t}$ and the instrumental variables (also called instruments) $\boldsymbol{Z_t}$ (EViews 7 User's Guide II, pp. 67-71). Two vectors are orthogonal to each other when their inner product is equal to zero. In our setting it means that the instrumental variables are uncorrelated to the error terms. The number of instruments has to be at least equal to or greater than the number of parameters to be estimated. When the number of instruments is greater than the number of parameters in a system of equations, we have overidentification. If the model is correctly specified the overidentifying restrictions should be close to zero. Overidentification makes it possible to evaluate how well a model is specified using the J – statistic.

GMM facilitates corrections for autocorrelation and conditional heteroscedasticity (Cochrane 2005 pp. 207-208). Using the HAC - Newey West method for estimating the covariance matrix $[\boldsymbol{Z_t}\ \boldsymbol{u_t}]$ we get estimators which are corrected for autocorrelation and heteroscedasticity.

Let the Euler equations, that is the first order optimization conditions of the utility maximization problem given a budget constraint, be written in the form

$E[f_i(\boldsymbol{\theta}, \boldsymbol{X_t})] = 0$	6-3

With the empirical counterpart being:

$\boldsymbol{\zeta} = \frac{1}{T}\sum_{t=1}^{T} E\left[f_i\left(\widehat{\boldsymbol{\theta}}, \boldsymbol{X_t}\right)\right] = 0$	6-4

The GMM estimator of $\boldsymbol{\theta}$ minimizes the expression

$$argmin\left\{\boldsymbol{\zeta}(\widehat{\boldsymbol{\theta}})'\boldsymbol{W}^{-1}\boldsymbol{\zeta}(\widehat{\boldsymbol{\theta}})\right\} \tag{6-5}$$

where W is a weighting matrix that has to be determined (Greene 1993, p. 376).

The empirical equivalent equations to be used in our tests are:

$$\left\|\frac{1}{T}\sum_{t=1}^{T}\mathfrak{m}_{t,t+1}\left(R_{t+1,i}\right)-1\right\|=0$$
$$\left\|\frac{1}{T}\sum_{t=1}^{T}\mathfrak{m}_{t,t+1}\left(R_{t+1,f}\right)-1\right\|=0 \tag{6-6}$$

Where

$\mathfrak{m}_t$ is the stochastic discount factor (pricing kernel).

$\hat{\alpha}$, $\hat{\gamma}$ and $\hat{\delta}$ are the estimators for the risk aversion coefficient $\gamma=1-\alpha$ and the subjective discount factor δ.

The use of GMM for testing consumption capital asset pricing models was pioneered by Hansen and Singleton (1982).

Parameter assumptions

We expect on theoretical grounds that the subjective discount factor $\delta<1$, $\delta\neq 0$ the risk aversion coefficient $\gamma<1$, $\gamma\neq 0$, $\alpha=1-\gamma>0$ and $\alpha\neq 1$. The rational for $\delta<1$ is that the time preference rate θ is a positive number so that $\delta=\frac{1}{1+\theta}<1$ meaning that a consumer prefers to consume today rather than tomorrow. The rational for $\gamma<1$ and $\alpha>0$ is based on the assumption of the consumer being risk averse. We want $\gamma\neq 0$ and $\alpha\neq 1$ because the utility functions are of the form $\frac{C^{\gamma}}{\gamma}$, alternatively written as $\frac{C^{1-\alpha}}{1-\alpha}$ or variants of them.

Preparing the data used in the regressions

We added together consumption of non-durables and services and adjust the numbers to capita dividing by the population. Then we calculated consumption growth dividing the consumption at time $t+1$ with consumption at time t. To adjust for inflation we multiplied with $\frac{CPI_t}{CPI_{t+1}}$ where CPI_t is the consumption price index.

The gross return for stocks is based on the Morgan Stanley Index for Norwegian stocks and includes dividends. This index is widely used. We downloaded quarterly data from the financial database Datastream. The calculation we did was dividing the stock index at time $t+1$ with the index at time t and adjusting for inflation. We calculated in a similar manner the gross return for three months Norwegian treasury bills using data from Bank of Norway (Norges Bank). Population data was manually extracted from the statistics year books of Statistics. Inflation data was downloaded from the financial database Datastream.
Using these datasets we calculated the Sharpe ratio. We estimated the HJ-bound, the risk aversion parameter and the relative risk aversion parameter in the econometric program EViews by means of the general method of moments (GMM).

The average adjusted for inflation Morgan Stanley total return index for Norwegian Stocks is 1,030. The standard deviation adjusted for inflation for the same index is 0,130.

The average adjusted for inflation three months Norwegian T-bills gross return is 1,007 and the respective standard deviation is 0,009.

The average adjusted for inflation Equity Premium is 0,023 and its standard deviation is 0,130.

In testing the hypothesis that the stochastic discount factor specified by the power utility preferences explains the risk premium and satisfies the HJ-bounds, we used CCAP-models with isoelastic utility functions. With isoelastic we mean a utility function with constant relative risk aversion (CRRA). This class of utility functions implies that risk aversion is independent of the wealth level.

6.4.4 Lucas' consumption capital asset pricing model

6.4.4.1 The theoretical model structure

As a reference model we test first the Lucas CCAPM (1978) which doesn't make any habit formation assumptions.

In 4-41 we formulated the dynamic consumption optimization problem in discrete time and derived the corresponding Euler equations (the first order condition) for this problem (4-46)

Lucas CCAP-model uses a power utility function of the form:

$U_t = E_t\left[\sum_{j=0}^{\infty} \delta^j \left(\frac{(C_{t+j})^{\gamma}}{\gamma}\right)\right]$	6-7

The periodic utility function is:

$$\frac{C^{\gamma}}{\gamma}$$

where C is the aggregate consumption of non-durables and services per capita and γ is the risk aversion coefficient.

Plugging the utility function in the Euler equation $[\mathfrak{m}R] - 1 = 0$, where the pricing kernel (that is the stochastic discount factor) $\mathfrak{m} = \delta\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1}$, we get the following model:

$E_t\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1} R_{t+1,m} - 1\right] = 0$ $E_t\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1} R_f - 1\right] = 0$	M - 15

where $R_{t+1,i}$ denotes the risky asset and R_f denotes the risk-free rate.

So

$E_t\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1} R_{t+1,m} - 1 - \delta\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1} R_f + 1\right] = 0 \rightarrow$ $E_t\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1} \left(R_{t+1,m} - R_f\right)\right] = 0$	6-8

The first and second derivatives of the period utility function used in Lucas model are:

$$U(C_t) = \frac{{C_t}^{\gamma}}{\gamma}$$

$$U'(C_t) = \frac{\gamma {C_t}^{\gamma-1}}{\gamma} = {C_t}^{\gamma-1}$$

$$U''(C_t) = (\gamma - 1){C_t}^{\gamma-1-1} = (\gamma - 1){C_t}^{\gamma-2}$$

The relative risk aversion coefficient RRA is:

$$RRA = -C_t \frac{U''(C_t)}{U'(C_t)} = -C_t \frac{(\gamma - 1){C_t}^{\gamma-2}}{{C_t}^{\gamma-1}} = -(\gamma - 1)\frac{C_t {C_t}^{\gamma-2}}{{C_t}^{\gamma-1}} = (1-\gamma)\frac{{C_t}^{\gamma-2+1}}{{C_t}^{\gamma-1}} = (1-\gamma)$$

The subjective discount factor is δ. The subjective discount rate which shows time preferences is ϑ in $\delta = \frac{1}{1+\vartheta}$.

The model assumes homogeneous preferences and state and time separable utility in a representative agent setting.

There are two ways to proceed.

1) Estimate δ and γ simultaneously from 6-7

or

2) Estimate γ from 6-8 and then use $\hat{\gamma}$ to estimate δ as a two stage process.

We opted for the first alternative. This procedure was used for all CCAP-models we tested.

We tested Lucas' CCAPM using in succession sets of three instrumental variables (in short IV) lagged one, two, three, four and five periods. We used also one set with all lagged periods. For instance, using instrumental variables lagged five periods we get the following system of Euler equations:

$$\begin{bmatrix} E_t\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1} R_{t+1,m}-1\right] \\ E_t\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1} R_{t+1,f}-1\right] \\ E\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1} R_{t+1,m}-1 \,\middle|\, \frac{C_{t-4}}{C_{t-5}}\right] \\ E\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1} R_{t+1,m}-1 \,\middle|\, R_{t-4,m}\right] \\ E\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1} R_{t+1,m}-1 \,\middle|\, R_{t-4,f}\right] \\ E\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1} R_{t+1,f}-1 \,\middle|\, \frac{C_{t-4}}{C_{t-5}}\right] \\ E\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1} R_{t+1,f}-1 \,\middle|\, R_{t-4,m}\right] \\ E\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1} R_{t+1,f}-1 \,\middle|\, R_{t-4,f}\right] \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

We will call such a system of Euler equations a model variant. Each model variant is going to comprise of common Euler equations of the form $E[\mathfrak{m}R_t] = 0$ and Euler equations of the form $E[\mathfrak{m}R_t|z] = 0$ constructed using the instrumental variables z which vary from model variant to model variant.

6.4.4.2 Empirical test results

Our sample consisted of quarterly data for the period 3rd quarter 1978 to 2nd quarter 2012. The test results are as follows:

Table 6-35: Estimates for γ, α and δ in the model variants of Lucas model

Model variant	Instrumental Variables (IV) C, R_m, R_f	$\hat{\delta}$ (t-value) [p-value]	$\hat{\alpha}$ (t-value) [p-value]	$\hat{\gamma}$ (t-value) [p-value]	Hansen J-statistic (J-value) [p-value]	N
M - 16	C, R_m, R_f lagged one period 3 IV	0,988968 (582,418500) [0,0000]	0,126176 (2,267973) [0,0241]	0,873824 (15,70669) [0,0000]	0,057457 (7,814213) [0,252033]	136
M - 17	C, R_m, R_f lagged two periods 3 IV	0,985572 (620,02950) [0,0000]	-0,069909 (-0,337477) [0,7360]	1,069909 (5,164875) [0,0000]	0,106150 (14,330250) [0,026157]	135
M - 18	C, R_m, R_f lagged three periods 3 IV	0,987563 (459,541100) [0,0000]	-0,117351 (-1,448437) [0,1487]	1,117351 (13,79121) [0,0000]	0,063757 (8,543438) [0,200930]	134
M - 19	C, R_m, R_f lagged four periods 3 IV	0,988405 (589,315300) [0,0000]	0,027917 (1,475551) [0,1413]	0,972083 (51,37885) [0,0000]	0,057720 (7,676760) [0,262750]	133
M - 20	C, R_m, R_f lagged five periods 3 IV	0,989645 (566,991800) [0,0000]	0,070661 (1,890774) [0,0598]	0,929339 (24,86768) [0,0000]	0,060594 (7,998408) [0,238222]	132
M - 21	C, R_m, R_f lagged one to five periods together 15 IV	0,984359 (34,986300) [0,0000]	0,008742 (0,528426) [0,5977]	0,991259 (59,92157) [0,0000]	0,152760 (20,164320) [0,912191]	132

We see from the above table that the estimates for γ, α and δ in the model variants M - 16, M - 20 satisfy the theoretical expectations for these parameters, that is $\gamma < 1$, $\gamma \neq 0$, $\alpha > 0$, $\alpha \neq 1$, $\delta < 1$ and $\delta \neq 0$ with 90% to 95% confidence interval. The probability value ("pval") of the $J - statistic$ for the models M - 16, M - 20 was 0,238 and 0,252 correspondingly. That means that we cannot reject the hypothesis that the models we used are valid with 90 % or higher confidence interval.

6.4.4.3 *Testing against the basic HJ-bound*

The diagram below shows the HJ-bound using Norwegian data for Lucas' CCAPM.

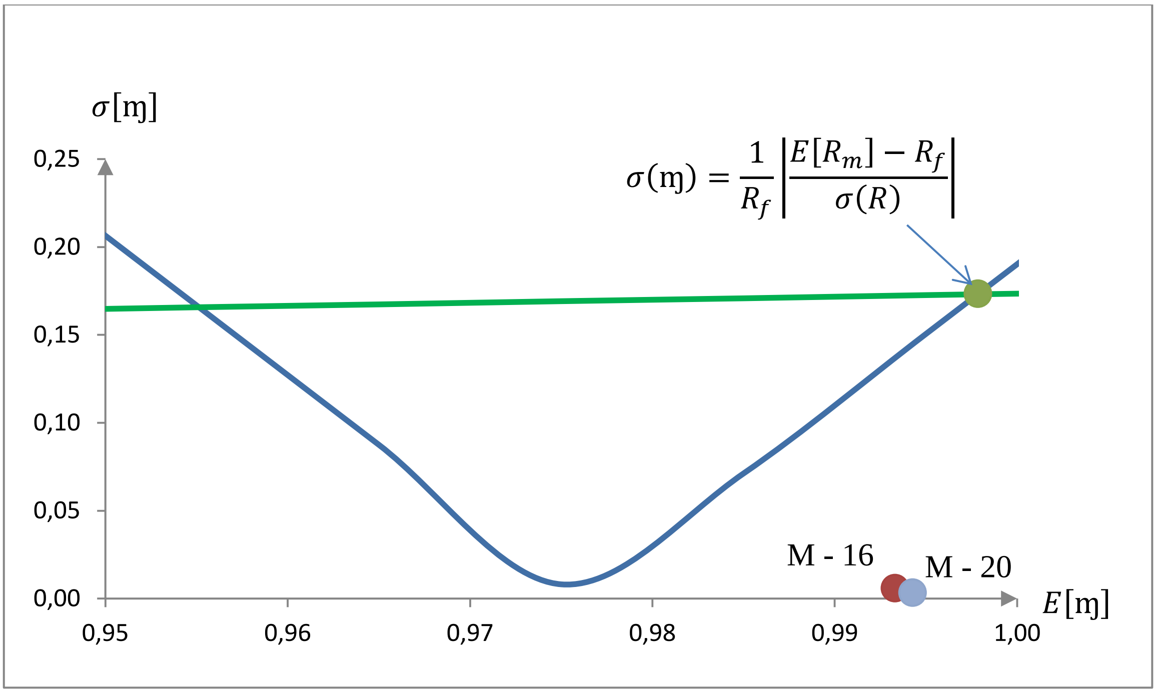


Figure 6-9: The basic HJ-bound using Norwegian data for Lucas' CCAPM

Plugging the estimates for γ and δ in the stochastic discount factor $\delta\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1}$ we calculated $E_t\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1}\right]$ and $\sigma\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1}\right]$ which we then plotted in their respective σ, E_t space for the model variants M - 16 and M - 20. The corresponding coloured dots lie below the green line, which means that Lucas' CCAPM doesn't satisfy the basic HJ-bound.

The green dot is the Sharpe ratio multiplied with $\frac{1}{R_f}$ which is the price of one unit of the risk free asset. At this point is

$$E_t\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1}\right] = \frac{1}{E_t\left[\left(R_{t+1,f}\right)\right]}$$

The blue line is constructed using the equation:

$$\sigma(ɱ) = \left|\left[1 - E[ɱ]E[R]\right]\frac{1}{\sigma(R)}\right|$$

The green line is constructed using the equation:

$$\sigma\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1}\right] = \frac{E_t\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1}\right] E_t\left[\left(R_{t+1,m} - R_{t+1,f}\right)\right]}{\sigma\left[\delta\left(R_{t+1,m}\right), \delta\left(R_{t+1,f}\right)\right]}$$

The basic HJ-bound is (Cochrane 2005 p.457):

$$\frac{E[R_e]}{\sigma(R_e)} \leq \frac{\sigma(\mathfrak{m})}{E[\mathfrak{m}]} \approx (1-\gamma)\sigma\left(\frac{C_{t+1}}{C_t}\right)$$

where

$$E[\mathfrak{m}] = \frac{1}{R_f}$$

Using the Norwegian data for consumption, stock market return and three months Treasury bills in the period 1978 to 2012, we found $R_f = 1{,}007189$, $E[\mathfrak{m}] = \frac{1}{R_f} = 0{,}992862$, $E[R_e] = 0{,}022653$ and $\sigma(R_e) = 0{,}129912$ (all numbers adjusted for inflation). In order to satisfy the HJ-bound the models should yield either $\sigma(\mathfrak{m}) = 0{,}173$ or $\gamma = -2{,}796246$

In EViews however we found γ to be either 0,873824 or 0,929339 whereas $\sigma(\mathfrak{m})$ is estimated to be either 0,005770 or 0,000398. So the theoretical prediction of the HJ-bound doesn't agree with the GMM-estimates. This demonstrates the equity premium puzzle with Norwegian data.

6.4.4.4 *Testing against the extended HJ-bound*

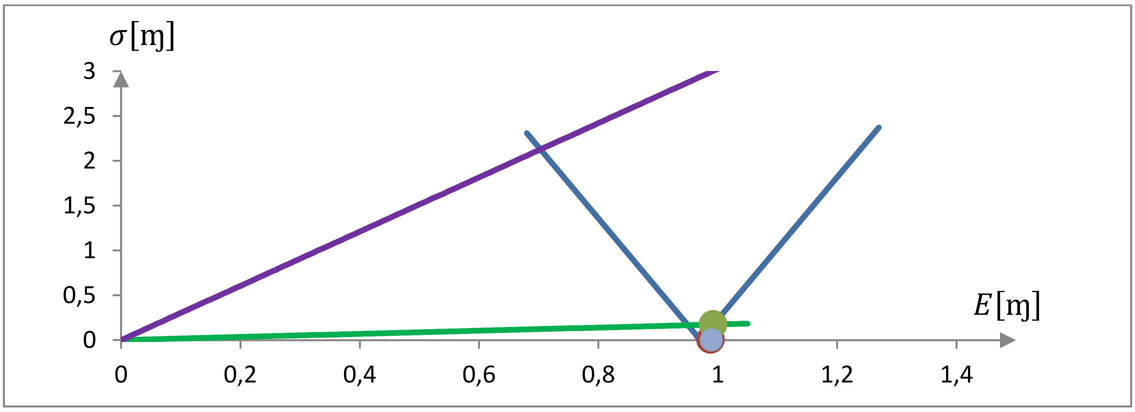


Figure 6-10: The correlation between the stochastic discount factor and the equity premium

Constructing the basic HJ-bound we implicitly assumed that

$$\left|\rho\left(\delta\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1}, R_{t+1,i} - R_{t+1,f}\right)\right| = 1.$$

The correlation between the stochastic discount factor and the equity premium for the valid model variant M - 16 is

$$\left|\rho\left(\delta\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1}, R_{t+1,m} - R_{t+1,f}\right)\right| = 0{,}057614$$

With valid variants we mean model variants which satisfy the conditions on γ, α, δ and are not misspecified. This applies to all tested CCAP-models.

Plugging the correlation number for ρ in the equation below

$$\sigma\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1}\right] = \frac{E_t\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1}\right]\left|E_t\left[\left(R_{t+1,m} - R_{t+1,f}\right)\right]\right|}{\sigma\left[\left(R_{t+1,m} - R_{t+1,f}\right)\right]} \times \frac{1}{\left|\rho\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1}, R_{t+1,m} - R_{t+1,f}\right]\right|}$$

we get the purple line in Figure 6-10. The coloured dots corresponding to model variants M - 16 and M - 20 lie well below the purple line, which means that Lucas' CCAPM doesn't satisfy the extended HJ-bound.

The correlation $\rho(\mathfrak{m}, R_e) = 0{,}057614$ in

$$\frac{E[R_e]}{\sigma(R_e)}\frac{1}{\rho(\mathfrak{m}, R_e)} \leq \frac{\sigma(\mathfrak{m})}{E[\mathfrak{m}]} \approx (1-\gamma)\sigma\left(\frac{C_{t+1}}{C_t}\right)$$

implies that $\gamma = -64{,}89101$

The above adjustment for correlation between consumption growth and the equity premium demonstrates the correlation puzzle with Norwegian data.

Based on the above calculations we see that the inequalities $\frac{E[R_e]}{\sigma(R_e)} \leq \frac{\sigma(\mathfrak{m})}{E[\mathfrak{m}]}$ and $\frac{E[R_e]}{\sigma(R_e)} \frac{1}{\rho(\mathfrak{m},R_e)} \leq \frac{\sigma(\mathfrak{m})}{E[\mathfrak{m}]}$ are not satisfied. Thus we don't find support for the H - 13 hypothesis that the stochastic discount factor specified by the power utility preferences explains the risk premium and complies with the HJ -bounds.

6.4.5 External habit ratio consumption capital asset pricing model

6.4.5.1 The theoretical model structure

Abel (1990, p.144) proposes an external habit model of the ratio form (Campbell et al. 1997, p. 328).

In this model the utility function is of the form:

$U_t = \sum_{j=0}^{\infty} \delta^j u(C_{t+j}, v_{t+j})$	6-9

$v_{t+j} \equiv [\mathcal{C}_{t-1}^{D} C_{t-1}^{1-D}]^{\kappa}$, $\kappa \geq 0$, $D \geq 0$

where

v_{t+j} is a preference variable

$\mathcal{C}_t$ is consumer's own consumption

C_t is the aggregate consumption per capita

D is a parameter that takes the values 1 for the internal habit variant of the model or 0 for the external habit variant of the model

In this thesis we examined the case $D = 0$ which implies that $v_{t+j} \equiv \mathcal{C}_{t-1}^{\kappa}$

Using the periodic utility function

$$u(C_{t+j}, v_{t+j}) = \frac{\left(\frac{C_{t+j}}{\mathcal{X}_{t+j}}\right)^{\gamma} - 1}{\gamma}$$

we get

$$U_t = \sum_{j=0}^{\infty} \delta^j \frac{\left(\frac{C_{t+j}}{\mathcal{X}_{t+j}}\right)^{\gamma} - 1}{\gamma}$$

Alternatively can be written as:

$$U_t = \sum_{j=0}^{\infty} \delta^j \frac{\left(\frac{C_{t+j}}{\mathcal{X}_{t+j}}\right)^{1-\alpha} - 1}{1-\alpha}$$

where $\mathcal{X}_{t+1} = C_t^{\kappa}$, κ is a parameter that shows the degree consumption preferences depend on past consumption, the risk aversion coefficient $\gamma = 1 - \alpha$, $\gamma < 1$ and $\alpha > 0$,.

The pricing kernel is

$\delta\left(\frac{C_t}{C_{t-1}}\right)^{\kappa(-\gamma)}\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1}$	6-10

The stochastic discount factor is the same as the pricing kernel. The subjective discount factor is δ. The subjective discount rate which shows time preferences is ϑ in $\delta = \frac{1}{1+\vartheta}$

The utility function and its first and second derivative is:

$$U(C_t, C_{t-1}^{\kappa}) = \frac{\left(\frac{C_t}{C_{t-1}^{\kappa}}\right)^{1-\alpha}}{1-\alpha} = \frac{{C_t}^{1-\alpha}\left(\frac{1}{C_{t-1}^{\kappa}}\right)^{1-\alpha}}{1-\alpha}$$

Assuming C_{t-1}^{κ} to be constant we get:

$$\frac{\partial U}{\partial C_t} = (1-\alpha)\frac{{C_t}^{1-\alpha-1}\left(\frac{1}{C_{t-1}^{\kappa}}\right)^{1-\alpha}}{1-\alpha} = {C_t}^{-\alpha}\left(\frac{1}{C_{t-1}^{\kappa}}\right)^{1-\alpha}$$

$$\frac{\partial^2 U}{\partial {C_t}^2} = -\alpha {C_t}^{-\alpha-1}\left(\frac{1}{C_{t-1}^{\kappa}}\right)^{1-\alpha} = -\alpha {C_t}^{-\alpha-1}\left(\frac{1}{C_{t-1}^{\kappa}}\right)^{1-\alpha}$$

The relative risk aversion coefficient RRA is:

$$RRA = -C_t\frac{-\alpha {C_t}^{-\alpha-1}\left(\frac{1}{C_{t-1}^{\kappa}}\right)^{1-\alpha}}{{C_t}^{-\alpha}\left(\frac{1}{C_{t-1}^{\kappa}}\right)^{1-\alpha}} = \alpha$$

The Euler equation for the risky asset is:

$$E_t\left[\delta\frac{\frac{\partial U}{\partial C_{t+1}}}{\frac{\partial U}{\partial C_t}}R_{m,t}+1\right]-1=0 \rightarrow E_t\left[\delta\frac{{C_{t+1}}^{-\alpha}\left(\frac{1}{C_t^{\kappa}}\right)^{1-\alpha}}{{C_t}^{-\alpha}\left(\frac{1}{C_{t-1}^{\kappa}}\right)^{1-\alpha}}R_{m,t+1}\right]-1=0 \rightarrow$$

$$E_t\left[\delta\frac{{C_{t+1}}^{-\alpha}}{{C_t}^{-\alpha}}\frac{C_t^{-\kappa(1-\alpha)}}{C_{t-1}^{-\kappa(1-\alpha)}}R_{m,t+1}\right]-1=0 \rightarrow E_t\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha}\left(\frac{C_t}{C_{t-1}}\right)^{-\kappa(1-\alpha)}R_{m,t+1}\right]-1=0 \rightarrow$$

$$E_t\left[\delta\left(\frac{C_t}{C_{t-1}}\right)^{\kappa(\alpha-1)}\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha}R_{m,t+1}\right]-1=0 \rightarrow E_t\left[\delta\left(\frac{C_t}{C_{t-1}}\right)^{\kappa(-\gamma)}\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1}R_{m,t+1}\right]-1=0$$

Campbell et al. (1997, p. 328) impose the condition $\kappa(\alpha - 1) > 0$ or $\kappa(-\gamma) > 0$. It tells us that an increase in yesterday's consumption increases the current marginal utility of consumption.

Alternatively, the Euler equations can be written as the following model:

$E_t\left[\delta\left(\frac{\mathcal{X}_{t+1}}{\mathcal{X}_t}\right)^{-(1-\alpha)}\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha} R_{m,t+1}\right] - 1 = 0$ $E_t\left[\delta\left(\frac{\mathcal{X}_{t+1}}{\mathcal{X}_t}\right)^{-(1-\alpha)}\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha} R_{f,t+1}\right] - 1 = 0$	M - 22

Abel's model accommodates nonseparability of utility. Otherwise the model assumes homogeneous preferences in a representative agent setting.

C_t^{κ} shows the effect of past aggregate consumption on today's utility. When $\kappa \to -\infty$ then $C_{t-1}^{\kappa} \to 0$ and when $\kappa \to \infty$ then $C_{t-1}^{\kappa} \to +\infty$. To satisfy the condition $\kappa(\alpha - 1) > 0$ under the assumption that $\alpha > 0$, $\alpha \neq 1$, then:

If $\alpha \in (0,1)$ then $\alpha - 1 < 0$. So we should have $\kappa < 0$.

If $\alpha > 1$ then $\alpha - 1 > 0$. So we should have $\kappa > 0$.

In terms of γ the condition becomes $\kappa(-\gamma) > 0$. Assuming $\gamma < 1$, $\gamma \neq 0$, then:

If $\gamma \in (0,1)$ then $-\gamma < 0$. So we should have $\kappa < 0$.

If $\gamma < 0$ then $-\gamma > 0$. So we should have $\kappa > 0$.

Plausible intervals for values of κ for C_t equal to 10 000 NOK are in the table below.

Table 6-36: Consumption raised to the nonseparability of utility parameter κ

C_t	κ	C_t^κ
10 000	1	10 000
10 000	0,1	3
10 000	0,001	1
10 000	0,00001	1
10 000	-0,00001	0,999908
10 000	-0,001	0,990832
10 000	-0,01	0,912011
10 000	-0,1	0,398107
10 000	-1	0,000100

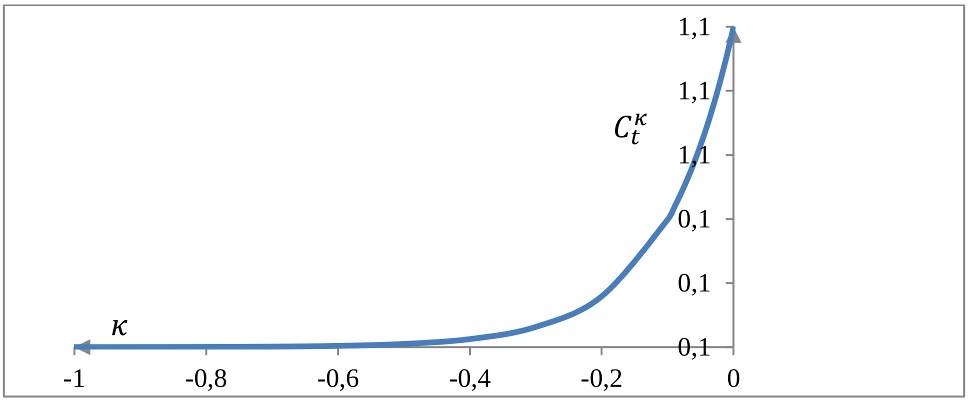


Figure 6-11: Utility of consumption from earlier period due to habit formation C_t^κ when $\kappa \in [-1{,}0)$

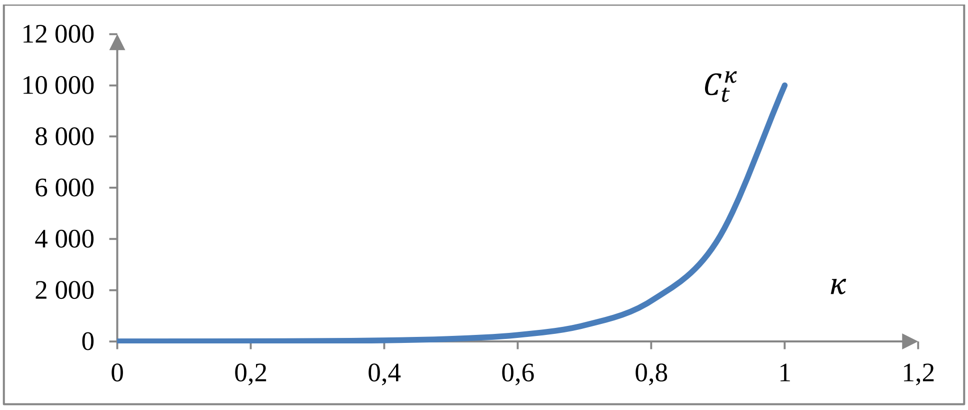


Figure 6-12: Utility of consumption from earlier period due to habit formation C_t^κ when $\kappa \in (0{,}1]$

For the external ratio habit model Abel (1990) examines the case $D = 0$ and $\kappa = 1$ in $v_{t+j} \equiv [C_{t-1}^{D} C_{t-1}^{1-D}]^{\kappa}$. In this thesis we investigated three cases: $\kappa = -1$, $\kappa = 1$ and $\kappa = -0{,}001$. The last one is for investigating the case where κ approaches 0.

6.4.5.2 Empirical Test results, the case $D = 0$ and $\kappa = -1$

The test results setting $D = 0$ and $\kappa = -1$ are the following:

Table 6-37: Estimates for γ, α and δ in the model variants of Abel's CCAPM

Model variant	Instrumental Variables (IV) C, R_m, R_f	$\hat{\delta}$ (t-value) [p-value]	$\hat{\alpha}$ (t-value) [p-value]	$\hat{\gamma}$ (t-value) [p-value]	Hansen J-statistic (J-value) [p-value]	$\kappa(\alpha - 1)$	N
M - 23	R_m, R_f lagged one period C lagged two periods 3 IV	0,995073 (338,834700) [0,0000]	0,990211 (10,623310) [0,0000]	0,009789 (0,105017) [0,91649]	0,041085 (5,546475) [0,475858]	0,009789	135
M - 24	C, R_m, R_f lagged two periods 3 IV	0,987540 (416,634800) [0,0000]	0,804420 (8,701636) [0,0000]	0,195580 (2,115648) [0,0353]	0,121248 (16,368480) [0,011907]	0,19558	135
M - 25	C, R_m, R_f lagged three periods 3 IV	0,984147 (278,638100) [0,0000]	0,022774 (0,176809) [0,8598]	0,977226 (7,586922) [0,0000]	0,066051 (8,850834) [0,182141]	0,977226	134
M - 26	C, R_m, R_f lagged four periods 3 IV	0,974109 (290,585000) [0,0000]	-0,651314 (-5,560328) [0,0000]	1,651314 (14,097360) [0,0000]	0,063404 (8,432732) [0,208081]	1,651314	133
M - 27	C, R_m, R_f lagged five periods 3 IV	1,003295 (285,431900) [0,0000]	1,954244 (13,490090) [0,0000]	-0,954244 (-6,587121) [0,0000]	0,062191 (8,209212) [0,223173]	-0,954244	132
M - 28	C lagged two to five periods, R_m, R_f lagged one to five periods, together 14 IV	0,979987 (558,701300) [0,0000]	0,408756 (10,802430) [0,0000]	0,591244 (15,625180) [0,0000]	0,194649 (25,693668) [0,589881]	0,591244	132

We are looking for variants of Abel's model with $D = 0$ and $\kappa = -1$ which satisfy the conditions $\kappa(\alpha - 1) > 0$, $\gamma < 1$, $\gamma \neq 0$, $\alpha > 0$, $\alpha \neq 1$, $\delta < 1$ and $\delta \neq 0$. The model variant which satisfy all the above criteria with 99% confidence interval is the M - 28. The probability value ("pval") of the $J - statistic$ for M - 28 is 0,589881. That means that we cannot reject the hypothesis that this model variant is not misspecified with 90 % or higher confidence interval.

6.4.5.3 *Testing against the basic HJ-bound*

The diagram below shows the HJ-bound using Norwegian data for Abel's CCAPM with non-separable utility parameter $\kappa = -1$.

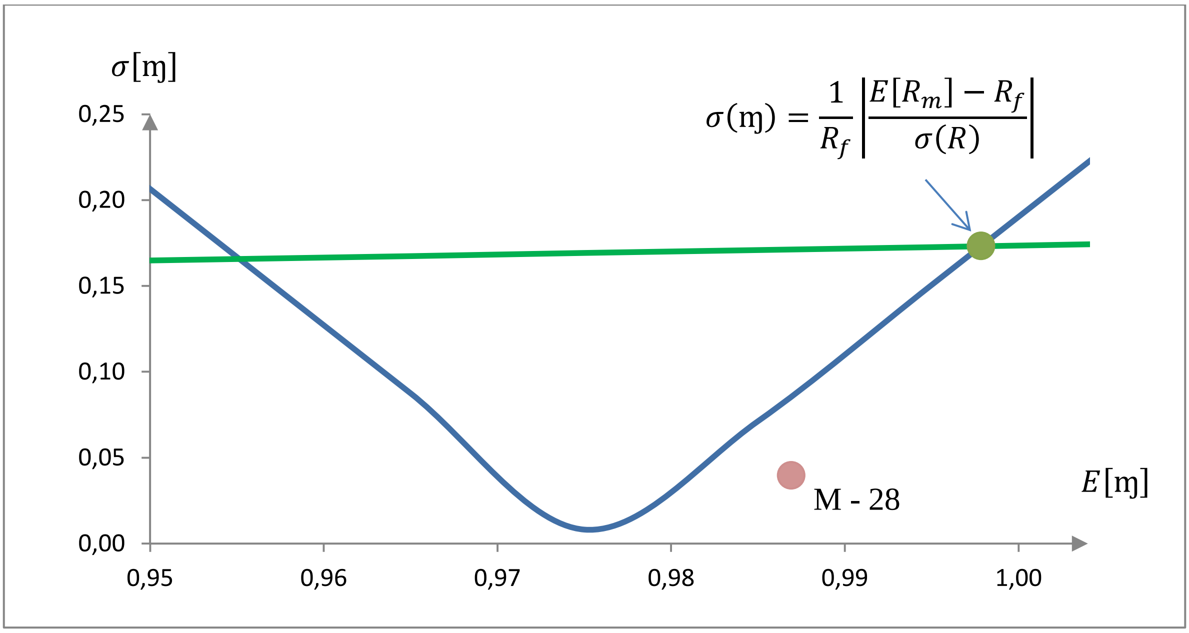


Figure 6-13: The basic HJ-bound using Norwegian data for Abel's CCAPM with $\kappa = -1$

Plugging the estimates for γ and δ in the stochastic discount factor $\delta\left(\frac{C_t}{C_{t-1}}\right)^{\kappa(-\gamma)}\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1}$ we calculated $E_t\left[\delta\left(\frac{C_t}{C_{t-1}}\right)^{\kappa(-\gamma)}\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1}\right]$ and $\sigma\left[\delta\left(\frac{C_t}{C_{t-1}}\right)^{\kappa(-\gamma)}\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1}\right]$ which we then plotted in their respective $\sigma[\mathfrak{m}], E[\mathfrak{m}]$ space for the model variant M - 28. The corresponding dot lies below the green line, which means that Abel's CCAPM with $\kappa = -1$ doesn't satisfy the basic HJ-bound.

The blue line, the green line and the green dot point in the bisection of the blue and green line are constructed in the same way as for Lucas' CCAPM.

In order to satisfy the basic HJ-bound the model variants should yield either $\gamma = -2{,}796246$ or $\sigma(\mathfrak{m}) = 0{,}173$ (both calculated under Lucas model). In EViews however we found γ to be 0,591244 whereas $\sigma(\mathfrak{m})$ is estimated to be between 0,039565. So the theoretical prediction of the HJ-bound doesn't agree with the GMM-estimates.

6.4.5.4 Testing against the extended HJ-bound

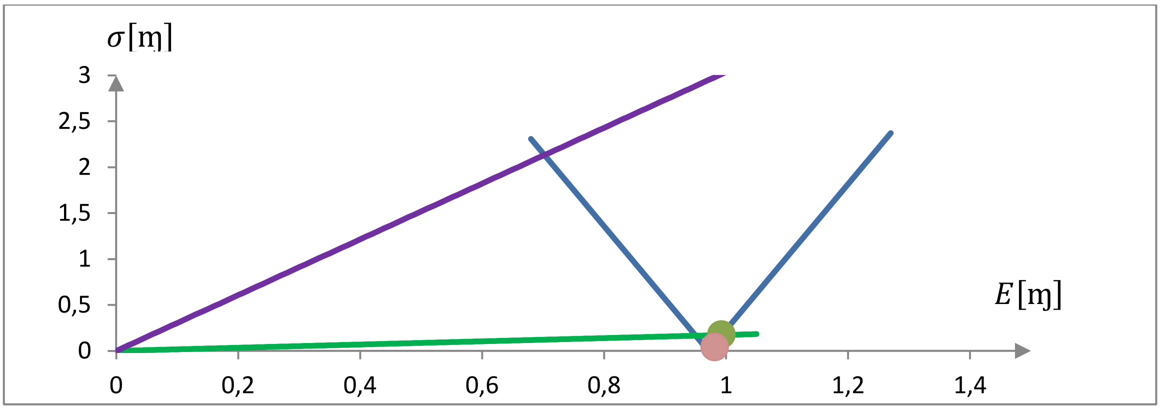


Figure 6-14: The correlation between the stochastic discount factor and the equity premium

Constructing the basic HJ-bound we implicitly assumed that

$$\left|\rho\left(\delta\left(\frac{C_t}{C_{t-1}}\right)^{\kappa(-\gamma)}\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1}, R_{t+1,m}-R_{t+1,f}\right)\right| = 1$$

The correlation between the stochastic discount factor and the equity premium for model M - 28 is

$$\left|\rho\left(\delta\left(\frac{C_t}{C_{t-1}}\right)^{\kappa(-\gamma)}\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1}, R_{t+1,m}-R_{t+1,f}\right)\right| = 0{,}053487$$

Plugging the correlation number for ρ in the equation below

$$\sigma\left[\delta\left(\frac{C_t}{C_{t-1}}\right)^{\kappa(-\gamma)}\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1}\right] =$$

$$\frac{E_t\left[\delta\left(\frac{C_t}{C_{t-1}}\right)^{\kappa(-\gamma)}\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1}\right]\left|E_t\left[\left(R_{t+1,m}-R_{t+1,f}\right)\right]\right|}{\sigma\left[\left(R_{t+1,i}-R_{t+1,f}\right)\right]} \times \frac{1}{\left|\rho\left[\delta\left(\frac{C_t}{C_{t-1}}\right)^{\kappa(-\gamma)}\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1}, R_{t+1,m}-R_{t+1,f}\right]\right|}$$

we get the purple line in Figure 6-14. The corresponding dot for the model variant M - 28 lies well below the purple line which means that Abel's CCAPM with $\kappa = -1$ doesn't satisfy the extended HJ-bound.

The correlation $\rho(ɱ, R_e) = 0{,}053487$ in

$$\frac{E[R_e]}{\sigma(R_e)}\frac{1}{\rho(ɱ, R_e)} \leq \frac{\sigma(ɱ)}{E[ɱ]} \approx (1-\gamma)\sigma\left(\frac{C_{t+1}}{C_t}\right)$$

implies that $\gamma = -69{,}97476$.

Thus we don't find support for the H - 13 hypothesis that the stochastic discount factor specified by the power utility preferences explains the risk premium and complies with the HJ bounds.

6.4.5.5 Empirical Test results, the case $D = 0$ and $\kappa = 1$

The test results setting $D = 0$ and $\kappa = 1$ are the following:

Table 6-38: Estimates for γ, α and δ in the model variants of Abel's CCAPM

Model variant	Instrumental Variables (IV) C, R_m, R_f	$\hat{\delta}$ (t-value) [p-value]	$\hat{\alpha}$ (t-value) [p-value]	$\hat{\gamma}$ (t-value) [p-value]	Hansen J-statistic (J-value) [p-value]	$\kappa(\alpha-1)$	N
M - 29	R_m, R_f lagged one period C lagged two periods 3 IV	0,995324 (336,9544) [0,0000]	0,985166 (7,849000) [0,0000]	0,014834 (0,118188) [0,9060]	0,038486 (5,195610) [0,518978]	-0,014834	135
M - 30	C, R_m, R_f lagged two periods 3 IV	0,989975 (460,7129) [0,0000]	0,884742 (8,522769) [0,0000]	0,115258 (1,110286) [0,2679]	0,120974 (16,331490) [0,012081]	-0,115258	135
M - 31	C, R_m, R_f lagged three periods 3 IV	0,990246 (360,8577) [0,0000]	0,049614 (0,342790) [0,7320]	0,950386 (6,566382) [0,0000]	0,096106 (12,878204) [0,045012]	-0,950386	134
M - 32	C, R_m, R_f lagged four periods 3 IV	0,995052 (418,0938) [0,0000]	0,340777 (13,600070) [0,0000]	0,659223 (26,30897) [0,0000]	0,037198 (4,947334) [0,550588]	-0,659223	133
M - 33	C, R_m, R_f lagged five periods 3 IV	0,996242 (419,1907) [0,0000]	0,677751 (33,946090) [0,0000]	0,322249 (16,14025) [0,0000]	0,042865 (5,658180) [0,462549]	-0,322249	132
M - 34	C lagged two to five periods, R_m, R_f lagged one to five periods, together 14 IV	0,987091 (964,6246) [0,0000]	0,539633 (46,60471) [0,0000]	0,460367 (39,75900) [0,0000]	0,183514 (24,223848) [0,669667]	-0,460367	132

We are looking for model variants of Abel's model with $D = 0$ and $\kappa = 1$ which satisfy the conditions $\kappa(\alpha - 1) > 0$, $\gamma < 1$, $\gamma \neq 0$, $\alpha > 0$, $\alpha \neq 1$, $\delta < 1$ and $\delta \neq 0$. None of the above model variants satisfy the criterium $\kappa(\alpha - 1) > 0$. Because of this we don't analyze further this case.

6.4.5.6 *Empirical Test results, the case $D = 0$ and $\kappa = -0,001$*

Table 6-39: Estimates for γ, α and δ in the model variants of Abel's CCAPM

Model variant	Instrumental Variables (IV) C, R_m, R_f	$\hat{\delta}$ (t-value) [p-value]	$\hat{\alpha}$ (t-value) [p-value]	$\hat{\gamma}$ (t-value) [p-value]	Hansen J-statistic (J-value) [p-value]	$\kappa(\alpha - 1)$	N
M - 35	R_m, R_f lagged one period C lagged two periods 3 IV	0,994354 (205,895500) [0,0000]	0,736478 (0,937410) [0,3494]	0,263521 (0,335417) [0,7376]	0,033103 (4,468905) [0,613482]	0,000264	135
M - 36	C, R_m, R_f lagged two periods 3 IV	0,985553 (620,128300) [0,0000]	-0,065772 (-0,318613) [0,7503]	1,065772 (5,162839) [0,0000]	0,106586 (14,389110) [0,025579]	0,001066	135
M - 37	C, R_m, R_f lagged three periods 3 IV	0,987563 (459,433100) [0,0000]	-0,117297 (-1,448093) [0,1488]	1,117297 (13,783580) [0,0000]	0,063717 (8,538078) [0,201271]	0,001117	134
M - 38	C, R_m, R_f lagged four periods 3 IV	0,988394 (589,592600) [0,0000]	0,027491 (1,452620) [0,1475]	0,972509 (51,387750) [0,0000]	0,057760 (7,682080) [0,262332]	0,000973	133
M - 39	C, R_m, R_f lagged five periods 3 IV	0,989622 (567,556000) [0,0000]	0,068926 (1,840444) [0,0668]	0,931074 (24,861290) [0,0000]	0,060695 (8,011740) [0,237242]	0,000931	132
M - 40	CC lagged two to five periods, R_m, R_f lagged one to five periods, together 14 IV	0,984424 (931,555000) [0,0000]	0,013325 (0,822088) [0,4118]	0,986675 (60,872550) [0,0000]	0,151993 (20,063076) [0,862155]	0,000987	132

We are looking for variants of Abel's model with $D = 0$ and $\kappa = 1$ which satisfy the conditions $\kappa(\alpha - 1) > 0$, $\gamma < 1$, $\gamma \neq 0$, $\alpha > 0$, $\alpha \neq 1$, $\delta < 1$ and $\delta \neq 0$. The model variant which satisfies all the above criteria with 90% confidence interval is M - 39. The probability value ("pval") of the $J - statistic$ for M - 39 was 0,237242. That means that we cannot reject the hypothesis that this variant is not misspecified with 90 % or higher confidence interval.

6.4.5.7 *Testing against the basic HJ-bound*

The diagram below shows the HJ-bound using Norwegian data for Abel's CCAPM with non-separable utility parameter $\kappa = -0{,}001$

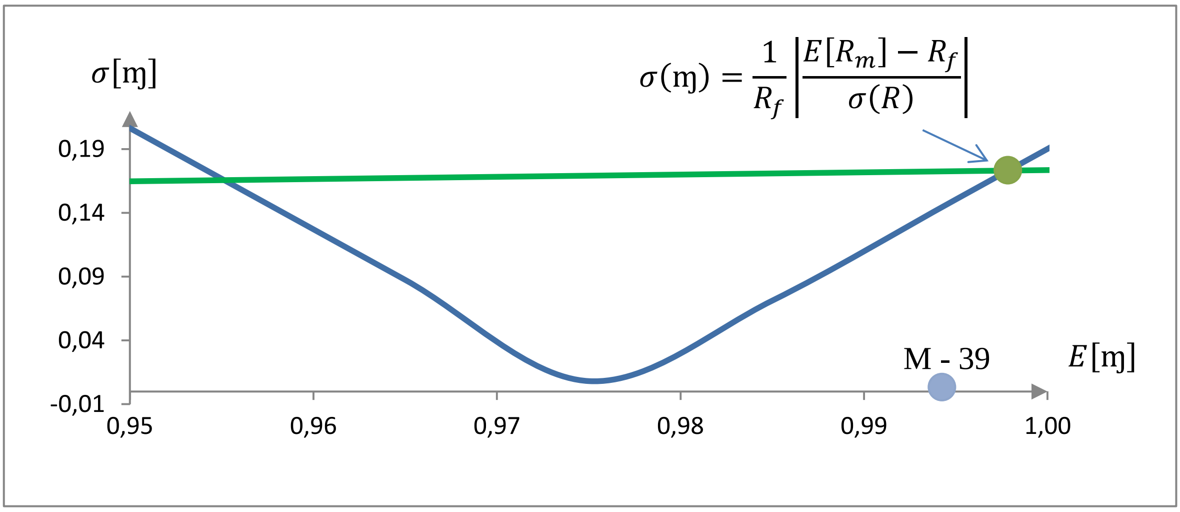


Figure 6-15: The basic HJ-bound using Norwegian data for Abel's CCAPM with $\kappa = -0{,}001$

Plugging the estimates for γ and δ in the stochastic discount factor $\delta\left(\frac{C_t}{C_{t-1}}\right)^{\kappa(-\gamma)}\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1}$ we calculated $E_t\left[\delta\left(\frac{C_t}{C_{t-1}}\right)^{\kappa(-\gamma)}\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1}\right]$ and $\sigma\left[\delta\left(\frac{C_t}{C_{t-1}}\right)^{\kappa(-\gamma)}\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1}\right]$ which we then plotted in their respective $\sigma[\mathfrak{m}], E[\mathfrak{m}]$ space for the model variant M - 39. The corresponding coloured dot lies below the green line, which means that Abel's CCAPM with $\kappa = -0{,}001$ doesn't satisfy the basic HJ-bound.

The blue line, the green line and the green dot point in the bisection of the blue and green line are constructed in the same way as for Lucas CCAP-model.

In order to satisfy the HJ-bound the models should yield either $\sigma(\mathfrak{m}) = 0{,}173$ or $\gamma = 2{,}796246$. In EViews however we found γ to be 0,931074 whereas $\sigma(\mathfrak{m})$ is estimated to be 0,003145. So the theoretical prediction of the HJ-bound doesn't agree with the GMM-estimates. This is because the HJ-bound requires either too high variation in consumption growth or too high risk aversion compared to real data. This demonstrates the equity premium puzzle with Norwegian data.

6.4.5.8 *Testing against the extended HJ-bound*

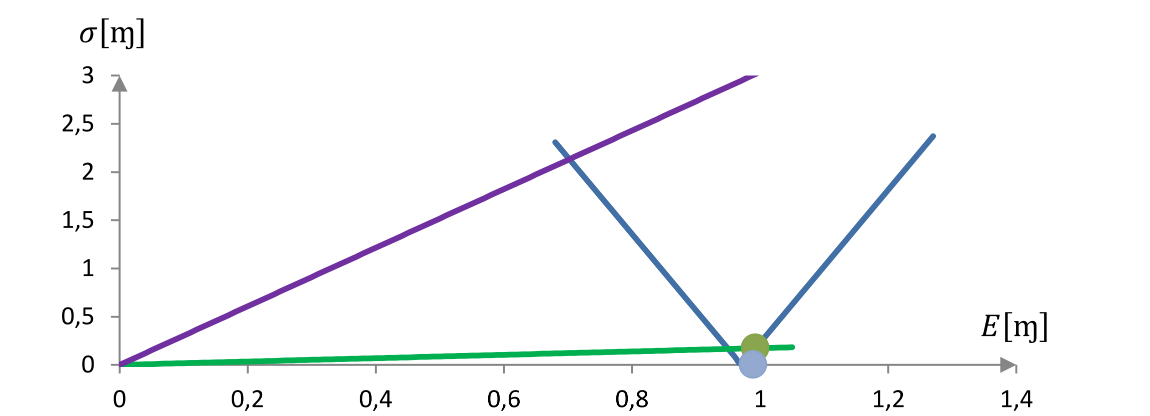


Figure 6-16: The correlation between the stochastic discount factor and the equity premium

Constructing the basic HJ-bound we implicitly assumed that

$$\left|\rho\left(\delta\left(\frac{C_t}{C_{t-1}}\right)^{\kappa(-\gamma)}\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1}, R_{t+1,m} - R_{t+1,f}\right)\right| = 1$$

The correlation between the stochastic discount factor and the equity premium for M - 39 is

$$\left|\rho\left(\delta\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1}, R_{t+1,m} - R_{t+1,f}\right)\right| = 0{,}057405$$

And plugging the correlation number for ρ in the equation below

$$\sigma\left[\delta\left(\frac{C_t}{C_{t-1}}\right)^{\kappa(-\gamma)}\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1}\right] =$$

$$\frac{E_t\left[\delta\left(\frac{C_t}{C_{t-1}}\right)^{\kappa(-\gamma)}\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1}\right]\left|E_t[(R_{t+1,m} - R_{t+1,f})]\right|}{\sigma[(R_{t+1,m} - R_{t+1,f})]} \times \frac{1}{\left|\rho\left[\delta\left(\frac{C_t}{C_{t-1}}\right)^{\kappa(-\gamma)}\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1}, R_{t+1,m} - R_{t+1,f}\right]\right|}$$

we get the purple line in Figure 6-16. The coloured dot corresponding to the model variant M - 39 lies well below the purple line, which means that Abel's CCAPM with $\kappa = -0{,}001$ doesn't satisfy the extended HJ-bound.

The correlation $\rho(\text{ɱ}, R_e) = 0{,}057405$ in

$$\frac{E[R_e]}{\sigma(R_e)}\frac{1}{\rho(\text{ɱ}, R_e)} \leq \frac{\sigma(\text{ɱ})}{E[\text{ɱ}]} \approx (1-\gamma)\sigma\left(\frac{C_{t+1}}{C_t}\right)$$

implies that $\gamma = -65{,}13129$

Based on the above calculations we see that the inequalities $\frac{E[R_e]}{\sigma(R_e)} \leq \frac{\sigma(\text{ɱ})}{E[\text{ɱ}]}$ and $\frac{E[R_e]}{\sigma(R_e)}\frac{1}{\rho(\text{ɱ},R_e)} \leq \frac{\sigma(\text{ɱ})}{E[\text{ɱ}]}$ are not satisfied. Thus we don't find support for the H - 13 hypothesis that the equity premium in Norway lies within the HJ-bounds.

6.4.6 External habit difference consumption capital asset pricing model

6.4.6.1 The theoretical model structure

An external habit difference model (Campbell and Cochrane 1999) has the utility function:

$U_t = E_t\left[\sum_{j=0}^{\infty} \delta^j \left(\frac{\left(C_{t+j} - X_{t+j}\right)^{\gamma} - 1}{\gamma}\right)\right]$	6-11

X_t can be interpreted as a habit or a subsistence level of consumption at time t and $C_t - X_t$ is the consumption surplus.

The periodic utility function is

$$U(C_t, X_t) = \frac{(C_t - X_t)^{\gamma} - 1}{\gamma}$$

The pricing kernel is

$\delta\left(\frac{\frac{C_{t+1} - X_{t+1}}{C_{t+1}}}{\frac{C_t - X_t}{C_t}}\right)^{\gamma-1}\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1}$	6-12

The stochastic discount factor is the same as the pricing kernel.

The subjective discount factor is

$$\delta = \frac{1}{1+\vartheta}$$

where ϑ is the subjective discount rate which shows time preference.

The risk aversion coefficient is $\gamma = 1 - \alpha$. The relative risk aversion coefficient RRA can be calculated as follows:

$$U(C_t, X_t) = \frac{(C_t - X_t)^{\gamma} - 1}{\gamma}$$

Holding habit $\mathcal{X}_t$ constant we get:

$$\frac{\partial U}{\partial C_t} = \frac{\gamma\left(C_t - X_t\right)^{\gamma-1}}{\gamma} = C^{\gamma-1}S^{\gamma-1}$$

$$\frac{\partial^2 U}{\partial {C_t}^2} = (\gamma - 1)(C_t - X_t)^{\gamma-2} = (\gamma - 1)(C_t)^{\gamma-2}(S_t)^{\gamma-2}$$

$$RRA = -C_t\frac{U''(C_t, X_t)}{U'(C_t, X_t)} \rightarrow RRA = -C_t(\gamma - 1)\frac{(C_t)^{\gamma-2}(S_t)^{\gamma-2}}{(C_t)^{\gamma-1}(S_t)^{\gamma-1}} \rightarrow$$

$$RRA = (1-\gamma)\frac{C_t (C_t)^{\gamma-2} (S_t)^{\gamma-2}}{(C_t)^{\gamma-1} (S_t)^{\gamma-1}} \rightarrow RRA = (1-\gamma)(S_t)^{\gamma-2-\gamma+1}(S_t)^{-1} \rightarrow RRA = \frac{(1-\gamma)}{S_t}$$

where

$$S_t = \frac{C_t - \mathcal{X}_t}{C_t}$$

The model assumes homogeneous and state and time separable preferences in a representative agent setting.

The Euler equation for asset i is:

$$E_t\left[\delta\left(\frac{\frac{C_{t+1}-\mathcal{X}_{t+1}}{C_{t+1}}}{\frac{C_t-\mathcal{X}_t}{C_t}}\right)^{-\alpha}\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha} R_{m,t+1}\right] - 1 = 0$$

or

$$E_t\left[\delta\left(\frac{\frac{C_{t+1}-\mathcal{X}_{t+1}}{C_{t+1}}}{\frac{C_t-\mathcal{X}_t}{C_t}}\right)^{\gamma-1}\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1} R_{m,t+1}\right] - 1 = 0$$

where $\gamma = 1 - \alpha$.

Alternatively can the Euler equations be written as a model of the following form:

$E_t\left[\delta\left(\frac{S_{t+1}}{S_t}\right)^{-\alpha}\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha} R_{m,t+1}\right] - 1 = 0$ $E_t\left[\delta\left(\frac{S_{t+1}}{S_t}\right)^{-\alpha}\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha} R_{f,t+1}\right] - 1 = 0$	M - 41

Data for the subsistence level of consumption per capita was collected from the Norwegian Institute for Research on Consumption (Statens institutt for Forbruksforskning, in short SIFO). Data from SIFO is for the period 1987 to 2012. The data set was extrapolated to 1978 using stochastic regression imputation based on the equation:

$$\frac{\mathcal{X}_t}{C_t} = \alpha + \beta C_t + r_1 \times \sqrt{MS}$$

Where $\mathcal{X}_t$ is the subsistence level of consumption per capita, C_t is consumption of non-durables and services per capita and r_1 is a random variable with a normal distribution, $r_1 \sim N(0,1)$.

The parameters used in the stochastic regression imputation are based on the regression

$$\frac{X_t}{C_t} = \alpha + \beta C_t + \varepsilon_t$$

which gave the following results:

Variable	*Coefficients*	*SE*	*t -Stat*	*P-value*
α	0,701984	0,008219	85,40615	0,0000
C_{t+1}	-8,84E-06	3,14E-07	-28,12137	0,0000

	df	*RSS*	*MS*	*F*
ANOVA	84	0,050694	0,0006035	790,8

$$RSS = \sum_{i=1}^{n} (x_i - \bar{x})^2$$

$$SE = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-k}}$$

$$MS = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-k}$$

where df denotes degrees of freedom, $RSS$is the sum of squared residual, SE is the standard error and MS is the mean squared error. The number of observations is denoted by n and the number of estimated parameters with k.

The result of the imputation is shown in the next figure.

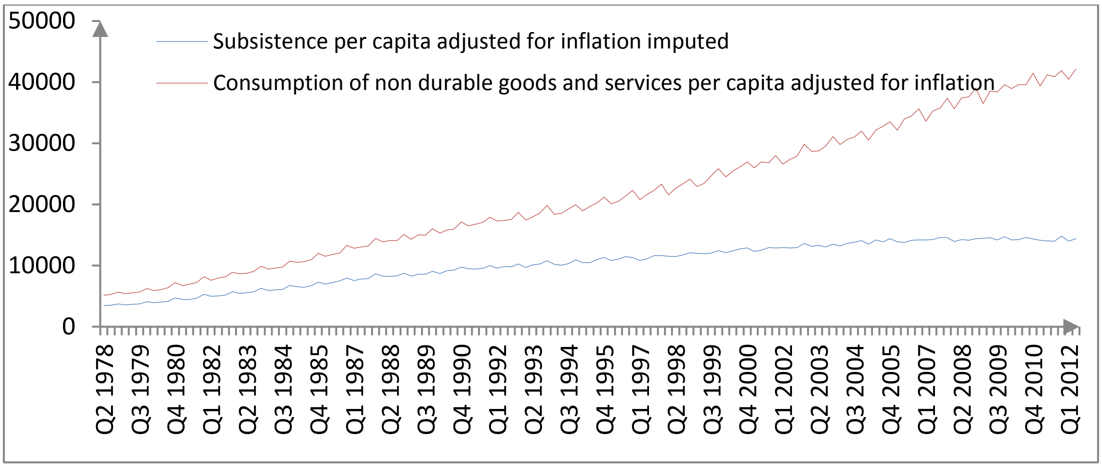


Figure 6-17: Consumption of non-durable goods and services and imputed subsistence per capita adjusted for inflation

The vertical axis shows consumption of non-durables and services per capita in NOK adjusted for inflation. The horizontal axis show time measured in quarters. The negative coefficient β shows that over time a lower percentage of the consumption of non-durables and services is used to subsistence consumption.

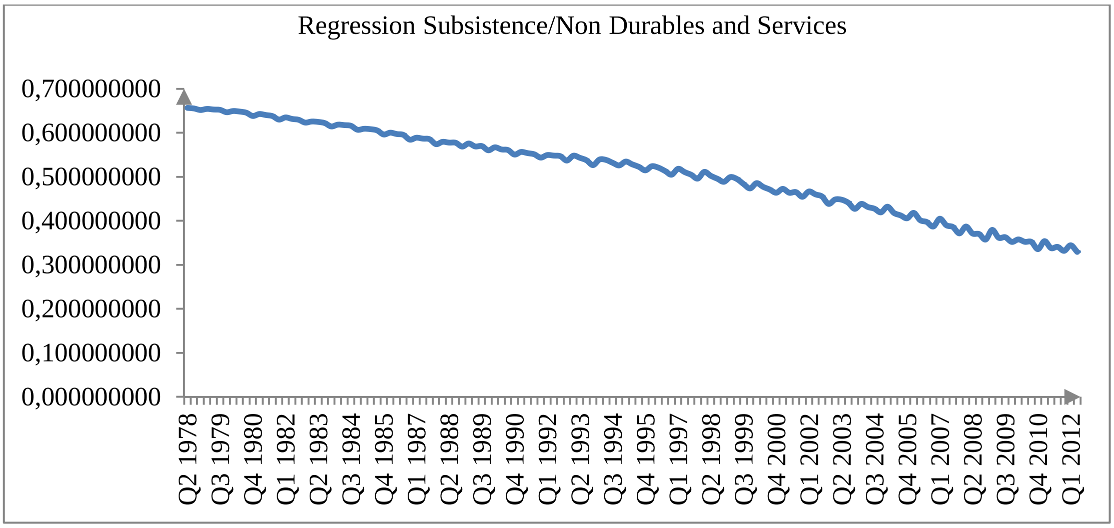


Figure 6-18: Subsistence per capita as a percentage of consumption of non-durables and services adjusted for inflation

The vertical axis shows subsistence consumption as a percentage of per capita consumption of non-durables and services adjusted for inflation. The negative slope means that subsistence consumption as a percentage of per capita consumption of non-durables and services is diminishing over time. This could be the result of real wage increases and/or the basic goods and services becoming cheaper over time like in USA (Johnson, Rogers, Tan, 2001, p. 32).

6.4.6.2 Empirical Test results

Table 6-40: Estimates for γ, α and δ in the model variants of Campbell and Cochrane's CCAPM

Model variant	Instrumental Variables (IV) C, S, R_i, R_f	$\hat{\delta}$ (t-value) [p-value]	$\hat{\alpha}$ (t-value) [p-value]	$\hat{\gamma}$ (t-value) [p-value]	Hansen J-statistic (J-value) [p-value]	N
M - 42	S, R_i, R_f lagged one period C lagged 2 periods 4 IV	0,049300 (666,0209) [0,0000]	0,049300 (2,095728) [0,0371]	0,950700 (40,41395) [0,0000]	0,066771 (8,947271) [0,346764]	134
M - 43	C, S, R_i, R_f lagged two periods 4 IV	0,987571 (635,7077) [0,0000]	-0,009807 (-0,144059) [0,8856]	1,009807 (14,83383) [0,0000]	0,082114 (11,00328) [0,201513]	134
M - 44	C, S, R_i, R_f lagged three periods 4 IV	0,987564 (477,9281) [0,0000]	-0,054189 (-0,873683) [0,3831]	1,054189 (16,99652) [0.0000]	0,071289 (9,481396) [0,303326]	133
M - 45	C, S, R_i, R_f lagged four periods 4 IV	0,987694 (687,1208) [0,0000]	0,011822 (1,03872) [0,2999]	0,988178 (86,82668) [0,0000]	0,081406 (10,74565) [0,216532]	132
M - 46	C, S, R_i, R_f lagged five periods 4 IV	0,989356 (662,8241) [0,0000]	0,039102 (2,24235) [0,0258]	0,960898 (55,10333) [0,0000]	0,064592 (8,461517) [0,389733]	131
M - 47	C, S, R_i, R_f lagged one to five periods together 19 IV	0,984071 (1439,083) [0,0000]	0,039608 (5,274818) [0,0000]	0,960392 (127,9001) [0,0000]	0,172403 (22,5848) [0,977662]	131

We are looking for variants of Campbell and Cochrane's model which satisfy the conditions, $\gamma < 1, \gamma \neq 0$, $\alpha > 0$, $\alpha \neq 1$, $\delta < 1$ and $\delta \neq 0$. The model variants which satisfy all the above criteria with 90% to 99% confidence interval are M - 42, M - 46 and M - 47. The probability values ("pval") of $J - statistic$ for the model variants M - 42, M - 46 and M - 47 were respectively 0,346764, 0,389733 and 0,977662. That means that we cannot reject the hypothesis that these variants are not misspecified with 90 % or higher confidence interval.

The diagram below shows the HJ-bound using Norwegian data for Campbell's CCAPM.

6.4.6.3 Testing against the basic HJ-bound

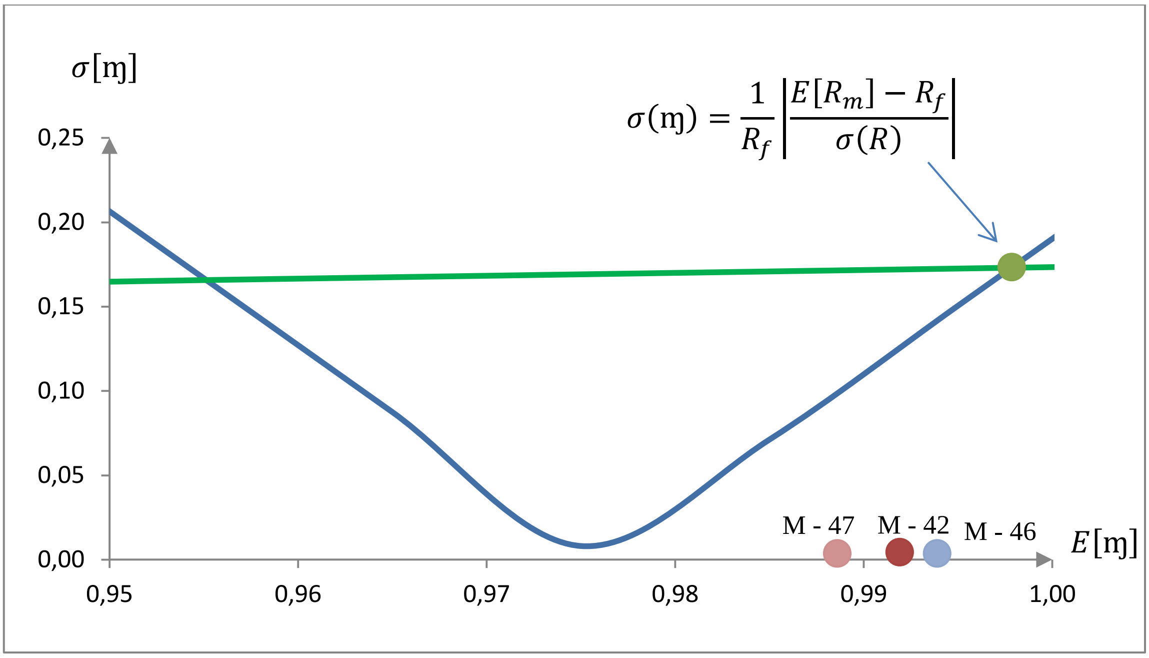


Figure 6-19: The basic HJ-bound using Norwegian data for Campbell's CCAPM

Plugging the estimates for γ and δ in the stochastic discount factor $\delta\left(\frac{\frac{C_{t+1}-X_{t+1}}{C_{t+1}}}{\frac{C_t-X_t}{C_t}}\right)^{-\alpha}\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha}$ we calculated $E_t\left[\delta\left(\frac{\frac{C_{t+1}-X_{t+1}}{C_{t+1}}}{\frac{C_t-X_t}{C_t}}\right)^{-\alpha}\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha}\right]$ and $\sigma\left[\delta\left(\frac{\frac{C_{t+1}-X_{t+1}}{C_{t+1}}}{\frac{C_t-X_t}{C_t}}\right)^{-\alpha}\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha}\right]$ which we then plotted in their respective $\sigma[\mathfrak{m}], E[\mathfrak{m}]$ space for the model variants M - 42, M - 46 and M - 47. The corresponding coloured dots lie below the blue line, which means that Campbell and Cochrane's CCAPM doesn't satisfy the basic HJ-bound.

The blue line, the green line and the green dot point in the bisection of the blue and green line are constructed in the same way as for Lucas' CCAPM.

In order to satisfy the HJ-bound the models should yield either $\sigma(\mathfrak{m}) = 0{,}173$ or $\gamma = 2{,}796246$. In EViews however we found γ to be either 0,950700, or 0,960898 or 0,960392, whereas the corresponding $\sigma(\mathfrak{m})$ is estimated to be either 0,004299, or 0,003415 or 0,003441. So the theoretical prediction of the HJ-bound doesn't agree with the GMM-estimates.

6.4.6.4 *Testing against the extended HJ-bound*

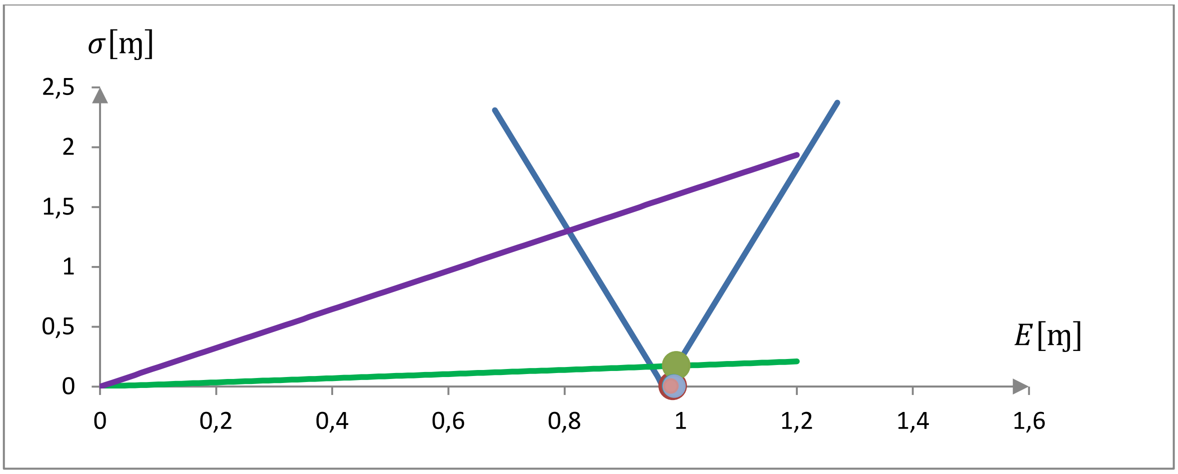


Figure 6-20: The correlations between the stochastic discount factor and the equity premium

Constructing the basic HJ-bound we implicitly assumed that

$$\left|\rho\left(\delta\left(\frac{\frac{C_{t+1}-X_{t+1}}{C_{t+1}}}{\frac{C_t-X_t}{C_t}}\right)^{-\alpha}\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha}, R_{t+1,m}-R_{t+1,f}\right)\right| = 1$$

The absolute values of the correlations between the stochastic discount factor and the equity premium for the model variant M - 42 of Campbell and Cochrane’s model was 0,108074.

$$\left|\rho\left(\delta\left(\frac{\frac{C_{t+1}-X_{t+1}}{C_{t+1}}}{\frac{C_t-X_t}{C_t}}\right)^{-\alpha}\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha}, R_{t+1,m}-R_{t+1,f}\right)\right| = 0{,}108074$$

Plugging the correlation number for ρ in the equation below:

$$\sigma\left[\delta\left(\frac{\frac{C_{t+1}-X_{t+1}}{C_{t+1}}}{\frac{C_t-X_t}{C_t}}\right)^{-\alpha}\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha}\right] =$$

$$\frac{E_t\left[\delta\left(\frac{\frac{C_{t+1}-X_{t+1}}{C_{t+1}}}{\frac{C_t-X_t}{C_t}}\right)^{-\alpha}\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha}\right]\left|E_t\left[\left(R_{t+1,m}-R_{t+1,f}\right)\right]\right|}{\sigma\left[\left(R_{t+1,m}-R_{t+1,f}\right)\right]} \times \frac{1}{\left|\rho\left[\delta\left(\frac{\frac{C_{t+1}-X_{t+1}}{C_{t+1}}}{\frac{C_t-X_t}{C_t}}\right)^{-\alpha}\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha}, R_{t+1,m}-R_{t+1,f}\right]\right|}$$

we get the purple line in the above diagram. The coloured dots corresponding to the model variants M - 42, M - 46 and M - 47 lie well below the purple line which means that Campbell and Cochrane’s CCAPM don't satisfy the extended HJ-bound.

The correlation $\rho(\mathfrak{m}, R_e) = 0{,}108074$ in

$$\frac{E[R_e]}{\sigma(R_e)}\frac{1}{\rho(\mathfrak{m}, R_e)} \leq \frac{\sigma(\mathfrak{m})}{E[\mathfrak{m}]} \approx (1-\gamma)\sigma\left(\frac{C_{t+1}}{C_t}\right)$$

implies that $\gamma = -34{,}12629$

Based on the above calculations we see that the inequalities $\frac{E[R_e]}{\sigma(R_e)} \leq \frac{\sigma(\mathfrak{m})}{E[\mathfrak{m}]}$ and $\frac{E[R_e]}{\sigma(R_e)}\frac{1}{\rho(\mathfrak{m},R_e)} \leq \frac{\sigma(\mathfrak{m})}{E[\mathfrak{m}]}$ are not satisfied. Thus we don't find support for the H - 13 hypothesis that the equity premium in Norway lies within the HJ-bounds.

6.4.7 Heterogeneous consumption capital asset pricing model

6.4.7.1 Methodology

This section contains the testing of Constantinides and Duffie's CCAPM (1996). The same methodology has been used for testing this model as the other CCAP-models with one exception: In this model it is required a time series for individual consumption of non-durables and services. This time series was simulated with a Monte Carlo simulation which generated 1000 cross sectional observations for each t. This was achieved using the random function in Excel on the aggregate consumption data of Statistics Norway. The reason for simulating the individual consumption data is lacking data for individual consumption for non-durables and services for Norwegian consumers.

Basic Monte Carlo methods entail the use of random number generation for simulating a stream of random numbers (Mcleish Monte Carlo 2005, p. 79). Checking a sequence of generated numbers for randomness can be done testing the null hypothesis that they are independent identically distributed variables. Conditional simulation generates a distribution of (y(t)) conditional on x(t) (Gourieroux and Monfort, 2002, p. 15and Schafer 1999, p. 5). In general as $\psi \to \infty$ for iid outcomes drawn from a right continuous distribution function F we have (Mittelhammer, Judge and Miller 2000, pp. 717 and 722):

$$\bar{g}_{\psi}(\theta) = \psi^{-1} \sum_{i=1}^{m} g\,(y_i; \theta) \xrightarrow{p} E[g(Y; \theta)] \qquad \forall\, \theta \in \Omega$$

Monte Carlo sampling can generate asymptotically consistent estimators by the Central Limit Theorem (CLT) and the Law of Large Numbers (LLN) if we can generate random numbers which sampling distribution reflects the properties of the assumed sampling distribution. The key assumptions for Monte Carlo are:

* The properties of the assumed sampling distribution F can be inferred from known data
* We can generate iid observations that behave as if they were drawn from F.

The random numbers for individual consumption of non-durables and services were generated using the Box-Muller method (Goodman 2005):

$$C_{j,t} = C_t + \sigma_{I_t} cos(2\pi r_2)\sqrt{-2ln(r_1)}$$

where r_1 and r_2, are random variables with normal distribution $\sim N(0,1)$. C_t is the time series of aggregate consumption per capita of non-durables and services and $\sigma_{I_t}$is a time series of the standard deviation of income per capita. So we assume that the standard deviation of income is a proxy for the standard deviation of consumption of non-durables and services.

σ_{I_t} is calculated by means of a stochastic regression imputation of the standard deviation of income regressed on income per capita based on observations for 1986-1996 and 2004-2011. The stochastic regression imputation is of the form:

$$\sigma_{I_t} = \alpha + \beta I_t + r_3 \times \sqrt{MS}$$

Where I_t is the income per capita for each period and r_3 is a random variable with a normal distribution, $r_3 \sim N(0,1)$.

The parameters used in the stochastic regression imputation are based on the regression $\sigma_{I_t} = \alpha + \beta I_t + \varepsilon_t$ which gave the following results:

Variable	*Coefficients*	*SE*	*t -Stat*	*P-value*
α	0,024725	0,000395	62,53473	0,0000
I_t	-7,89E-08	6,70E-09	-11,77254	0,0000

	df	*RSS*	*MS*	*F*
ANOVA	74	0,000163	0,0000022	0,0014841

$$RSS = \sum_{i=1}^{n} (x_i - \bar{x})^2$$

$$SE = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n - k}}$$

$$MS = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n - k}$$

Where RSS is the sum of squared residual, SE is the standard error and MS is the mean squared error. The number of observations is denoted by n and the number of estimated parameters with k.

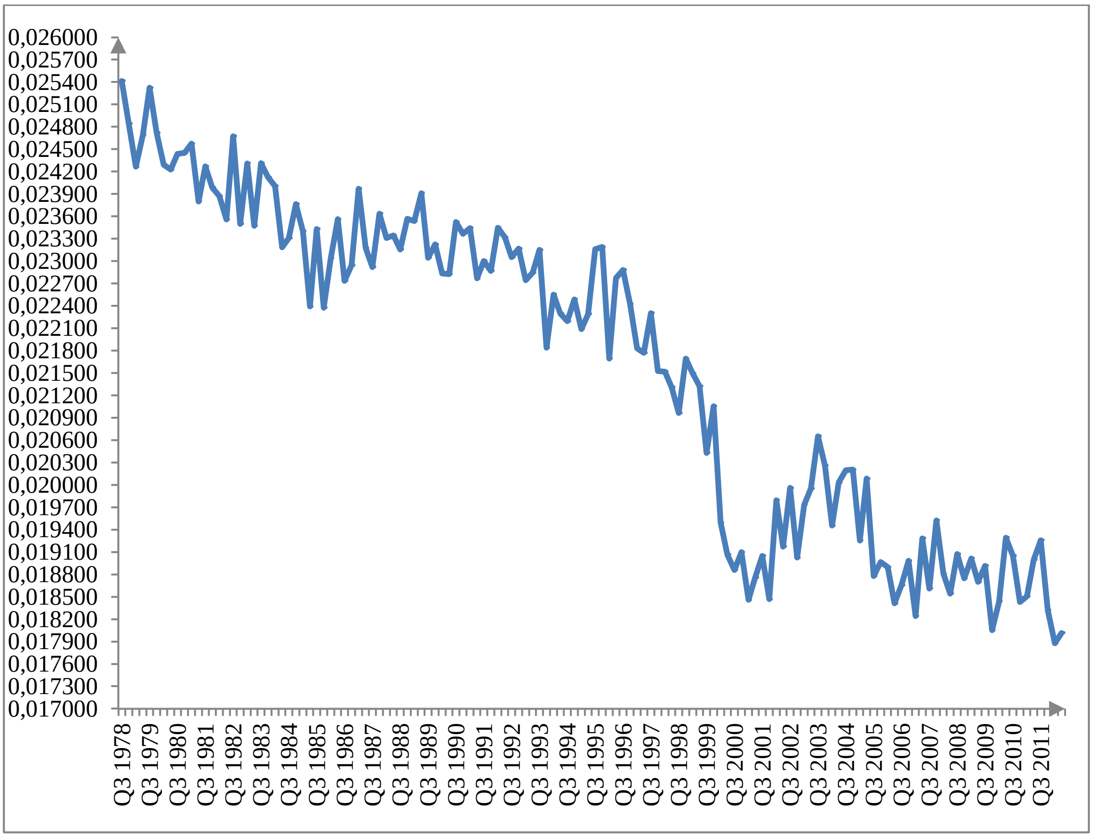


Figure 6-21: Stochastic regression imputation of standard deviation of income as a function of income per capita adjusted for inflation

The vertical axis in Figure 6-21 shows the standard deviation of income per capita used as a proxy for standard deviation of consumption of non-durables and services per capita. The horizontal axis shows time measured in quarters.

The downward sloping means that the income inequalities are reduced over time. We consider this to be a reasonable characteristic of Norwegian socioeconomics (OECD 2011 squared coefficient of income variation, p. 45 and Nolan 1987, p. 54).

6.4.7.2 *The theoretical Constantinides and Duffie's model structure*

Constantinides and Duffie (1996, p. 229) suggest a model as follows:

The utility function is time and state separable and has the form

$$U_t = E\left[\frac{1}{\gamma}\sum_{t=0}^{\propto} e^{-\vartheta} {C_{j,t}}^{\gamma} \mid \mathcal{F}_t\right]$$	6-13

The periodic utility function is

$$U(C_{j,t}) = \frac{{C_{j,t}}^{\gamma}}{\gamma}$$

The pricing kernel is

$$e^{-\vartheta}\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1} exp^{\frac{(1-\gamma)(1-\gamma+1)}{2}\sigma^2\left(\log\left(\frac{\frac{C_{j,t+1}}{C_{t+1}}}{\frac{C_{j,t}}{C_t}}\right)\right)}$$	6-14

The stochastic discount factor is the same as the pricing kernel

The subjective discount factor is $e^{-\vartheta}$

The subjective discount rate which shows time preferences is ϑ.

The risk aversion coefficient is $\gamma = 1 - \alpha$

The relative risk aversion coefficient RRA is:

$$RRA = -C_{j,t}\frac{U''}{U'} \rightarrow RRA = -C_{j,t}(\gamma - 1)\frac{{C_{j,t}}^{\gamma-2}}{{C_{j,t}}^{\gamma-1}} \rightarrow RRA = 1 - \gamma = \alpha$$

The Euler equation for asset i is:

$$E_t\left[e^{-\vartheta}\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha} exp^{\frac{\alpha(\alpha+1)}{2}\sigma^2\left(\log\left(\frac{\frac{C_{j,t+1}}{C_{t+1}}}{\frac{C_{j,t}}{C_t}}\right)\right)} R_{m,t+1}\right] - 1 = 0$$

Alternatively can be stated as:

$$E_t\left[e^{-\vartheta}\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1} exp^{\frac{(1-\gamma)(1-\gamma+1)}{2}\sigma^2\left(\log\left(\frac{\frac{C_{j,t+1}}{C_{t+1}}}{\frac{C_{j,t}}{C_t}}\right)\right)} R_{m,t+1}\right] - 1 = 0$$

Where $\gamma = 1 - \alpha$.

$C_{j,t+1}$ is the consumption of individual consumer at time $t+1$.

$\sigma^2\left(\log\left(\frac{\frac{C_{j,t+1}}{C_{t+1}}}{\frac{C_{j,t}}{C_t}}\right)\right)$ is the cross sectional variance of the logarithm of growth of individual consumption with respect to the average consumption.

The model is:

$E_t\left[e^{-\vartheta}\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1} exp^{\frac{(1-\gamma)(1-\gamma+1)}{2}\sigma^2\left(\log\left(\frac{\frac{C_{j,t+1}}{C_{t+1}}}{\frac{C_{j,t}}{C_t}}\right)\right)} R_{m,t+1}\right]-1=$ $E_t\left[e^{-\vartheta}\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1} exp^{\frac{(1-\gamma)(1-\gamma+1)}{2}\sigma^2\left(\log\left(\frac{\frac{C_{j,t+1}}{C_{t+1}}}{\frac{C_{j,t}}{C_t}}\right)\right)} R_{f,t+1}\right]-1=$	M - 48

Constantinides and Duffie's model is a heterogeneous CCAP-model because an individual's choice of consumption today influences the utility of consumption in the near future. This model accommodates heterogeneity of investors with respect to consumption, but assumes otherwise homogeneous and time and state separable preferences.

Parameter assumptions

We expect on theoretical grounds that the subjective discount factor $= e^{-\vartheta} < 1$, $\delta \neq 0$ the risk aversion coefficient $\gamma < 1$, $\gamma \neq 0$ and $\alpha = 1 - \gamma > 0$ and $\alpha \neq 1$. The rational for $\delta < 1$ is that the time preference rate θ is a positive number so that $\delta = e^{-\vartheta} < 1$ means that a consumer prefers to consume today rather than tomorrow. The rational for $\gamma < 1$ and $\alpha > 0$ is based on the assumption of the consumer being risk averse. We want $\gamma \neq 0$ and $\alpha \neq 1$ because the utility functions are of the form $\frac{C^\gamma}{\gamma}$, alternatively written as $\frac{C^{1-\alpha}}{1-\alpha}$ or variants of them.

6.4.7.3 Empirical test results

Table 6-41: Estimates for γ, α and δ in the model variants of Constantinides and Duffie's CCAPM

Model variant	Instrumental Variables (IV) C, C_j, R_i, R_f	$\hat{\delta}$ (t-value) [p-value]	$\hat{\alpha}$ (t-value) [p-value]	$\hat{\gamma}$ (t-value) [p-value]	Hansen J-statistic (J-value) [p-value]	N
M - 49	C, C_j, R_m, R_f lagged one period 4 IV	0,988132 (7,301893) [0,0000]	0,133384 (2,566434) [0,0108]	0,866531 (16,664260) [0,0000	0,066343 (8,889926) [0,351667]	134
M - 50	C, C_j, R_m, R_f lagged two periods 4 IV	0,985312 (10,870050) [0,0000]	-0,033050 (-0,197914) [0,8433]	1,031037 (6,179495) [0,0000]	0,114449 (15,221760) [0,054974]	133
M - 51	C, C_j, R_m, R_f lagged three periods 4 IV	0,987577 (6,288427) [0,0000]	-0,100436 (-1,307601) [0,1922]	1,100302 (14,338980) [0,0000]	0,073443 (9,694503) [0,287126]	132
M - 52	C, C_j, R_m, R_f lagged four periods 4 IV	0,987618 (9,979860) [0,0000]	0,021845 (1,262970) [0,2077]	0,978124 (56,570510) [0,0000]	0,083300 (10,912240) [0,206721]	131
M - 53	C, C_j, R_m, R_f lagged five periods 4 IV	0,989847 (6,671846) [0,0000]	0,107169 (3,192943) [0,0016]	0,892709 (26,585820) [0,0000]	0,073432 (9,546217) [0,29833]	130
M - 54	C, C_j, R_m, R_f lagged one to five periods together 20 IV	0,984043 (22,985300) [0,0000]	-0,033987 (-2,648277) [0,0086]	1,033999 (80,578310) [0,0000]	0,174365 (22,667490) [0,987548]	130

We are looking for variants of Constantinides and Duffie's model which satisfy the conditions, $\gamma < 1$, $\gamma \neq 0$, $\alpha > 0$, $\alpha \neq 1$, $\delta < 1$ and $\delta \neq 0$. The model variants which satisfy all the above criteria with 95% to 99% confidence interval are M - 49 and M - 53. The probability value ("pval") of the $J - statistic$ for the model variants M - 49 and M - 53 were 0,351667and 0,29833 respectively. That means that we cannot reject the hypothesis that these variants are not misspecified with 90 % or higher confidence interval.

6.4.7.4 *Testing against the basic HJ-bound*

The diagram below shows the HJ-bound using Norwegian data for Constantinides and Duffie's CCAPM.

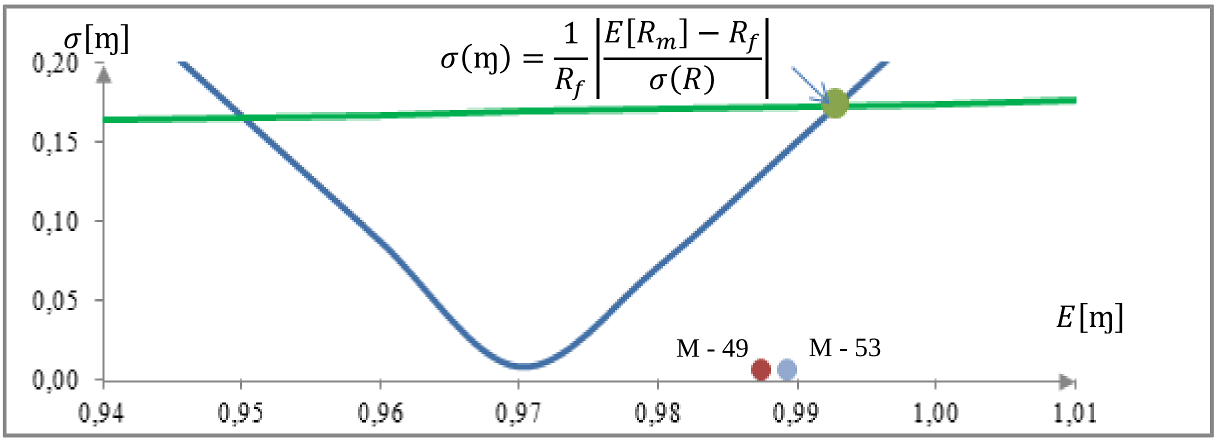


Figure 6-22: The basic HJ-bound using Norwegian data for Constantinides and Duffie's CCAPM.

Plugging the estimates for γ and δ in the stochastic discount factor

$$e^{-\vartheta}\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1} exp^{\frac{(1-\gamma)(1-\gamma+1)}{2}\sigma^2\left(\log\left(\frac{\frac{C_{j,t+1}}{C_{t+1}}}{\frac{C_{j,t}}{C_t}}\right)\right)}$$

we calculated $E_t\left[e^{-\vartheta}\left(\frac{C_{t+1}}{C_1}\right)^{\gamma-1} exp^{\frac{(1-\gamma)(1-\gamma+1)}{2}\sigma^2\left(\log\left(\frac{\frac{C_{j,t+1}}{C_{t+1}}}{\frac{C_{j,t}}{C_t}}\right)\right)}\right]$ and

$\sigma\left[e^{-\vartheta}\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1} exp^{\frac{(1-\gamma)(1-\gamma+1)}{2}\sigma^2\left(\log\left(\frac{\frac{C_{j,t+1}}{C_{t+1}}}{\frac{C_{j,t}}{C_t}}\right)\right)}\right]$ which we then plotted in their respective

$\sigma[\mathfrak{m}], E[\mathfrak{m}]$ space for model variants M - 49 and M - 53. The corresponding coloured dots lie below the blue line, which means that Constantinides and Duffie's model doesn't satisfy the basic HJ-bound.

The blue line, the green line and the green dot point in the bisection of the blue and green line are constructed in the same way as for Lucas' CCAPM. In order to satisfy the HJ-bound the models should yield either $\sigma(\mathfrak{m}) = 0{,}173$ or $\gamma = 2{,}796246$. In EViews however we found γ to be either 0,8666531 or 0,892709 whereas the corresponding $\sigma(\mathfrak{m})$ is estimated to be either 0,006107 or 0,004914. So the theoretical predictions of the HJ-bound don't agree with the GMM-estimates.

6.4.7.5 *Testing against the extended HJ-bound*

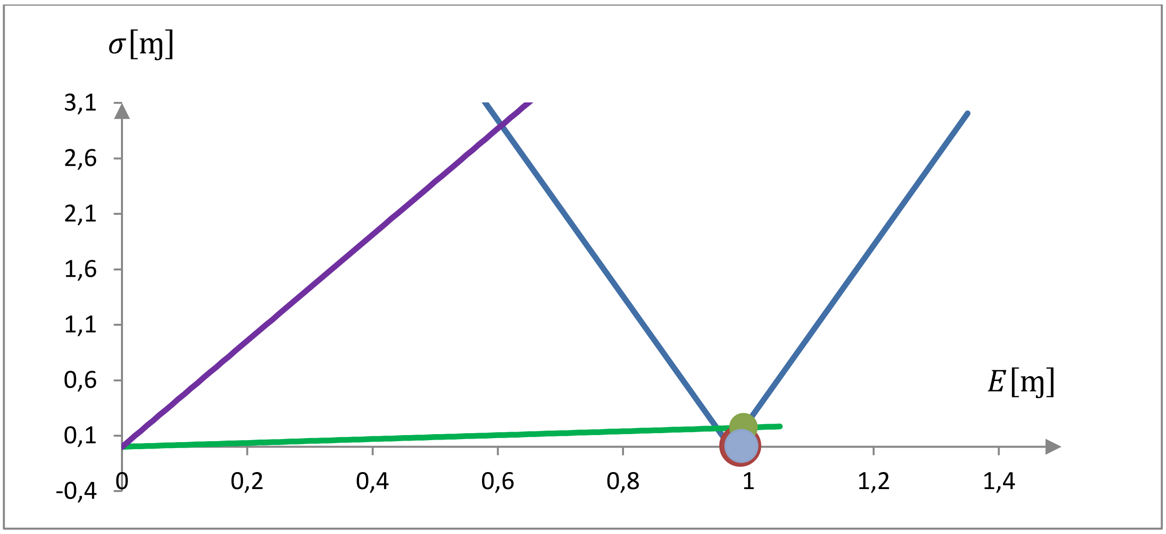


Figure 6-23: The correlations between the stochastic discount factor and the equity premium

Constructing the basic HJ-bound we implicitly assumed that

$$\left|\rho\left(e^{-\vartheta}\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1} exp^{\frac{(1-\gamma)(1-\gamma+1)}{2}\sigma^2\left(\log\left(\frac{\frac{C_{j,t+1}}{C_{t+1}}}{\frac{C_{j,t}}{C_t}}\right)\right)}, R_{t+1,m} - R_{t+1,f}\right)\right| = 1$$

The absolute values of the correlations between the stochastic discount factor and the equity premium for the model variant M - 49 is

$$\left|\rho\left(e^{-\vartheta}\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1} exp^{\frac{(1-\gamma)(1-\gamma+1)}{2}\sigma^2\left(\log\left(\frac{\frac{C_{j,t+1}}{C_{t+1}}}{\frac{C_{j,t}}{C_t}}\right)\right)}, R_{t+1,m} - R_{t+1,f}\right)\right| = 0{,}036464261 \text{ and}$$

plugging the correlation number for ρ in the equation below

$$\sigma\left[e^{-\vartheta}\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1} exp^{\frac{(1-\gamma)(1-\gamma+1)}{2}\sigma^2\left(\log\left(\frac{\frac{C_{j,t+1}}{C_{t+1}}}{\frac{C_{j,t}}{C_t}}\right)\right)}\right] =$$

$$
\frac{E_t\left[e^{-\vartheta}\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1} exp^{\frac{(1-\gamma)(1-\gamma+1)}{2}\sigma^2\left(\log\left(\frac{\frac{C_{j,t+1}}{C_{t+1}}}{\frac{C_{j,t}}{C_t}}\right)\right)}\right]\left|E_t\left[\left(R_{t+1,m}-R_{t+1,f}\right)\right]\right|}{\sigma\left[\left(R_{t+1,i}-R_{t+1,f}\right)\right]}
$$

$$
\times \frac{1}{\left\|\rho\left[e^{-\vartheta}\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1} exp^{\frac{(1-\gamma)(1-\gamma+1)}{2}\sigma^2\left(\log\left(\frac{\frac{C_{j,t+1}}{C_{t+1}}}{\frac{C_{j,t}}{C_t}}\right)\right)}, R_{t+1,m}-R_{t+1,f}\right]\right\|}
$$

we get the purple line in Figure 6-23 denoting the extended HJ-bound. The coloured dots corresponding to models M - 49 and M - 53 lie well below the purple line which means that Constantinide's and Duffie's CCAPM doesn't satisfy the extended HJ-bound.

The correlation $\rho(\mathfrak{m}, R_e) = 0{,}036464261$ in

$$
\frac{E[R_e]}{\sigma(R_e)}\frac{1}{\rho(\mathfrak{m}, R_e)} \leq \frac{\sigma(\mathfrak{m})}{E[\mathfrak{m}]} \approx (1-\gamma)\sigma\left(\frac{C_{t+1}}{C_t}\right)
$$

implies that $\gamma = -106{,}28$.

6.4.8 Summing up the test results of CCAP-models

We sum up the results against the basic HJ-bound since it is the measure used most frequently in the literature we reviewed. When we say basic HJ-bound we mean the HJ-bound derived when there is perfect correlation between the stochastic discount factor and the equity premium.

The prediction of the basic HJ-bound is that:

$$\frac{E[R_e]}{\sigma(R_e)} \leq \frac{\sigma(\mathfrak{m})}{E[\mathfrak{m}]}$$

Our test results show the following:

For Lucas' CCAPM (1978) using the EViews parameters we find that the basic HJ-bound is $\frac{E[R_e]}{\sigma(R_e)} = 0{,}174370$ while $\frac{\sigma(\mathfrak{m})}{E[\mathfrak{m}]} = 0{,}005837$, so $\frac{E[R_e]}{\sigma(R_e)} \geq \frac{\sigma(\mathfrak{m})}{E[\mathfrak{m}]}$

If we use $E(\mathfrak{m}) = \frac{1}{R_f}$ then $\frac{\sigma(\mathfrak{m})}{E[\mathfrak{m}]}$=0,00581 and still $\frac{E[R_e]}{\sigma(R_e)} \geq \frac{\sigma(\mathfrak{m})}{E[\mathfrak{m}]}$

The extended HJ-bound is 3,026526.

For Abel's CCAPM (1990) with the parameter that shows time nonseparability of consumption preferences $\kappa = -1$, using the EViews parameters we discover that the basic HJ-bound is $\frac{E[R_e]}{\sigma(R_e)} = 0{,}174370$ while $\frac{\sigma(\mathfrak{m})}{E[\mathfrak{m}]} = 0{,}040293$, so $\frac{E[R_e]}{\sigma(R_e)} \geq \frac{\sigma(\mathfrak{m})}{E[\mathfrak{m}]}$

If we use $E(\mathfrak{m}) = \frac{1}{R_f}$ then $\frac{\sigma(\mathfrak{m})}{E[\mathfrak{m}]}$=0,039849 and still $\frac{E[R_e]}{\sigma(R_e)} \geq \frac{\sigma(\mathfrak{m})}{E[\mathfrak{m}]}$

The extended HJ-bound is 3,260035.

For Abel's CCAPM (1990) with the parameter that shows time nonseparability of consumption preferences $\kappa = 1$, using the EViews parameters we find that none of the model variants satisfy the condition $\kappa(\alpha - 1) \geq 0$. Because of this we don't analyze this case further.

For Abel's CCAPM (1990) with the parameter the shows time nonseparability of consumption preferences $\kappa = -0{,}001$, using the EViews parameters we note that the HJ-bound is $\frac{E[R_e]}{\sigma(R_e)} = 0{,}174370$ while $\frac{\sigma(\mathfrak{m})}{E[\mathfrak{m}]} = 0{,}003178$ so $\frac{E[R_e]}{\sigma(R_e)} \geq \frac{\sigma(\mathfrak{m})}{E[\mathfrak{m}]}$

If we use $E(\mathfrak{m}) = \frac{1}{R_f}$ then $\frac{\sigma(\mathfrak{m})}{E[\mathfrak{m}]}$=0,003167 and still $\frac{E[R_e]}{\sigma(R_e)} \geq \frac{\sigma(\mathfrak{m})}{E[\mathfrak{m}]}$

The extended HJ-bound is 3,037562.

For Campbell and Cochrane's CCAPM (1998) using the EViews parameters we establish the basic HJ-bound to be $\frac{E[R_e]}{\sigma(R_e)} = 0{,}174370$ while $\frac{\sigma(\text{ɱ})}{E[\text{ɱ}]} = 0{,}004356$, so $\frac{E[R_e]}{\sigma(R_e)} \geq \frac{\sigma(\text{ɱ})}{E[\text{ɱ}]}$

If we use $E(\text{ɱ}) = \frac{1}{R_f}$ then $\frac{\sigma(\text{ɱ})}{E[\text{ɱ}]}$=0,00433 and still $\frac{E[R_e]}{\sigma(R_e)} \geq \frac{\sigma(\text{ɱ})}{E[\text{ɱ}]}$

The extended HJ-bound is 1,613431.

For Constantinides and Duffie's CCAPM (1996) using the EViews parameters we nail the basic HJ-bound to $\frac{E[R_e]}{\sigma(R_e)} = 0{,}174370$ while $\frac{\sigma(\text{ɱ})}{E[\text{ɱ}]} = 0{,}006184$, so $\frac{E[R_e]}{\sigma(R_e)} \geq \frac{\sigma(\text{ɱ})}{E[\text{ɱ}]}$

If we use $E(\text{ɱ}) = \frac{1}{R_f}$ then $\frac{\sigma(\text{ɱ})}{E[\text{ɱ}]}$=0,0061507 and still $\frac{E[R_e]}{\sigma(R_e)} \geq \frac{\sigma(\text{ɱ})}{E[\text{ɱ}]}$

The extended HJ-bound is 4,781951.

The problem is exacerbated with a magnitude order of 9 or higher when $|\rho(\text{ɱ}, R_e)| < 1$., i.e. when the correlation between the stochastic discount factor and the equity premium is less than one in absolute value.

Based on the above we note that the inequalities

$$\frac{E[R_e]}{\sigma(R_e)} \leq \frac{\sigma(\text{ɱ})}{E[\text{ɱ}]}$$

and

$$\frac{E[R_e]}{\sigma(R_e)} \frac{1}{\rho(\text{ɱ}, R_e)} \leq \frac{\sigma(\text{ɱ})}{E[\text{ɱ}]}$$

are not satisfied. Thus we don’t find support for the H - 13 hypothesis that the stochastic discount factor specified by the power utility preferences explains the risk premium and complies with the HJ bounds. This is because the variation of consumption growth is too low, the risk aversion coefficient required too high or the correlation between the stochastic discount factor and the equity premium too low, compared to real data.

Our calculated quarterly equity premium with Norwegian data adjusted for inflation is 2,3 % for the period Q2 1978 to Q2 2012. Our calculations are based on the Morgan Stanley total return index in Datastream and the three months T-bills rates extracted off the web pages of the national bank of Norway. The annualized equity premium is 9,52 %. Pennacchi (2008) states that the equity premium for the US should be approximately 1%. Using the US theoretical benchmark of approximately 1% equity premium annually, it seems that the equity premium in Norway is way too high.

6.4.9 Discussion

We inspected visually the residuals in the model variants in each CCAPM with the best results, that is the lowest distance to the basic HJ-bound (Cochrane 2005 p. 456).

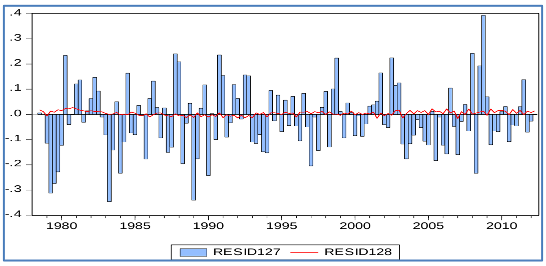


Figure 6-24: Residuals in Lucas CCAPM, model variant M - 16.

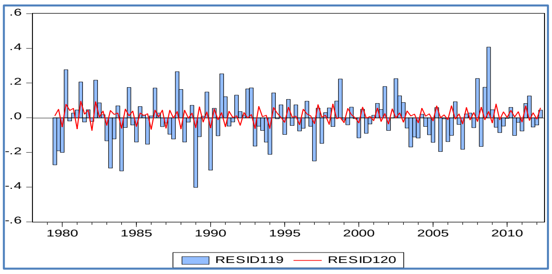


Figure 6-25: Residuals in Abel's CCAPM, model variant M - 28 ($= -1$)

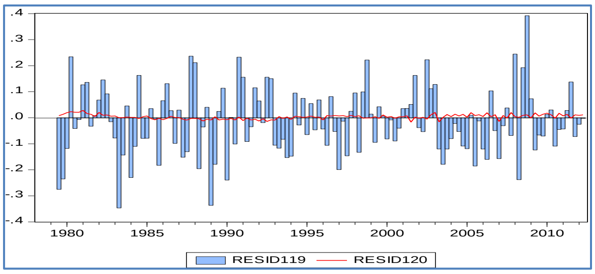


Figure 6-26: Residuals in Abel's CCAPM, model variant M - 39 ($\kappa = -0{,}001$)

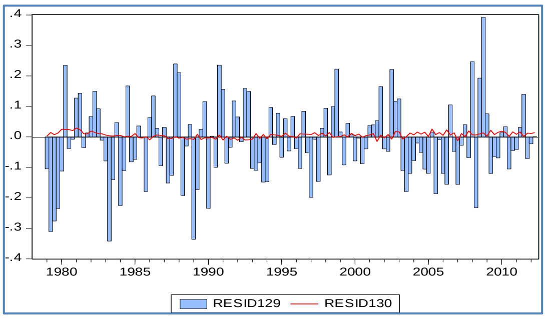


Figure 6-27: Residuals in Campbell and Cochrane model M - 42

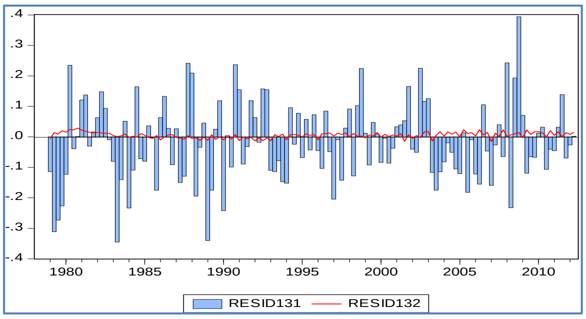


Figure 6-28: Residuals in Constantinides and Duffie model M - 49

The blue line shows the residuals of the Euler equation containing the market return. The red line shows the residuals of the Euler equation containing the risk free return. Abel's model M - 28 with the time nonseparability of utility parameter $\kappa = -1$ creates the greatest residual variance in the Euler equation containing the risk free return. The above diagrams show also that the sample means of the residuals in the consumption capital asset pricing models we tested is close to zero.

Table 6-42: The distance between the HJ-bounds and gamma

Diff. basic HJ-bound, gamma						
	M-16	M-17	M-18	M-19	M-20	M-21
Lucas	-3,670070	-3,866155	-3,913597	-3,768329	-3,725585	-3,787504
	M-23	M-24	M-25	M-26	M-27	M-28
Abel kappa = -1	-2,806035	-2,991826	-3,773472	-4,447560	-1,842002	-3,387490
	M-29	M-30	M-31	M-32	M-33	M-44
Abel kappa = 1	-2,811080	-2,911504	-3,746632	-3,455469	-3,118495	-3,256613
	M-35	M-36	M-37	M-38	M-39	M-40
Abel kappa = -0,001	-3,059767	-3,862018	-3,913543	-3,768755	-3,727320	-3,782921
	M-42	M-43	M-44	M-45	M-46	M-47
Campbell and Cochrane	-3,746946	-3,806053	-3,850435	-3,784424	-3,757144	-3,756638
	M-49	M-50	M-51	M-52	M-53	M-54
Constantinides and Duffie	-3,662777	-3,827283	-3,896548	-3,774370	-3,688955	-3,830245

Diff. extended HJ-bound, gamma						
	M-16	M-17	M-18	M-19	M-20	M-21
Lucas	-65,764835	-65,960920	-66,008362	-65,863094	-65,820350	-65,882269
	M-23	M-24	M-25	M-26	M-27	M-28
Abel kappa = -1	-65,141289	-65,327080	-66,108726	-66,782814	-64,177256	-65,722744
	M-29	M-30	M-31	M-32	M-33	M-44
Abel kappa = 1	-65,412302	-65,512726	-66,347854	-66,056691	-65,719717	-65,857835
	M-35	M-36	M-37	M-38	M-39	M-40
Abel kappa = -0,001	-66,080382	-66,139489	-66,183871	-66,117860	-66,090580	-66,090074
	M-42	M-43	M-44	M-45	M-46	M-47
Campbell and Cochrane	-35,076988	-35,136095	-35,180477	-35,114466	-35,087186	-35,086680
	M-49	M-50	M-51	M-52	M-53	M-54
Constantinides and Duffie	-103,975198	-104,139704	-104,20896	-104,086791	-104,001376	-104,142666

(see also appendix **Feil! Fant ikke referansekilden.**).

The red colour denotes estimates in models which don't pass one or more of the following criteria: $\gamma<1$, $\alpha>0$, $\delta<1$ and p-value of the J-statistic>0,1

The green colour denotes estimates in models that pass these criteria.

The blue colour denotes the estimates in models where the distance to the basic and the extended HJ-bound is minimized.

Abels' model performs best in the basic HJ-bound while Campbell and Cochrane's model perform best in the extended HJ-bound where the effect of correlation between the stochastic discount factor and the equity premium comes into place.

The model by Lucas (1978) is a neat bench mark model against which other models can be compared. Lucas model is the base of many other consumption capital asset pricing models and has been over time extended and expanded in many directions including habit formation and heterogeneity.

The model by Abel (1990) provides parameter results closest to fulfill the demands set by the Hansen-Jagannathan bound. This is in line with the results of Ferson and Harvey (1992) and Brown, Constantinides and Ferson (1993) which find that non-time separable preferences improve the fit of the models tested. Abel's model yields the highest residual volatility in the Euler equation containing the risk free rate of return. In this model there are three parameters to estimate, the stochastic discount factor ɱ, the subjective discount rate δ and the parameter κ which denotes the degree the past consumption influences the utility of current consumption. We had at our disposal the Euler equations $E[ɱR_m] - 1 = 0$ and $E\left[ɱR_f\right] - 1 = 0$ where ɱ is the stochastic discount factor, R_m is the market return and R_f is the risk free rate of return. The values of κ used in our tests were based on Abel (1990) and are chosen so that $κ(1 - \gamma) > 0$ where γ is the risk aversion parameter. However, the choice of κ is still rather arbitrary and constitutes a weakness of the test results.

The model by Campbell and Cochrane requires data on the maintenance level of consumption. An issue in testing this model is the lack of a complete set of data for subsistence consumption in Norway. To deal with this we extrapolated data on subsistence consumption from the Norwegian National Institute for Consumer Research (SIFO in Norwegian) for the period 1987 to 2012 to the period back to 1978. This was done using stochastic imputation regression. The subsistence data series produced with imputation has the feature of a diminishing percentage of the maintenance consumption as a percentage of the total consumption of non-durables and services per capita. This can be due to real wage increasing faster than the real prices of the basic necessities, which seems to be a reasonable explanation. Johnson, Rogers and Tan (2001 p. 29) find for instance that the household budget in USA between 1919 and 1999 has increased in real terms; but not as much as changes in per capita gross domestic product.

In Constantinides and Duffie's model (year 1996) we used Monte Carlo simulation. By the law of large numbers and the central limit theorem, Monte Carlo yields asymptotically consistent estimators as long as the expectation and the variance are well defined and the sequence of random variables used in the simulation is iid (Glynn 2011). We used the volatility of panel data on income as proxy for the volatility of individual consumption of non-durables and services. We do not claim that the volatility of individual income exactly maps the volatility of individual consumption. Nevertheless it is plausible that there is a positive correlation between the volatility of consumption and the volatility of income. Dogra and Gorbachev (2013) for instance find that between 1980 and 2004 the volatility of income in USA increased by 50 percent and the volatility of household consumption by 33 percent. Using the volatility of income as a proxy for the volatility of consumption means that the volatility numbers we used are probably biased somewhat upwards since consumption is less volatile than income. In order to account for this we tone down the quantitative aspects and lay more emphasis on the qualitative characteristics of the test results. In the model by Constantinides and Duffie the volatility of individual consumption is an extra element which adds to the total variability of the stochastic discount factor.

Ferson and Harvey (1992) are critical to the use of seasonally adjusted consumption data pointing out that it can lead to spurious rejections of consumption based asset pricing. They find that data with seasonal effects gives better results. This is because seasonal adjustment reduces the variability of consumption. We need the variability of consumption for explaining the equity premium. The consumption data we used in our tests is not seasonally adjusted.

We observe that a feature in the consumption capital asset pricing models tested is that the first model variant gives best results followed closely by the fifth model variant. This means that the instrumental variables are best correlated with the model variables one period and five periods back. As instrumental variables we used the regressive terms of the model variables from one to five lags back in the fashion of Hansen and Singleton (1982). The correlation effect between the model variables and the instrumental variables five lags back seem reasonable due to seasonal effects given that we used quarterly data. The correlation effect between the model variables and the instrumental variables one lag back is due to time proximity.

The GMM version we employed in the econometric program EViews makes use of a weighting matrix which is robust to heteroscedasticity and autocorrelation (EViews guide II 2007, p. 429). Hayashi (2000, p. 215) points out that the efficient weighting matrix is a function of fourth moments and may require large sample sizes. Because of this it is sometimes advised in small samples to use the identity weighting matrix as well and compare the results thus obtained with the results provided using the efficient weighting matrix. This could be a possible extension to our tests in the future.

Van Praag and Booij (2003) are doing a survey of individual responses to betting questions in order to derive a simultaneous estimate of the relation between relative risk aversion and the time preference discount rate. They find a moderate negative correlation. That implies a moderate positive correlation between the subjective discount factor and relative risk aversion. Based on this criterion are Abel’s CCAPM M - 28 and Constantinides and Duffie’s CCAPM M - 49 which yield results in line with Van Praag and Booij’s prediction (see appendix A - xxiii).

Aase (2012) proposes a representative agent model with recursive utility which claims remedying the empirical deficiencies of consumption capital asset pricing models. A feature often encountered in CCAPM is that the reciprocal of the relative risk aversion is equal to the elasticity of intertemporal substitution of consumption. Aase's model disentangles this relation into two distinct parameters and adds new terms in the equilibrium equation of interest rate. The mechanism of the model is such that it yields both low risk aversion and low risk free interest rate even for low correlation values between the consumption growth and asset returns.

6.4.10 Contribution to research

Our tests contribute to research as follows:

i) We estimated the stochastic discount factor, the Hansen-Jagannathan bound, the risk aversion, the relative risk aversion and the subjective discount factor, using models which relax the assumption of time separability of utility (Abel 1990), relax the assumption of homogeneity of consumption (Constantinides and Duffie 1996) and take in to account the subsistence level of consumption (Campbell and Cochrane 1999). To our knowledge the last three models have not been previously tested with Norwegian data.

ii) We found that the model which relaxes the assumption of time separability of utility (Abel 1990) creates the greatest residual variance in the Euler equation containing the risk free return.

iii) Another finding is that the model using the subsistence level of consumption (Campbell and Cochrane 1999) has the greatest correlation between the stochastic discount factor and the equity premium.

6.4.11 Conclusion

In this bundle of tests we considered the stochastic discount factor as a means to explain the equity premium and achieve the demands of the Hansen-Jagannathan bounds.

We employed an arsenal of consumption capital asset pricing models which were analyzed so that the nuts and bolts of their construction were revealed. We chose consciously which models to analyze in order to obtain a diversity of traits, including a bench mark model, a model of non-separable utility, a model with subsistence consumption and a model of heterogeneous consumption.

We discovered that the consumption capital asset pricing models with the feature of a non-separable utility function exhibited qualitative characteristics which helped to explain the equity premium up to a point. The model with heterogeneous consumption yields higher volatility of the consumption growth but lower correlation between the stochastic discount factor and the equity premium. The model with the subsistence level of consumption generates the highest correlation between the stochastic discount factor and the equity premium. All models have a long way to go before fulfilling the predictions defined by the Hansen-Jagannathan bounds.

Consumption based asset pricing provides an elegant connection between macroeconomic risk and asset prices. The low variability of the consumption of non-durables and services makes it challenging to establish a relation that complies with the basic Hansen-Jagannathan bound. Relaxing the assumptions of time separability of the utility function produced results that closed down this gap. In the extended Hansen-Jagannathan bound the advantage is with models which generate high correlation between the stochastic discount factor and the equity premium. The model by Campbell and Cochrane comes out best as far as correlation is concerned.

Lack of available data is a hurdle for testing models of more complicated structure. Using simulation allows a qualitative analysis of the results yielded by the models but interdicts drawing sturdy conclusions from a quantitative comparison of the results.

Testing consumption capital asset pricing models which combine characteristics such as nonseparability of utility, recursive utility, subsistence consumption and heterogeneity, with Norwegian data, would be appropriate extensions for deepening our understanding of the equity premium in Norway.

6.4.12 Testing the regression assumptions

Assumption I: Quantitativeness:

The descriptive statistics show that all variables are quantitative.

The descriptive statistics for the variables used in our tests of difference of opinions were as following

Table 6-43: Descriptive statistics for variables

Variables	AI_MSCI_BR_R	AI_TBILLS_3M_BR_R	AI_NON_DUR_SERV_GR	AI_CT_XT_CT_GR	AI_CR_SEC_VAR_1000
Mean	1,029547	1,007120	1,006833	0,995824	0,000178
Median	1,034697	1,007508	1,015142	0,997279	0,000186
Maximum	1,360427	1,027976	1,111349	1,062945	0,000235
Minimum	0,615832	0,974048	0,886490	0,910312	0,000121
Std. Dev.	0,129373	0,009288	0,045818	0,031596	0,000033
Skewness	-0,150474	-0,450426	-0,388472	-0,202361	-0,159691
Kurtosis	3,256715	3,644120	2,710744	2,584632	1,600661
Jarque-Bera	0,8932	7,0008	3,9234	1,905874	11,58837
Probability	0,639802	0,030185	0,140620	0,385607	0,003045
Sum	141,048000	137,975400	137,936100	135,4320	0,024034
Sum Sq. Dev.	2,276270	0,011731	0,285510	0,134770	0,000000147
N	137	137	137	136	135

Where

ai_msci_br_r shows the gross return R_m of the Morgan Stanley Index for Norwegian stocks adjusted for inflation.

ai_tbills_3m_br_r shows the gross return R_f of Norwegian three months treasury bills adjusted for inflation.

ai_non_dur_serv_gr shows non-durables and services growth $\frac{C_{t+1}}{C_t}$ adjusted for inflation.

$ai\_ct\_xt\_ct\_gr$ shows the consumption growth above a subsistence level $\frac{\frac{C_{t+1}-X_{t+1}}{C_{t+1}}}{\frac{C_t-X_t}{C_t}}$ adjusted for inflation.

$ai\_cr\_sec\_var\_1000$ shows the cross sectional variance of the logarithm of individual consumption growth $\sigma^2\left(\log\left(\frac{\frac{C_{j,t+1}}{C_{t+1}}}{\frac{C_{j,t}}{C_t}}\right)\right)$ for 1000 different consumers using Monte Carlo simulation, adjusted for the aggregate consumption growth and inflation.

All the consumption variables are per capita.

Assumption II: Variance

The descriptive statistics show that all variables have some variance.

Assumption III: Identification

Let θ_1 and θ_2 be two different parameter vectors. For any number of observations n and number of parameters K so that $n \geq K$ there existsdata sets $\bar{m}_n(\theta_1) \neq \bar{m}_n(\theta_2)$ (Greene 2012, pp. 475-476). That means that the probability limit is uniquely minimized. An alternative formulation is that the number of orthogonality conditions is greater than the number of parameters to be estimated (Hayashi 2000, p. 202).

See test under assumption V.

Assumption IV: Orthogonality conditions

All the instrumental variables are orthogonal to the current error term (Hayashi 2000, p.198).

See test under assumption V.

Assumption V: Asymptotic normality

At each set of values for the i independent variables, $x_i, x_{i-1}, \ldots, x_1$, $E(\varepsilon_i|\varepsilon_{i-1}, \ldots, \varepsilon_1, x_i, x_{i-1}, \ldots, x_1) = 0$ (i.e. the conditional expected mean value of the error term is zero) (Hayashi 2000, pp. 202-203). The conditional and the unconditional expectations are equal when the error term is independent from the current and past instruments.

Assumptions III, IV and V were tested by means of the Hansen J-statistic. The null hypothesis is that the model variants are well specified.

Table 6-44:

CCAPM	Model variant	Hansen J-statistic	J-value	p-value
Lucas	M - 16	0,057457	7,814152	0,252033
Lucas	M - 17	0,10615	14,33025	0,026157
Lucas	M - 18	0,063757	8,543438	0,20093
Lucas	M - 19	0,05772	7,67676	0,26275
Lucas	M - 20	0,060594	7,998408	0,238222
Lucas	M - 21	0,15276	20,16432	0,912191
Abel, $\kappa = -1$	M - 23	0,041085	5,546475	0,475858
Abel, $\kappa = -1$	M - 24	0,121248	16,36848	0,011907
Abel, $\kappa = -1$	M - 25	0,066051	8,850834	0,182141
Abel, $\kappa = -1$	M - 26	0,063404	8,432732	0,208081
Abel, $\kappa = -1$	M - 27	0,062191	8,209212	0,223173
Abel, $\kappa = -1$	M - 28	0,194649	25,69367	0,589881
Abel, $\kappa = -0{,}001$	M - 35	0,033103	4,468905	0,613482
Abel, $\kappa = -0{,}001$	M - 36	0,106586	14,38911	0,025579
Abel, $\kappa = -0{,}001$	M - 37	0,063717	8,538078	0,201271
Abel, $\kappa = -0{,}001$	M - 38	0,05776	7,68208	0,262332
Abel, $\kappa = -0{,}001$	M - 39	0,060695	8,01174	0,237242
Abel, $\kappa = -0{,}001$	M - 40	0,151993	20,06308	0,862155
Campbell and Cochrane	M - 42	0,066771	8,947271	0,346764
Campbell and Cochrane	M - 43	0,082114	11,00328	0,201513
Campbell and Cochrane	M - 44	0,071289	9,481396	0,303326
Campbell and Cochrane	M - 45	0,081406	10,74565	0,216532
Campbell and Cochrane	M - 46	0,064592	8,461517	0,389733
Campbell and Cochrane	M - 47	0,172403	22,5848	0,977662
Constantinides and Duffie	M - 49	0,066343	8,889926	0,351667
Constantinides and Duffie	M - 50	0,114449	15,221760	0,054974
Constantinides and Duffie	M - 51	0,073443	9,694503	0,287126
Constantinides and Duffie	M - 52	0,083300	10,912240	0,206721
Constantinides and Duffie	M - 53	0,073432	9,546217	0,29833
Constantinides and Duffie	M - 54	0,174365	22,667490	0,987548

The p-values show that we reject the null hypothesis for the model variants M - 17, M - 24, M - 36, M - 37 and M - 50.

Assumption VI: Ergodic Stationarity

Let x_i be the K-dimensional vector of instruments, and let w_i be the unique and non-constant elements of (v_i, z_i, x_i). $\{w_i\}$ is jointly stationary and ergodic (Hayashi 2000, p.198). In a stationary and ergodic sequence the time average converges to the ensemple (expected) average as the sample size increases (Zivot 2013, p. 11).

The null hypothesis is that the residuals in each system of equations have a unit root.

Table 6-45: Unit root tests

CCAPM	Model variant	Unit root tests using Levin, Lin & Chu (LLC) t-star statistic	Unit root tests using Im, Pesaran and Shin (IPS) W-statistic	Unit root tests using ADF - Fisher (ADF-F) Chi-square	Unit root tests using PP – Fisher (PP-F) Chi-square
Lucas	M - 16	-5,61882 [0,0000] N=267	-7,24983 [0,0000] N=267	66,7321 [0,0000] N=267	137,377 [0,0000] N=270
Abel, $\kappa = -1$	M - 28	0,09955 [0,5397] N=255	-8,70294 [0,0000] N=255	78,2691 [0,0000] N=255	136,006 [0,0000] N=262
Abel, $\kappa = -0{,}001$	M - 39	-5,72677 [0,0000] N=259	-7,95157 [0,0000] N=259	71,2591 [0,0000] N=259	121,141 [0,0000] N=262
Campbell and Cochrane	M - 42	-6,08055 [0,0000] N=263	-7,17317 [0,0000] N=263	65,7025 [0,0000] N=263	127,984 [0,0000] N=266
Constantinides and Duffie	M - 49	-5,24745 [0,0000] N=263	-7,22483 [0,0000] N=263	66,0919 [0,0000] N=263	138,986 [0,0000] N=266

We tested the residuals in the systems of Euler equations we used for a common unit root. For Lucas CCAPM, Campbell and Cochrane's CCAPM, Constantinides and Duffie's CCAPM and Abel's CCAPM with $\kappa = -0{,}001$, the LLC , IPS, ADF – F and PP-F tests reject the null hypothesis of unit root of the Euler equations system. That means that the system of Euler equations is stationary. For Abel's model the LLC test shows that we can't reject the null hypothesis of a unit root when the parameter of time nonseparability of consumption preferences κ is equal to -1 while the unit root tests of IPS, ADF-F and PP-F reject the null hypothesis of a unit root. The Breitung unit root test also rejects the null hypothesis of a unit root with $t = -2{,}05908$, $p = 0{,}0197$. Since four out of five tests reject the null hypothesis of a unit root for Abel's model with $\kappa = -1$ we conclude that the test results show stationary processes also for this model.

LLN and CLT

In order to have asymptotically consistent estimators it is also assumed that the law of large numbers (LLN) and central limit theorem (CLT) apply:

*** Convergence of the empirical moments**

The empirical moments converge by the law of large numbers in probability to their expectation $E[\overline{m}_n(\theta_0)] = 0$ so that $\overline{m}_n(\theta_0) = \frac{1}{N}\sum_{i=1}^{N} m_i\,(\theta_0) \xrightarrow{p} 0$ (Greene 2012, pp. 474-475).

*** Asymptotic distribution of the empirical moments**

The empirical moments converge in distribution by central limit theorem to a normal distribution so that $\sqrt{n}\overline{m}_n(\theta_0) \xrightarrow{d} N[0, \Phi]$ (Greene 2012, pp. 476-477).

Similar conditions on asymptotically consistent estimators are applying also to ordinary least squares (OLS) (Greene 2012, pp. 65-67).

7 Total Conclusion

In this master thesis we set out with an expectation through empirical research to find explanations to the trading volume, the volatility of returns, the dispersion of the stock returns and the equity premium. We hoped to accomplish this under a heterogeneity perspective in the setting of the Norwegian stock market. The connection between these tests is the relaxation of the assumption of homogeneity in the context of our tests. The heterogeneity examined in this master involves dispersion of beliefs and heterogeneity in consumption.

Heterogeneity in the form of dispersion of beliefs forms the base of sentiment risk which in this thesis is the change in belief dispersion. We used the analysts' beliefs concerning price targets or targets on earnings as a proxy for sentiment. The results showed that the change in the standard deviation of analysts' beliefs is a useful explanatory variable of trading volume and volatility of stock returns, thus confirming the predictions of the models by Xiouros (2009) and Iori (2002).

Our empirical tests showed that the relation between the dispersion of stocks returns and market return is non-linear .This contradicts the prediction of the rational expectations CAPM of a linear relation. A non-linear negative relation is an illustration of herding behavior. We don't find evidence of herding. The coefficient for the quadratic term is positive and is interpreted as divergence of opinions which is a signal of heterogeneity. The nonlinearity is pronounced in the majority of high and low states of the stock market for volume, volatility and market returns despite directional asymmetry between certain high and low states.

In consumption asset pricing we substantiated the anticipation of heterogeneity leading to higher volatility in consumption of non-durables and services. Campbell et al. (1997, p. 329) conjectures that time nonseparability of preferences is likely to make the riskless real interest rate more variable. We find that the model relaxing the assumption of time separability of preferences produces higher variability in the Euler equation concerning the risk free rate of return while the model with the subsistence level of consumption generates the highest correlation between the stochastic discount factor and the equity premium. Heterogeneity in consumption produces higher volatility in the consumption growth, which is an advantage, but lower correlation between the stochastic discount factor and the equity premium, which is a disadvantage. So, idiosyncratic consumption has its own merits and flaws in explaining the equity premium puzzle.

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A - i: Definitions

Arbitrage

- Arbitrage: The opportunity to make a profit with no risk by exploiting deviations between the market price and the value based on available information (Davies, Lowes and Pass 1988, pp. 16-17).

Another definition is arbitrage as the purchase and sale at the same time of a security and a nearly perfect substitute in two different markets for different prices (Sharpe and Alexander 1990 cited in Shleifer and Vishny (1997, p. 35, see appendix A - iii).

Yet another definition is arbitrage as an opportunity to finance a consumption plan c with a trading strategy (α, θ) with the properties of:

$\alpha(0)B(0) + \theta(0)^T\big(S(0) + X(0)\big) \leq 0$

$c \geq 0$

$c(a_t, t) > 0$

$a_t \in \mathcal{F}_t$

where $c(a_t, t)$ denotes consumption at time, $a_t$is an event, $\mathcal{F}_t$is a sigma algebra of the information set F at time t, B and S are the prices of a stock and its substitute, α is the number of B stocks, θ is the number of S stocks and X is the dividend paid at time t (Huang and Litzenberger 1988, p.226).

A sigma algebra is a set $\mathcal{F}_t$that contains the empty set, the complement $X^c$of any set X in $\mathcal{F}_t$ and the union of the sequences X_n (Øksendal 2000, p.7). A sigma algebra shows the information generated up to time t.

- Limited arbitrage: The inability to take full advantage of arbitrage opportunities as a result of fundamental risk, noise trader risk and implementation costs (Ackert and Deaves 2010, pp.72-74). The fundamental risk pertains to the risk of new information arriving before the arbitrage transaction is completed. The noise trader risk concerns the risk of discrepancy between an asset's intrinsic value and the asset's market price. Implementation costs refer to costs incurred when trades are actualized.

- Excess return: The difference between an asset's return and the return on some reference asset. It is also the payoff on an arbitrage portfolio that goes long in the first asset and short in the reference asset with no net investment at the initial date (Campbell et al. 1997, p. 12)

Behavior

- Behavioral finance: Is the body of financial theory which attempts to explain the development of asset prices on psychological grounds and obstacles to smart money (Shiller 2003).

- Herding: Is the alignment of individual behavior by imitation (Jegadeesh and Kim 2010).

- Rational behavior: Behavior that is free for emotional bias and consistent with financial facts rather than rumours. It is often associated with risk aversion and utility maximization.

- Bounded rationality: Constrained rationality due to acknowledgement that information is costly to acquire and process (Jones 1999).

- Near rational behavior: Behavior steered by bounded rationality.

Beliefs

Beliefs are probability distributions about the states of the world adapted to the observation of private signals (Barucci 2003, p. 219). Many authors are using opinions and beliefs interchangeably (Miller 1977, Varian 1980). In this master thesis we chose to use divergence of opinions and dispersion of beliefs interchangeably.

Bubbles and Crashes

- Bubble: A steep climbing of security prices above their value based on fundamentals, often as a result of a collective euphoria.

- Crash: A steep fall of security prices, often as a result of panic behavior.

Cost types

- Transaction costs: Costs incurred in an economic exchange (Ball and Brown 1968).

- Information costs: Costs of acquiring and processing information in order to exploit arbitrage opportunities (Grossman and Stiglitz 1980, see appendix A - iv, Brock and Hommes 1998, Hommes 2006).

Efficient Market Hypothesis

- A hypothesis that security prices reflect the available relevant information. The efficiency comes from competition to exploit arbitrage opportunities when the security price deviates from its intrinsic value based on that information. Weak market efficiency applies to past information, semi-strong applies to past and present public information and strong applies to past, present public and inside information (Davies, Lowes and Pass, 1988, p. 160).

Equilibrium in finance

Here we follow the presentation by Barucci (2003).

- Equilibrium asset price

An equilibrium asset price is obtained when arbitrage opportunities are precluded (Neftci 2000, p.13 and Pliska 2005, p.67). Given a pair of a stock price-dividend (S, D) adapted process there exists an economy generating S as an equilibrium price process (Barucci 2003, pp.164-166). With adapted process is meant a process B_s which entails all information generated up to time t (Øksendal 2000, p. 25).

- The Arrow – Debreu equilibrium

Assume a complete market, i.e. a market with a contingent claim for each state of the world. The pairs of prices and goods which are the solutions to the agents' utility maximizations given their endowment constraints constitute the Arrow - Debrew equilibrium (Debreu 1959, Arrow 1968).

- The Radner equilibrium

Radner (1982, p. 932) defines a rational expectation equilibrium under uncertainty. Assume the agents' preferences, their initial endowments and the asset returns being common knowledge. If the agents in addition have homogeneous price expectations and perfect foresight, the same prices can be associated to the same events.

- The Green - Lucas equilibrium

Assuming asymmetric information and heterogeneous beliefs, the agents are making use of prices as information tools for updating their beliefs on the probability of events. By the market clearing condition that demand equals supply the assets fundamental value can be derived (Barucci 2003, pp. 219 and 226-227).

- Other Equilibrium models

An equilibrium model of asset pricing in relation to principal-agent issues is derived by Ou-Yang (2005). He sets up a model where the expected rate of return is a function of the idiosyncratic (non-systematic) risk and the management's compensation scheme as well.

Equilibrium in strategies

- Nash equilibrium: An equilibrium in a zero sum game where each player's strategy is optimal given the other players strategy. A subgame perfect equilibrium is a strategy set which is a Nash equilibrium for every subgame (Bierman and Fernandez 1998, p.133).

- Tipping: The transition at macro level from one equilibrium state to another on the grounds of preferences at micro level (Schelling 1971).

Expectations

- Homogeneous expectations: Individual's sharing of the same beliefs on the relative likelihoods of different states occurring is termed homogeneous expectations. It implies that the set of state probabilities is common knowledge (Copeland, Weston and Shastri 2005, p. 82). Agents associate the same probabilities and prices to the same events (Barucci 2003, p.84).

- Heterogeneous expectations: Differences in beliefs about future prices (Brock and Hommes 1998).

- Adaptive expectations: The expected value of an economic variable P_t formed adaptively by

$P_{t+1} - P_t = b(P_{t+1} - P_t)$

Integrating forwards the equation becomes

$P_{t+1} = bP_{t+1} + (1-b)P_t = bP_{t+1} + b(1-b)P_t + b(1-b)^2 P_{t-1} \dots$

(Chow 2011)

- Rational Expectations: An agent's expectation of the value of a variable v at time t + k, where k $\geq$ 0, given the information set I, such that:

$E_{t-1}v_{t+k} = E[v_{t+k}|I_{t-1}] + \varepsilon_t$, where ε_t is an error term satisfying $E\left[\varepsilon_{t+k}\middle|I_{t-1,i}\right] = 0$. (Tesfatsion 2011, p.2)

- Iterated Expectations, the law of: The expected value given an information set I_t of the expected value given information set J_t where $I_t \subset J_t$ is equal to the expected value given an information set I_t.

$E[X|I_t] = E[E[X|J_t]|I_t]$, where I and J are information sets and $I_t \subset J_t$.

The above equation means that we cannot make a forecast given the information set I_t that is better than the forecast we could make given the information set J_t.

(Campbell et al. 1997, p. 24 and Cochrane 2005, p.133)

Information types

- Symmetric information: The same information is shared equally by all agents.

- Asymmetric information: Information that is known only to some of the players in a zero sum game.

Market microstructure

- The study of security trading and pricing under the influence of trading rules and the strategic behavior among trading parties (O'Hara 1995, p.1).

Motion and models of stock prices

- Random walk: A stochastic process in discrete time in which future development cannot be predicted:

$P_t = \mu + P_{t-1} + \varepsilon_t$

There are several versions of random walk. One versions assumes the error term increments are independent and identically distributed, $\varepsilon_t \sim IID\ (0, \sigma^2)$. Another version assumes INID error term increments, i.e. independent but not identically distributed. A third version assumes uncorrelated but dependent increments, for instance $Cov[\varepsilon_t, \varepsilon_{t-k}] = 0$ for all $k \neq 0$ but where $Cov[\varepsilon_t^2, \varepsilon_{t-k}^2] \neq 0$ for some $k \neq 0$ (Campbell et al. 1997, p.32-33). Edges in a probability distribution which are thicker than in a normal distribution are called fat tails which is a symptom of excess volatility (Dash 2004, p.284).

- Autoregressive models: Time series models in which the value of a variable depends on its historical values.

- Autoregressive model AR (1): It is a time series model where the lagged value at time t-1 can be used as independent variable. Because the independent variable is a previous value of the dependent variable, the errors are not independent between periods.
(Solberg 1992, p.357)

- Martingale: A stochastic process M_t adapted to the information set $\mathcal{F}_s$ such as:
$E[M_t|\mathcal{F}_s] = M_s$ for all $s \leq t$
(Benth 2002, p.43)

- Submartingale: A stochastic process M_t adapted to the information set $\mathcal{F}_s$ such as:
$E[M_t|\mathcal{F}_s] \geq M_s$ for all s $\leq t$.
(Øksendal 2000, p.298)

- Brownian motion: A motion modelled by a Wiener process, that is a stochastic dynamic process in continuous time defined by expected average and variance (Borowski and Borwein 1989, p.635). It is customary to denote it with B.
The parameter H in $|B_{t+1} - B_t|^{2H}$, $0 \leq H \leq 1$ is called the Hurst exponent. For H = $\frac{1}{2}$ is B the familiar Brownian motion. For H$\neq \frac{1}{2}$ we get fractional Brownian motions. For H > $\frac{1}{2}$ are the increments positively correlated and for H < $\frac{1}{2}$ are the increments negatively correlated (Voss 1989, p.24). A mathematical operator that can be used on fractional Brownian motions is the Wick product which is the product $F \lozenge G = \sum_{n,m} I_{n+m}(f_n \widehat{\otimes} g_n)$ of two square integrable random variables where $F = \sum_{n=0}^{\infty} I_n(f_n)$ and $G = \sum_{n=0}^{\infty} I_n(g_n)$, (Nualart and Taqqu 2006). Square integrable stochastic variables have finite variance (Neftci 2000, p.126). Fractional Brownian motion with H > $\frac{1}{2}$ can be derived from a Polya urn process (Hammond and Sheffield 2011). A Polya urn process is a time dependent stochastic process where the probability of picking a ball of a certain colour from an urn depends on the number of balls of the same colour already in the urn (Zhu 2009, p.3).

- Chaos: A dynamic process which resembles a random walk but is deterministic. Small changes in the initial conditions have a great influence on the development of the process (Hsieh 1991).

- Fat tails: The distribution of a random variable X is said to have a fat tail if
$Pr[X > x] \sim x^{-\alpha}$ as $x \rightarrow \infty, \alpha > 0$
(Fama 1965a)

Opinions

Barucci (2003, p. 219) is making a distinction between opinions and beliefs. He defines opinions as probability distributions about the states of the world not adapted to the observation of private signals. Many authors are using opinions and beliefs interchangeably without making this distinction (Miller 1977, Varian 1980). In this master thesis we chose to use divergence of opinions and dispersion of beliefs interchangeably.

Preferences

- Homogeneous preferences: Similar preferences to risk or consumption, regarding the level or the timeline.

- Heterogeneous preferences: Differences in preferences to risk or consumption, regarding the level or the timeline (Tran and Zeckhauser 2011).

Trader types

- Noise traders: Security trade agents who act on erratic or irrational grounds and introduce noise in the security prices (Black 1986).

- Chartists: Security trade agents who believe that the prices can be predicated by analysing charts of past movements.

- Liquidity traders: Security trade agents who act on their preferences to consumption sooner than later or their need to cash.
- Market makers: Security trade agents who harmonize supply and demand through price adjustments and provide liquidity to the market (Davies, Lowes and Pass, 1988, p. 314).

- Fundamentalists: Security trade agents who judge security prices by analysing the fundamentals of companies, i.e. the degree of soundness of their current and prospect financial positions.

- Insiders: Agents who have privileged company information in advance of publicizing. Transaction of securities based on this type of information is called insider trading and stands to financial gains through exploitation of arbitrage opportunities (Davies, Lowes and Pass, 1988, p. 253).

A - ii: De Long, Shleifer, Summers and Waldmann (1990): Noise Trader Risk in Financial markets

$R_{t,s}^{e} = E[R_t]$	Ap - 1

"Smart" traders, expected return smart trader fundamentalist

$R_{t,n}^{e} = E[R_t] + \rho_t \quad \rho \sim N\,(\rho^{*}, \sigma_\rho^2\ )$	Ap - 2

"Noise" traders, chartists, ρ_t noise term

$U(w_t) = E[w_t] - \gamma\, var[w_t],$	Ap - 3

Utility function, all agents

$p_f = 1$	Ap - 4

Price of risk free asset

$d_{t+1} = r$	Ap - 5

Dividends of the risky asset

p_t is the price of risky asset

The excess demand of risky asset, "smart" traders is

$q_t^s = \dfrac{E[p_{t+1}^e + d_{t+1} - p_{t\,(1+r)}]}{2\gamma\sigma_t^2}$	Ap - 6

Constant absolute risk aversion (CARA) utility function

$q_t^s = \frac{p_{t+1}^e + r - p_{t\,(1+r)}}{2\gamma\sigma_t^2} = \frac{R_t^e}{2\gamma\sigma_t^2} \rightarrow \sigma_t^2 = var[p_{t+1} + d_{t+1}] = var[p_{t+1} + r] = var[p_{t+1}]$	Ap - 7

Excess demand of risky asset, "noise" traders;

$q_t^n = \dfrac{p_{t+1}^e + r - p_{t\,(1+r)} + \rho_t}{2\gamma\sigma_t^2} = \dfrac{R_t^e + \rho_t}{2\gamma\sigma_t^2}$	Ap - 8

$(1-\mu)\, q_t^s + \mu q_t^n = 1$	Ap - 9

μ fraction of noise traders

$(1-\mathrm{\mu})\dfrac{p_{t+1}^e + r - p_t\,(1+\mathrm{r})}{2\gamma\sigma_t^2} + \mu\dfrac{p_{t+1}^e + r - p_t\,(1+\mathrm{r}) + \rho_t}{2\gamma\sigma_t^2} = 1$ $p_t\dfrac{(1+r)}{2\gamma\sigma_t^2}(1-\mu) + (1-\mu)\dfrac{p_{t+1}^e}{2\gamma\sigma_t^2} - p_t\dfrac{(1+r)}{2\gamma\sigma_t^2}\,\mu$ $+\mu\dfrac{p_{t+1}^e + \rho_t}{2\gamma\sigma_t^2} + \dfrac{r}{2\gamma\sigma_t^2} = 1 \xrightarrow{\times(-1)}$ $p_t\dfrac{(1+r)}{2\gamma\sigma_t^2}(1-\mu) + p_t\dfrac{(1+r)}{2\gamma\sigma_t^2}\,\mu + (\mu-1)\dfrac{p_{t+1}^e}{2\gamma\sigma_t^2} - \mu\dfrac{p_{t+1}^e + \rho_t}{2\gamma\sigma_t^2} - \dfrac{r}{2\gamma\sigma_t^2} = -1$ $\xrightarrow{2\gamma\sigma_t^2} p_t(1+r) + (\mu-1)p_{t+1}^e - \mu(p_{t+1}^e + \rho_t) - r = -2\gamma\sigma_t^2 \rightarrow$ $p_t(1+r) = -(\mu-1)p_{t+1}^e + \mu(p_{t+1}^e + \rho_t) + r - 2\gamma\sigma_t^2 \rightarrow$ $p_t(1+r) = -\mu\, p_{t+1}^e + p_{t+1}^e + \mu\, p_{t+1}^e + \mu\,\rho_t + r - 2\gamma\sigma_t^2 \rightarrow$ $p_t(1+r) = p_{t+1}^e + \mu\,\rho_t + r - 2\gamma\sigma_t^2 \rightarrow$ $p_t = \dfrac{1}{1+r}\,(p_{t+1}^e + \mathrm{r} + \mathrm{\mu}\rho_t - 2\gamma\sigma_t^2) \rightarrow$ $\xrightarrow{E_{t+1}} E\,[p_{t+1}] = E\,[\dfrac{1}{1+r}\,(p_{t+2} + \mathrm{r} + \mathrm{\mu}\rho_{t+1} - 2\gamma\sigma_{t+1}^2)]$	Ap - 10

The variance is constant

$p_{t+1}^e = \left(\dfrac{1}{1+r}\right)[E(p_{t+2} + \mathrm{r} + \mathrm{\mu}\rho_{t+1} - 2\gamma\sigma_t^2)] \rightarrow$ $p_{t+1}^e = \left(\dfrac{1}{1+r}\right)[p_{t+1}^e + \mathrm{r} + \mathrm{\mu}\rho^* - 2\gamma\sigma_t^2)] \rightarrow$ $p_t = \left(\dfrac{1}{1+r}\right)[\left(\dfrac{1}{1+r}\right)(p_{t+2}^e + \mathrm{r} + \mathrm{\mu}\rho^* - 2\gamma\sigma_t^2) + r + \mathrm{\mu}\rho_t - 2\gamma\sigma_t^2\,] \rightarrow$	

$\xrightarrow{\left(\frac{1}{1+r}\right)(p^e_{t+2}+r+\mu\rho^*-2\gamma\sigma^2_t=p^e_{t+1}}$ $p_t=\left(\frac{1}{1+r}\right)[\left(\frac{1}{1+r}\right)p^e_{t+2}+\ \mathrm{r}(\frac{1}{1+r}+1)$ $+\mu\rho^*(\frac{1}{1+r})+\mu\rho_t-2\gamma\sigma^2_t(\frac{1}{1+r}+1)\ ]$	Ap - 11

$\xrightarrow{\lim_{n\to\infty}\frac{1}{(1+r)^n}}\ p_t=\underbrace{p^e_{t+2}\frac{1}{(1+r)^2}}_{0}{}_t+r\left(\frac{1}{(1+r)^2}+\frac{1}{1+r}\right)$ $+\mu\rho^*\frac{1}{(1+r)^2}+\mu\rho_t\frac{1}{1+r}\ -2\gamma\sigma^2_t\left(\frac{1}{(1+r)^2}+\frac{1}{1+r}\right)$	Ap - 12

Use ordinary annuity:

$\sum_1^n\frac{1}{(1+r)^n}=(\frac{1}{(1+r)^n}+\cdots+\frac{1}{(1+r)^2}+\frac{1}{1+r})=\frac{1}{r}$	Ap - 13

$r\frac{1}{r}+\ \mu\rho^*\frac{1}{r(1+r)}\ +\ \mu\rho_t\frac{1}{1+r}-2\gamma\sigma^2_t\ \frac{1}{r}\rightarrow$ $=1+\ \mu\frac{1}{1+r}(\rho^*\ \frac{1}{r}+\rho_t)-2\gamma\sigma^2_t\ \frac{1}{r}$	Ap - 14

$E[p_t]=\mathrm{E}\ \left[1+\mu\frac{1}{1+r}\left(\frac{1}{r}\rho^*+\rho_t\right)-2\gamma\sigma^2_t\ \frac{1}{r}\right]\rightarrow$ $E[p_t]=\ \mathrm{E}\ \left[\mu\frac{1}{1+r}\frac{1}{r}\rho^*\right]+\ \mathrm{E}\ \left[\frac{1}{1+r}\ \rho_t\ \mu\right]=\ \mu\frac{1}{1+r}\frac{1}{r}\rho^*+\frac{1}{1+r}\rho^*\ \mu\rightarrow$	

$$= \mu\rho^{*}(\frac{1}{(1+r)r} + \frac{r1}{r(1+r)}) = \mu\rho^{*}\left(\frac{1+r}{r(1+r)}\right) = \mu\rho^{*}\frac{1}{r}$$ Ap - 15

$$E[p_t] = 1 + \mu\rho^{*}\frac{1}{r} - 2\gamma\sigma_t^2\ \frac{1}{r} \xrightarrow{\mathrm{E}[\rho_t]=\bar{p}} \quad \bar{p} = 1 + \mu\rho^{*}\frac{1}{r} - 2\gamma\sigma_t^2\ \frac{1}{r}$$ Ap - 16

$$p_t = 1 + \mu\frac{1}{1+r}\left(\rho^{*}\ \frac{1}{r} + \rho_t\right) - 2\gamma\sigma_t^2\ \frac{1}{r} \quad \rightarrow$$

$$p_t = 1 + \frac{1}{1+r}\ \mu\rho_t + \frac{1}{r(1+r)}\ \mu\rho^{*} - \frac{2\gamma\sigma_t^2}{r}$$ Ap - 17

$$\xrightarrow{add\ a\ period} p_{t+1} = 1 + \frac{1}{1+r}\mu\rho_{t+1} + \frac{1}{r(1+r)}\ \mu\rho^{*} - \frac{2\gamma\sigma_{t+1}^2}{r}$$ Ap - 18

$$\sigma_{t+1}^2 = \sigma_t^2$$ Ap - 19

Variance is assumed to be constant over time

$$p_{t+1} - \bar{p} = 1 + \frac{1}{1+r}\ \mu\rho_{t+1} + \frac{1}{r(1+r)}\ \mu\rho^{*} - \frac{2\gamma}{r}\sigma_{t+1}^2 - 1 - \frac{1}{r}\ \mu\rho^{*} + \frac{2\gamma}{r}\sigma_{t+1}^2 \rightarrow$$

$$= \frac{1}{1+r}\mu\rho_{t+1} + \frac{1}{r(1+r)}\ \mu\rho^{*} - \frac{1}{r}\ \ \mu\rho^{*} = \frac{r.1}{r(1+r)}\mu\rho_{t+1} + \frac{1}{r(1+r)}\ \mu\rho^{*} - \frac{(r+1).1}{r(1+r)}\ \mu\rho^{*} \rightarrow$$

$$= \frac{r.\mu.\rho_{t+1} + \mu\rho^{*} - r\mu\rho^{*} - \mu\rho^{*}}{r(1+r)} = p_{t+1} = \frac{r(\mu\rho_{t+1} - \mu\rho^{*})}{r(1+r)} = \mu(\frac{\rho_{t+1} - \rho^{*}}{(1+r)})$$ Ap - 20

$\left.\begin{array}{l}\sigma_t^2 = \text{var}[p_{t+1}] = \text{E}[(p_{t+1} - \bar{p})^2] \\ p_{t+1} - \bar{p} = \mu(\frac{\rho_{t+1} - \rho^*}{(1+r)})\end{array}\right\} \to E[(p_{t+1} - \bar{p})^2] = \to \text{E}[\,\mu(\frac{\rho_{t+1} - \rho^*}{(1+r)})]^2 = \to$ $E[\,\mu^2 \frac{(\rho_{t+1} - \rho^*)^2}{(1+r)^2}] = \mu^2 \frac{1}{(1+r)^2} \text{E}[(\rho_{t+1} - \bar{\rho})^2] \to$ $= \mu^2 \frac{1}{(1+r)^2} \sigma_\rho^2 = \mu^2 \frac{\sigma_\rho^2}{(1+r)^2} \to \sigma_t^2 = \mu^2 \frac{\sigma_\rho^2}{(1+r)^2}$	Ap - 21

$\sigma_\rho^2 = 0 \to p_t = p_f$

$\forall\ t, p$ is the price of an asset

$\left.\begin{array}{l} q_t^s = \frac{p_{t+1}^e + r - p_{t\,(1+r)}}{2\gamma\sigma_t^2} = \frac{R_t^e}{2\gamma\sigma_t^2} \dashrightarrow 2\gamma\sigma_t^2\, q_t^s = p_{t+1}^e + r - p_{t\,(1+r)} \\ q_t^n = \frac{p_{t+1}^e + r - p_{t\,(1+r)} + \rho_t}{2\gamma\sigma_t^2} = \frac{R_t^e + \rho_t}{2\gamma\sigma_t^2} \dashrightarrow 2\gamma\sigma_t^2\, q_t^n = p_{t+1}^e + r - p_{t\,(1+r)} + \rho_t \end{array}\right\} \to$ $q_t^n - q_t^s = \frac{\rho_t}{2\gamma\sigma_t^2}$	Ap - 22

$\Delta R = (q_t^n - q_t^s)\left(r + p_{t+1}^e - p_t(1+r)\right)$	Ap - 23

Where r is the return on risk free asset

$p_{t+1}^e - p_t(1+r)$	Ap - 24

The above equation shows the excess return on a risky asset.

$p_t = \frac{1}{1+r}(p_{t+1}^e + \text{r} + \mu\rho_t - 2\gamma\sigma_t^2) \to$ $p_t(1+r) = p_{t+1}^e + \text{r} + \mu\rho_t - 2\gamma\sigma_t^2 \to$ $2\gamma\sigma_t^2 = p_{t+1}^e + \text{r} + \mu\rho_t - p_t(1+r) \to 2\gamma\sigma_t^2 - \mu\,\rho_t = p_{t+1}^e + r - p_t(1+r) \to$ $2\gamma\sigma_t^2 - \mu\,\rho_t = r + p_{t+1}^e - p_t(1+r)$	Ap - 25

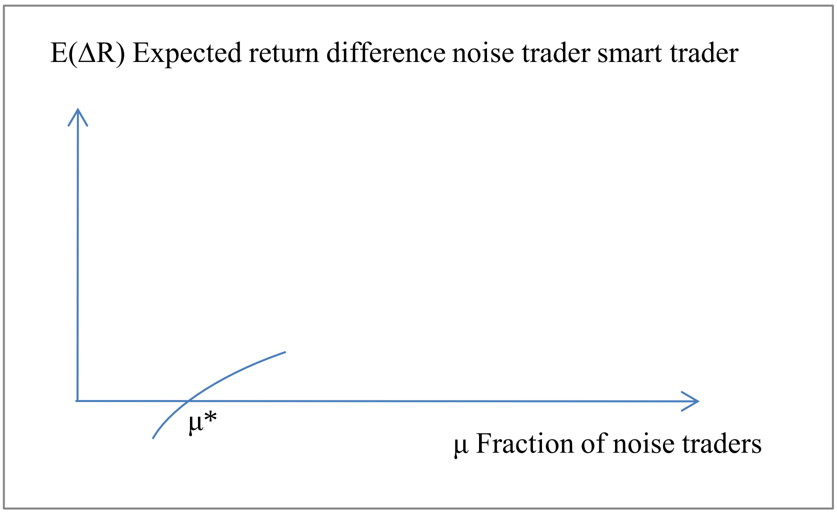
$$\left.\begin{array}{r} 2\gamma\sigma_t^2 - \mu\,\rho_t = r + p_{t+1}^e - p_t(1+r) \\ \Delta R = (q_t^n - q_t^s)\left(r + p_{t+1}^e - p_t(1+r)\right) \end{array}\right\} \rightarrow \Delta R = (q_t^n - q_t^s)(2\gamma\sigma_t^2 - \mu\rho_t)$$	Ap - 26

$$\left.\begin{array}{r} q_t^n - q_t^s = \dfrac{\rho_t}{2\gamma\sigma_t^2} \\ \Delta R = (q_t^n - q_t^s)\,(2\gamma\sigma_t^2 - \mu\rho_t) \end{array}\right\} \rightarrow \Delta R = \frac{\rho_t}{2\gamma\sigma_t^2}(2\gamma\sigma_t^2 - \mu\rho_t)$$ $$\rightarrow \Delta R = \frac{\rho_t}{2\gamma\sigma_t^2}(2\gamma\sigma_t^2 - \mu\rho_t) = \frac{\rho_t}{2\gamma\sigma_t^2}2\gamma\sigma_t^2 - \frac{\rho_t}{2\gamma\sigma_t^2}\mu\rho_t = \rho_t - \frac{\mu\rho_t^2}{2\gamma\sigma_t^2}$$	Ap - 27

$$E[\Delta R] = E\left[\rho_t - \frac{\mu\rho_t^2}{2\gamma\sigma_t^2}\right] \rightarrow E[\Delta R] = E[\rho_t - E\left[\frac{\mu\rho_t^2}{2\gamma\sigma_t^2}\right] \rightarrow$$ $$E[\Delta R] = \rho^* - \frac{E[\mu\rho_t^2]}{E[2\gamma\sigma_t^2]} = \rho^* - \frac{\mu E[\rho_t^2]}{2\gamma\sigma_t^2}\rho^* - \frac{\mu\left(\rho^{*\,2} + \sigma_\rho^2\right)}{2\gamma\sigma_t^2} \rightarrow$$ $$\left.\begin{array}{r} E[\Delta R] = \rho^* - \dfrac{\mu\left(\rho^{*\,2} + \sigma_\rho^2\right)}{2\gamma\sigma_t^2} \\ \sigma_t^2 = \mu^2\,\dfrac{\sigma_\rho^2}{(1+r)^2} \end{array}\right\} \rightarrow E[\Delta R] = \rho^* - \frac{\mu\left(\rho^{*\,2} + \sigma_\rho^2\right)}{2\gamma\mu^2\,\dfrac{\sigma_\rho^2}{(1+r)^2}} \rightarrow$$ $$\rho^* - \frac{(1+r)^2\mu\left(\rho^{*\,2} + \sigma_\rho^2\right)}{2\gamma\mu^2\sigma_\rho^2} = \rho^* - \frac{(1+r)^2\left(\rho^{*\,2} + \sigma_\rho^2\right)}{2\gamma\mu\,\sigma_\rho^2}$$	Ap - 28

if $\rho^* - \frac{(1+r)^2\left(\rho^{*\,2}+\sigma_\rho^2\right)}{2\gamma\mu\,\sigma_\rho^2} < 0$ then noise traders make less profit than smart traders

if $\rho^* - \frac{(1+r)^2\left(\rho^{*\,2}+\sigma_\rho^2\right)}{2\gamma\mu\,\sigma_\rho^2} > 0$ then noise traders can make more profit than smart traders depending on the relation between the other parameters. For example, other things equal, the higher μ, i.e. the higher fraction of noise traders, the higher profits for noise traders.



Appendix Figure - 1: Expected return difference noise trader smart trader as a function of fraction of noise traders

$$E[\Delta R] = \rho^* - \frac{(1+r)^2\left(\rho^{*\,2} + \sigma_\rho^2\right)}{2\gamma\mu\,\sigma_\rho^2} = 0 \rightarrow$$

$$\rightarrow \frac{2\gamma\mu\,\sigma_\rho^2\rho^* - (1+r)^2\left(\rho^{*\,2} + \sigma_\rho^2\right)}{2\gamma\mu\,\sigma_\rho^2} = 0 \rightarrow$$

$$\rightarrow 2\gamma\mu\,\sigma_\rho^2\rho^* - (1+r)^2\left(\rho^{*\,2} + \sigma_\rho^2\right) = 0$$

$$\mu^* = \frac{(1+r)^2\left(\rho^{*\,2} + \sigma_\rho^2\right)}{2\gamma\sigma_\rho^2\rho^*}$$

Ap - 29

$\mu > \mu^*$ $\rightarrow$ the noise traders make higher returns than smart traders.

A - iii: Shleifer - Vishny model of short termism (1990)

Let the interaction between the costs of borrowing and the information about a firm's future prospects is revealed. The demand by noise traders for the equity of a firm engaged in project $i = s$ or $i = l$ where s stands for short term with $t = 1$ and l for long term with $t = 1,2$ is

$q(NT, i) = (V_t - S_t)/P_t$	Ap - 30

where NT denotes noise traders, V is the asset's fundamental value and $S > 0$ is a pessimism shock.

The demand curve of smart money is

$q(SM, i) = n_t b/P_t$	Ap - 31

where n is the arbitrageurs number and b are the money borrowed to invest on asset i.

There is one unit supply of each asset i so equilibrium is given by

$1 = q(SM, i) + q(NT, i)$	Ap - 32

and, hence, using Ap - 30and Ap - 31, the equilibrium price for each asset is given by

$P_t^e = V_t - S_t + n_t b$	Ap - 33

In period 0 the arbitrageur buys $\frac{b}{(\mathrm{V}-S+nb)}$ shares of the asset. The net return NR_s in period 1 over the borrowing cost of bR for the short term asset is

$NR_s = \frac{V_s b}{P_s^e} - bR = \frac{bV_s}{(\mathrm{V}_s - S_s + n_s b)} - bR$	Ap - 34

The net return NR_l in period 1 over the borrowing cost of bR for the long term asset is

$NR_l = \frac{V_l b}{P_l^e} - bR = \frac{bV_l}{\mathrm{R}(\mathrm{V}_l - S_l + n_l b)} - bR$	Ap - 35

Arbitrage becomes less lucrative as R rises.

The noise traders can be irrationally pessimistic (or optimistic) about future returns. This noise trader risk implies that equilibrium price can permanently deviate from the fundamentals price due to limits to arbitrage. The longer the time it takes to reveal to the market the success of the firm's investment decisions, the greater the mispricing. This encourages short-termism in investment project.

A - iv: Grossman and Stiglitz (1980): On the Impossibility of Informationally Efficient Market

Important Conjectures from Grossman and Stiglitz:

● The greater the number of individuals who are informed, the more informative is the price system.

● The higher the cost of information, the smaller will be the equilibrium percentage of individuals who are informed.

● The greater the magnitude of noise, the less informative will the price system is. In equilibrium, the greater the magnitude of noise, the larger the proportion of informed individuals.

● Other things being equal, markets will be thinner under those conditions in which the percentage of individuals who are informed is either near zero or near one... markets will be thin when there is very little noise in the system, or when costs of information are very low.

- Thin market:

Few bids to buy and few offers to sell. Prices in thin markets are more volatile than in markets with great liquidity because the few trades that take place affect prices significantly. Institutional investors, who buy and sell large blocks of stock, tend to avoid thin markets, because getting in or out of position affect's the stock's price.

- Results:

● An increase in the quality of information increases the informativeness of the system.

● A decrease in the cost of information increases the informativeness of the system.

● A decrease in risk aversion leads informed individuals to take larger positions, increases informativeness of the system.

● An increase in noise decrease the informativeness of the prices, ceteris paribus, but increase the proportion of informed trades who find it profitable to trade. The net effect is that the informativeness of the system remains unchanged.

● As the preciseness of information decrease, prices become less informative, ceteris paribus, but increases the proportion of traders who find it profitable to trade. The net effect is no change in the informativeness of the system.

$\tilde{u}$:a risky asset

$\tilde{u} = \tilde{\theta} + \tilde{\varepsilon}$	Ap - 36

where $\tilde{\theta}$ is observable at cost c; $\tilde{\varepsilon}$ is unobservable.

The budget constraint is:

$$M_i + PX_i = W_{0i} \equiv \bar{M}_i + P\bar{X}_i \qquad \text{Ap - 37}$$

Securities: $\bar{M}_i$ denotes a riskless security in the initial endowment, $\bar{X}_i$ denotes a risky security in the initial endowment.

1= price of riskless security

P : price of risky security

Total "endowment" at time 0 for trader i:

$$W_{0i} = \bar{M}_i + P\bar{X}_i \qquad \text{Ap - 38}$$

"Trading" at time 0 (change proportions of risky and riskless assets):

Investment at $t = 0 \rightarrow M_i + \mathrm{P}X_i$ (Ap - 39)

Time 1 pay- offs:

$$M_i\mathrm{R} + X_i\tilde{u} = W_{1i} \qquad \text{Ap - 40}$$

Utility function:

$$V(W_{1i}) = -e^{-aW_{1i}} \qquad \text{Ap - 41}$$

The utility is defined over $t = 1$

Pick W_{1i} to max. expected utility (value of wealth at t=1)

$$maxE(\,V(W_{1i})) = maxE(-e^{-aW_{1i}}) = \; max\;[-E(e^{-aW_{1i}})] \qquad \text{Ap - 42}$$

Distributional assumption:

Uncorrelated→ $\tilde{\theta}, \tilde{\varepsilon}, \sim N(\cdot,\cdot);\; E[\tilde{\varepsilon}] = 0, E\left[\tilde{\varepsilon}\,,\tilde{\theta}\right] = 0$ (Ap - 43)

Since $\tilde{u}$ is a linear combination of normals, $\tilde{u} = \tilde{\theta} + \tilde{\varepsilon}$, $\tilde{u}$ is normal too.

$$W_{1i} = M_i R + X_i \tilde{u} \qquad \text{Ap - 44}$$

Lognormal distribution, useful properties: $\ln(\Psi) \sim N(\mu, \sigma^2) \rightarrow \Psi$ has lognormal distribution;

$$E[\Psi] = e^{\mu+\frac{1}{2}\sigma^2}\ ;\ var(\Psi) = e^{2\mu}\left(e^{2\sigma^2} - e^{\sigma^2}\right) \rightarrow$$

$$E[\Psi] = e^{E[ln(\Psi)]+\frac{1}{2}var(ln(\Psi))}$$

Let $\Psi = e^{-aW_{1i}}$

$$E[\ln(\Psi)] = \mathrm{E}[\ln(e^{-aW_{1i}})] \rightarrow$$

$$\mathrm{E}[-\mathrm{a}W_{1i}] = \mathrm{E}[-\mathrm{a}\,(M_i\ R + X_i\ \tilde{u})] = -a(M_i\ R + X_i\ E[\tilde{u}]) \qquad \text{Ap - 45}$$

$$Var\big(ln(\Psi)\big) = var\ (ln(e^{-aW_{1i}})) = var\ (-a \cdot W_{1i}) =$$

$$= \mathrm{var}\big(-\mathrm{a}(M_i\ R + X_i\ \tilde{u})\big) = \mathrm{var}\ \big(-\mathrm{a}(X_i\ \tilde{u})\big) = (-a)^2\ X_i\ ^2\mathrm{var}(\tilde{u}) \qquad \text{Ap - 46}$$

$$E[\Psi] = -E[e^{-a}W_{1i}] = -e^{-a(M_i\ R+X_i\ E[\tilde{u}])+\frac{1}{2}a^2\ X_i\ ^2\ var(\tilde{u})} \qquad \text{Ap - 47}$$

Maximization problem: Maximize expected utility subject to budget constraint:

$$\underbrace{Max}_{\{M_i,X_i\}}\ \mathrm{E}[e^{-a}W_{1i}] = -e^{-\mathrm{a}(M_i\ R+X_i\ E[\tilde{u}])+\frac{1}{2}a^2\ X_i\ ^2\ \mathrm{var}(\tilde{u})} \qquad \text{Ap - 48}$$

Subject to

$$M_i + \mathrm{P}X_i = \bar{M}_i + \mathrm{P}\bar{X}_i = W_{0i} \qquad \text{Ap - 49}$$

where M_i , fraction of risk free asset; X_i= fraction of risky asset

Solution: substitute budget constraint into objective function after solving for one “decision” variable f.eg. M_i ;

$M_i = W_{0i} - \mathrm{P}X_i$	Ap - 50

The F.O.C. demand of risky asset for agent is:

$-E[e^{-aW_{1i}}] = -e^{-a(M_i R+X_i E[\tilde{u}])+\frac{1}{2}a^2 X_i^2 var(\tilde{u})} =$ $= -e^{-a[(W_{0i}- PX_i)] R+X_i E[\tilde{u}])+\frac{1}{2}a^2 X_i^2 var(\tilde{u})}$ $\frac{\partial}{\partial X_I}\left(-e^{-a[(W_{0i}- PX_i)] R+X_i E[\tilde{u}])+\frac{1}{2}a^2 X_i^2 var(\tilde{u})}\right) =$ $= -e^{-a(-a[(W_{0i}- PX_i)] R+X_i E[\tilde{u}])+\frac{1}{2}a^2 X_i^2 var(\tilde{u}))} \times$ $\left(-a\left((-PR + E[\tilde{u}]) + \frac{1}{2} a^2 2X_i var(\tilde{u})\right)\right) =$ $\underbrace{-e^{-a(-a[(W_{0i}- PX_i)] R+X_i E[\tilde{u}])+\frac{1}{2}a^2 X_i^2 var(\tilde{u}))}}_{\neq 0}\left(-a\left(-PR + E[\tilde{u}] - aX_i var(\tilde{u})\right)\right) = 0$ $\rightarrow X_i = \frac{E[\tilde{u}] - PR}{avar(\tilde{u})}$	Ap - 51

Information: Now "add" the information about random return, $\tilde{u}= \tilde{\theta}+\tilde{\varepsilon}$; $\tilde{\theta}$ is observable at cost c.

Behavior of informed

Let X_I be the informed individual's demand for the risky security. Then

$$E[\tilde{u}] = E\,[\tilde{\theta} + \tilde{\varepsilon} \mid \tilde{\theta}] = \tilde{\theta} + E\,[\tilde{\varepsilon}\,] = \tilde{\theta} + 0 = \tilde{\theta}$$

Ap - 52

$$var(\tilde{u}) = var\big(\tilde{\theta} + \tilde{\varepsilon}\,\big) = 0 + var(\tilde{\varepsilon}\,) + \underbrace{2\,cov\,\big(\tilde{\theta}, \tilde{\varepsilon}\,\big)}_{=0} = var(\tilde{\varepsilon}\,) = \sigma_{\tilde{\varepsilon}}^2$$

$$X_I = \frac{E[\tilde{u}] - PR}{a var(\tilde{u})} \overset{var(\tilde{u}) = \sigma_{\tilde{\varepsilon}}^2}{\Rightarrow} X_I = \frac{\theta - P.R}{a \cdot \sigma_{\tilde{\varepsilon}}^2}$$

if $\tilde{\theta} < PR \rightarrow X_I < 0$

Ap - 53

Behavior of uninformed

$$E[\tilde{u}] = \mathrm{E}\big[\tilde{u} \mid \tilde{P}\big(\tilde{\theta}, x\big) = P\,\big]$$

Ap - 54

$$var(\tilde{u}) = var(\tilde{u} \mid \tilde{P}(\tilde{\theta}, x) = P)$$

$$X_i = \frac{E[\tilde{u}] - PR}{avar(\tilde{u})} \overset{var(\tilde{u}) = var(\tilde{u} \mid \tilde{P}(\tilde{\theta}, x) = P)}{\Rightarrow} X_U = \frac{E\big[\tilde{u} \mid \tilde{P}\big(\tilde{\theta}, x\big) = P\,\big] - P \cdot R}{avar(\tilde{u} \mid \tilde{P}(\tilde{\theta}, x) = P)}$$

Ap - 55

$\tilde{P}\big(\tilde{\theta}, x\big)$is the price of the risky asset as a function of the information $\tilde{\theta}$and the supply x of the risky asset.

Let λ = fraction of informed traders, $\tilde{P}_\lambda(\tilde{\theta}, x)$.

Equilibrium condition: Demand of the risky asset = Supply of the risky asset:

$$\lambda X_I + (1 - \lambda) X_U = x \rightarrow$$

$$\lambda X_I(\,P_\lambda(\tilde{\theta}, x) \mid \tilde{\theta}\,) + (1 - \lambda)\,X_U(\,P_\lambda^*(\theta, x) \mid \tilde{P}_\lambda(\tilde{\theta}, x)) = x$$

Ap - 56

$P_\lambda^*(\theta, x)$is a statistical equilibrium. What $\tilde{P}_\lambda(\tilde{\theta}, x)$ satisfies the equilibrium condition?

Let x be random, $\tilde{x}$, i.e. let total supply to be uncertain:

$\lambda X_I\left(P_\lambda^*(\theta, x) \mid \tilde{\theta} \right) + (1-\lambda)\, X_U(P_\lambda^*(\tilde{\theta}, \tilde{x}) \ \mid \tilde{P}_\lambda(\tilde{\theta}, \tilde{x})) \ = \ \tilde{x}$	Ap - 57

Theorem I:

If $\tilde{\theta}$, $\tilde{\varepsilon}$, $\tilde{x}$ have a non-degenerate joint normal distribution such that they are mutually independent, then there exists $P_\lambda^*(\tilde{\theta}, \tilde{x})$ as the solution to Ap - 57. Non-degenerate means that the covariance matrix of the multivariate normal distribution is positive definite. A matrix B is real positive definite if and only if for all $x \neq 0$, $x^T \mathrm{B} x > \ 0$.

$P_\lambda^*(\tilde{\theta}, \tilde{x})$ is the equilibrium price function.

Let: $\omega_\lambda\left(\tilde{\theta}, \tilde{x}\right) = \begin{Bmatrix} \tilde{\theta} - \frac{a\sigma_{\tilde{\varepsilon}}^2}{\lambda} \ (\tilde{x} \ - \ \mathrm{E}[\tilde{x}]) & \text{if}\ \lambda \ > \ 0 \\ \tilde{x} & \text{if}\ \lambda = \ 0 \end{Bmatrix}$ Then $P_\lambda^*(\tilde{\theta}, \tilde{x}) = \ a_1 \ + \ a_2 \omega_\lambda(\tilde{\theta}, \tilde{x})$	Ap - 58

Where a_1 , a_2 are real numbers that may depend on λ ; $a_2 > 0$. If $\lambda = 0$ the price has no information about $\tilde{\theta}$.

Theorem I discussion:

A price system is fully informative if observing $\widetilde{P}$ is the same as observing $\tilde{\theta}$

$Var(\widetilde{P}\,) = \ var(\,\widetilde{\omega}_\lambda(\tilde{\theta}, x))$	Ap - 59

if $\mathrm{Var}\left(\widetilde{P}\,\right) = \ \mathrm{var}\left(\ \widetilde{\omega}_\lambda\left(\tilde{\theta}, x\right)\right) = 0 \rightarrow$ by observing $\widetilde{P}$ is equivalent to observing $\tilde{\theta}$;

if $\mathrm{Var}\left(\widetilde{P}\,\right) = \ \mathrm{var}\left(\ \widetilde{\omega}_\lambda\left(\tilde{\theta}, x\right)\right) > 0 \rightarrow$ by observing $\widetilde{P}$ only gives a noisy signal about $\tilde{\theta}$;

Let $\lambda > 0$ $var\left(\ \widetilde{\omega}_\lambda\left(\tilde{\theta}, x\right)\right) = \ var\left(\tilde{\theta} - \frac{a\sigma_{\tilde{\varepsilon}}^2}{\lambda}\ (\,x - \ E[\tilde{x}])\right)$ $= \ var\left(\tilde{\theta} - \frac{a\sigma_{\tilde{\varepsilon}}^2}{\lambda} \ x \ + \frac{a\sigma_{\tilde{\varepsilon}}^2}{\lambda} \ E[\tilde{x}]\right) = \ var(\frac{a\sigma_{\tilde{\varepsilon}}^2}{\lambda} \ \ \tilde{x}) = \left(\frac{a\sigma_{\tilde{\varepsilon}}^2}{\lambda} \right)^2 var(\tilde{x}\,)$	Ap - 60

$var(\ \tilde{x}\ )$ is the uncertainty about the supplyof the risky asset

$\left(\frac{a\sigma_{\tilde{\varepsilon}}^2}{\lambda}\right)^2$ is the responsiveness of the price to the signal about $\tilde{\theta}$;

when a increases, responsiveness decreases

when $\sigma_{\tilde{\varepsilon}}^2$ increases, responsiveness decreases

when λ increases, responsiveness increases

when $var(\tilde{x}\ )$ increases, responsiveness decreases

Previously λ "fixed"; now let λ (the choice to become informed) be endogenous:

$W_{1i} = W_{0i}\,R + X_i\ (\tilde{u} - PR)$	Ap - 61

Informed trades:

$W_{Ii}^{\lambda} = (\ W_{0i} - c)R + (\tilde{u}\ -\ R\ P_{\lambda}(\tilde{\theta}, x))\ X_I\ (P_{\lambda}(\tilde{\theta}, x), \tilde{\theta})$	Ap - 62

Uninformed trades:

$W_{Ui}^{\lambda} = W_{0i}R\ +\ (\tilde{u}\ -\ RP_{\lambda}(\tilde{\theta}, x))\ X_U(P_{\lambda}(x), \widetilde{P_{\lambda}}\ )$	Ap - 63

When all are informed:

$\lambda = 1$, with $\mathrm{E}\ [\mathrm{v}\ (W_{Ii}^{\lambda})] > E\left[\mathrm{v}\left(W_{Ui}^{\lambda}\right)\right]$	Ap - 64

When all are uninformed:

$\lambda = 0$, with $\mathrm{E}\ [\mathrm{v}\ (W_{Ui}^{\lambda})] < E\left[\mathrm{v}\left(W_{Ii}^{\lambda}\right)\right]$	Ap - 65

When some informed and some uninformed;

$1 > \lambda > 0$, $E\ [v\ (W_{Ui}^{\lambda})] = \mathrm{E}\left[\mathrm{v}\left(W_{Ii}^{\lambda}\right)\right]$	Ap - 66

Theorem II: Given assumptions of theorem I, and if $\bar{X}_i$ independent of $(\tilde{\theta}, \tilde{\varepsilon}, \tilde{x})$, then

$\frac{E\left[v\left(W_{Ui}^{\lambda}\right)\right]}{E\left[v\left(W_{Ii}^{\lambda}\right)\right]} = e^{ac}\sqrt{\frac{var(\tilde{u} \mid \tilde{\theta})}{var(\tilde{u} \mid \omega_{\lambda})}} \quad = \gamma(\lambda)$	Ap - 67

if $1 > \lambda > 0$ *then in equilibrium* $\gamma(\lambda) = 1$

Discussion of theorem II: determine how information affects prices;

Informativeness of price can also be measured by the correlation between the price and $\tilde{\theta}$, $\rho_{\tilde{\theta}}$ = corr.$(\tilde{P}_{\lambda}, \tilde{\theta})$

Let $m = \left(\frac{a\sigma_{\tilde{\varepsilon}}^2}{\lambda}\right)^2 \frac{\sigma_{x}^2}{\sigma_{\tilde{\theta}}^2}$	Ap - 68

m is inversely related to the informativeness of prices,

$\rho_{\tilde{\theta}}^2 = \frac{1}{1+m}$	Ap - 69

$n = \frac{\sigma_{\tilde{\theta}}^2}{\sigma_{\tilde{\varepsilon}}^2}$	Ap - 70

Where n measures the quality of the informed trader's information

For $0 < \lambda < 1$, $1 - \rho_{\tilde{\theta}}^2 = \frac{\boldsymbol{e^{2ac}-1}}{\boldsymbol{n}}$	Ap - 71

Characterizes the informativeness of the price-system.

If n ↑, the informativeness of price ↑; if a, c ↑, the informativeness ↓

Theorem III:

Let 1 > λ > 0

When n $\uparrow$ or c $\downarrow$ or a $\downarrow$ the equilibrium informativeness $\rho^2_{\tilde{\theta}}$ $\uparrow$; $\rho^2_{\tilde{\theta}}$ unchanged if σ^2_x changes or if σ^2_u changes with n

The equilibrium percentage of informed trades $\uparrow$ if σ^2_x $\uparrow$ or σ^2_u $\uparrow$, for a fixed n or c $\downarrow$.

Let:

$$\frac{e^{2ac}-1}{\bar{n}-(e^{2ac}-1)}=\frac{\mathrm{n}}{n+1}$$	Ap - 72

If $n > \bar{n}$, then when n $\uparrow$, λ $\downarrow$.

If $n < \bar{n}$, then when n $\uparrow$, λ $\uparrow$.

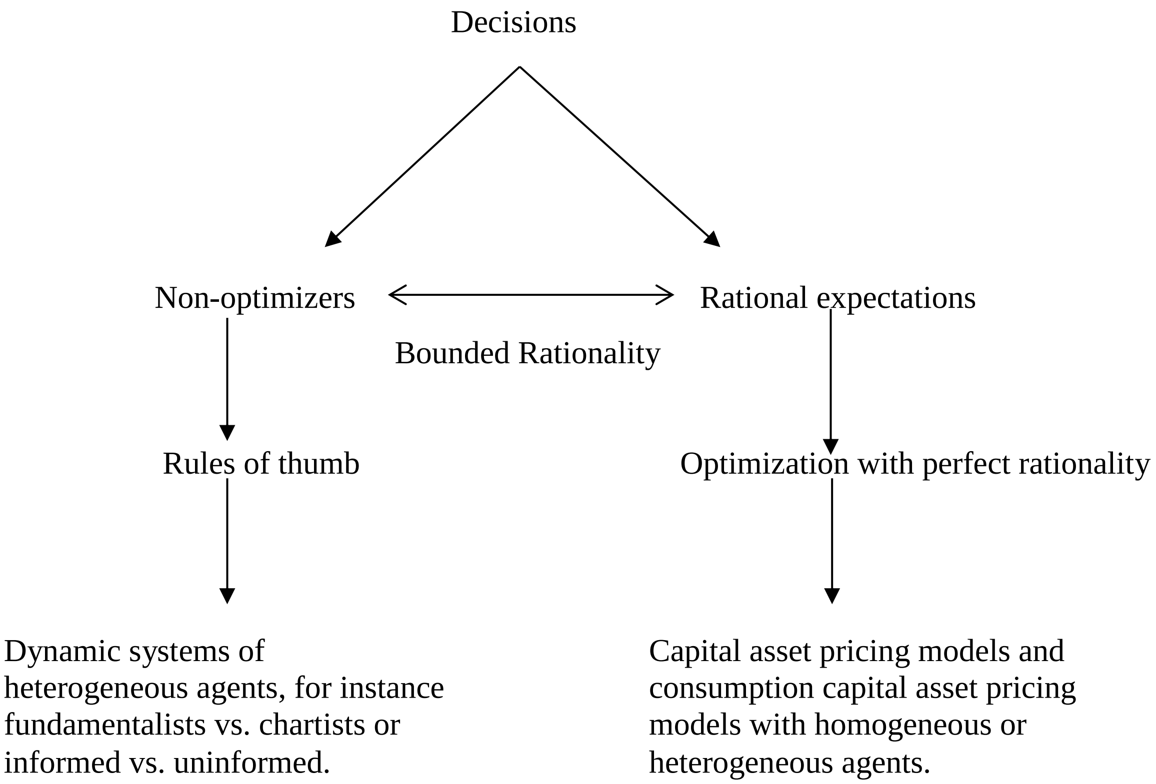
Perfect information:

if c small , $\sigma^2_x = 0$; $n = \infty$, $\sigma^2_{\tilde{\varepsilon}} = 0$ do not exist competitive equilibrium

Theorem IV:

$if\ e^{ac} < \sqrt{1+n}\ and\ \sigma_x = 0 \rightarrow$ no competitive equilibrium

A - v: Sketch graph of key relationships of agents in Finance



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A - vii: List of hypotheses

H - 1: The absolute values of stock returns are proportional to the sentiment risk expressed as the absolute value of differencing of analysts' stock price targets.
H - 2: The absolute values of stock returns are proportional to the sentiment risk expressed as the absolute value of differencing of analysts' EBITDA estimates.
H - 3: The absolute values of stock returns are proportional to the changes of sentiment risk expressed as the absolute value of the second differencing of analysts' EPS estimates.
H - 4: Stock returns' volatility is proportional to the sentiment risk expressed as the absolute value of differencing of analysts' stock price targets.
H - 5: Trade volume of stock is proportional to the sentiment risk expressed as the absolute value of differencing of analysts' stock price targets.
H - 6: Trade volume of stock is proportional to stock returns' volatility.
H - 7: The cross sectional absolute deviation of returns has a quadratic relation to the market return in times of higher market return.
H - 8: The cross sectional absolute deviation of returns has a quadratic relation to the market return in times of lower market return.
H - 9: The cross sectional absolute deviation of returns has a quadratic relation to the market return in times of higher trade volume of stocks.
H - 10: The cross sectional absolute deviation of returns has a quadratic relation to the market return in times of lower trade volume of stocks.
H - 11: The cross sectional absolute deviation of returns has a quadratic relation to the market return in times of higher volatility of the stock market.
H - 12: The cross sectional absolute deviation of returns has a quadratic relation to the market return in times of lower volatility of the stock market.
H - 13: The stochastic discount factor specified by the power utility preferences explains the equity premium and satisfies the HJ-bounds.

A - viii: List of models

M - 1

$$\left|\sum_{i=0}^{N}\left[\left(\frac{MV_{i,t}}{\sum_{j=0}^{N} MV_{j,t}}\right)\left(r_{i,t}\right)\right]\right| = \alpha + \mathbf{1}\gamma \left|d^1 \sum_{i=0}^{N}\left[\left(\frac{MV_{i,t}}{\sum_{j=0}^{N} MV_{j,t}}\right)\sigma\left(P_{\tau,i,t}\right)\right]\right| + \varepsilon_{\,t,t-1}$$

M - 2

$$\left|\sum_{i=0}^{N}\left[\left(\frac{EV_{i,t}}{\sum_{j=0}^{N} EV_{j,t}}\right)\left(r_{i,t}\right)\right]\right| = \alpha + \mathbf{1}\gamma \left|d^1 \sum_{i=0}^{N}\left[\left(\frac{EV_{i,t}}{\sum_{j=0}^{N} EV_{j,t}}\right)\sigma\left(P_{\tau,i,t}\right)\right]\right| + \varepsilon_{\,t,t-1}$$

M - 3

$$\left|\sum_{i=0}^{N}\left[\left(\frac{EV_{i,t}}{\sum_{j=0}^{N} EV_{j,t}}\right)\left(r_{i,t}\right)\right]\right| = \alpha + \mathbf{1}\gamma \left|d^1 \sum_{i=0}^{N}\left[\left(\frac{EV_{i,t}}{\sum_{j=0}^{N} EV_{j,t}}\right)\sigma\left(EBITDA_{\tau,i,t}\right)\right]\right| + \varepsilon_{\,t,t-1}$$

M - 4

$$\left|\sum_{i=0}^{N}\left[\left(\frac{EV_{i,t}}{\sum_{j=0}^{N} EV_{j,t}}\right)\left(r_{i,t}\right)\right]\right| = \alpha + \mathbf{1}\gamma \left|d^2 \sum_{i=0}^{N}\left[\left(\frac{EV_{i,t}}{\sum_{j=0}^{N} EV_{j,t}}\right)\sigma\left(EPS_{\tau,i,t}\right)\right]\right| + \varepsilon_{\,t,t-1,t-2}$$

M - 5

$$\sum_{i=0}^{N}\left[\left(\frac{MV_{i,t}}{\sum_{j=0}^{N} MV_{j,t}}\right)\sqrt{\left(r_{i,t} - r_{i,t-1}\right)^2}\,\right] = \alpha + \mathbf{1}\gamma \left|d^1 \sum_{i=0}^{N}\left[\left(\frac{MV_{i,t}}{\sum_{j=0}^{N} MV_{j,t}}\right)\sigma\left(P_{\tau,i,t}\right)\right]\right| + \varepsilon_{\,t,t-1}$$

M - 6

$$\sum_{i=0}^{N}\left[\left(\frac{EV_{i,t}}{\sum_{j=0}^{N} EV_{j,t}}\right)\sqrt{\left(r_{i,t} - r_{i,t-1}\right)^2}\,\right] = \alpha + \mathbf{1}\gamma \left|d^1 \sum_{i=0}^{N}\left[\left(\frac{EV_{i,t}}{\sum_{j=0}^{N} EV_{j,t}}\right)\sigma\left(P_{\tau,i,t}\right)\right]\right| + \varepsilon_{\,t,t-1}$$

M - 7

$$\sum_{i=0}^{N}\left[\left(\frac{MV_{i,t}}{\sum_{j=0}^{N} MV_{j,t}}\right)\left(k_{i,t}\right)\right] = \alpha + \mathbf{1}\gamma \left|d^1 \sum_{i=0}^{N}\left[\left(\frac{MV_{i,t}}{\sum_{j=0}^{N} MV_{j,t}}\right)\sigma\left(P_{\tau,i,t}\right)\right]\right| + \varepsilon_{\,t,t-1}$$

M - 8

$$\sum_{i=0}^{N}\left[\left(\frac{MV_{i,t}}{\sum_{j=0}^{N} MV_{j,t}}\right)\left(k_{i,t}\right)\right] = \alpha + \mathbf{1}\gamma \sum_{i=0}^{N}\left[\left(\frac{MV_{i,t}}{\sum_{j=0}^{N} MV_{j,t}}\right)\sqrt{\left(r_{i,t} - r_{i,t-1}\right)^2}\,\right] + \varepsilon_t$$

M - 9

$$CSAD_{t,\mathrm{MVW},hr} = \alpha + \gamma_{1,high\,return}\left|R_{m,t,\mathrm{MVW},high\,return}\right| + \gamma_{2,high\,return}\left(R_{m,t,\mathrm{MVW},high\,return}\right)^2 + \varepsilon_t$$

M - 10

$$CSAD_{t,\mathrm{MVW},lr} = \alpha + \gamma_{1,low\,return}\left|R_{m,t,\mathrm{MVW},low\,return}\right| + \gamma_{2,low\,price}\left(R_{m,t,\mathrm{MVW},low\,return}\right)^2 + \varepsilon_t$$

M - 11

$$CSAD_{t,\mathrm{MVW},hv} = \alpha + \gamma_{1,high\,volume}\left|R_{m,t,\mathrm{MVW},high\,volume}\right| + \gamma_{2,high\,volume}\left(R_{m,t,\mathrm{MVW},high\,volume}\right)^2 + \varepsilon_t$$

M - 12

$$CSAD_{t,\mathrm{MVW},lv} = \alpha + \gamma_{1,low\,volume}\left|R_{m,t,\mathrm{MVW},low\,volume}\right| + \gamma_{2,low\,volume}\left(R_{m,t,\mathrm{MVW},low\,volume}\right)^2 + \varepsilon_t$$

M - 13

$$CSAD_{t,\mathrm{MVW},h\sigma^2} = \alpha + \gamma_{1,high\,\sigma^2}\left|R_{m,t,\mathrm{MVW},high\,\sigma^2}\right| + \gamma_{2,high\,\sigma^2}\left(R_{m,t,\mathrm{MVW},high\,\sigma^2}\right)^2 + \varepsilon_t$$

M - 14

$$CSAD_{t,\,MVW,l\sigma^2} = \alpha + \gamma_{1,low\,\sigma^2}\left|R_{m,t,\mathrm{MVW},low\,\sigma^2}\right| + \gamma_{2,low\,\sigma^2}\left(R_{m,t,\mathrm{MVW},\;low\,\sigma^2}\right)^2 + \varepsilon_t$$

M - 15

$$E_t\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1} R_{t+1,m} - 1\right] = 0$$

$$E_t\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1} R_f - 1\right] = 0$$

M - 16
C, R_m, R_f lagged one period, 3 IV (IV stands for instrumental variables)

M - 17
C, R_m, R_f lagged two periods, 3 IV

M - 18
C, R_m, R_f lagged three periods, 3 IV

M - 19
C, R_m, R_f lagged four periods, 3 IV

M - 20
C, R_m, R_f lagged five periods, 3 IV

M - 21
C, R_m, R_f lagged one to five periods together, 15 IV

M - 22

$$E_t\left[\delta\left(\frac{\mathcal{X}_{t+1}}{\mathcal{X}_t}\right)^{-(1-\alpha)}\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha}R_{m,t+1}\right]-1=0$$

$$E_t\left[\delta\left(\frac{\mathcal{X}_{t+1}}{\mathcal{X}_t}\right)^{-(1-\alpha)}\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha}R_{f,t+1}\right]-1=0$$

$$\mathcal{X}_{t+1}=C_t^{\kappa}$$

In M - 22 is $\mathcal{X}_{t+1}=C_t^{\kappa}$. The case $\kappa=-1$ applies to model variants M - 23 to M - 28:
M - 23
C lagged two periods, R_m, R_f lagged one period, 3 IV

M - 24
C, R_m, R_f lagged two periods ,3 IV

M - 25
C, R_m, R_f lagged three periods, 3 IV

M - 26
C, R_m, R_f lagged four periods, 3 IV

M - 27
C, R_m, R_f lagged five periods, 3 IV

M - 28
C lagged two to five periods, R_m, R_f lagged one to five periods together 14 IV

In M - 22 is $\mathcal{X}_{t+1} = C_t^{\kappa}$. The case $\kappa = 1$ applies to model variants M - 29 to M - 34:
M - 29
C lagged two periods, R_m, R_f lagged one period, 3 IV

M - 30
C, R_m, R_f lagged two periods, 3 IV

M - 31
C, R_m, R_f lagged three periods, 3 IV

M - 32
C, R_m, R_f lagged four periods, 3 IV

M - 33
C, R_m, R_f lagged five periods, 3 IV

M - 34
C lagged two to five periods, R_m, R_f lagged one to five periods together 14 IV

In M - 22 is $\mathcal{X}_{t+1} = C_t^{\kappa}$. The case $\kappa = 0{,}001$ applies to model variants M - 35 to M - 40:
M - 35
C lagged two periods, R_m, R_f lagged one period, 3 IV

M - 36
C, R_m, R_f lagged two periods 3 IV

M - 37
C, R_m, R_f lagged three periods 3 IV

M - 38
C, R_m, R_f lagged four periods 3 IV

M - 39
C, R_m, R_f lagged five periods 3 IV

M - 40
C lagged two to five periods, R_m, R_f lagged one to five periods together 14 IV

M - 41

$$E_t\left[\delta\left(\frac{S_{t+1}}{S_t}\right)^{-\alpha}\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha}R_{m,t+1}\right]-1=0$$

$$E_t\left[\delta\left(\frac{S_{t+1}}{S_t}\right)^{-\alpha}\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha}R_{f,t+1}\right]-1=0$$

M - 42

S, R_i, R_f lagged one period C lagged two periods 4 IV

M - 43

C, S, R_i, R_f lagged two periods 4 IV

M - 44

C, S, R_i, R_f lagged three periods 4 IV

M - 45

C, S, R_i, R_f lagged four periods 4 IV

M - 46

C, S, R_i, R_f lagged five periods 4 IV

M - 47

C lagged two to five periods, S, R_i, R_f lagged one to five periods together 19 IV

M - 48

$$E_t\left[e^{-\vartheta}\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1} exp^{\frac{(1-\gamma)(1-\gamma+1)}{2}\sigma^2\left(\log\left(\frac{\frac{C_{j,t+1}}{C_{t+1}}}{\frac{C_{j,t}}{C_t}}\right)\right)} R_{m,t+1}\right]-1=$$

$$E_t\left[e^{-\vartheta}\left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1} exp^{\frac{(1-\gamma)(1-\gamma+1)}{2}\sigma^2\left(\log\left(\frac{\frac{C_{j,t+1}}{C_{t+1}}}{\frac{C_{j,t}}{C_t}}\right)\right)} R_{f,t+1}\right]-1=$$

M - 49

C, C_j, R_m, R_f lagged one period 4 IV

M - 50

C, C_j, R_m, R_f lagged two periods 4 IV

M - 51

C, C_j, R_m, R_f lagged three periods 4 IV'

M - 52

C, C_j, R_m, R_f lagged four periods 4 IV

M - 53

C, C_j, R_m, R_f lagged five periods 4 IV

M - 54

C, C_j, R_m, R_f lagged one to five periods together 20 IV

A - ix: Girsanov theorem

Here is an example of transforming a probability measure $\mathcal{P}$ to $\mathcal{Q}$ so that the risk free rate r_f can be used for finding the asset price. Let the asset price following a stochastic process:

$dP = \mu dt + \sigma dB_{\mathcal{P}}$	Ap - 73

Say we want to transform the above equation to

$dP = r_f dt + \sigma d\hat{B}$	Ap - 74

where $\mathcal{P}$ and $\mathcal{Q}$ are two equivalent probability measures.

Then we should have:

$\mu dt + \sigma dB_{\mathcal{P}} = r_f dt + \sigma d\hat{B} \rightarrow \sigma dB_{\mathcal{P}} = (\mu - r_f)dt + \sigma d\hat{B} \rightarrow$ $dB_{\mathcal{P}} = \frac{\mu - r_f}{\sigma} dt + d\hat{B}$	Ap - 75

Inserting the above equation in dP we have:

$dP = r_f dt + \sigma \left(\frac{\mu - r_f}{\sigma} dt + d\hat{B}\right) \rightarrow dP = r_f dt + \sigma \frac{\mu - r_f}{\sigma} dt + \sigma d\hat{B} \rightarrow$ $dP = r_f dt + \mu dt - r_f dt + \sigma d\hat{B} \rightarrow dY = r_f dt + \sigma d\hat{B}$	Ap - 76

A non-trivial application of the fundamental theorem of finance is the derivation of the partial differential equation that can be used to derive the Black and Scholes formula for pricing European options.

A - x: The Efficient Market Hypothesis

Bachelier in a Ph.D. dissertation (1900) on the theory of speculation is one of the first to describe Brownian motion and the expected payoff of the speculator as a fair game. Samuelson in 1965 publishes an article on the random fluctuation of anticipated prices. The seminal Ball and Brown study of 1968 shows that annual income reports contains information relevant to security prices. Approximately 85 to 90 percent of the information content is incorporated into prices by prompter channels. Annual statements confirmed what was already known. Fama in 1970 puts forward the idea that prices in an efficient market reflect all available information at all times.

The efficiency comes from competition to exploit arbitrage opportunities when the security price deviates from its intrinsic value. There are different degrees of market efficiency. The weak version of market efficiency states that only past information is reflected in the stock price. The semi strong version of market efficiency includes also public available information. The strong version of market efficiency adds non-public information into the known information set (Malkiel 1991, pp.195-198 and Davies, Lowes and Pass 1988, p. 160).

The efficient market hypothesis (EMH) has been the dominant paradigm in main stream financial theory. EMH is according to Copeland, Weston and Shastri (2005, p. 354) resting on the following assumptions:

i) Investors have rational expectations

ii) Prices adjust instantaneously and reflect fully all relevant information

Shleifer (2000, p. 2) portrays EMH as being founded on three progressively weaker assumptions:

a) Investors have rational expectations

b) The trades of non-rational investors are random and cancel each other out

c) The influence of non-rational investors on prices is wiped off through the exploitation of arbitrage by rational investors

The above assumptions have been contested on both empirical and theoretical grounds.

Grossman and Stiglitz (1980) argue that there is an intrinsic inconsistency in asset prices being informationally efficient, i.e. fully conveying all available information at all times.

Their argument goes as follows: Had asset prices been informationally efficient, security analysts shouldn't do their job because there wouldn't be the opportunity of a profit but then again, if security analysts didn't do their work, asset prices wouldn't be informationally efficient. Milgrom and Stokey (1982) argue that each agent has pieces of information which are aggregated and revealed in equilibrium prices where there is no further trade. Noise traders can make the process of diffusion of information less feasible (Xiouros 2009, p.109).

Instantaneous adjustment of prices to new information occurs when the information shared among the market participants, the information disseminated by the companies and the information transmitted by the media is symmetric (Marisetty 2003). There is empirical evidence that this is not the case. For instance, the insider portfolio of a Norwegian financial newspaper (Finansavisen) has beaten the market index by a good margin in the last 15 years (in letter of 16.03.2011 from Oslo Stock Exchange to the Norwegian department of finance).

Sentiment and psychology can create non-rational expectations and cause moves of asset prices away from their fundamental value without the arrival of new information (Keynes 1936, Kahneman and Tversky 1979, Shleifer 2000, p.21). The exploitation of arbitrage opportunities is not risk-free, so the neutralization of the effect of noise trading on asset prices is not ensured (Shleifer 2000, p.24).

If all available information is epitomized in the market price then the motive for speculative trade is eliminated. This brings us to the no - trade theorem. However, the question pops up: How prices mirror all information if nobody trades on information? This contradiction leads to the information paradox pointed out by Grossman and Stiglitz (Hens and Rieger 2010, p. 289).

A - xi: Random Walk Hypotheses

The unpredictability hypothesis of asset prices has been called the random walk hypothesis. The unpredictability of prices can be illustrated by using the law of iterated expectations (Campbell et al. 1997, p. 24):

$E_t[P_{t+1} - P_t] = E\left[E_{t+1}[V^*\|\mathcal{F}_t] - E[V^*\|\mathcal{F}_t]\right] = 0$	Ap - 77

where $V^*$is the fundamental value of an asset given an information set $\mathcal{F}_t$.

There have been a multitude of stochastic processes called for random walks. Campbell et al. (1997, pp.31-33) identify for instance three types, the random walk, martingale and fair game hypotheses. As far as these are concerned we follow for most part the presentation by Campbell et al. (ibid, pp. 28-33) and Copeland, Weston and Shastri (2005, pp. 366-370).

The asset price process can be modeled as a fair game, as a martingale, as a submartingale or as a random walk and their variants in discrete time. Asset pricing can also be modeled in continuous time.

The fair game hypothesis assumes no arbitrage and unpredictability of asset prices (Guerrien and Ozgur 2011). Samuelson's fair game theorem (1965) states that " there is no way of making an expected profit by extrapolating past changes in the future price, by chart or any other esoteric devices of magic or mathematics". Fair game means that everybody has an equal chance to win a game. In finance it implies that a trader can't beat the market.

Fair game models satisfy the following properties:

$\varepsilon_{t+1} = P_{t+1} - E(P_{t+1}\|\mathcal{F}_t)$	Ap - 78

$E(\varepsilon_{t+1}) = 0$	Ap - 79

where *P* is the asset price (Le Roy 1989, Copeland, Weston and Shastri 2005, pp. 367-371).

Martingales are fair games. Glosten and Milgrom (1985) assert that the sequence of transaction prices is following a martingale process. A martingale is a stochastic process M_t adapted to the information set $\mathcal{F}_s$ so that:

$E[M_t \vert \mathcal{F}_s] = M_s$ for all $s \leq t$	Ap - 80

The martingale hypothesis states that the expected value of an asset price is equal to today's price, given the information generated up to time t, for instance the price history:

$E_Q(P_{t+1} \vert \mathcal{F}_t) = P_t$	Ap - 81

or

$E_Q(P_{t+1} \vert \mathcal{F}_t) - P_t = 0$	Ap - 82

where E_Q is expectation under the equivalent martingale measure Q.

A measure is one of the main components of a probability space, the others being a filtration information set and an event space. An equivalent martingale measure is a measure under which the discounted price of an asset is a martingale (Pliska 2005, p.241). It is equivalent to the original probability measure P because P and Q agree on which event can happen or not but assign different probabilities to the events. Martingales mean that the past history doesn't matter. Tomorrow's price is expected to be the same as today's price given an information set $\mathcal{F}_t$. The price process is then called a $\mathcal{F}_t$ −adapted process. A fascinated journey to the origins of the word martingale can be found in Mansuy (2009).

Investors would require a risk premium as compensation for investing on a risky asset. A submartingale is a process where prices are expected to increase over time (Copeland et al. 2005, pp. 367, 368):

$E_P(P_{t+1} \vert \mathcal{F}_t) > P_t$	Ap - 83

where E_P is the expectation under the original probability measure P.

Random walks and martingales have many versions which are captured by the orthogonality condition:

$Cov[f(r_t), g(r_{t+k})] = 0$	Ap - 84

Where $f(r_t)$ and $g(r_{t+k})$ are two arbitrary functions of random variables and r_t and r_{t+k} are share prices at different dates independent of each other (Campbell et al. 1997, p. 28).

Campbell et al. (1997, pp. 31-33) classify random walks into three versions. The first version is random walk with identical and independently distributed (IID) error terms:

$P_{t+1} = \mu + P_t + \varepsilon_{t+1}, \qquad \varepsilon_{t+1} \sim IID\ N(0, \sigma^2)$	Ap - 85

The above equation says that share price at t+1 is equal to the expected price change μ plus the price at time t. From the above equation we get:

$P_{t+1} - P_t = \mu + \varepsilon_{t+1}$	Ap - 86

In order to avoid negative share prices, one can use the natural logarithm of P_{t+1}, which gives:

$lnP_{t+1} = \mu + lnP_t + \varepsilon_{t+1}, \qquad \varepsilon_{t+1}\ IID\ N(0,\ \sigma^2)$	Ap - 87

The assumption of IID is not realistic because risk changes over time. The second version of random walk is using the assumption of independent but not identically distributed error terms (INID). This means that variance is a variable instead of a constant:

$P_{t+1} = \mu + P_t + \varepsilon_{t+1}, \qquad \varepsilon_{t+1} \sim INID\ N\left(0, \sigma^2(t)\right)$	Ap - 88

A third version of the random walk hypothesis is that the error terms are uncorrelated but not independent:

$Cov[\varepsilon_t, \varepsilon_{t-k}] = 0\ for\ all\ \ k \neq 0, Cov[\varepsilon_t^2, \varepsilon_{t-k}^2] \neq 0\ for\ some\ \ k \neq 0$	Ap - 89

Martingales constitute a more general class of stochastic processes than random walks because they don't have to satisfy the property that increments are independent or uncorrelated (Copeland et al. 2005, pp. 367-369).

LeRoy (1989), based on Lucas (1978), sets up an asset price equilibrium model in an exchange economy as:

$P_t U_t' = (1+\rho)^{-1} E_t(P_{t+1} + d_{t+1}) U_{t+1}'$	Ap - 90

where $1+\rho$ is a stochastic discount factor, d is the dividend, U' is the marginal utility and P is the asset price.

LeRoy points out, as Lucas does, that asset prices in this setting are not martingales unless the utility of consumption at time t+1 is equal to the utility of consumption at time t.

We carried through a survey in the literature of models used to describe the properties of asset price process. The description of the price model and its error term is in most cases adequate for portraying the price process. These models were adjusted by us in order to get a uniform notation. We sort them out into two groups. One group consists of models which describe processes that don't depend on the past history. The other group is made of models which describes processes that depend on the past history. Models in discrete time are presented together with their counterparts in continuous time whenever this was possible.

Appendix table - 1: Stochastic pricing processes not depending on past history, discrete time

Model	Discrete time	Assumption on error term
Fair Game	$E[(P_{t+1} - P_t)\|\mathcal{F}_t] = E[\varepsilon_{t+1}\|\mathcal{F}_t]$ (LeRoy 1989, p. 1589)	$E[\varepsilon_{t+1}\|\mathcal{F}_t] = 0$
Fair Game as return	$E\left[\left(\frac{P_{t+1} - P_t}{P_t}\right)\|\mathcal{F}_t\right] = E[\varepsilon_{t+1}\|\mathcal{F}_t]$	$E[\varepsilon_{t+1}\|\mathcal{F}_t] = 0$
Martingale	$E[P_{t+1}\|\mathcal{F}_t] = P_t + E[\varepsilon_{t+1}\|\mathcal{F}_t]$ (Campbell et al. 1997, p. 30)	$E[\varepsilon_{t+1}\|\mathcal{F}_t] = 0$
Submartingale	$E[P_{t+1}\|\mathcal{F}_t] > P_t + E[\varepsilon_{t+1}\|\mathcal{F}_t]$ (Salge 1997, p. 89)	$E[\varepsilon_{t+1}\|\mathcal{F}_t] = 0$
Random Walk I	$P_{t+1} = \mu + P_t + \varepsilon_{t+1}$ (Campbell et al. 1997, p. 31)	$\varepsilon_{t+1} \sim IID\ N(0, \sigma^2)$ (Campbell et al. 1997, p. 31)
Geometric Random Walk	$ln\left(\frac{P_{t+1}}{P_t}\right) = \mu + \varepsilon_{t+1}$ (Campbell et al. 1997, p. 36)	$\varepsilon_{t+1} \sim IID\ N(0, \sigma^2)$ (Campbell et al. 1997, p. 36)
Mean Reverting (MR)	$P_{t+1} - P_t = \mu(1 - \mathrm{e}^{-\varphi t}) + (\mathrm{e}^{-\varphi t} - 1)P_t + \varepsilon_{t+1}$ (Dixit and Pindyck 1994, p. 76)	$\varepsilon_{t+1} \sim N\left(0, \frac{\sigma}{2\varphi}^2 (1 - e^{-2\varphi t})\right)$ (Meucci 2010, p. 21)

Campbell et al. (1997) outline two general processes which they call RW II and RW III. RW II is a process $P_{t+1} = \mu + P_t + \varepsilon_{t+1}$ where $\varepsilon_{t+1} \sim INID\ N(0, \sigma^2(t))$, i.e. the variance of the error term is a variable (Campbell et al. 1997, p. 32-33). RW III (ibid, p. 33) has independent but not uncorrelated increments so that $Cov[\varepsilon_{t+1}^n, \varepsilon_{t+1-k}^n] \neq 0$ for some $n > 1$ and some $k \neq 0$. RW III traces processes depending on past history and is exemplified by autoregressive and similar models (see the table for stochastic processes depending on past history).

Appendix table - 2: Stochastic pricing processes not depending on past history, continuous time

Stochastic process	Continuous time	Assumption on error term
Brownian motion without drift	$dP_t = \sigma dB_t$	$(B_t - B_s) \sim IID\ N(0, t-s)$, $s < t$ (Chang 1999, p. 5-3)
Brownian motion with drift	$dP_t = \mu dt + \sigma dB_t$	$(B_t - B_s) \sim IID\ N(0, t-s)$, $s < t$ (Chang 1999, p. 5-3)
Geometric Brownian Motion	$dP_t = \mu P_t dt + \sigma P_t dB_t$	$(B_t - B_s) \sim IID\ N(0, t-s)$, $s < t$ (Chang 1999, p. 5-3)
Geometric Brownian Motion with compensation scheme to management	$dP_t = \mu_t P_t dt + (P_t + a_t)\sigma P_t dB_t$ (Ou-Yang 2005, p. 1281)	$(B_t - B_s) \sim IID\ N(0, t-s)$, $s < t$
Ohrnstein - Uhlenbeck, Continuous Mean Reverting (CMR)	$dP_t = \varphi(\mu - P_t)dt + \sigma dB_t$ (Dixit and Pindyck 1994 p. 74)	$(B_t - B_s) \sim IID\ N(0, t-s)$, $s < t$

The notation in the Appendix table - 1and Appendix table - 2 is as follows:

- P is an asset price
- B is a standard Brownian motion which starts at 0, has expectation 0 and has covariance function $E[B(t) - B(s)] = 0$
- $\mathcal{F}_t$ is an information set generated up to time t
- μ is a drift term
- ε is an error term
- $t - s$ is the time interval between two points in time where $s < t$
- a_t is the value of the manager's cash compensation at time t.

• φ is the speed of reversion

When a reference is not given, either the formulas are well known relations or are our own evaluations on the issue at hand.

The models in the above tables have been using for testing EMH and for modeling in theoretical models the law of motion of stock prices.

From Appendix table - 1 we have:

$P_{t+1} = \mu + P_t + \varepsilon_{t+1}$, where $\varepsilon_{t+1} \sim IID\ N(0, \sigma^2)$	Ap - 91

The above equation is called by Campbell et al. (1997, p.32) Random Walk I.

The mean and variance of P_{t+1} are:

$E_t[\,P_{t+1}] = \mu t$	Ap - 92

$Var[\,P_{t+1}] = \sigma^2 t$	Ap - 93

We observe that the stochastic process P_t with drift has the following normally distributed increments:

$(P_t - P_s) \sim N(\mu t, t\sigma^2)$	Ap - 94

From Ap - 92, Ap - 93and Ap - 94 we conclude that random walk I is related to the Brownian motion with drift (Campbell et al. 1997, p. 32). In fact Brownian motion can be informally described (Privault 2012, p. 66) as the limiting case of a random walk when the time intervals $t - s$ go to zero. The increments ΔB_t are given by the squ re root of Δt. The geometric random walk is related in a similar way to the geometric Brownian motion. Price processes modelled as Brownian motions can be generalized by allowing μ and σ to vary as functions of the stock price P_t and time. Brownian motions can be constructed by means of stepwise indicator functions of the form $f(t) = \sum_{i=1}^{n} a_i \mathbf{1}_{(t_{i-1}, t_i]}(t)$, (Privault 2012, p. 70).

Stating again the arithmetic Brownian motion:

$dP_t = \mu dt + \sigma dB_t$	Ap - 95

Candidate solution:

$P_t = P_0 + \left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma B_t$	Ap - 96

To verify that Ap - 96 is the solution we use Itô's lemma:

$P_t(B_t, t)$ $dP_t = \frac{\partial P_t(B_t,t)}{\partial B_t} dB_t + \frac{\partial P_t(B_t,t)}{\partial t} dt + \frac{1}{2} \frac{\partial^2 P_t(B_t,t)}{\partial B_t^2} \sigma^2 dB^2 \rightarrow$ $dP_t = \sigma dB_t + \left(\mu - \frac{1}{2}\sigma^2\right) dt + \underbrace{\sigma dB_t dt}_{dB_t dt = 0} + \frac{1}{2}\sigma^2 \underbrace{dB^2}_{dB^2 = dt} = \sigma dB_t + \mu dt$	Ap - 97

The arithmetic Brownian motion is used for modeling earnings before interest and taxes (Genser 2006, p. 75-121). However, because the arithmetic Brownian motion allows negative values it is not a realistic model for stock prices. The geometric Brownian motion takes care of this issue:

$dP_t = \mu P_t dt + \sigma P_t dB_t$	Ap - 98

Can be rewritten as:

$\frac{dP_t}{P_t} = \mu dt + \sigma dB_t$	Ap - 99

A candidate solution is:

$P_t = P_0 e^{\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma B_t} \leftrightarrow$ $ln(P_t) = ln(P_0) + \left(\mu - \frac{1}{2}\sigma^2\right) t + \sigma B_t$	Ap - 100

So if P_t is lognormally distributed, $ln(P_t)$ is normally distributed.

To verify that Ap - 100 is the solution we use Itô's lemma:

$d\big(ln(P_t)\big) = \frac{\partial\big(ln(P_t)\big)}{\partial B_t} dB_t + \frac{\partial\big(ln(P_t)\big)}{\partial t} dt + \frac{1}{2}\, \frac{\partial^2\big(ln(P_t)\big)}{\partial B_t^2} \sigma^2 dB_t^2 \rightarrow$ $d\big(ln(P_t)\big) = \sigma dB_t + \left(\mu - \frac{1}{2}\sigma^2\right) dt + \underbrace{\sigma dB_t dt}_{dB_t dt = 0} + \frac{1}{2}\sigma^2 \underbrace{dB^2}_{dB^2 = dt} = \sigma dB_t + \mu dt$	Ap - 101

Appendix table - 3: Stochastic pricing processes depending on past history, discrete time

Stochastic process	Discrete time	Assumption on error term
Autoregressive (AR)	$P_{t+1} = \beta_0 + \beta_1 P_t + .. + \beta_k P_{t-k} + \varepsilon_{t+1}$ (Solberg 1992, p.357)	$E[\varepsilon_{t+1}] = 0, Var(\varepsilon_{t+1}) = \sigma^2, Cov(\varepsilon_{t+i}\varepsilon_t) = 0,$ $\varepsilon_{t+1} \sim WN(0, \sigma^2)$ (Brockwell and Davis 2002, pp. 16, 17)
Moving Average (MA)	$P_{t+1} = \theta_0 + \theta_1 \varepsilon_t + \cdots + \theta_q \varepsilon_{t+1-q} + \varepsilon_{t+1}$ (Brockwell and Davis 2002, p.50)	$\varepsilon_{t+1} \sim WN(0, \sigma^2)$ (Brockwell and Davis 2002, p.50)
Autoregressive Moving Average (ARMA)	$P_{t+1} = \beta_1 P_t + \cdots + \beta_k P_{t+1-k} + \varepsilon_{t+1} + \theta_0 + \theta_1 \varepsilon_t + \cdots + \theta_q \varepsilon_{t+1-q}$ (Hamilton 1994, p.59 and Brockwell and Davis 2002, p.83)	$\varepsilon_{t+1} \sim WN(0, \sigma^2)$ (Brockwell and Davis 2002, p.55)
Autoregressive Integrated Moving Average (ARIMA)	$p_{t+1} = \beta_1 p_t + \cdots + \beta_k p_{t+1-k} + \varepsilon_{t+1} + \theta_0 + \theta_1 \varepsilon_t + \cdots + \theta_q \varepsilon_{t+1-q}$ $p_{t+1} = \Delta^d P_{t+1} = (P_{t+1} - P_t)^d$, d is an integer number (Brockwell and Davis 2002, pp.180-210, Reschenhofer 2009, p.5)	$\varepsilon_{t+1} \sim WN(0, \sigma^2)$ (Brockwell and Davis 2002, p.180)
Autoregressive Fractional Integrated Moving Average Model (ARFIMA)	$p_{t+1} = \beta_1 p_t + \cdots + \beta_k p_{t+1-k} + \theta_0 + \varepsilon_{t+1} + \theta_1 \varepsilon_t + \cdots + \theta_q \varepsilon_{t+1-q}$ $p_{t+1} = \Delta^d P_{t+1} = (P_{t+1} - P_t)^d$, d is a non-integer number (Brockwell and Davis 2002, pp.361-365, Reschenhofer 2009, p.6)	$\varepsilon_{t+1} \sim WN(0, \sigma^2)$ $Var(P_{t+1}) = \sigma^2 \frac{\Gamma(1-2d)}{\Gamma^2(1-2d)}, \Gamma(1-2d) \sim \int_0^\infty x^{-2d} e^{-x} dx$ (Brockwell and Davis 2002, pp.361-363, Borowski and Borwein 1989, p, 239)
Autoregressive Conditional Heteroscedasticity Model (ARCH)	$P_{t+1} = P_t + \mu + u_{t+1}$ $u_{t+1} = \sigma_{t+1} \varepsilon_{t+1}$ $\sigma_{t+1} = \sqrt{\beta_0 + \sum_{i=1}^{q} \beta_i u_{t+1-i}^2}$	$E[\varepsilon_{t+1} \vert \varepsilon_t] = 0 , Var[\varepsilon_{t+1} \vert \varepsilon_t] = 1 , \varepsilon_{t+1} \sim WN(0,1)$ $\sigma_{t+1} \sim \left(0, \beta_0 + \sum_{i=1}^{q} \beta_i u_{t+1-i}^2\right)$ $E(u_{t+1} \vert u_t, \ldots) = 0 , Var[u_{t+1} \vert u_t] = \beta_0 + \sum_{i=1}^{q} \beta_i u_{t+1-i}^2$ (Ruppert 2004, pp.363-370, Zivot 2005 p.84)
Generalized Autoregressive Conditional Heteroscedasticity Model (GARCH)	$r_{t+1} = \mu + u_{t+1} , r_{t+1} = \frac{P_{t+1} - P_t}{P_t}$ (Reider, 2009) $u_{t+1} = \sigma_{t+1} \varepsilon_{t+1}$ $\sigma_{t+1} = \sqrt{\beta_0 + \sum_{i=1}^{q} \beta_i u_{t+1-i}^2 + \sum_{i=1}^{p} \vartheta_i \sigma_{t+1-i}^2}$	$E[\varepsilon_{t+1} \vert \varepsilon_t] = 0 , Var[\varepsilon_{t+1} \vert \varepsilon_t] = 1$ $\varepsilon_{t+1} \sim WN(0,1), \sigma(r_{t+1}) \sim \left(0, \beta_0 + \sum_{f=1}^{q} \beta_f u_{t+1-f}^2 + \sum_{m=1}^{u} \vartheta_m \sigma_{t+1-m}^2\right)$ $E(u_t \vert u_{t-1}, \ldots) = 0$, $Var[u_{t+1} \vert u_t] = \beta_0 + \sum_{f=1}^{q} \beta_f u_{t+1-f}^2 + \sum_{m=1}^{u} \vartheta_m \sigma_{t+1-m}^2$ (Ruppert 2004, pp.363-.370)

Appendix table - 4: Stochastic pricing processes depending on past history, continuous time

Stochastic process	Continuous time	Assumption on error term
Continuous Autoregressive (CAR)	The CAR(k) process Y(t) is the solution of $Y^{(k)}(t) = B(t) - \beta_k Y(t) - \beta_{k-1} Y'(t) - \cdots - \beta_1 Y^{(k-1)}(t)$ $d\boldsymbol{P}(\boldsymbol{t}) = \boldsymbol{\beta P}(\boldsymbol{t})dt + \sigma \boldsymbol{b} dB(t)$ $\boldsymbol{Y}(\boldsymbol{t}) = \boldsymbol{\theta}^{\boldsymbol{T}} \boldsymbol{P}(\boldsymbol{t})$ $\boldsymbol{\beta} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \dots & 0 & 1 \\ -\beta_k & -\beta_{k-1} & \dots & \dots & -\beta_1 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$, $\boldsymbol{P}(\boldsymbol{t}) = \begin{bmatrix} P(t)^{(0)} \\ P(t)^{(1)} \\ \vdots \\ P(t)^{(k-1)} \\ P(t)^{(k)} \end{bmatrix}$, $\boldsymbol{\theta} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$ (adapted from Roux 2002, pp. 32-33)	$[B(t) - B(s)] \sim IID\ N(0, t-s), s < t$
Continuous Moving Average (CMA)	The CMA(q) process Y(t) is the solution of $Y^{(q)}(t) = B(t) + \theta_1 B'(t) + \cdots + \theta_q B^{(q)}(t)$ $d\boldsymbol{P}(\boldsymbol{t}) = \boldsymbol{\beta P}(\boldsymbol{t})dt + \sigma \boldsymbol{b} dB(t)$ $\boldsymbol{Y}(\boldsymbol{t}) = \boldsymbol{\theta}^{\boldsymbol{T}} \boldsymbol{P}(\boldsymbol{t})$ $\boldsymbol{\beta} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \dots & 0 & 1 \\ 0 & 0 & \dots & \dots & 0 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$, $\boldsymbol{P}(\boldsymbol{t}) = \begin{bmatrix} P(t)^{(0)} \\ P(t)^{(1)} \\ \vdots \\ P(t)^{(q-1)} \\ P(t)^{(q)} \end{bmatrix}$, $\boldsymbol{\theta} = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_q \end{bmatrix}$ (adapted from Roux 2002, pp. 36-37)	$[B(t) - B(s)] \sim IID\ N(0, t-s), s < t$
Continuous Autoregressive Moving Average (CARMA)	The CARMA(k,q) process Y(t) is the solution of $Y^{(k)}(t) = B(t) + \theta_1 B'(t) + \cdots + \theta_q B^{(q)}(t) - \beta_k Y(t) - \beta_{k-1} Y'(t) .. - \beta_1 Y^{(k-1)}(t)$ $d\boldsymbol{P}(\boldsymbol{t}) = \boldsymbol{\beta P}(\boldsymbol{t})dt + \sigma \boldsymbol{b} dB(t)$ $\boldsymbol{Y}(\boldsymbol{t}) = \boldsymbol{\theta}^{\boldsymbol{T}} \boldsymbol{P}(\boldsymbol{t})$ $\boldsymbol{\beta} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \dots & 0 & 1 \\ -\beta_k & -\beta_{k-1} & \dots & \dots & -\beta_1 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$, $\boldsymbol{P}(\boldsymbol{t}) = \begin{bmatrix} P(t)^{(0)} \\ P(t)^{(1)} \\ \vdots \\ P(t)^{(k-1)} \\ P(t)^{(k)} \end{bmatrix}$, $\boldsymbol{\theta} = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_q \end{bmatrix}$ (adapted from Roux 2002, pp. 28-29, and Brockwell and Davis 2002, pp. 357-361)	$[B(t) - B(s)] \sim IID\ N(0, t-s), s < t$
Continuous Autoregressive Fractional Integrated Moving Average (CARIMA)	The CARIMA(k,q) process Y(t) is the solution of $Y^{(k)}(t) = B(t) + \theta_1 B'(t) + \cdots + \theta_q B^{(q)}(t) - \beta_k Y(t) - \beta_{k-1} Y'(t) .. - \beta_1 Y^{(k-1)}(t)$ $dY^{(j-1)}(t) = Y^{(j)}(t)$ $d\boldsymbol{P}(\boldsymbol{t}) = \boldsymbol{\beta P}(\boldsymbol{t})dt + \sigma \boldsymbol{b} d\mathcal{B}(t)$ $\boldsymbol{Y}(\boldsymbol{t}) = \boldsymbol{\theta}^{\boldsymbol{T}} \boldsymbol{P}(\boldsymbol{t})$ $\boldsymbol{\beta} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \dots & 0 & 1 \\ -\beta_k & -\beta_{k-1} & \dots & \dots & -\beta_1 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$, $\boldsymbol{P}(\boldsymbol{t}) = \begin{bmatrix} P(t)^{(0)} \\ P(t)^{(1)} \\ \vdots \\ P(t)^{(k-1)} \\ P(t)^{(k)} \end{bmatrix}$, $\boldsymbol{\theta} = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_q \end{bmatrix}$	$[B(t) - B(s)] \sim IID\ N(0, t-s), s < t$
Continuous Autoregressive Fractional Integrated Moving Average (CARFIMA)	The CARFIMA(k,q) process Y(t) is the solution of $Y^{(k)}(t) = B_H(t) + \theta_1 B_H'(t) + \cdots + \theta_q {B_H}^{(q)}(t) - \beta_k Y(t) - \beta_{k-1} Y'(t) .. - \beta_1 Y^{(k-1)}(t)$ $dY^{(j-1)}(t) = Y^{(j)}(t)$ $d\boldsymbol{P}(\boldsymbol{t}) = \boldsymbol{\beta P}(\boldsymbol{t})dt + \sigma \boldsymbol{b} d\mathcal{B}_H(t)$ $\boldsymbol{Y}(\boldsymbol{t}) = \boldsymbol{\theta}^{\boldsymbol{T}} \boldsymbol{P}(\boldsymbol{t})$ $\boldsymbol{\beta} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \dots & 0 & 1 \\ -\beta_k & -\beta_{k-1} & \dots & \dots & -\beta_1 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$, $\boldsymbol{P}(\boldsymbol{t}) = \begin{bmatrix} P(t)^{(0)} \\ P(t)^{(1)} \\ \vdots \\ P(t)^{(k-1)} \\ P(t)^{(k)} \end{bmatrix}$, $\boldsymbol{\theta} = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_q \end{bmatrix}$ (adapted from Tsai 2009, p. 181)	$[B_H(t) - B_H(s)] \sim (0, (t-s)^{2H})$, $s < t$ (Meucci 2010, p. 23, Voss 1985 p. 23, Dieker 2004, p. 6) $E[B^2(t)] = t^{2H}$ (Krzywda 2011, p. iii) $E[P^2(t)] = \sigma^2 t^{2H}$ (Krzywda 2011, p. 1)

The notation in Appendix table - 3and Appendix table - 4 is as follows:

● β is the coefficient of the lagged terms

● θ is a moving average parameter (Harvey 1993, p. 3) used as the coefficient of the error term in MA, ARMA, ARIMA and ARFIMA and their counterparts in continuous time.

● μ is a drift term

● d is the order of differencing. In ARFIMA and CARFIMA processes it takes fractional, i.e. non-integer, values

● j is a j-fold differentiation

● k is the number of lags of the auto-regressed variable

● q is the number of lags of the error terms

● Ɓ is a standard fractional Brownian motion which starts at 0 and has expectation 0 and has variance $[B_H(t) - B_H(s)]^2 = (t-s)^{2H}$ (Voss 1989, p. 23). Its covariance function is $E[B_H(t) - B_H(s)] = \frac{1}{2}[|t|^{2H} + |s|^{2H} - |t-s|^{2H}]$ (Tsai 2009, pp. 179-180).

● H is called the Hurst exponent and takes values between 0 and 1. For $H = \frac{1}{2}$ the process is not fractional. When $H > \frac{1}{2}$ the process is persistent and exhibits positive autocorrelation. When $H < \frac{1}{2}$ the process is antipersistent and exhibits negative autocorrelation.

AR models is used for processes where lagged terms matter. MA models are used to smooth out a random component, thus making trends more visible. ARMA combines AR with MA. Processes that need to be differenced in order to make them stationary are modeled with ARIMA (Tsay 2010, p. 74). A stationary process is a process with no tendency for its spread to change over time (Harvey 1993, p. 3). ARCH models are used for volatilities which vary over time (Harvey 1993, pp. 269-270). The volatilities themselves are processes represented with AR models (Brockwell and Davis 2002, p. 349). In GARCH the volatilities are represented with ARMA models (Brockwell and Davis 2002, p. 352).

Martingales and continuous stochastic processes are connected together through the Feynman-Kac formula which is used to set up generators of stochastic processes from martingales:

Let $\hat{f}\big(t, r(t)\big) = E_{P,t}\left[e^{-\int_t^u g(r(s))ds} f\big(r(u)\big)\right]$

Then $\frac{\partial \hat{f}}{\partial t} = A\hat{f} - g\big(r(t)\big)\hat{f}$

Where A is an operator so that it generates the following stochastic process:

$$A\hat{f} = a(t)\frac{\partial \hat{f}}{\partial r(t)} + \frac{1}{2}\sigma(t)^2\frac{\partial^2 \hat{f}}{\partial r(t)^2}$$

The generator of a stochastic process of the form $dX_t = \mu(t, X_t)dt + \sigma(t, X_t)dB_t$ is a second order partial differential operator (Øksendal 2000, p. 115). For a financial instrument the Feynman-Kac formula derives a partial differential equation which can be solved numerically, if not analytically (Neftci 2000, p.487).

A - xii: The connection between EMH and RWH

EMH and RWH are connected through testing with relevant models.

Notwithstanding the convention of using the term RWH, more general and flexible models have been used for testing EMH than the random walk with iid increments of the error term. In what follows we keep ourselves mainly to the presentations by Fama (1970 and 1991).

Weak form tests:

Since Bachelier (1900) is the notion of a fair game considered as a suitable model to test asset prices against. Bachelier stated that the behavior of asset prices should follow a fair game with the expected profits of a speculator equal to zero. Samuelson (1965) provided a proof that anticipated prices should fluctuate randomly.

Fair games and submartingales are considered as suitable models to test the weak form market efficiency (Fama 1970, pp. 385-386):

$E\left(\tilde{x}_{j,t+1} \vert \mathcal{F}_t\right) = 0$ (fair game)	Ap - 102

where $\tilde{x}_{j,t+1} = P_{j,t+1} - P_{j,t}$

$E\left(\tilde{r}_{j,t+1} \vert \mathcal{F}_t\right) \geq 0$ or $E\left(\tilde{p}_{j,t+1} \vert \mathcal{F}_t\right) \geq p_{j,t}$, $\tilde{r}$ is a submartingale	Ap - 103

The weak form market efficiency can be tested through a statistical test of independence of the increments of market returns or stock price changes. Alternatively can one test that past returns cannot be used to construct filter rules which yield abnormal returns after adjusting for transaction costs.

Other tests of weak market efficiency are tests for return predictability using the value to book ratio $\frac{M}{B}$, the earnings over price ratio $\frac{E}{P}$ or dividend to price ratio $\frac{D}{E}$.

The total intrinsic, i.e. fundamental, value of a company is:

$V_t = B_t + \sum_{j=t+1}^{\infty} \frac{RI_j}{(1+r)^j}$	Ap - 104

where B is the book value of the operating assets and RI is the residual income (Investopedia). Residual income is the net income generated after subtracting the cost of capital including equity costs and amortization costs.

One could think that processes that do not depend on the past history are more likely to accommodate the idea of the EMH. However, Lucas (1978, p. 1444) claims that the martingale property of stock prices doesn't cast light on market efficiency and rational expectations. Why's that? The reason can be that the underlying macroeconomic fundamentals of the economy introduce a predictable element to the stock price process.

Any tests of the EMH suffer from the joint hypotheses issue (Roll 1977) which concerns the validity of the model used.

RI is the residual income:

$RI_{t+1} = C_{t+1} - A_{t+1} - rB_t$	Ap - 105

where C is the cash flow, A is the amortization cost and r is the required rate of return.

In a well-functioning market the total intrinsic, i.e. fundamental value is equal to the total market value:

$M_t = B_t + \sum_{j=t+1}^{\infty} \frac{RI_j}{(1+r)^j}$	Ap - 106

Dividing both sides by B_t we get:

$\frac{M_t}{B_t} = 1 + \frac{\sum_{j=t+1}^{\infty} \frac{RI_j}{(1+r)^j}}{B_t}$	Ap - 107

The Market to Book ratio is calculated practically by dividing the market price of shares outstanding by the book value of equity. The component $\frac{\sum_{j=t+1}^{\infty} \frac{RI_j}{(1+r)^j}}{B_t}$ depends on the residual income RI, the discounted factor r and the book value at time *t*.

Fluctuations of $\frac{M_t}{B_t}$ are due to business cycles which affect all firms. When an economy is in a high state one expects increases in the price to book ratio. The book value does not reflect growing opportunities in which case it is lower than the market value. The opposite is the true for an economy in a low state. The value of a company's stock changes on a daily basis due to the arrival of new information, while the change in the value of the assets and liabilities entered in the books takes place ex-post. This makes likely a discrepancy between market price and book value even in the case of fair value accounting.

Fama and French (1991) find that book to market ratios are associated with expected returns. This contradicts the prediction of CAPM that a stocks beta suffices for predicting the cross section of expected returns (Fama 1991). Fama and French attribute this association to a possible correlation between $\frac{M_t}{B_t}$ and the true β.

Semi-strong form tests:

To which extent do asset prices reflect all publicly available information? This is the question tests of the semi strong form of market efficiency have to answer. The following regression equation can be used to test for semi-strong market efficiency:

$$\tilde{r}_j = \alpha_j + \beta_j \tilde{r}_M + \gamma_j \mathbf{1}_D + \tilde{u}_{j,t+1} \qquad \text{Ap - 108}$$

where γ shows the abnormal return (Binder 1985, Karafiath 1988).

We added the term $\mathbf{1}_D$ as an indicator function equal to 1 on the event date and 0 otherwise.

Event tests are widely used to test semi strong market efficiency. With event is meant an occurred state of nature with relevance for an asset price. In a semi strong efficient market, an agent trading on an event ex post should not be able to reap abnormal earnings.

Equity analysts base their evaluations on publically available information. An investor should not be able to make abnormal returns following analyst's recommendations after adjusting for transaction and information costs.

Strong form tests:

Test of strong form market efficiency should concentrate on abnormal returns done by group of investors such as senior managers and exchange specialists. Fama (1991) suggests using the more descriptive title "tests for private information".

A possible regression equation for testing the strong form market efficiency is the following:

$E(\tilde{r}_{j,t+1}\|\mathcal{F}_t, r_{m,t+1}) = r_{f,t+1}[1-\beta_j\|\mathcal{F}_t] + r_{m,t+1}\beta_j\|\mathcal{F}_t$	Ap - 109

This is a joint hypothesis test, i.e. a test of both market efficiency and correct model specification. If the hypothesis that the capital asset market is efficient is rejected one doesn't know if it is because the capital asset market is not efficient or due to model misspecification.

Fama (1970) uses the performance of mutual funds managers against a norm given by the capital asset pricing model, as a test. This is based implicitly on the assumption that mutual fund managers have access to relevant private information. However, Fama doesn't provide a convincing argument why testing the performance of mutual fund managers is a suitable test for the strong form of market efficiency. On what grounds do the mutual fund managers possess inside information?

Senior managers have access to privileged information. However, security market exchange rules place constraints to trading the assets of the firms they work in. In Norway trade is not allowed unless the information the trade is based on has become publically known.

Exchange specialists can take advantage of order information to their own benefit. This is done either by buying or by selling for their own accounts in advance of filling customer's buy or sell orders. The knowledge of placed orders creates an arbitrage opportunity since substantial orders, will change the asset price. Buy orders will raise the asset price while sell orders will lead to a reduction of the asset price. This practice is called front running.
Using orders from clients to gain advantage for one's own transactions is illegal if this information is not publically known.

Broker houses have analysts which come up with estimates such as target prices for stock prices. In the Norwegian Finance newspaper (Finansavisen of 13th July 2012, p. 8 and

Adresseavisen of 13th July, p. 18) there's a story of ABG Sundal Collier (a broker house) adjusting downwards the target price of Funcom, a Norwegian company registered in Oslo Stock Exchange, the day before. The new estimate was ca. 13 Nok while the previous estimate was 30 NOK. The estimate change lead to a drop of the stock price by 25%. Having known the new target price in advance an arbitrageur would make a profit shortselling the Funcom stocks before the announcement of the new estimate. Short selling is legal in Norway as long as short selling is covered, which means that the shortseller borrows the stocks. This causes rent costs so that the money one makes by shortselling should exceed rent and transaction costs. Another strategy is uncovered short selling which eliminates rent costs. There is an ongoing trial in Norwegian courts of the legitimacy of uncovered short selling for foreign investors trading through broker houses outside of Norway (Dagens Næringsliv of 13th July 2012, p. 4).

A - xiii: Adaptive expectations vs. rational expectations

Adaptive Expectations

Adaptive expectations was introduced by Friedman (1957) and was widely used in macroeconomic models, for instance in the Phillip's curve. It is based on the assumption of a forecast error. Lucas (1976) criticized the soundness of a forecast error remaining unexploited despite it being computable. So he proposed the alternative called rational expectations. Adaptive expectations are not compatible with the efficient market hypothesis since the forecast error represents an arbitrage opportunity which is not taken advantage of.

Rational Expectations

Following Lucas argument, the workers' expectation about inflation is formed by looking ahead anticipating the central bank's optimal inflationary policy. Under the assumptions of perfect information and deterministic variables, is perfect foresight feasible. When the variables are stochastic, rational expectations are correct on average. Forecasts based on rational expectations of stochastic variables are called unbiased. In a setting of a dynamic game with perfect information, the optimal strategy is based on rational expectations. Perfect or unbiased forecast is the subgame perfect equilibrium (Bierman and Fernandez 1998, pp.175-177).

The rational expectation hypothesis says that today's price is a good predictor for tomorrow's price. That means that the expectations of agents at time t are adjusted with the information available up to this time. The expectations don't need to be precise but the agents' forecasting errors are neither systematically biased nor predictable. The rational expectations equilibrium assumes that the forecasts are not different from the resulted equilibrium. Only unanticipated information can have an influence on formed expectations and an effect on asset prices (Davies, Lowes and Pass 1988, p.443). According to Sargent (1993, p.6), the rational expectations equilibrium presupposes consistent beliefs. This implies correct evaluation of the distribution of the variables which the agents are trying to evaluate.

The rational expectation hypothesis is related to the efficient market hypothesis since both rational expectations and market efficiency assume informational efficiency (Copeland, et al. 2005, pp. 360-364). So to the strong form of market efficiency corresponds the strong form rational expectations, to the semi-strong market efficiency corresponds the semi-strong form rational expectations and to the weak form market efficiency corresponds the weak form rational expectations. The degree of market efficiency is proportional to the degree of rational expectations.

Rational expectations are conditioned to agents making optimal use of all available information. Making optimal use of available information can take excessively long time (Daskalakis, Goldberg, and Papadimitriou 2006).

A - xiv: The fundamental value of an asset and the relation between prices, dividends and returns

Let's start assuming that $E_t[r_{t+1}] = r_t$. That implies that $E_t[r_{t+1}]$ is constant.

The return at time $t+1$ is equal to the sum of price and dividends at $t+1$ minus the price at time t , divided by the price at time t:

$r_{t+1} = \dfrac{P_{t+1} + d_{t+1} - P_t}{P_t}$	Ap - 110

Taking expectations at both sides, substituting $E_t[R_{t+1}]$ with R_t and solving for P_t we get:

$$E_t[r_{t+1}] = E_t\left[\frac{P_{t+1} + d_{t+1} - P_t}{P_t}\right] \to r_t = E_t\left[\frac{P_{t+1} + d_{t+1}}{P_t} - \frac{P_t}{P_t}\right] \to$$

$$r_t = E_t\left[\frac{P_{t+1} + d_{t+1}}{P_t} - 1\right] \to r_t = E_t\left[\frac{P_{t+1} + d_{t+1}}{P_t}\right] - 1 \to$$

$$r_t + 1 = E_t\left[\frac{P_{t+1} + d_{t+1}}{P_t}\right] \to P_t(1 + r_t) = P_t E_t\left[\frac{P_{t+1} + d_{t+1}}{P_t}\right] \to$$

$$P_t(1 + r_t) = E_t[P_{t+1} + d_{t+1}] \to P_t = \frac{E_t[P_{t+1} + d_{t+1}]}{1 + r_t} \qquad \text{Ap - 111}$$

We observe that Ap - 111 is not a martingale since $E_t[P_{t+1}] = (1 + r_t)P_t - E_t[d_{t+1}]$ (Campbell et al. 1997, pp. 256 and 257). In order to have a martingale one has to reinvest all dividends in the stock.

Integrating forward using the law of iterated expectations we find the following expression for P_t:

$P_t = E_t\left[\sum_{i=1}^{k}\left(\frac{1}{1 + r_{t+i}}\right)^i d_{t+i}\right] + E_t\left[\left(\frac{1}{1 + r_{t+k}}\right)^k P_{t+k}\right]$	Ap - 112

Equation Ap - 112 says that the present price of an asset is equal to the sum of the expected dividend flows and the terminal price discounted with the demanded return rate.

As the time horizon goes to infinity, the limit of the last component of Ap - 112 goes to zero:

$\lim_{k\to\infty} E_t\left[\left(\frac{1}{1 + r}\right)^k P_{t+k}\right] = 0$	Ap - 113

Equation Ap - 113 is called the no-bubble assumption.

Then from Ap - 112 we see that:

$P_t = E_t\left[\sum_{i=1}^{k}\left(\frac{1}{1+r}\right)^i d_{t+i}\right]$	Ap - 114

The above equation is known as the fundamental value of an asset.

Assuming a constant growth rate g we get:

$E_t[d_{t+i}] = (1+g)E_t[d_{t+i-1}] = (1+g)^i d_t$	Ap - 115

Inserting Ap - 115 in Ap - 114 and assuming $g < r$ we get the following formula:

$P_t = \frac{E_t[d_{t+1}]}{r-g} = \frac{(1+g)d_t}{r-g} = \frac{d_{t+1}}{r-g}$	Ap - 116

Equation Ap - 116 is called the Gordon growth formula and is a fundamental result of the classical asset pricing theory.

Ths stock's sensitivity to changes in d, r and g is:

$$\frac{\partial P}{\partial d} > 0, \frac{\partial P}{\partial r} < 0\ , \frac{\partial P}{\partial g} > 0$$

Equation Ap - 116 shows that future dividends and growth are proportional to current prices while returns are inversely proportional to current prices.

One can derive a dynamic version of Ap - 116 starting with the identity:

$r_{t+1} \equiv log\left(\frac{P_{t+1} + d_{t+1}}{P_t}\right) = log(P_{t+1} + d_{t+1}) - log(P_t) =$ $log(P_{t+1}) - log(P_t) + log(P_{t+1} + d_{t+1}) - log(P_{t+1}) =$ $log(P_{t+1}) - log(P_t) + log\left(\frac{P_{t+1} + d_{t+1}}{P_{t+1}}\right) =$ $p_{t+1} - p_t + log\left(1 + e^{log\left(\frac{d_{t+1}}{P_{t+1}}\right)}\right)$	Ap - 117

By taking a first order Taylor expansion we get:

$r_{t+1} \approx k + \rho p_{t+1} + (1-\rho)log(d_{t+1}) - p_t$	Ap - 118

where

$p_t = log(P_t)$,

$\rho = \frac{1}{1+e^{\overline{log\left(\frac{d}{P}\right)}}}, \rho \in (0,1)$

$k = -log(\rho) - (1-\rho)log\left(\frac{1}{\rho} - 1\right)$

Equation Ap - 108 shows similar relations like equation Ap - 116. In order for expectations to be rational they have to associate the relations between future dividends, growth and current prices in the same way as the equations predict (Campbell 1997, pp. 261-262). The advantages of equation Ap - 118 over equation Ap - 116 is that the former one doesn't assume constant expected returns and complies better with empirical data.

Because p is greater than $log(d)$, ρ is greater than $(1-\rho)$ which is reasonable since a proportional change in the price has to have a greater effect on r than a proportional change in $log(d)$.

The logarithm of the stock price P_t is p_t. The expected discounted value of the logarithm of future dividends is p_{d_t} whereas the expected discounted value of future log stock returns is

p_{r_t}. In this setting are ρ and k the parameters of linearization and $log\overline{\left(\frac{d}{P}\right)}$ is the logaritm of the average ratio of dividend divided by the stock price (Campbell et al 1997, p. 262). Solving forward for p_t and imposing the condition $\lim_{n\to\infty}\rho^j p_{t+j} = 0$ which excludes rational bubbles (Campbell et al 1997, p. 263) we get:

$p_t = \dfrac{k}{1-\rho} + E_t\left[\sum_{j=0}^{\infty}\rho^j[(1-\rho)log(d_{t+1+j}) - r_{t+j+1}]\right]$	Ap - 119

The above equation is called the dynamic Gordon growth (Campbell et al 1997 p. 262):

Using a simpler notation:

$p_t = \dfrac{k}{1-\rho} + p_{d_t} - p_{r_t}$	Ap - 120

where

$$p_{d_t} = E_t\left[\sum_{j=0}^{\infty}\rho^j(1-\rho)\log(d_{t+1+j})\right] = E_t\left[\frac{1}{1-\rho}(1-\rho)\log(d_{t+1+j})\right] = E_t[\log(d_{t+1+j})]$$

$$p_{r_t} = E_t\left[\sum_{j=0}^{\infty}\rho^j\log(r_{t+j+1})\right] = E_t\left[\frac{1}{1-\rho}\log(r_{t+j+1})\right]$$

Solving with respect to $log\left(\frac{d_t}{P_t}\right)$ we obtain:

$log\left(\dfrac{d_t}{P_t}\right) = -\dfrac{k}{1-\rho} + E_t\left[\sum_{j=0}^{\infty}\rho^j\left(-\Delta log(d_{t+1+j}) + log(r_{t+1+j})\right)\right] =$ $= -\dfrac{k}{1-\rho} + E_t\left[\dfrac{1}{1-\rho}\left(-\Delta log(d_{t+1+j}) + log(r_{t+1+j})\right)\right]$	Ap - 121

The asset returns can be written as a linear expression of expected dividends and returns:

$log(r_{t+1}) - E_t[log(r_{t+1})]$ $= E_{t+1}\left[\sum_{j=0}^{\infty}\rho^j \Delta\log(d_{t+1+j})\right] - E_t\left[\sum_{j=0}^{\infty}\rho^j \Delta\log(d_{t+1+j})\right]$ $-\left(E_{t+1}\left[\sum_{j=1}^{\infty}\rho^j log(r_{t+1+j})\right] - E_t\left[\sum_{j=1}^{\infty}\rho^j log(r_{t+1+j})\right]\right)$	Ap - 122

In a simpler notation:

$log(r_{t+1}) - E_t[log(r_{t+1})] = \eta_{t+1} = \eta_{d,t+1} - \eta_{r,t+1}$	Ap - 123

where η_{t+1} is the unexpected stock return, $\eta_{d,t+1}$ is the change in expectations of future dividends and $\eta_{r,t+1}$ is the change in expectations of future returns.

Let the log stock return be:

$E_t[log(r_{t+1})] = r + \zeta_t$	Ap - 124

where r is a constant and ζ_t is an AR(1) process so that:

$\zeta_{t+1} = \phi\zeta_t + \xi_{t+1}\ , \phi \in (-1,1)$ $\zeta_{t+1} \sim (\phi\zeta_t, \sigma_\zeta^2),\ \xi \sim (0, \sigma_\xi^2)$	Ap - 125

where $\sigma_\xi^2 = (1-\phi^2)\sigma_\zeta^2$ and ϕ is the persistency coefficient.

Starting at:

$p_{r_t} = E_t\left[\sum_{j=0}^{\infty}\rho^j \log(r_{t+j+1})\right] \to p_{r_t} = \left[\sum_{j=0}^{\infty}\rho^j (r + \phi^j\zeta_t)\right] \to$ $p_{r_t} = \left[\sum_{j=0}^{\infty}\rho^j\, r\right] + \left[\sum_{j=0}^{\infty}\rho^j\, \phi^j\zeta_t\right] \to p_{r_t} = \frac{1}{1-\rho}r + \frac{1}{1-\rho\phi}\zeta_t$	Ap - 126

So we obtain

$$p_{r_t} = \frac{r}{1-\rho} + \frac{\zeta_t}{1-\rho\phi}$$	Ap - 127

The greater ϕ, the more persistent is the process ζ_t, and the higher p_{r_t} under the assumption that ρ is close to 1. That implies that the variability on this stock price is high even if the variability on the required rate of return is low Figure 8-1: Prediction of the dynamic Gordon Growth Model on the Behavior of Stock Price.

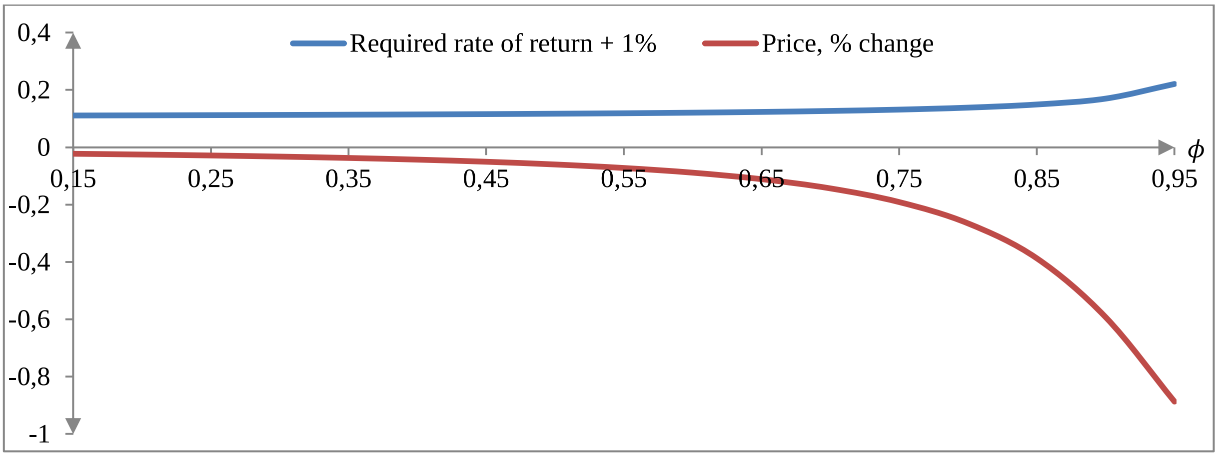


Figure 8-1: Prediction of the dynamic Gordon Growth Model on the Behavior of Stock Price

Since

$$log(r_{t+1}) - E_t[log(r_{t+1})] = \eta_{d,t+1} - \eta_{r,t+1} \rightarrow$$ $$log(r_{t+1}) = E_t[log(r_{t+1})] + \eta_{d,t+1} - \eta_{r,t+1} \rightarrow$$ $$log(r_{t+1}) = r + \zeta_t + \eta_{d,t+1} - \left(E_{t+1}\left[\sum_{j=1}^{\infty} \rho^j log(r_{t+1+j})\right] - E_t\left[\sum_{j=1}^{\infty} \rho^j log(r_{t+1+j})\right]\right)$$ $$\rightarrow log(r_{t+1}) = r + \zeta_t + \eta_{d,t+1} - \frac{\rho\xi_{t+1}}{1-\rho\phi} \rightarrow$$	Ap - 128

Under the assumption that $\eta_{d,t+1}$ and ξ_{t+1} are uncorrelated, ρ is close to 1 and using

$$\sigma_\xi^2 = (1-\phi^2)\sigma_\zeta^2$$	Ap - 129

the variance of $log(r_{t+1})$ becomes:

$var[log(r_{t+1})] = var(\eta_{d,t+1}) + var(\zeta)\left[\frac{1+\rho^2-2\rho\phi}{(1-\rho\phi^2)}\right]$	Ap - 130

where $var[\eta_{d,t+1}] = var(d)$

The equations above can be used for testing if the stochastic process followed by stock returns is an ARMA process (Campbell et al. 1997 p. 266).

A - xv: Limits to Arbitrage

Consider one of the assumptions of EMH, namely the exploitation of arbitrage opportunities to render the notion of prices reflecting all available information. Taking advantage of every incongruity between the market prices and the fundamental values would bring the market prices of the securities back to their fundamental values. Arbitrageurs represent smart money who exploits this misalignment. However, arbitrage is constrained by fundamental risk, noise trader risk and implementation costs. These restrictions lead to limited arbitrage.

A postulate that lies underneath the efficient market hypothesis is that of a perfect competition environment. In this setting, a large number of arbitrageurs is taking a position against mispriced securities. In reality there is a small number of specialized professionals who act as agents representing a few wealthy principals, for instance banks. Because of the constant changes in underlying economic fundamentals, arbitrage is not risk free (Shleifer and Vishny 1997, Scott 2011, pp. 201-203). Limits to arbitrage arise due to the complexity of the investment decision.

Arbitrageur's role is to eliminate inefficiencies through the exploitation of misalignments. Their trading rule is selling the substitute of a security with the same risk and return whenever the price of the security falls below its substitute and buying the security. Conversely, the arbitrageurs are buying the substitute of the security whenever the price of the security rises above its substitute and are selling the security. The arbitrage is risk free given the existence of perfect substitutes. This works fine with derivatives. For other securities it's difficult to find perfect substitutes. In a situation where the arbitrageur considers the market price of a stock to be low compared to its fundamental value, the strategy of being long on the stock is not risk free (Shleifer, Summers 1990).

The stock price can be calculated using for instance the dividend discount model or the Feltham-Ohlson model. The dividend discount model calculates the present value of the firm's equity as the discounted sum of future cash flows:

$MV_t = \dfrac{D}{r-g}$	Ap - 131

where D denotes dividends, r denotes return on equity and g denotes growth.

According to the Feltham - Ohlson model the stock price is equal to the book value plus the discounted abnormal returns:

$$MV_t = Y_t + \sum_{\tau=1}^{\infty} \frac{E[X_{t+\tau}^a]}{(R_k)^\tau} = Y_t + \sum_{\tau=1}^{\infty} \frac{E[X_t - (R_k - 1)]Y_{t-1}}{(R_k)^\tau} \qquad \text{Ap - 132}$$

Where $\mathrm{MV_t}$ is the market value of equity, $\mathrm{Y_t}$ and $\mathrm{Y_{t-1}}$are book values of equity at different points of time, $\mathrm{X_{t+\tau}^a}$ shows abnormal earnings and $\mathrm{X_t}$ denotes net-earnings.$\mathrm{R_k} = 1 + r$ is the discount factor.

Let stocks selling above the stock price calculated after either of the aforementioned models. By selling these stocks one runs the risk of the dividend realization being better than expected. Because the arbitrageur sells short, he has to cover the stock dividends as well. If new information arrives of other positive non-anticipated events, the stock price will raise even further, inducing further cash crunches. In these cases the arbitrageur incurs losses. These hazards constitute the fundamental risk.

Another eventuality is the sustainment and even the deepening of the discrepancy between the fundamental value and the market value due for instance to a prevailing biased conception. This can happen in periods where noise traders dominate the market (De Long, Shleifer, Summers and Waldmann 1990, see appendix A - ii). In this case, the security price would drift in the short run even further apart from its fundamentals. Whether the arbitrageur manages his own money or other people's money, he might be forced to liquidate at a loss despite the arbitrage trade being profitable in the longer run. The need to liquidate can arise either because of running out of own money or because the capital owners demand fast results. These factors can shorten down substantially the time horizon of the arbitrageur. This is the core of the noise risk.

Yet another risk is related to the implementation costs. Short selling is not always feasible because of illiquidity or because short selling becomes prohibitively expensive. Implementation costs impede the impact of each arbitrageur by their stock holdings and short position confinements (Abreu and Brunnermeier 2003).

Because of the probability of losses the exploitation of arbitrage will be hampered. Even more, due to capital costs, the arbitrageur's time horizon for taking advantage of an arbitrage opportunity cannot be particularly long. Another reason for short time horizons is the frequent performance evaluation of money managers. When the accomplishments are considered to be under par the arbitrageur runs the risk of being sacked, which limits the horizon of arbitrage.

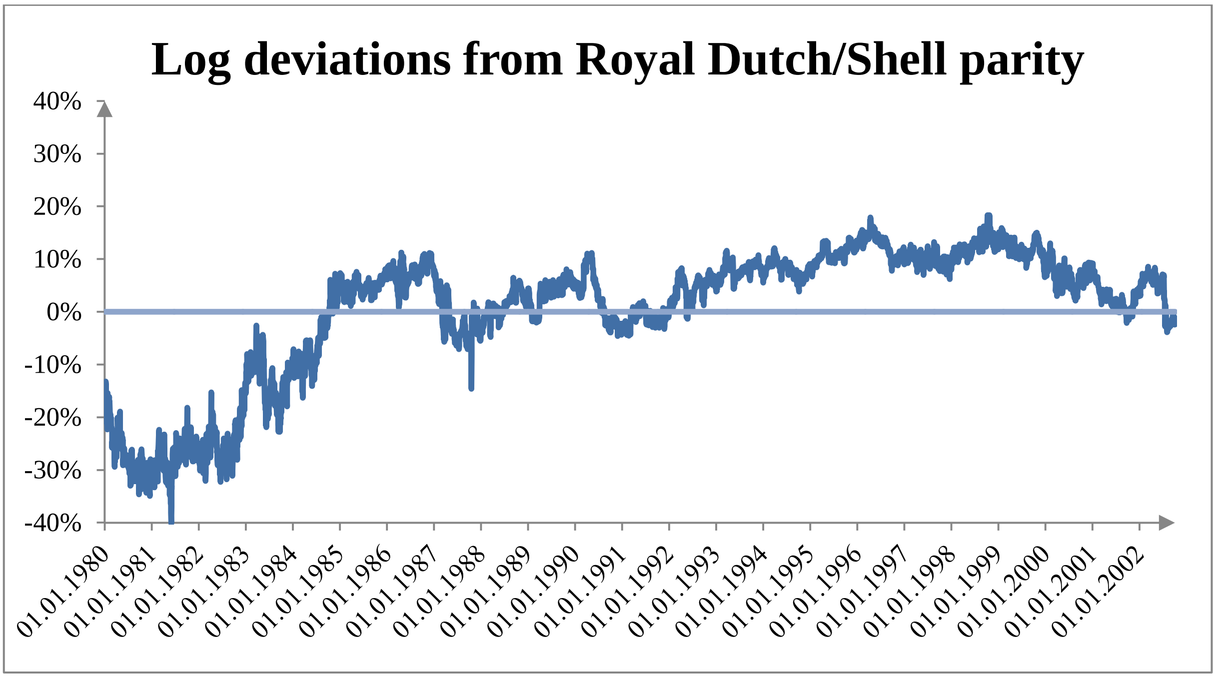
Limits to arbitrage imply that noise traders are not necessary eliminated through the exploitation of arbitrage opportunities by fundamentalists

Twin Shares

Twin shares, i.e. shares of firms merged together while remaining separate entities can be used to exemplify the limits to arbitrage. Consider for instance Royal Dutch and Shell transport which merged together in 1907 under the provision of Royal Dutch maintaining a claim of 60% of the total cash flows and Shell a claim of 40%. In an efficient market should the market value of Royal Dutch be 1,5 times the market value of Shell. Froot and Dabora (1999) examined the ratio of Royal Dutch to Shell equity for the period 1907-1995. They found deviations from the predicted market price from -35% to + 15%. This is attributed by Barberis and Thaler (2003, pp.1051-1121) to noise risk.

Long- term capital management (LCTM) was a hedge fund company that tried to exploit the arbitrage opportunity of the equity differentials beyond the 1,5 ratio . LCTM invested 2,3 billion dollars in the summer of 1997 on a long position in Shell and a short position in Royal Dutch. The reason for this was Royal Dutch trading at that time at a price premium of 8%. In the autumn of 1998, LTCM had to liquidate its positions due to heavy losses on Russian debt which defaulted. At the time LTCM unwounded its positions on Royal Dutch and Shell, the Royal Dutch equity price premium had increased to 22%. LCTM incurred losses amounted to 286 million dollars, which more than half was attributed to the Royal Dutch/Shell trade (Lowenstein 2000, p. 234).

Arbitrage positions have to be on hold until the twin equity prices converge. However, the converging date is not known ex ante. Because the arbitrageur has limited horizon it exists a substantial risk of closing the position with a loss if the prices don't converge. The equity price premium of Royal Dutch drifted further apart from the price under rational expectations, during the time horizon the arbitrageur seems fit waiting. This made LCTM's attempt to exploit the arbitrage opportunity presented by the Royal Dutch/Shell price equity differentials, considerably risky.



Appendix Figure - 2: Log deviations from Royal Dutch/Shell parity
Based on data provided by De Jong, Rosenthal and Van Dijk (2009).

Shleifer (2000, p. 32) makes the point that even when a security has a perfect substitute, as in the case of 1 Royal Dutch to 1,5 Shell security, due to risky arbitrage, can substantial deviations from the fundamental value be maintained over longer time intervals.

The differentials between the Royal Dutch/Shell parity and the prevailing market prices can neither be explained with the national tax rates nor with differences in the cash flow risk (Froot and Dabora 1999, Shleifer 2000, p.30).

De Jong, Rosenthal and Van Dijk (2009) investigate arbitrage exploitation of deviations from theoretical price parity for 12 dual-listed companies (DLCs) over the period 1980-2002. They find that it can take up to nine years before the equity prices converge. In between the price differentials can amplify. The arbitrageur is going to incur a loss if for whatever reason he is forced to liquidate his position before the convergence of prices. This can happen for instance in the case of margin calls which occurs when the investor buys securities with borrowed cash and the margin requirement gets uncovered, due to an adverse change in the market prices of the leveraged assets.

A - xvi: Program code used in EViews

```
'run pairwise least square regressions between series
for !i=1 to groupname.@count-1
  %iname = groupname.@seriesname(!i)
  for !j=!i+1 to groupname.@count
    %jname = groupname.@seriesname(!j)
    equation eq_{%iname}_{%jname}.ls {%iname} c {%jname}
  next
next

name and generate series from groups
for !i=1 to groupname.@count
      %name=groupname.@seriesname(!i)
      %newname = "X" + %name ‘X is the change in name
      series {%newname}=equation ‘specify the equation to be used
next

'transfer data from excel to eviews
%filename = "path"   'file name of the file to be opened
%sheetnames = @tablenames(%filename)   'find the names of the sheets in that file
%sheetname = @word(%sheetnames,1)   'get the first sheet name
wfopen(wf=panel,page=%sheetname) %filename range=%sheetname    'open the first sheet as a new workfile (with name=panel, and pagename=the first sheet name)
'loop through the remaining sheets, loading them into the workfile one at a time
for !i=2 to @wcount(%sheetnames)
  %sheetname = @word(%sheetnames,!i)  'get the name of the next sheet
  pageload(page=%sheetname) %filename range=%sheetname       'load the next sheet
next

'change names in series from groups
      for !i =1 to groupname.@count
      %oldname = groupname.@seriesname(!i)
      %newname = %oldname + "X" ‘X is the change in the old name
      rename {%oldname} {%newname}
```

```
next
'grab company code, name series and generate regressions
	for !i=1 to groupname.@count
	%code = @right(groupname.@seriesname(!i), 6)  'grab the last 6 digits of the series
	name.  This should be the company code
	%groupname1= "X1" + %code ‘X1 is the name change in groupname1
	%groupname2 = "X2" + %code ‘X2 is the name change in groupname2
	equation equationname{%code}.ls {%groupname1} c {%groupname2}
next

'recode zeroes to NA
for !i=1 to groupname.count
	%name = groupname.@seriesname(!i)
	{%name} = @recode({%name}=0,NA,{%name}) ‘NA stands for not available
next
```

A - xvii: List of companies used in our tests

Appendix table - 5: Datastream

Acta Holding	Blom	Ekornes	Infratek	North energy	Rocksource	Statoil
AF Gruppen	Bouvet	Eltek	InterOil Exploration	Norway Pelagic	Rieber & Søn	Telenor
AGR Group	Bridge Energy	Electromagnetic GeoServices	Itera	Norway Royal Salmon	Reservoir Exploration technology,	Telio Holding
Aktiv Kapital	BWG Homes	EOC	Intex Resources	Norske Skogindustrier	Saga Tankers	TGS-NOPEC Geophysical
Aker BioMarine	BW Offshore Limited	Evry	Jinhui Shipping and Transportation	TTS group	SalMar	Tomra Systems
Aker	Badger Explorer	Fairstar Heavy Transport	Jason Shipping	Odfjell ser, A	SAS AB	Transeuro Energy Corp,
Aker Seafoods	Cecon	Farstad Shipping	Kongsberg Automotive Holding	Olav Thon Eiendom	SeaBird Exploration	Veidekke
Aker Solutions	Cermaq	FLEX LNG	Kongsberg Gruppen	Opera Software	Schibsted	Vizrt
AKVA Group	Clavis Pharma	Fred, Olsen Energy	Lerøy Seafood Group	Orkla	Scana Industrier	Wilson
Algeta	Codfarmers	Fred, Olsen Production	Medi-Stim	ORIGIO	Seadrill	Wentworth Resources
Awilco LNG	Comrod Communication	Frontline	Marine Harvest	Petrolia Drilling	S D Standard drilling	Wilh, Wilhelmsen
American Shipping ,,,	Copeinca	Funcom	Morpol	Panoro Energy	Sevan Marine	Wilh, Wilhelmsen Holding
Asia Offshore Drilling	ContextVision	Gjensidige Forsikring	Norwegian Air Shuttle	Petroleum Geo-Service	Seven Drilling	Yara International
Apptix	Data Respons	Goodtech	Northland Resources	Photocure	Statoil Fuel & Retail	
Archer	Deep Sea Supply	Golden Ocean Group	Navamedic	Polarcus	SINO	
ABG Sundal Collier Holding	Det norske oljeselskap	Golar LNG	Norse Energy Corp	Pronova BioPharma	Semi Offshore	
Atea	DiaGenic	Ganger Rolf	Norsk Hydro	Prospector Offshore Drilling	SNI	
Austevoll Seafood	DnB NOR	Green Reefers	Northern Logistic Property	Protector Forsikring	Solstad Offshore	
Avocet Mining	DNO International	Grieg Seafood	Norwegian Car Carriers	Prosafe	Solvtrans Holding	
Awilco Drilling	Dockwise	Havila Shipping	Nordic Semiconductor	PSI Group	Songa Offshore	
Bakkafrost	DOF	Hexagon Composites	Norwegian Energy Company	Questerre Energy	Spectrum	
Belships	Dolphin Interconn ,,,	Hoegh Long Holdings	Northern Offshore	Q Free	SpareBank	
Bergen Group	Domstein	Hafslund ser, A	Nordic Mining	Royal Caribbean Cruises	The Scottish Salmon Company	
Biotec Pharmacon	Eitzen Chemical	Imarex	Norwegian Property	Renewable Energy	Siem Shipping INC,	
Birdstep Technology	Eidesvik Offshore	I,M, Skaugen	Norda	REM Offshore	Storebrand	

Appendix table - 6: Factset

ABG Sundal Collier Holding	Byggma	Fara	Kitron	Olav Thon Eiendomsselskap	Siem Offshore Inc,	Trefoil Ltd Usd 0,6667
Acta Holding	Canargo Energy Corp Com	Farstad Shipping	Komplett	Opera Software	Siem Shipping Inc	Trolltech Nok 0,04
AF Gruppen	Captura Nok 0,38	Fast Search And Transfer	Kongsberg Automotive Holding	ORIGIO A/S	Simrad Optronics Nok 0,518197814	TTS Group
AGR Group	Cermaq	Fesil Nok 10	Kongsberg Gruppen	Orkla	SinOceanic Shipping	Unison Forsikr Nok 5,00
Aker	Clavis Pharma	Fosen Nok 25	Kverneland	Otrum Nok 1	Sinvest	Veidekke
Aker BioMarine	Codfarmers	Fred, Olsen Energy	Leroy Seafood Group	PA Resources AB	Skiens Aktiemolle	Vizrt Ltd.
Aker Drilling	Component Software Nok 3,00	Frontline Ltd.	Luxo Nok 5	Petrobank Energy & Resources Ltd.	Software Innovatio Nok0,34	Vmetro Nok 0,50
Aker Floating Prod Nok 1,00	Confirmit Nok 0,175	Funcom N.V.	Mamut	Petrojack As Nok0,25	Solstad Offshore	Voss Veksel og Landmandsbank
Aker Seafoods	Conseptor Nok 0,1	Ganger Rolf	Marine Farms As Nok2	Petrojarl	Solvang	Wentworth Resources Ltd.
Aker Solutions	ContextVision AB	GC Rieber Shipping	Marine Harvest	Petroleum Geo-Services	Songa Offshore SE	Wilh, Wilhelmsen Holding
Aktiv Kapital	Crew Gold Corp Com Stk (Rfd 01/01/	Golar LNG Ltd.	Medistim	Petrolia	Spits Nok 0,40	Wilhelm Wilhelmsen Holding
AKVA Group	Crew Gold Corp.	Golden Ocean Group Ltd.	Namsos Trafikkselskap	PhotoCure	Statoil	Wilson
Altinex	Data Respons	GoodTech	Navamedic	Polaris Media	Stavanger Aftenbla Nok1	Yara International
American Shipping Co,	Deep Ocean	Green Reefers	Nio Inc,	Powel Nok1,00	Steen & Strom	
Apl Nok 0,25 (Post Split	Deep Sea Supply PLC	Grenland Group Nok 2,00	Norda	Profdoc Nok 0,50	StepStone	
Apptix	Det Norske Oljeselskap	Gyldendal	Nordic Semiconductor	Prosafe SE	Stolt-Nielsen Ltd.	
Arendals Fossekompani	DiaGenic	Hafslund	Norgani Hotels	PSI Group	StoreBrand	
Atea	DNB	Hafslund «A» Nok1	Norman Nok 0,385	Q-Free	STX Europe	
Austevoll Seafood	DNO International	Havila Shipping	Norse Energy Corp,	Questerre Energy Corp.	Subsea 7 Inc Shs	
Awilco Offshore	DOF	Hexagon Composites	Norsk Hydro	Renewable Energy Corp,	Subsea 7 S.A.	
Axis-Shield PLC	DOF Subsea	Hjellegjerde Nok 0,05	Norsk Vekst Nok 10	Reservoir Exploration Technology	Superoffice As Nok 0,7	
B&H OCEAN CARRIERS LTD COM NEW	Dolphin Group	Hurtigruten	Norske Skogindustrier	Revus Energy	Synnove Finden Nok 2	
Belships	Domstein	I.M. Skaugen SE	Norstat Nok 0,07	Rieber & Son	Tandberg	
Bionor Pharma	Eastern Drilling	IGE Resources AB	Northland Resource Com Npv (Temp Share)	Rocksource	Tandberg Data	
Biotec Pharmacon	Eidesvik Offshore	Ignis	Northland Resources S.A.	Roxar Nok1	Tandberg Storage Nok 0,01	
Birdstep Technology	Eitzen Chemical	IMAREX	Norwegian Air Shuttle	Royal Caribbean Cruises Ltd,	Tandberg Television	
Bjorge	Ekornes	Inmeta Crayon	Norwegian Car Carriers	SAS AB	Teco Maritime Nok 0,50	
Blom	Eltek	Intelecom Group Nok 0,50	Norwegian Property	Scana Industrier	Telecomputing Nok 0,1334	
Bonheur	EVRY	InterOil Exploration & Production	Ocean Rig	Schibsted	Telenor	
Borgestad	Exense Nok 0,10	Intex Resources	Odfjell B	Scorpion Offshore Ltd Shs	Telio Holding	
BW Gas	Expert	Itera	Odfjell Invest	SeaBird Exploration PLC	TGS-NOPEC Geophysical Co.	
BW Offshore Ltd.	Fairstar Heavy Transport N.V.	Jason Shipping	Odfjell SE	Seadrill Ltd.	Tide	
BWG Homes	Faktor Eiendom	Jinhui Shipping & Transportation Ltd.	Odim	Sevan Marine	Tomra Systems	

A - xviii: Sentiment, summary of articles

Appendix table - 7: Sentiment - Dispersion of beliefs - Divergence of Opinions

	Sentiment	Heterogeneity	Asset prices
Representativeness bias	Overreaction	Agent type	- Above the asset's fundamental value given good news. - Below the asset's fundamental value given bad news.
Conservatism bias	Underreaction	Agent type	- Below the asset's fundamental value given good news. - Above the asset's fundamental value given bad news.
Psychological bias	The rate of change of divergence of opinions	Individual assigns different probabilities to the same state of nature without using information	Above or below the asset's fundamental value.
Rational expectations	The rate of change of dispersion of beliefs	Individual assigns different probabilities to the same state of nature based on commonly known information	Dispersion of beliefs a risk which requires a premium. Auction theory of asset prices, asset prices increase with the dispersion of beliefs reflecting the optimistic investors.
Rational expectations	The rate of change of divergence of opinions	Individual assigns different probabilities to the same state of nature based on commonly known information(Varian)	Divergence of opinions a risk which requires a premium
Measures	- Dispersion of analyst's targets. - Dispersion of analyst's recommendations. - Consumer confidence indicator. - Economic sentiment indicator. - Put Call parity. - The VIX index. - Composite index made of closed-end fund discount, the stock market's share turnover, the number and average first-day returns on IPOs, the equity share in new issues and the dividend premium. - Twitter's positive brand twits.		

A - xix: Herding. Summary of articles

Appendix table - 8: Herding

	Information cascade	Reputation	Compensation schemes	Search costs	Herding measures and instrument for detecting herding	Directional asymmetry
Market open				Herding	-Patterson, Sharma	
Market close	Herding				-Patterson, Sharma	
Small cap	Herding				-Patterson, Sharma - Chang, Chen and Khorana	- Various market states
High cap				Herding	-Patterson, Sharma - Chang, Chen and Khorana	- Bull market - Various market states
Intraday, high frequency				Herding	-Patterson, Sharma	
Low frequency	Herding				- Chang, Chen and Khorana	- Various market states
Fund managers		Herding	Herding		-Lakonishok, Shleifer, Vishny	
Analysts t		Herding	Herding		- Jegadeesh and Kim	

A - xx: Asset pricing puzzles, summary of articles

Appendix table - 9: Asset pricing puzzles

Puzzles	Explanations proposed	Hansen-Jagannatahan bound
Equity premium	- External Habit formation - Internal Habit formation - Non-separable time preferences - Recursive utility - Myopic loss aversion - Heterogeneity	- Basic - Extended
Risk free rate	- Habit formation - Recursive utility - Myopic loss aversion - Heterogeneity	- Basic
Correlation	- Recursive utility - Myopic loss aversion	- Extended

A - xxi: CSAD Reviewed articles

Appendix table - 10: Reviewd articles

	PTSD	**CSAD**	**CCAPM**
Heterogeneity	- *Sentiment* - *Dispersion of Beliefs* - *Divergence of Opinions*	*Herding*	- *Puzzles* - *Equity premium* - *Correlation puzzle* - *Hansen Jagannathan Bound* - *Stochastic Discount Factor*
Theoretical articles	- Xiouros (Ph.D. dissertation) - Milgrom, Stokey (3878) - Miller (1729) - Cvitanic et al.(18) - Shleifer (2071) - Iori (147) - Hommes (563) - Rubinstein (208) - Varian (266) - Baker, Wurgler (586) - Barberis, Thaler (1765) - Lee, Shleifer, Thaler (1283) - Tirole (705) - Odean (1370) - Chordia, Roll, Subrahmanyam (49) - Hens, Rieger (18) - Abreu, Brunnermeier (745) - Shefrin (971) - Delong, Shleifer, Summers, Waldmann (575) - Shleifer, Vishny (2595) - Hong, Stein (2061) - Hong, Stein (526) - Lakonishok, Shleifer, Vishny (1242) - Daniel, Hirschleifer, Subrahmanyam (3201) - Miller (1729) - Daniel, Hirshleifer (3223)	- Beja & Goldman (201) - Delong et al (3525) - Cont and Bouchaud (572) - Alfarano, Lux and Wagner (124) - Lux 1995 (646) - Jegadeesh, Kim (81) - Trueman (601) - Lin, Tsai, Sun (6) - Scharfstein, Stein (2399) - Lakonishok, Shleifer and Vishny (1242) - Gebka, Wohar (0) - Abreu and Brunnermeier (745) - Ackert, Charupat, Church and Deaves (66) - McQueen et al. (217) - Richards (38) - Vayanos, Wang (124)	- Lucas (4079) - Abel (1701) - Campbell and Cochrane (2812) - Constantinides and Duffie (858) - Kahneman and Tversky (25578) - Aase (0) - Barberis and Thaler (1765) - Blanchard and Watson (1024) - Cutler, Poterba and Summers (850) - Duffie (3306) - Øksendal (5120) - Sottinen 2003 (20) - Bhamra and Uppal (34) - Cochrane (34) - Pennacchi (49) - Ackert and Deaves (11) - Ferson and Constantinides (550) - Tallarini and Zhang (29) - Cvitanic et al.2011 (32) - Mehra (21)
Empirical articles	- Xiouros (Ph.D. dissertation) - Diether, Malloy, Scherbina (898) - Baker, Wurgler (586) - De Bondt and Thaler (4620) - Bernard and Thomas (1270) - Jegadeesh and Titman (5111) - Schwert (570) - Fama and French (4061)	- Lakonishok, Shleifer, Vishny (1242 - Bikhchandani and Sharma (634) - Jegadeesh and Kim (81 - Lin, Tsai, Sun (6) - Patterson, Sharma (5) - Chang, Chen and Khorana (313) - Tan, Chiang, Mason and Nelling (80) - Economou, Kostakis, Philippas (7) - Kallinterakis, Lodetti (1) - Al-Shboul (1) - Araghi, Mavi and Alidoost (0) - Prosad, Kapoor and Sengupta (0) - Gebka, Wohar (0)	- Mehra and Prescott (4658) - Barberis, Huang and Santos (1259) - Ackert et al (66) - Hansen and Singleton (2069) - Ødegaard (Handbook)
Simulation articles	- Shleifer (2071) - Iori (147)		- Kahneman and Tversky 1979(25578) - Benartzi and Thaler (1794)
Empirical articles Norway	- Grigaliūnienė, Cibulskienė (0) - Stenstad and Rabben (master thesis) - Jubbega (master thesis)	- Lindhe (master thesis) - Amundsen, Bay (master thesis)	- Dimson et al. (143) - Ødegaard 2011(Lecture notes) - Donadelli, Prosperi (4) - Christensen (master thesis) - Båtvik (master thesis) - Mellingen and Kleiven (master thesis)

PTSD stands for price target standard deviation, CSAD stands for cross sectional absolute deviation and CCAPM stands for consumption capital asset pricing models.

In parentheses are the numbers of citations in Google Scholar.

A - xxii: CSAD test for the Norwegian stock market as a whole

Appendix table - 11: CSAD test for the Norwegian stock market 0120007-072012

Dependent Variable: CSAD_MVW
Method: Least Squares
Date: 04/18/13 Time: 14:05
Sample (adjusted): 1/02/2007 7/12/2012
Included observations: 1391 after adjustments
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed
bandwidth = 8.0000)
CSAD_MVW=C(1)+C(2)*@ABS(RM_MVW)+C(3)*(RM_MVW)^2

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.010488	0.000327	32.07124	0.0000
C(2)	0.139533	0.037117	3.759229	0.0002
C(3)	1.499589	0.545284	2.750109	0.0060

R-squared	0.289809	Mean dependent var	0.012886
Adjusted R-squared	0.288786	S.D. dependent var	0.005775
S.E. of regression	0.004871	Akaike info criterion	-7.809072
Sum squared resid	0.032926	Schwarz criterion	-7.797775
Log likelihood	5434.209	Hannan-Quinn criter.	-7.804847
F-statistic	283.2023	Durbin-Watson stat	1.355075
Prob(F-statistic)	0.000000		

where CSAD is the cross sectional absolute deviation of stock returns, RM is the stock market return and MVW stands for market value weighted.

A - xxiii: Multicollinearity, relative risk aversion (SDF) and the subjective discount factor in CCAP-Models

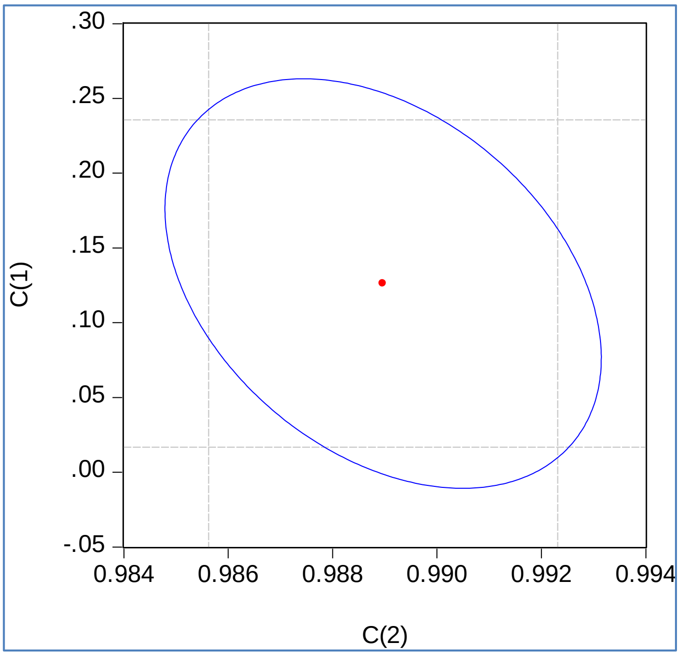


Figure 8-2: Multicollinearity between RRA denoted by C(2)
and the subj. discount factor denoted by C(1)
in Lucas CCAPM, model M - 16

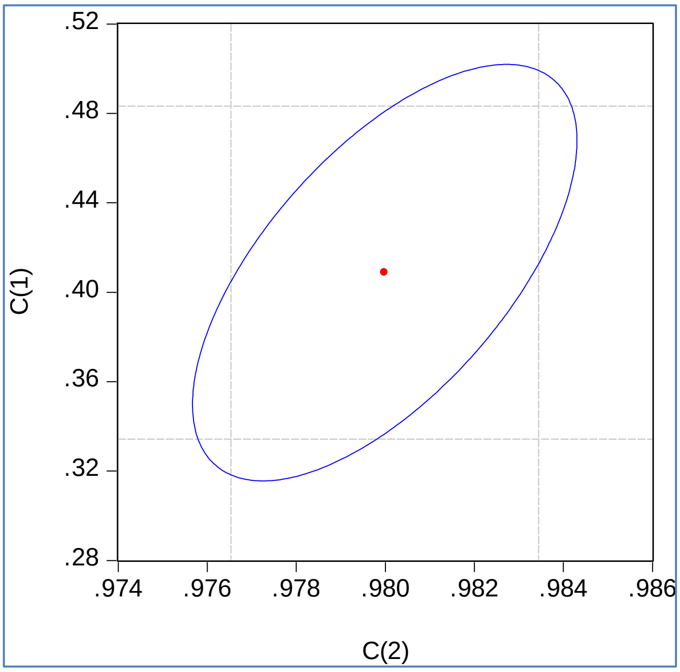


Figure 8-3: Multicollinearity between RRA denoted by C(2)
and the subj. discount factor denoted by C(1)
in Abel's CCAPM, model M - 28($\kappa = -1$)

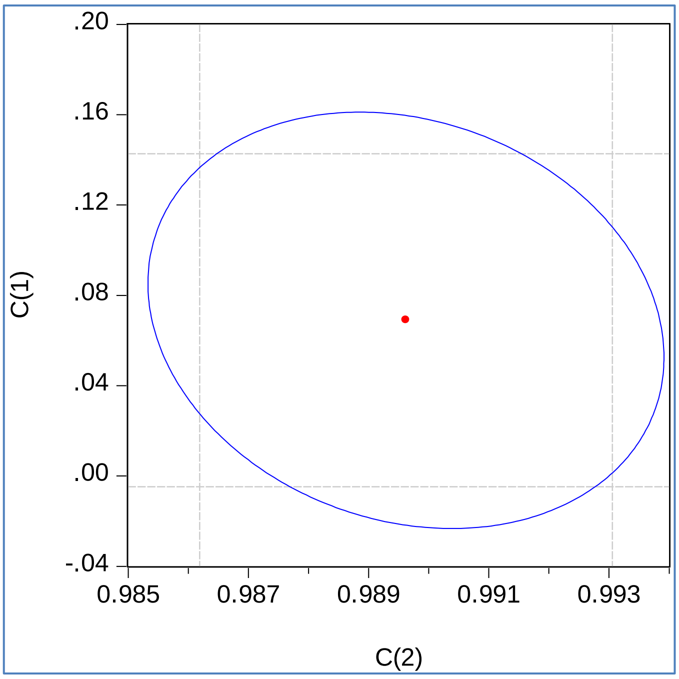


Figure 8-4: Multicollinearity between RRA denoted by C(2)
and the subj. discount factor denoted by C(1)
in Abel's CCAPM, model M - 39 ($\kappa = -0{,}001$)

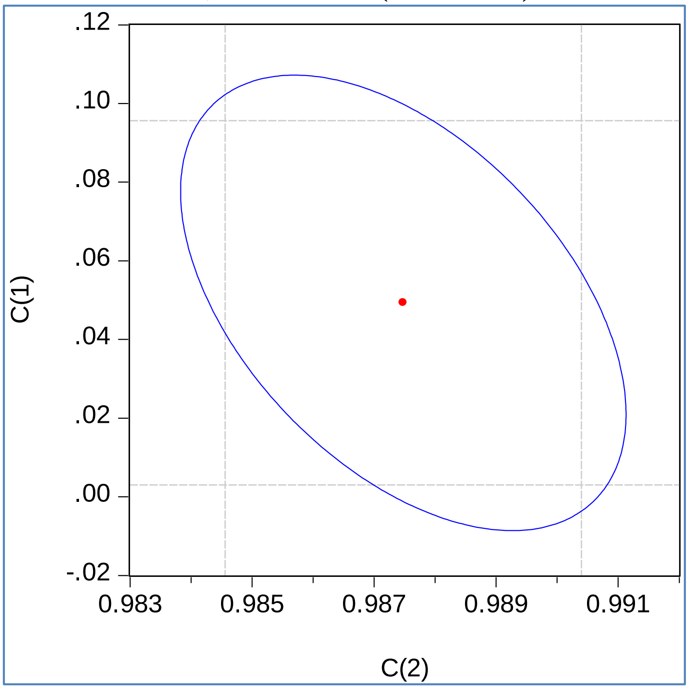


Figure 8-5: Multicollinearity between RRA denoted by C(2)
and the subj. discount factor denoted by C(1)
in Campbell and Cochrane's CCAPM, model M - 42

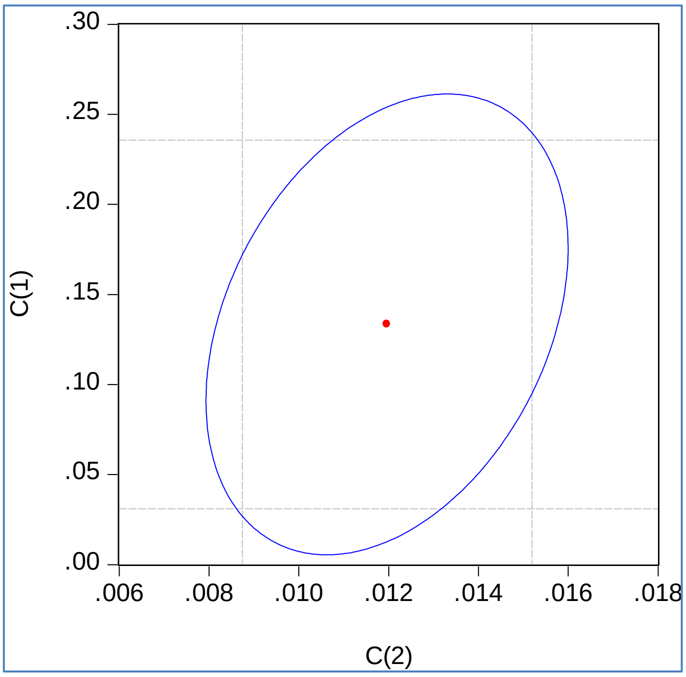


Figure 8-6: Multicollinearity between RRA denoted by C(2)
and the subj. discount factor denoted by C(1)
in Constantinides and Duffie’s CCAPM, model M - 49

A - xxiv: CCAPM models and the HJ-bounds

HJ-bound, basic:						
	-2,796246	-2,796246	-2,796246	-2,796246	-2,796246	-2,796246
Model variant	M-16	M-17	M-18	M-19	M-20	M-21
Lucas	-2,796246	-2,796246	-2,796246	-2,796246	-2,796246	-2,796246
	M-23	M-24	M-25	M-26	M-27	M-28
Abel kappa = -1	-2,796246	-2,796246	-2,796246	-2,796246	-2,796246	-2,796246
	M-29	M-30	M-31	M-32	M-33	M-44
Abel kappa = 1	-2,796246	-2,796246	-2,796246	-2,796246	-2,796246	-2,796246
	M-35	M-36	M-37	M-38	M-39	M-40
Abel kappa = -0,001	-2,796246	-2,796246	-2,796246	-2,796246	-2,796246	-2,796246
	M-42	M-43	M-44	M-45	M-46	M-47
Campbell and Cochrane	-2,796246	-2,796246	-2,796246	-2,796246	-2,796246	-2,796246
	M-49	M-50	M-51	M-52	M-53	M-54
Constantinides and Duffie	-2,796246	-2,796246	-2,796246	-2,796246	-2,796246	-2,796246
gamma estimates						
Model variant	M-16	M-17	M-18	M-19	M-20	M-21
Lucas	0,873824	1,069909	1,117351	0,972083	0,929339	0,991258
	M-23	M-24	M-25	M-26	M-27	M-28
Abel kappa = -1	0,009789	0,195580	0,977226	1,651314	-0,954244	0,591244
	M-29	M-30	M-31	M-32	M-33	M-44
Abel kappa = 1	0,014834	0,115258	0,950386	0,659223	0,322249	0,460367
	M-35	M-36	M-37	M-38	M-39	M-40
Abel kappa = -0,001	0,263521	1,065772	1,117297	0,972509	0,931074	0,986675
	M-42	M-43	M-44	M-45	M-46	M-47
Campbell and Cochrane	0,950700	1,009807	1,054189	0,988178	0,960898	0,960392
	M-49	M-50	M-51	M-52	M-53	M-54
Constantinides and Duffie	0,866531	1,031037	1,100302	0,978124	0,892709	1,033999
Diff. basic HJ-bound, gamma						
	M-16	M-17	M-18	M-19	M-20	M-21
Lucas	-3,670070	-3,866155	-3,913597	-3,768329	-3,725585	-3,787504
	M-23	M-24	M-25	M-26	M-27	M-28
Abel kappa = -1	-2,806035	-2,991826	-3,773472	-4,447560	-1,842002	-3,387490
	M-29	M-30	M-31	M-32	M-33	M-44
Abel kappa = 1	-2,811080	-2,911504	-3,746632	-3,455469	-3,118495	-3,256613
	M-35	M-36	M-37	M-38	M-39	M-40
Abel kappa = -0,001	-3,059767	-3,862018	-3,913543	-3,768755	-3,727320	-3,782921
	M-42	M-43	M-44	M-45	M-46	M-47
Campbell and Cochrane	-3,746946	-3,806053	-3,850435	-3,784424	-3,757144	-3,756638
	M-49	M-50	M-51	M-52	M-53	M-54
Constantinides and Duffie	-3,662777	-3,827283	-3,896548	-3,774370	-3,688955	-3,830245

The red colour denotes estimates in models which don't pass one or more of the following criteria

$\gamma<1$, $\alpha>0$, $\delta<1$ and p-value of the J-statistic>0,1 where γ is the risk aversion parameter, α is the relative risk aversion parameter and δ is the subjective discount factor.

The green colour denotes estimates in models that pass these criteria.

The blue colour denotes the estimates in models where the distance to the basic HJ-bound is minimized.

HJ-bound, extended						
Model variant	M-16	M-17	M-18	M-19	M-20	M-21
Lucas	-64,891011	-64,853119	-64,845345	-64,870859	-64,879337	-64,867200
	M-23	M-24	M-25	M-26	M-27	M-28
Abel kappa = -1	-65,131500	-63,972864	-93,543049	-199,922924	-82,867047	-69,974764
	M-29	M-30	M-31	M-32	M-33	M-44
Abel kappa = 1	-65,397468	-67,307456	-94,511959	-297,664224	-84,638285	-154,406301
	M-35	M-36	M-37	M-38	M-39	M-40
Abel kappa = -0,001	-65,129682	-64,650269	-64,736837	-65,497467	-65,131286	-66,254440
	M-42	M-43	M-44	M-45	M-46	M-47
Campbell and Cochrane	-34,126288	-34,188154	-34,234774	-34,165486	-34,136944	-34,136415
	M-49	M-50	M-51	M-52	M-53	M-54
Constantinides and Duffie	-103,108667	-103,021428	-102,989159	-103,049014	-103,094212	-103,020967
gamma estimates						
Model variant	M-16	M-17	M-18	M-19	M-20	M-21
Lucas	0,873824	1,069909	1,117351	0,972083	0,929339	0,991258
	M-23	M-24	M-25	M-26	M-27	M-28
Abel kappa = -1	0,009789	0,195580	0,977226	1,651314	-0,954244	0,591244
	M-29	M-30	M-31	M-32	M-33	M-44
Abel kappa = 1	0,014834	0,115258	0,950386	0,659223	0,322249	0,460367
	M-35	M-36	M-37	M-38	M-39	M-40
Abel kappa = -0,001	0,950700	1,009807	1,054189	0,988178	0,960898	0,960392
	M-42	M-43	M-44	M-45	M-46	M-47
Campbell and Cochrane	0,950700	1,009807	1,054189	0,988178	0,960898	0,960392
	M-49	M-50	M-51	M-52	M-53	M-54
Constantinides and Duffie	0,866531	1,031037	1,100302	0,978124	0,892709	1,033999
Diff. extended HJ-bound, gamma						
	M-16	M-17	M-18	M-19	M-20	M-21
Lucas	-65,764835	-65,960920	-66,008362	-65,863094	-65,820350	-65,882269
	M-23	M-24	M-25	M-26	M-27	M-28
Abel kappa = -1	-65,141289	-65,327080	-66,108726	-66,782814	-64,177256	-65,722744
	M-29	M-30	M-31	M-32	M-33	M-44
Abel kappa = 1	-65,412302	-65,512726	-66,347854	-66,056691	-65,719717	-65,857835
	M-35	M-36	M-37	M-38	M-39	M-40
Abel kappa = -0,001	-66,080382	-66,139489	-66,183871	-66,117860	-66,090580	-66,090074
	M-42	M-43	M-44	M-45	M-46	M-47
Campbell and Cochrane	-35,076988	-35,136095	-35,180477	-35,114466	-35,087186	-35,086680
	M-49	M-50	M-51	M-52	M-53	M-54
Constantinides and Duffie	-103,975198	-104,139704	-104,208969	-104,086791	-104,001376	-104,142666

Aase 179; 188; 285
Abel 145; 147; 149; 150; 151; 152; 153; 154; 155; 173; 175; 176; 177; 179; 184; 185; 188; 285; 287; 288; 290; 291
Abreu 19; 188; 275; 285
Ackert 41; 78; 188; 207; 285
aggregate consumption 65; 66; 137; 145; 147; 164; 182; 190
Alfarano 27; 188; 285
Alidoost 74; 106; 188; 285
Amundsen 73; 188; 285
Araghi 74; 106; 188; 285
assumption 9; 10; 18; 24; 25; 33; 40; 45; 58; 60; 61; 62; 65; 67; 95; 97; 99; 100; 101; 102; 103; 104; 119; 121; 122; 124; 125; 126; 127; 128; 129; 133; 135; 147; 168; 179; 183; 187; 224; 249; 263; 265; 268; 272
Baker 20; 23; 70; 188; 285
Barberis...18; 23; 62; 63; 77; 189; 276; 285
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