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A robust model predictive control with constraint modification for gas lift allocation optimization

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ABSTRACT

This paper presents a novel approach for implementing a robust real-time optimization framework under the presence of parametric uncertainty. Conservativeness is an inevitable drawback of a robust control approach. Therefore we aimed to provide a simple and efficient method to mitigate the conservativeness while the robust fulfillment of the constraints is still preserved. The proposed method in this paper is based on the worst-case realization of the uncertainties, however, with constraint modification. The mismatch between measured and predicted output is used directly to modify the active constraint in the optimization problem. The superiority of the method in terms of conservativeness and computational time has been demonstrated in comparison with the other robust optimization counterparts, such as traditional min–max and multi-stage MPC. The promising advantage of the proposed method is that not only it reduces the conservativeness significantly, but also the computational price for this achievement is considerably cheaper than closed-loop optimization methods such as multi-stage MPC.

1. Introduction

Model Predictive Control (MPC) is an advanced optimization strategy with remarkable merits that has received a great deal of attention, especially in the process control community. It is a convenient tool for dealing with multiple-input multiple-output processes. Moreover, it is capable of handling the constraints directly, whether they are on the states, control actions, or outputs. However, the fact that it uses a mathematical model to forecast the future behavior of the process can make it susceptible to poorer performance in practical applications since a perfect mathematical model simply does not exist in most cases.

Parametric uncertainty, unmodeled dynamics, exogenous disturbances, measurement noise, etc., are some well-known sources of uncertainty that introduce a mismatch between the model and the actual process. This mismatch can deteriorate the prediction part of MPC and consequently lead to poor performance. This situation becomes even more challenging when there are hard constraints that should strictly be satisfied throughout the operation. Therefore it is almost inevitable to consider the effect of uncertainty in practical applications.

A conventional remedy to mitigate the effect of uncertainty is the robust approach, where the uncertainty is assumed to belong to a bounded set, and the controller is designed to guarantee robust requirements for the worst-case situation. This ensures that if any other realization within the bounded uncertainty region happens, the controller is able to handle it. The worst-case formulation, which is also known as the min–max formulation, was originally proposed in [1] and later in the context of MPC in [2]. Traditional min–max MPC formulation as well as standard MPC are open-loop optimizations in the sense that they solve an open-loop optimal control problem at each sampling time. An open-loop optimization fashion does not take into account the explicit notion of feedback in the formulation of the optimization problem, although the new information will be available in the next time instance. The main drawback of open-loop optimization is that it leads to an overly conservative solution; therefore, the controller will be significantly sub-optimal, and all resources available in the process may not be fully utilized.

To address the problem of conservativeness, the notion of feedback has been explicitly introduced in the closed-loop min–max framework as in [3,4]. This means that the optimization will be solved over control policies rather than a single control sequence. This allows the future decisions to depend directly on the future measurements. In other words, it introduces some extra degrees of freedom to the optimization problem that reduces the conservativeness. However, the general formulation which leads to dynamic programming suffers from the curse of dimensionality and will not be practically implementable. Hence, optimization over state feedback policies [5], affine policies parameterized on

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the uncertainty [6,7], and deep neural network [8] have been proposed to approximate the general problem.

Tube-based MPC method is another alternative in the framework of robust approach for both linear [9] and nonlinear systems [10]. The basic idea of this method is to split the control problem into two parts. First is an ancillary controller, which is responsible for maintaining the real uncertain system within an invariant set around the nominal trajectory. Second, a deterministic standard MPC with tightened constraints based on the nominal trajectory which steers the bundle of trajectories (known as tube) to the desired state. Since the invariant set can be designed offline, the method does not impose too much extra online computational cost. However, constructing such an invariant set might not be simple, especially for complicated nonlinear systems. Different modifications of the method have been presented in [11–14].

Another possibility to implement closed-loop optimization in the framework of robust MPC is to use multi-stage MPC [15]. This method approximates the general formulation of dynamic programming by considering only a finite realization of the uncertainty, which is represented by a scenario tree. The scenario tree makes it possible to solve the optimization problem over different control trajectories; hence it will reduce the conservativeness. However, the computational cost is still expensive because the number of scenarios and consequently the size of the problem grows exponentially with the length of the prediction horizon and the number of uncertainty realizations considered. Therefore applicability of the method is still limited.

The main challenge in this regard is that the robust methods are inherently conservative, and the existing methods to reduce the conservativeness are either computationally heavy or they deteriorate the robust performance [16]. Therefore, this paper aims to address the problem of conservativeness, particularly by looking into a case study to maximize the oil production from a gas-lifted oil network under the presence of parametric uncertainty. Previous studies [17,18] have shown that parametric uncertainties must be considered in the optimization problem; otherwise, the constraints would be violated. It has also been shown that robust formulations are overly conservative or computationally expensive. Conservativeness, which in this case can be interpreted as an unexploited possibility for more production, is an inevitable price that should be paid to ensure robust performance. However, this paper aims to provide a simple and efficient method to mitigate conservativeness while the robust fulfillment of the constraints is preserved.

The proposed method in this paper is based on the worst-case realization of the uncertainties with constraint modification. More specifically, the mismatch between measured and predicted output is used directly to modify the active constraint in the optimization problem. Since the design is based on the worst-case situation, there will be no mismatch between the prediction and measurement when the worst-case realization of the uncertainty occurs. Under such conditions, the method reduces to traditional min–max MPC. However, for the other realizations of uncertainty, the constraint modification leads to a higher production rate and thus results in a less conservative operation.

Although the output error was employed in the early versions of predictive control [19] and later in more recent versions of adaptive model predictive control [20,21], the fundamental distinction between the proposed method of this paper and previous works lies in the role of measurements. Despite the adaptive approach, which makes use of output error to estimate the uncertainty and utilizes the estimated values of uncertainty in the optimization problem, in the proposed method of this paper, the output error is used directly to reconstruct the boundary on constraints in the optimization problem, meaning the method does not contain any estimation algorithm. It is well known that in an adaptive approach, the constraints can be violated dynamically during the transient periods due to the lag in the parameter estimation step [16]. However, the proposed method of this paper does not estimate the parameters. Instead, the measurement is used to modify the constraint boundaries directly. The second major difference is that contrary to the adaptive approach, the optimization problem in the proposed method is based on the worst-case realization of the uncertainty, which enables this method to fulfill the constraints robustly for all the realizations of uncertainty within the considered bounded set.

Although the proposed method has been developed based on special features of gas-lifted oil fields, it can be generalized to be applicable to a class of systems with the same features. The superiority of the method in terms of conservativeness and computational time has been demonstrated in comparison with the other robust optimization counterparts, such as traditional min–max and multi-stage MPC. The main contribution of this work is that it not only significantly reduces conservatism but also the price for such achievement is considerably cheaper than closed-loop optimization methods such as multi-stage MPC. This puts the proposed method superior to the original min–max MPC since the proposed method is less conservative with the same level of complexity and robustness. The advantage of the proposed method over multi-stage MPC is that it is simpler and computationally more efficient, and it reduces the conservativeness even better than multi-stage MPC.

The rest of the paper is organized as follows. Section 2 briefly describes mathematical modeling of the gas-lifted oil field system. The control design and simulation results are presented in Sections 3 and 4, respectively before concluding in Section 5.

2. Mathematical model of gas-lifted oil field

2.1. Process description

The gas lift mechanism is a well-known artificial lifting method to increase or revive the production from oil fields by reducing the fluid mixture density in the well's tubing. A gas-lifted oil field consists of multiple oil wells that share a common lift gas source. Different components of a single oil well have been shown schematically in Fig. 1. A gas-lifted oil well simply works by injecting high-pressurized natural gas into the well's annulus. For each well, a gas lift choke valve controls the gas flow rate from the common gas distribution pipeline into the annulus. The injected gas finds its way towards the tubing at some points located at proper depths and mixes with the multiphase fluid from the reservoir. As a result, the density of the mixture in the tubing will be reduced. Consequently, the hydrostatic pressure of the column of fluid above the injection point and the flowing pressure losses in the tubing will be reduced. Therefore, the pressure gradient between the reservoir and top side will be sufficient to overcome the resistance in the well and pushes the reservoir fluid to the surface.

First principle modeling of gas-lifted oil fields has been investigated for flow stabilization [22–24], control and production optimization purposes [17,18,25]. All these models are derived based on the mass balance of different fluid phases in the tubing and annulus. It has been shown that the first principle models based on mass conservation are accurate enough to be used for control purposes [26].

2.2. Governing equations

In this section, we only briefly present the governing equations of the process as derived in [18] because mathematical modeling is not the objective of this paper. The readers are also referred to [17,25] for further details. We considered a gas-lifted oil field

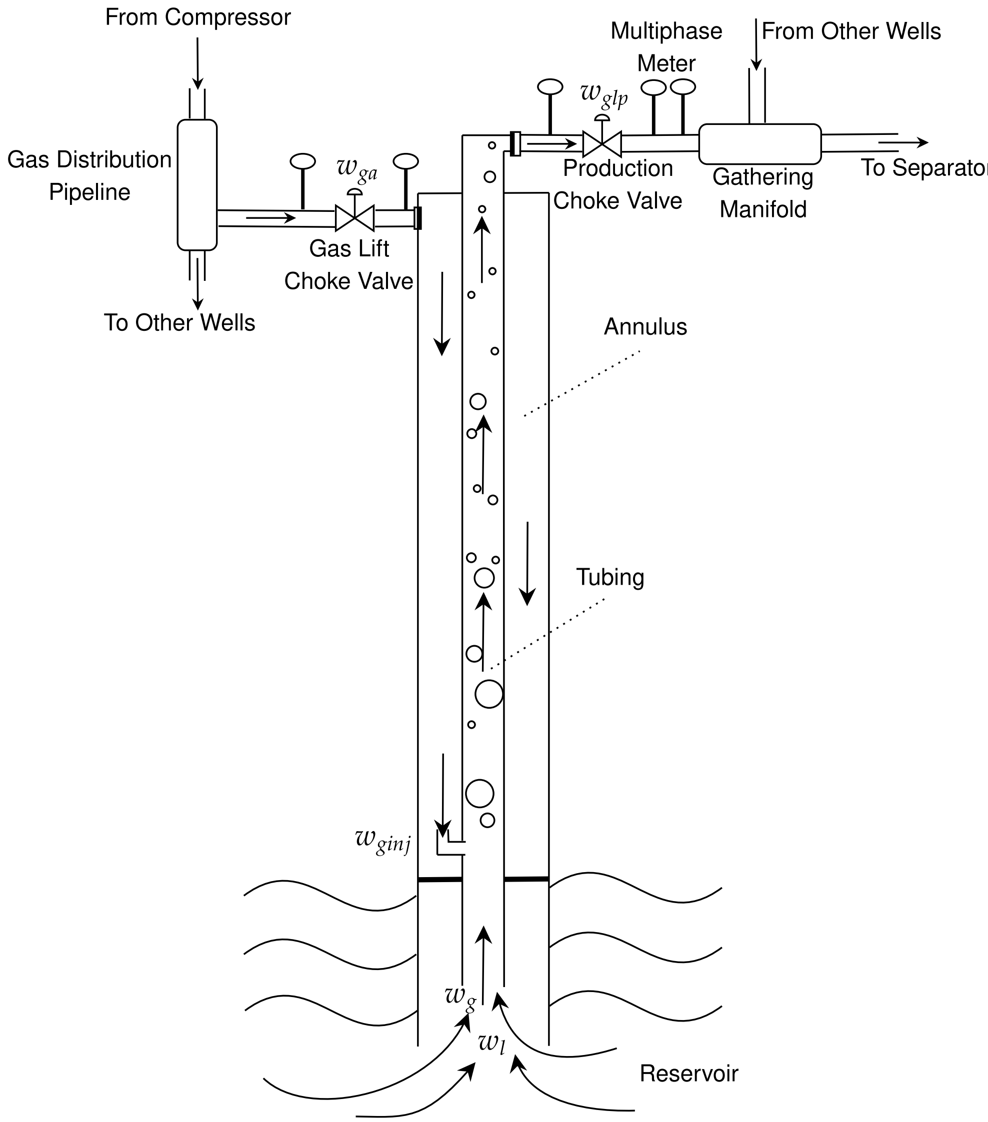

Fig. 1. Schematic diagram of a single gas lift oil well.

with two oil wells that share a gas distribution pipeline and a gathering manifold. The superscript i refers to the ith oil well. Three states are considered for each well, namely, the mass of lift gas in annulus m^i_{ga}, the mass of gas phase in the tubing above the injection point m^i_{gt}, and the mass of liquid phase (mixture of oil and water) in the tubing above the injection point m^i_{lt}. Three corresponding differential equations derived using the law of mass balance are given by:

$$\dot{m}^i_{ga} = w^i_{ga} - w^i_{ginj} \tag{1}$$

$$\dot{m}^i_{gt} = w^i_{ginj} + w^i_{gr} - w^i_{gp} \tag{2}$$

$$\dot{m}^i_{lt} = w^i_{lr} - w^i_{lp} \tag{3}$$

w^i_{ga} is the mass flow rate of the injected lift gas into each well from the gas lift choke valve (control input). w^i_{ginj} is the mass flow rate of the gas injection from the annulus into the tubing. w^i_{gp} and w^i_{lp} are the mass flow rates of the produced gas and liquid phase fluid from the production choke valve, respectively. w^i_{gr} and w^i_{lr} are the gas and liquid mass flow rates from the reservoir into the well. w^i_{glp} is the total mass flow rate of all phases from the production choke valve, and w^i_{op} is the oil compartment of the w^i_{lp}. All the flow equations are given by:

$$w^i_{ginj} = K^i Y^i_2 \sqrt{\rho^i_{ga} \max(P^i_{ainj} - P^i_{tinj}, 0)} \tag{4}$$

$$w^i_{gp} = \frac{m^i_{gt}}{m^i_{gt} + m^i_{lt}} w^i_{glp} \tag{5}$$

$$w^i_{lp} = \frac{m^i_{lt}}{m^i_{gt} + m^i_{lt}} w^i_{glp} \tag{6}$$

$$w^i_{lr} = PI^i \max(P_r - P^i_{wf}) \tag{7}$$

$$w^i_{or} = \frac{\rho_o}{\rho_w}(1 - WC^i) w^i_{lr} \tag{8}$$

$$w^i_{gr} = GOR^i w^i_{or} \tag{9}$$

$$w^i_{glp} = C_v Y^i_3 \sqrt{\rho^i_m \max(P^i_{wh} - P_s, 0)} \tag{10}$$

$$w^i_{op} = \frac{\rho_o}{\rho_w}(1 - WC^i) w^i_{lp} \tag{11}$$

P^i_a is the pressure of lift gas in the annulus downstream of the gas lift choke valve. P^i_{ainj} is the pressure upstream of the gas injection valve in the annulus. P^i_{tinj} denotes the pressure downstream the gas injection valve in the tubing. P^i_{wh} and P^i_{wf} are the wellhead and bottom hole pressure, respectively. All the pressures are given by:

$$P^i_a = \frac{zm^i_{ga}RT^i_a}{MA^i_aL^i_{a\_tl}} \tag{12}$$

$$P^i_{ainj} = P^i_a + \frac{m^i_{ga}}{A^i_aL^i_{a\_tl}}gL^i_{a\_vl} \tag{13}$$

$$P^i_{tinj} = \frac{zm^i_{gt}RT^i_t}{MV^i_G} + \frac{\rho^i_m gL^i_{t\_vl}}{2} \tag{14}$$

$$P^i_{wh} = \frac{zm^i_{gt}RT^i_t}{MV^i_G} - \frac{\rho^i_m gL^i_{t\_vl}}{2} \tag{15}$$

$$P^i_{wf} = P^i_{tinj} + \rho^i_l gL^i_{r\_vl} \tag{16}$$

ρ^i_{ga} is the average density of gas in the annulus. ρ^i_{gl} is the density of the liquid phase (mixture of the oil and water). ρ^i_m denotes the average density of the multi-phase mixture (oil, water, and gas) in the tubing above the injection point. Y^i_2 and Y^i_3 are the gas expandability factors for the gas that passes through the gas injection valve and production choke valve, respectively. V^i_G is the volume of gas present in the tubing above the gas injection point, and C_v is the production choke valve characteristics. All the densities and other algebraic variables are given by:

$$\rho^i_{ga} = \frac{M(P^i_a + P^i_{ainj})}{2zRT^i_a} \tag{17}$$

$$\rho^i_l = \rho_w WC^i + \rho_o(1 - WC^i) \tag{18}$$

$$\rho^i_m = \frac{m^i_{gt} + m^i_{lt}}{A^i_tL^i_{t\_tl}} \tag{19}$$

$$Y^i_2 = 1 - \alpha_Y \frac{P^i_{ainj} - P^i_{tinj}}{\max(P^i_{ainj}, P^{\min}_{ainj})} \tag{20}$$

$$Y^i_3 = 1 - \alpha_Y \frac{P^i_{wh} - P_s}{\max(P^i_{wh}, P^{\min}_{wh})} \tag{21}$$

$$V^i_G = A^i_tL^i_{t\_tl} - \frac{m^i_{lt}}{\rho^i_l} \tag{22}$$

It should be noted that the algebraic variables given by the Eqs. (4) to (22) can be eliminated by substitution. So the explicit set of ordinary differential equations (ODE) in compact form can be written as:

$$\dot{x} = f(x, u, \theta) \tag{23a}$$

$$y_1 = h_1(x, \theta) \tag{23b}$$

$$y_2 = h_2(x, \theta) \tag{23c}$$

$x \in \mathbb{X} \subset \mathbb{R}^6$ and $u \in \mathbb{U} \subset \mathbb{R}^2$ are the states and control inputs as shown in Eqs. (24) and (25). $y_1 \in \mathbb{Y}_1 \subset \mathbb{R}$ and $y_2 \in \mathbb{Y}_2 \subset \mathbb{R}$ in Eqs. (26) and (27) are two desired outputs denoting total produced oil and total produced fluid respectively. Finally, $\theta \in \Theta \subset \mathbb{R}^6$ in Eq. (28) is the vector of uncertain parameters of the process that includes productivity index, gas to oil ratio, and water cut of each well.

$$x = \begin{bmatrix} m^1_{ga} & m^2_{ga} & m^1_{gt} & m^2_{gt} & m^1_{lt} & m^2_{lt} \end{bmatrix}^{\mathrm{T}} \tag{24}$$

$$u = \begin{bmatrix} w^1_{ga} & w^2_{ga} \end{bmatrix}^{\mathrm{T}} \tag{25}$$

$$y_1 = \sum_{i=1}^{2} w^i_{op} \tag{26}$$

$$y_2 = \sum_{i=1}^{2} w^i_{glp} \tag{27}$$

$$\theta = \begin{bmatrix} PI^1 & PI^2 & GOR^1 & GOR^2 & WC^1 & WC^2 \end{bmatrix}^{\mathrm{T}} \tag{28}$$

2.3. Uncertainty description

According to the sensitivity analysis in [18] three uncertain parameter has been considered for each well. Productivity index PI denotes the reservoir's ability to deliver fluids to the wellbore. The gas to oil ratio GOR is defined as the mass ratio of produced gas to produced oil, and the water cut WC is defined as the volumetric flow rate of water to the total produced liquid. These uncertain parameters are upper and lower bounded and can take any value within their bounds.

$$\theta_i = \theta^{nom}_i \pm \theta^{dev}_i, \qquad i = 1, 2, \dots, 6 \tag{29}$$

For PI, GOR, and WC of each oil well, a deviation of 10%, 5%, and 15% from their nominal values are considered, respectively, based on expert knowledge. The nominal values of the uncertain parameters and all the other parameters are provided in Table 1.

3. Controller design

3.1. Classical min-max MPC

In this section, the original formulation of open-loop min–max MPC will be presented. The design procedure assumes the uncertain parameters are constant and bounded, as described in Section 2.3. Nevertheless, the controllers are also tested against time-varying parameters to show the capability of handling any parameter change within the uncertainty region. The primary objective is to find the optimal distribution of lift gas between two wells that maximizes the total oil production (output y_1 from Eq. (26)) from the field, subject to some operational constraints. Therefore the objective function includes the total oil production from the field y_1 with the negative sign to pose it as a minimization problem. Additionally, the injected lift gas u and its rate of change Δu can be incorporated into the objective function to penalize excessive lift gas utilization and fluctuations in the control signal. Hence for $k \in \{0, 1, \dots, N-1\}$ where N is the length of the prediction horizon and Q, R, and S denote proper tuning weights, the objective function is given by:

$$J(x, u, \theta) = \sum_{k=0}^{N-1} \left(-Q(y_{1,k})^2 + R\sum_{i=1}^{2} u_k(i)^2 + S\sum_{i=1}^{2} \Delta u_k(i)^2 \right) \tag{30}$$

The most important operational constraints in the problem arise from separator capacity and the total available lift gas. In particular, the total amount of produced fluid (mixture of oil, water, and gas) should be less than the separator capacity, and the total gas needed for injection should not exceed the total available lift gas. So, the optimal control problem formulation throughout the prediction horizon $\mathcal{K} = \{0, \dots, N-1\}$ is given by:

$$\min_{x,u} \; J(x, u, \theta) \tag{31a}$$

$$\text{s.t.} \quad x_{k+1} = f(x_k, u_k, \theta), \qquad k \in \mathcal{K} \tag{31b}$$

$$\sum_{i=1}^{2} u_k(i) \le W^{\max}_{gc,k}, \qquad k \in \mathcal{K} \tag{31c}$$

$$y_{2,k} \le W^{\max}_s, \qquad k \in \mathcal{K} \tag{31d}$$

$$u_{LB} \le u_k \le u_{UB}, \qquad k \in \mathcal{K} \tag{31e}$$

$$\Delta u_{LB} \le \Delta u_k \le \Delta u_{UB}, \qquad k \in \mathcal{K} \tag{31f}$$

Eq. (31b) denotes the discretized dynamic model and is imposed as state continuity constraint. The constraint on the used lift gas is denoted in (31c) where $W^{\max}_{gc,k}$ represents the total

Table 1
List of the parameters and their corresponding nominal values.

Parameter	Well 1	Well 2	Units	Comments
PI^{nom}	2.51	1.63	[kg/s bar]	Nominal productivity index
GOR^{nom}	0.08	0.07	[kg/kg]	Nominal gas to oil ratio
WC^{nom}	0.15	0.15	[m^3/m^3]	Nominal water cut
L_{a_tl}, L_{t_tl}	2758	2559	[m]	Total length above injection point
L_{a_vl}, L_{t_vl}	2271	2344	[m]	Vertical length above injection point
L_{r_vl}	114	67	[m]	Vertical length below injection point
A_t	0.0194	0.0194	[m^2]	Tubing cross section area
A_a	0.0174	0.0174	[m^2]	Annulus cross section area
K	68.43	67.82	[$\frac{\sqrt{\frac{kgm^3}{bar}}}{hr}$]	Gas injection valve constant
T_a, T_t	280	280	[K]	Annulus/tubing temperature
P_r	150	150	[bar]	Reservoir pressure
P_s	30	30	[bar]	Separator pressure
α_Y	0.66	0.066	–	Constant
C_v	8190	8190	–	Valve characteristics
ρ_o	800	800	[kg/m^3]	Density of oil
ρ_w	1000	1000	[kg/m^3]	Density of water
M	0.020	0.020	[kg/mol]	Molar mass
z	1.3	1.3	[–]	Compressibility factor

available lift gas. The constraint on the total produced fluid is enforced in (31d), where y_2 comes from Eq. (27) and $W_s^{\max}$ stands for the maximum capacity of the separator. The lower and upper bounds on the control signal and the rate of change of control inputs are also implemented in (31e) and (31f), respectively.

Due to the uncertainty in the parameters, the problem defined in (31) cannot be solved directly. However, classical open-loop min–max formulation considers the worst-case realization of the uncertainty. In other words, it finds the appropriate decision variables that minimize the maximum of objective functions over all the possible realizations of θ. Classical open-loop min–max MPC formulation is given by:

$$\min_{x,u} \quad \max_{\theta} \quad J(x,u,\theta) \tag{32a}$$

$$\text{s.t.} \quad x_{k+1} = f(x_k, u_k, \theta), \qquad k \in \mathcal{K}, \forall \theta \in \Theta \tag{32b}$$

$$\sum_{i=1}^{2} u_k(i) \le W_{gc,k}^{\max}, \qquad k \in \mathcal{K}, \forall \theta \in \Theta \tag{32c}$$

$$y_{2,k} \le W_s^{\max}, \qquad k \in \mathcal{K}, \forall \theta \in \Theta \tag{32d}$$

$$u_{LB} \le u_k \le u_{UB}, \qquad k \in \mathcal{K}, \forall \theta \in \Theta \tag{32e}$$

$$\Delta u_{LB} \le \Delta u_k \le \Delta u_{UB}, \qquad k \in \mathcal{K}, \forall \theta \in \Theta \tag{32f}$$

Solving the original problem defined in (32) is not always straightforward since the worst-case realization of the uncertainty is not trivial. However, for the application considered in the paper, it is well known that the worst-case scenario occurs when the *PI* and *GOR* of all the wells take their maximum realization and the *WC* of all wells take their minimum realization, simultaneously [17]. Therefore we can simply take the a-priori computed worst-case values of all parameters, and the optimization problem reduces to:

$$\min_{x,u} \quad J(x,u,\theta_w) \tag{33a}$$

$$\text{s.t.} \quad x_{k+1} = f(x_k, u_k, \theta_w), \qquad k \in \mathcal{K} \tag{33b}$$

$$\sum_{i=1}^{2} u_k(i) \le W_{gc,k}^{\max}, \qquad k \in \mathcal{K} \tag{33c}$$

$$y_{2,k} \le W_s^{\max}, \qquad k \in \mathcal{K} \tag{33d}$$

$$u_{LB} \le u_k \le u_{UB}, \qquad k \in \mathcal{K} \tag{33e}$$

$$\Delta u_{LB} \le \Delta u_k \le \Delta u_{UB}, \qquad k \in \mathcal{K} \tag{33f}$$

θ_w in (33) stands for the worst-case realization of uncertainty, and it is equal to the maximum values of all *PI*s and *GOR*s and minimum values for all *WC*s.

3.2. Proposed constraint modification

The proposed method in this section is a modified version of the original min–max MPC to reduce the conservativeness of the classical open-loop min–max in a computationally efficient manner. Since the proposed method does not solve the optimization problem over control policies, it does not increase the computational costs; however, it decreases the conservativeness even better than the closed-loop optimization techniques. The main idea behind this novel method relies on the fact that the output constraint is upper bounded, and the conservativeness arises from overestimating outputs in the prediction part. Therefore a simple innovative method has been developed to compensate for this overestimation by modifying the active output constraint. The most important requirement of the method is that the output (constraint) should be directly measurable, which is an admissible requirement for several chemical processes since the constraints are mostly on pressures or temperatures or flows. Therefore, the method can be generalized to be applicable to a class of systems where this requirement is fulfilled, although it has been developed based on a gas-lifted oil field as the case study.

Since the design is based on robust worst-case optimization like (33), while the active constraint will be modified using measurements, the new method can be formulated as:

$$\min_{x,u} \quad J(x,u,\theta_w) \tag{34a}$$

$$\text{s.t.} \quad x_{k+1} = f(x_k, u_k, \theta_w), \qquad k \in \mathcal{K} \tag{34b}$$

$$\sum_{i=1}^{2} u_k(i) \le W_{gc,k}^{\max}, \qquad k \in \mathcal{K} \tag{34c}$$

$$y_{2,k} \le W_s^{\max} + \delta W, \qquad k \in \mathcal{K} \tag{34d}$$

$$u_{LB} \le u_k \le u_{UB}, \qquad k \in \mathcal{K} \tag{34e}$$

$$\Delta u_{LB} \le \Delta u_k \le \Delta u_{UB}, \qquad k \in \mathcal{K} \tag{34f}$$

The correction factor δW in (34d) reduces the conservativeness by modifying the constraint. It should be emphasized that δW is not a slack variable which is calculated by the optimizer.

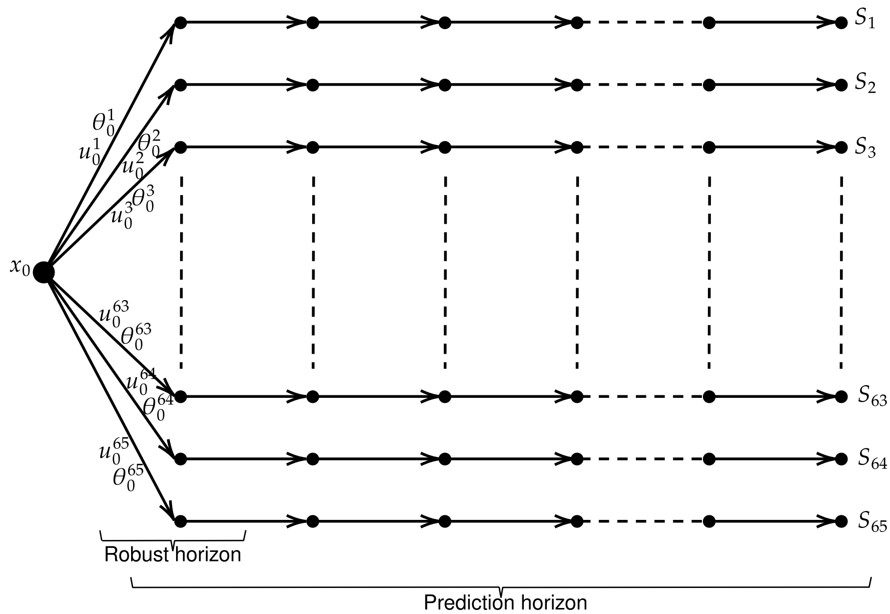

Fig. 2. Scenario tree representation of the uncertainties considered in this study with 65 scenarios and the robust horizon equals 1.

However, it represents how much mismatch exists between the forecast model in the controller and the real process, and it can be calculated by subtracting the measured output (constraint) from the predicted output (constraint) at the current time ($k = 0$) as:

$$\delta W = y_2(x_k, \theta_w) - y_{2,k}^{meas} \tag{35}$$

where $y_2(x_k, \theta_w)$ is the calculated total produced fluid based on the worst-case realization of the parameters and $y_{2,k}^{meas}$ is the measured total produced fluid at the current time $k = 0$. When the parameters of the actual process take their worst-case realization, there will be no mismatch between the prediction and measurement; therefore, the method would be equivalent to a min–max MPC applied in the worst-case situation of the process. Otherwise, the mismatch between the prediction and measurement modifies the upper bound of the constraint and decreases the conservativeness.

3.3. Multi-stage MPC

Multi-stage MPC is used in this paper as a competing alternative to demonstrate the promising features of the novel method presented in this paper. The method behind multi-stage MPC is well documented in the literature [15,27]; therefore, only a condensed explanation of what has been used in this work is presented, and the readers are referred to [28] for more details on the multi-stage MPC for gas-lifted oil network.

Considering the boundaries of six uncertain parameters, there are 2^6 combinations of uncertainty realizations (branch) that adds up to 65 with nominal values. However, since the number of scenarios grows exponentially with the number of branches and time steps, the robust horizon is chosen to be 1. Therefore branching stops after the first node, as shown in Fig. 2 and 65 distinct scenarios are considered overall.

The optimization problem should be formulated over all the discrete scenarios of the scenario set $\mathcal{S} = \{1, \dots, S\}$ throughout the prediction horizon $\mathcal{K} = \{0, \dots, N-1\}$. Therefore, for $\forall j \in \mathcal{S}$, and $\forall k \in \mathcal{K}$ and the tuning weights ω_j, the multi-stage MPC is formulated as:

$$\min_{\mathrm{x,u}} \quad \sum_{j=1}^{S} \omega_j J_j \tag{36a}$$

$$\text{s.t.} \quad x_{k+1}^j = f\left(x_k^{p(j)}, u_k^j, \theta_k^{r(j)}\right), \qquad j \in \mathcal{S}, k \in \mathcal{K} \tag{36b}$$

$$\sum_{i=1}^{2} u_k^j(i) \le W_{gc,k}^{\max}, \qquad j \in \mathcal{S}, k \in \mathcal{K} \tag{36c}$$

$$y_{2,k}^j \le W_s^{\max}, \qquad j \in \mathcal{S}, k \in \mathcal{K} \tag{36d}$$

$$u_{LB} \le u_k^j \le u_{UB}, \qquad j \in \mathcal{S}, k \in \mathcal{K} \tag{36e}$$

$$\Delta u_{LB} \le \Delta u_k^j \le \Delta u_{UB}, \qquad j \in \mathcal{S}, k \in \mathcal{K} \tag{36f}$$

$$u_k^j = u_k^l \quad \text{if} \quad x_k^{p(j)} = x_k^{p(l)}, \qquad \forall j \& l \in \mathcal{S}, k \in \mathcal{K} \tag{36g}$$

J_j in (36a) denotes the objective function for scenario j. Tuning weights ω_j represent the relative likelihood of occurring each scenario. The constraint (36b) denotes the equation of the states. It means that the states at time $t = k + 1$ in scenario j are a function of their parental state $x_k^{p(j)}$ and the corresponding control u_k^j and uncertainty realization $\theta_k^{r(j)}$. The non-anticipativity constraints introduced in (36g) reflects the fact that at each time instance k, controls u_k^j and x_k^l from scenarios j and l with the same parental state $x_k^{p(j)} = x_k^{p(l)}$ have to be the equal. In our case, branching happens only once; therefore, $u_0^1 = u_0^2 = \cdots = u_0^{65}$ is the only set of non-anticipativity constraints because all these controls are branched from the same parental node x_0 as shown in Fig. 2. According to the receding horizon strategy, this first control action is the one that will be applied to the target system, and the non-anticipativity constraint guarantees that this value is unique.

4. Results and discussion

4.1. Simulation setup

The proposed novel method of this paper, classical min–max MPC, and multi-stage MPC method, have been applied to a gas lifted field with two oil wells. All the parameters of the wells are presented in Table 1. For all three methods, the tuning wights Q, R, and S that reflect the importance of each term in the objective function (30) are chosen to be 1, 0.5, and 50, respectively. All the sixty-five weights ω_j for sixty-five scenarios in the multi-stage method (36) are considered to be equally one because all the scenarios are equally likely to occur.

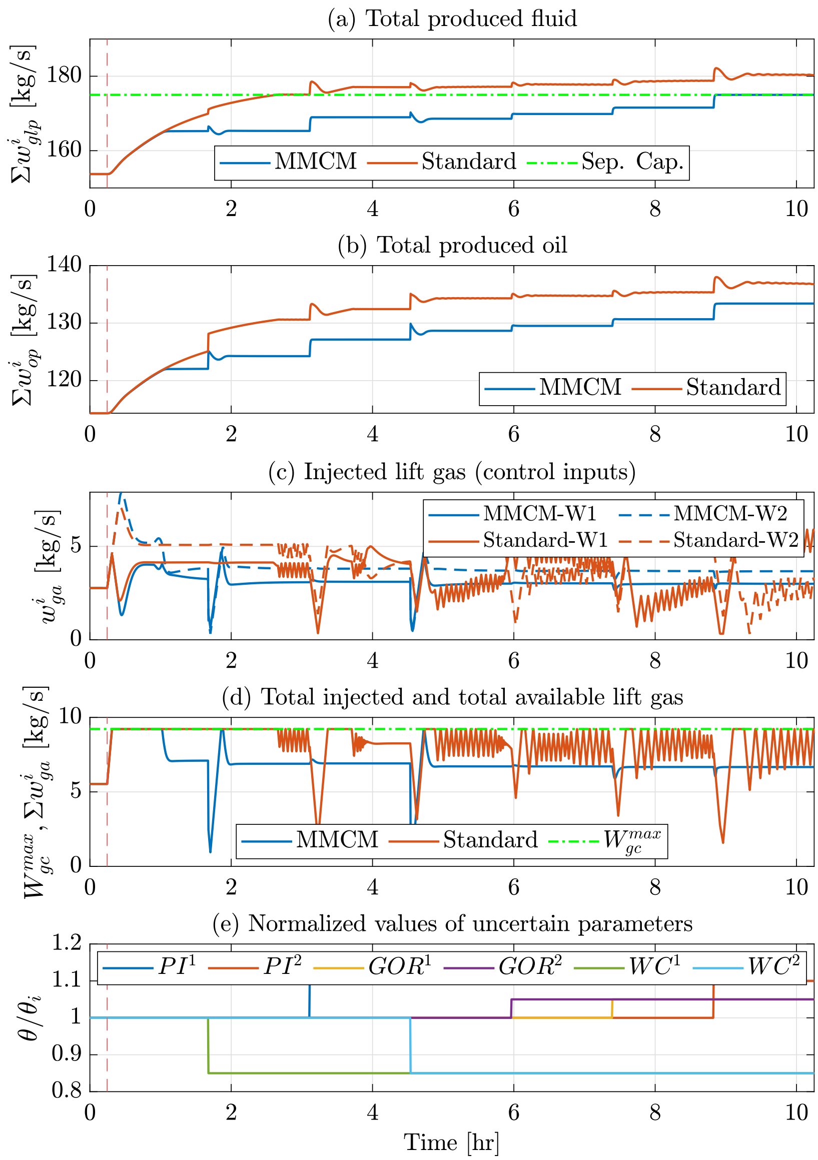

Fig. 3. Total produced fluid, total produced oil, injected lift gas, and normalized values of the uncertain parameters when the proposed method i.e. min–max with constraint modification (MMCM) and standard MPC based on the nominal model, are applied to the plant with varying parameters.

Classical min–max MPC is implemented by solving the optimization problem in (33) in a receding horizon fashion. The proposed method and multi-stage MPC are implemented respectively by solving optimization problems in (34) and (36). In all simulations, the total available lift gas $W_{gc}^{\max}$ varies during simulation to indicate the change in operational condition, while the constraint on the maximum capacity of the separator is assumed to be constant with $W_s^{\max} = 175$ [kg/s]. The lower and upper bounds of the gas injection flow rates (control signal) are 0.323 and 11.66 [kg/s]. The rate of control change is limited between ±0.15 [kg/s]. A sampling time of 20 s and a prediction horizon with 25 sampling times (≈8.3 min) is used for all methods.

The dynamic optimization problem is discretized using the direct multiple shooting method in CasADi v.3.5.5, an open-source tool for nonlinear optimization and algorithmic differentiation [29]. The simulations were implemented in MATLAB R2022b, using a 1.8 GHz laptop with 16 GB memory. The IPOPT v.3.14.1 solver has been used to solve the problem [30].

4.2. Simulation results

The applicability of the proposed approach is demonstrated and compared to competing approaches such as classical min–max and multi-stage MPC in terms of conservativeness and execution time.

The first simulation case has been conducted to show the robustness of the proposed method and the shortcoming of standard MPC in compensating for the parametric uncertainty. To do so, the proposed method and a standard MPC based on nominal values of parameters have been applied to the process with varying parameters. The result is plotted in Fig. 3. The first subplot (a) demonstrates the total produced fluid from the field and its upper bound. The second subplot (b) shows the total produced oil. Control inputs (injected lift gas) are shown in the third subplot (c). The fourth subplot (d) depicts the total lift gas used and its upper bound. And the last subplot (e) shows the uncertain parameters, which are normalized with their nominal values.

In all the subplots, a vertical dashed red line shows the time when the real-time optimizer is activated. It can be seen that the uncertain parameters take their nominal values at the beginning, and then they change to their worst-case realization in random order.

Subplot (a) in Fig. 3 demonstrates that standard MPC based on the nominal model is not able to fulfill the constraint and the total produced fluid exceeds its upper bound. However, this constraint is robustly respected by our method all the time, even in the presence of sudden changes in parameters. It should be noted that the constraint on the output (separator capacity) becomes active only at the end of the simulation time when all the parameters have taken their worst-case realization. However, at other times when at least some of the parameters are not taking their worst-case realization, the controller prefers not to use all the available lift gas even though the constraint on the separator part is not active and even when there is a possibility of higher production. This price that has to be paid to guarantee robust satisfaction of the constraint is typically known as conservativeness.

In the subsequent three simulation cases, it has been shown that the proposed method is superior to the traditional min–max and multi-stage MPC in terms of conservativeness. In other words, the proposed method has the same performance in the worst-case situation, while it is considerably less conservative at other times.

In case (I), all three methods, namely traditional min–max MPC (MM), multi-stage MPC (MS), and the proposed min–max with constraint modification method (MMCM), are applied to the plant with the worst-case realization of uncertain parameters. Total produced fluid and the corresponding constraint, the total produced oil, injected lift gas to each well, and the total injected and available lift gas are plotted in Fig. 4 for all three methods. The first and fourth subplots show that the production will be increased by utilizing all the available gas in the beginning until the constraint on the separator side becomes active. Then the controller decreases the amount of injected lift gas since it has been penalized in the objective function. After almost four hours, all three controllers decreased the total used lift gas even further to respect the constraint on the amount of available gas. The simulation shows that all three competing methods are able to cope with the worst-case realization of the uncertainty; therefore, they are robust.

In case (II), all three methods are applied to the plant with the nominal realization of uncertain parameters to investigate the conservativeness of the methods. The results are plotted in Fig. 5 for comparison. As expected, all three methods are conservative to some extent in the sense that they do not utilize all the available lift gas to increase production. However, the maximum production rate is not touched yet. Nevertheless, their level of conservativeness is not the same. The traditional min–max MPC as an open-loop optimization method is the most conservative one. Multi-stage MPC as a closed-loop optimization method increases the oil production by around 0.1%, which is justifiable considering six uncertain parameters in the process. The proposed novel method is the least conservative method among these three competitors. It increases the oil production by 1.46% with respect to standard min–max MPC, while it does not increase the complexity of the problem compared with open-loop min–max MPC.

The third and last case is case (III), where the three methods are applied to the process in which the *PI* and *GOR* of all the wells take their minimum realization and the *WC* of all wells take their maximum realization, simultaneously. The special fact about the considered case is that all the parameter realizations are exactly the opposite with respect to the worst-case realization. This means if the constraint was lower bounded as well (which

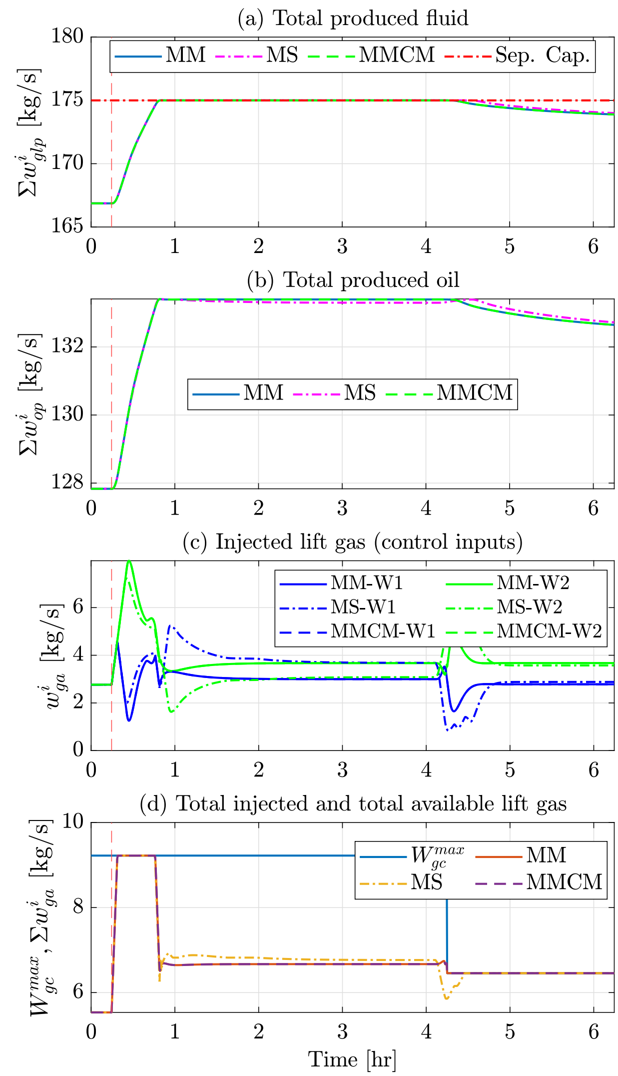

Fig. 4. Case (I): Applying standard min–max MPC (MM), multi-stage MPC (MS), and the proposed method i.e. min–max with constraint modification (MMCM) to the plant with the worst-case realization of uncertain parameters.

is not the case in our process), this combination of uncertain parameters would be the worst-case realization corresponding to the lower bound of the constraint. In other words, this case is the farthest distance to the worst-case realization of uncertainties; therefore, it is preferred to call it the safe case. All the other realizations of parameters within the uncertainty range put the process between these two extremes. The simulation result for case (III) is presented in Fig. 6. It can be seen that the improvement made by the proposed novel method is more significant. The method increases the oil production by 3.44% with respect to standard min–max MPC, while multi-stage MPC increases the production only by 0.2%.

The other advantage of the method proposed is low computational costs. In contrast to multi-stage MPC, which reduces the conservativeness by solving the optimization problem over control policies, the proposed novel method is still an open-loop optimization method. Therefore, its computational cost remains at the same level as open-loop min–max MPC. This has been validated by comparing the execution time of the methods for the three discussed cases. The execution time for each iteration

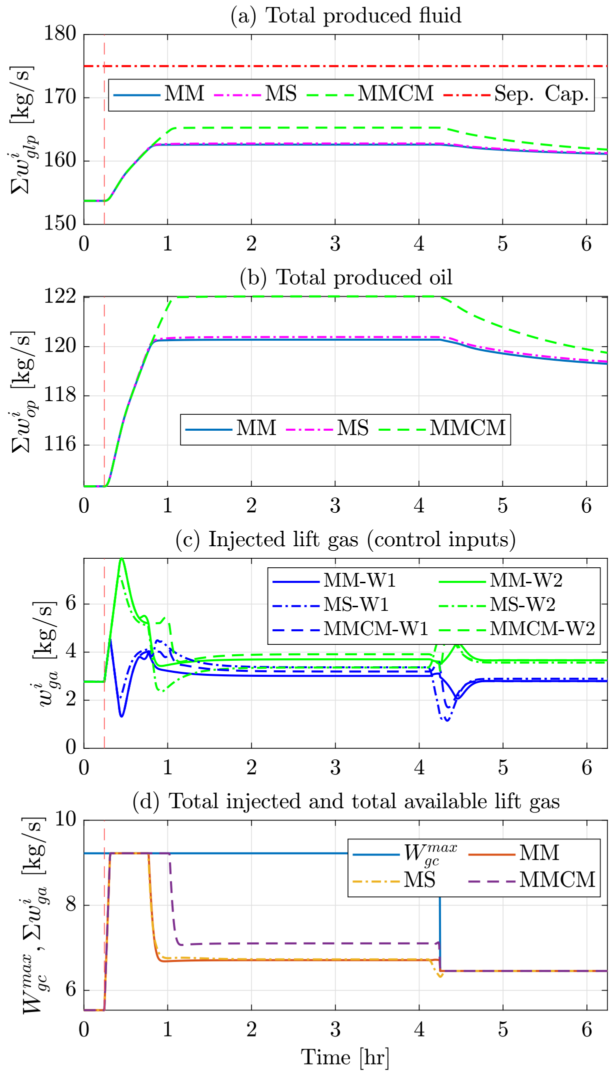

Fig. 5. Case (II): Applying standard min–max MPC (MM), multi-stage MPC (MS), and the proposed method min–max with constraint modification (MMCM) to the plant with the nominal realization of uncertain parameters.

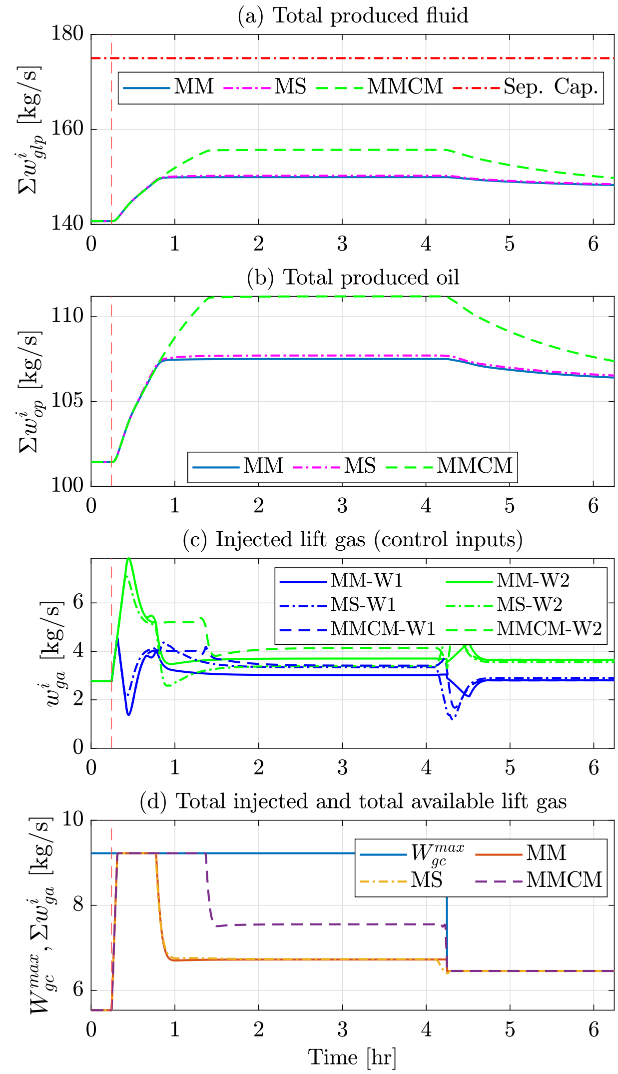

Fig. 6. Case (III): Applying standard min–max MPC (MM), multi-stage MPC (MS), and the proposed method min–max with constraint modification (MMCM) to the plant with the safe realization of uncertain parameters.

Table 2
The mean execution time of iterations for three cases (I), (II), and (II).

	Min–max	Multi-stage	Proposed novel method
Case (I)	0.030	3.01	0.029
Case (II)	0.042	2.85	0.038
Case (III)	0.037	3.38	0.029

has been plotted in Fig. 7. It shows that the computational costs for min–max MPC and the proposed novel method are more or less the same; however, the multi-stage MPC is computationally expensive. The average execution time for each method is also presented in Table 2 and reflects the same fact. Table 2 apparently shows that the proposed novel method is approximately 100 times faster than multi-stage MPC, yet it is less conservative.

5. Conclusion

All the robust methods are inherently conservative when the uncertainties take other values rather than their worst-case realization. Therefore, an efficient, robust nonlinear model predictive framework, particularly for real-time production maximization of the gas lifted well network under the presence of parametric uncertainty was presented in this paper to mitigate the problem of conservativeness. The conservativeness in our problem can be interpreted as the unutilized resources to increase production. So, the performance of the proposed framework is evaluated in this regard through several simulation cases.

The proposed optimal control framework of this paper consists of traditional open-loop min–max MPC with constraint modification. In particular, the error between measured and predicted output is used as a correction factor to modify the upper bound of the constraint. The design is based on the worst-case realization of the uncertainty; therefore, when the parameters take their worst-case realization, the error between measurement and prediction is zero, which means that the formulation reduces to a standard min–max MPC. Otherwise, the error term modifies the constraint that, consequently, leads to a reduction in conservativeness.

Several simulation cases have been conducted to demonstrate the promising advantages of the proposed novel method over open-loop min–max MPC and multi-stage MPC. All the competing methods are applied to a gas-lifted oil field with two oil wells in three simulation cases. It has been shown that when the uncertain parameters of the process take their worst-case

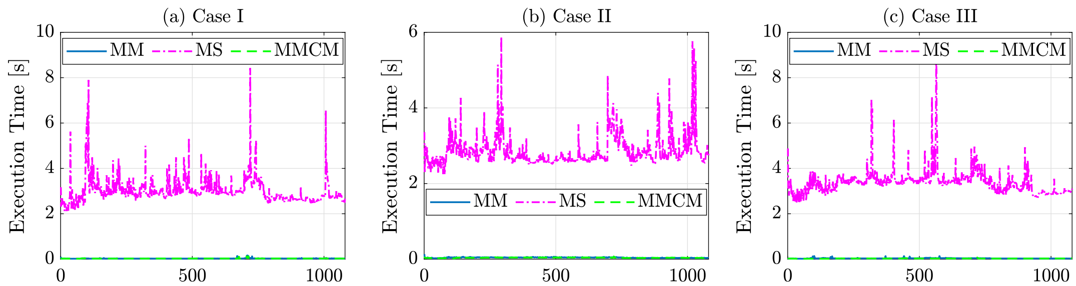

Fig. 7. Execution time for three cases. Case(I): worst-case realization of uncertainty, Case(II): nominal realization of uncertainty, Case(III): safe case realization of uncertainty.

realizations, all three methods are able to satisfy the constraints, i.e., all methods are robust within the uncertainty region. However, when the uncertain parameters take other realizations, the proposed method is significantly less conservative than min–max and multistage methods; therefore, the proposed method is superior to both min–max and multistage methods because it preserves the robust performance and reduces the conservativeness significantly. The method's superiority in terms of complexity is also investigated by comparing the execution times. It has been shown that the complexity of the method is at the level of min–max MPC, and it is considerably more straightforward and more efficient than multistage MPC.

Despite the significant benefits of the developed method, there are some limitations that give rise to further improvement in future works. First, some a priori knowledge about the process has been used to simplify the min–max MPC; however, the worst-case realization of the parameter might not be known a priori for other processes. Although this is a valid argument, it should be noted that the method at least does not impose any further complexity on the original min–max formulation. While on the contrary, multi-stage MPC introduces too much complexity with less reward in terms of conservativeness.

The next and most critical limitation which should be addressed in future works is that the method needs the active constraints to be directly measurable. The future direction in this regard is to generalize this work for the cases where constraints are not directly measurable by using other outputs/measurements to modify the constraints.

In fact, this method falls somewhere between the adaptive and robust methods. This is because, in the adaptive approach, the measurements are used to estimate the uncertain parameters, and then the estimated parameters will be used in a certainty equivalence deterministic MPC. However, the measurements in our method are used to directly modify the active constraints.

CRediT authorship contribution statement

Nima Janatian: Conceptualization, Methodology, Software, Writing – original draft. **Roshan Sharma:** Conceptualization, Supervision, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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References

[1] H. Witsenhausen, A minimax control problem for sampled linear systems, IEEE Trans. Automat. Control 13 (1) (1968) 5–21.
[2] P.J. Campo, M. Morari, Robust model predictive control, in: 1987 American Control Conference, IEEE, 1987, pp. 1021–1026.
[3] J.H. Lee, Z. Yu, Worst-case formulations of model predictive control for systems with bounded parameters, Automatica 33 (5) (1997) 763–781.
[4] P.O. Scokaert, D.Q. Mayne, Min–max feedback model predictive control for constrained linear systems, IEEE Trans. Automat. Control 43 (8) (1998) 1136–1142.
[5] A. Bemporad, Reducing conservativeness in predictive control of constrained systems with disturbances, in: Proceedings of the 37th IEEE Conference on Decision and Control (Cat. No. 98CH36171), IEEE, 1998, pp. 1384–1389.
[6] J. Lofberg, Approximations of closed-loop minimax MPC, in: 42nd IEEE International Conference on Decision and Control (IEEE Cat. No. 03CH37475), IEEE, 2003, pp. 1438–1442.
[7] P.J. Goulart, E.C. Kerrigan, J.M. Maciejowski, Optimization over state feedback policies for robust control with constraints, Automatica 42 (4) (2006) 523–533.
[8] J.A. Paulson, A. Mesbah, Approximate closed-loop robust model predictive control with guaranteed stability and constraint satisfaction, IEEE Control Syst. Lett. 4 (3) (2020) 719–724.
[9] D.Q. Mayne, M.M. Seron, S. Raković, Robust model predictive control of constrained linear systems with bounded disturbances, Automatica 41 (2) (2005) 219–224.
[10] D.Q. Mayne, E.C. Kerrigan, E. Van Wyk, P. Falugi, Tube-based robust nonlinear model predictive control, Internat. J. Robust Nonlinear Control 21 (11) (2011) 1341–1353.
[11] P. Falugi, D.Q. Mayne, Getting robustness against unstructured uncertainty: a tube-based MPC approach, IEEE Trans. Automat. Control 59 (5) (2013) 1290–1295.
[12] S. Raković, W.S. Levine, B. Açikmese, Elastic tube model predictive control, in: 2016 American Control Conference, ACC, IEEE, 2016, pp. 3594–3599.
[13] X. Lu, M. Cannon, Robust adaptive tube model predictive control, in: 2019 American Control Conference, ACC, IEEE, 2019, pp. 3695–3701.
[14] J. Köhler, R. Soloperto, M.A. Müller, F. Allgöwer, A computationally efficient robust model predictive control framework for uncertain nonlinear systems, IEEE Trans. Automat. Control 66 (2) (2020) 794–801.
[15] S. Lucia, T. Finkler, S. Engell, Multi-stage nonlinear model predictive control applied to a semi-batch polymerization reactor under uncertainty, J. Process Control 23 (9) (2013) 1306–1319.
[16] D. Krishnamoorthy, B. Foss, S. Skogestad, Steady-state real-time optimization using transient measurements, Comput. Chem. Eng. 115 (2018) 34–45.

[17] D. Krishnamoorthy, B. Foss, S. Skogestad, Real-time optimization under uncertainty applied to a gas lifted well network, Processes 4 (4) (2016) 52.

[18] N. Janatian, K. Jayamanne, R. Sharma, Model based control and analysis of gas lifted oil field for optimal operation, in: The First SIMS EUROSIM Conference on Modelling and Simulation, SIMS EUROSIM 2021, and 62nd International Conference of Scandinavian Simulation Society, SIMS 2021, Linköping Electronic Conference Proceedings, 2022, pp. 241–246.

[19] C.E. Garcia, A.M. Morshedi, Quadratic programming solution of dynamic matrix control (QDMC), Chem. Eng. Commun. 46 (1–3) (1986) 73–87.

[20] D.A. Copp, T.A. Nguyen, R.H. Byrne, Adaptive model predictive control for real-time dispatch of energy storage systems, in: 2019 American Control Conference, ACC, IEEE, 2019, pp. 3611–3616.

[21] P.A. Delou, J.P.A. de Azevedo, D. Krishnamoorthy, M.B. de Souza Jr., A.R. Secchi, Model predictive control with adaptive strategy applied to an electric submersible pump in a subsea environment, IFAC-PapersOnLine 52 (1) (2019) 784–789.

[22] G.O. Eikrem, O.M. Aamo, H. Siahaan, B. Foss, Anti-slug control of gas-lift wells-experimental results, IFAC Proc. Vol. 37 (13) (2004) 799–804.

[23] O.M. Aamo, G.O. Eikrem, H.B. Siahaan, B. Foss, Observer design for multiphase flow in vertical pipes with gas-lift—theory and experiments, J. Process Control 15 (3) (2005) 247–257.

[24] L. Sinegre, N. Petit, P. Menegatti, Predicting instabilities in gas-lifted wells simulation, in: 2006 American Control Conference, ACC, IEEE, 2006, pp. 5530–5537.

[25] R. Sharma, K. Fjalestad, B. Glemmestad, Modeling and control of gas lifted oil field with five oil wells, in: 52nd International Conference of Scandinavian Simulation Society, SIMS, 2011, pp. 47–59.

[26] E. Jahanshahi, S. Skogestad, Simplified dynamic models for control of riser slugging in offshore oil production, Oil Gas Facil. 3 (6) (2014) 80–88.

[27] R. Marti, S. Lucia, D. Sarabia, R. Paulen, S. Engell, C. de Prada, Improving scenario decomposition algorithms for robust nonlinear model predictive control, Comput. Chem. Eng. 79 (2015) 30–45.

[28] N. Janatian, R. Sharma, Multi-stage scenario-based MPC for short term oil production optimization under the presence of uncertainty, J. Process Control 118 (2022) 95–105.

[29] J.A. Andersson, J. Gillis, G. Horn, J.B. Rawlings, M. Diehl, CasADi – A software framework for nonlinear optimization and optimal control, Math. Program. Comput. 11 (1) (2019) 1–36, http://dx.doi.org/10.1007/s12532-018-0139-4.

[30] A. Wächter, L.T. Biegler, On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming, Math. Program. 106 (1) (2006) 25–57, http://dx.doi.org/10.1007/s10107-004-0559-y.