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Modelling and Control of a Typical High Head Hydropower Plant in Norway

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***Abstract*—This paper describes an effective mathematical model of a hydropower plant and how a decentralized control strategy for frequency and terminal voltage can be simulated. Several dynamic equations are presented for each element of a typical high head hydropower with ODEs (ordinary differential equations), except penstock model is carried out with hydraulic PDEs (partial differential equations), while a fourth order of synchronous generator with exciter model is presented as modelling of generated electrical power and terminal voltage. This paper merged these two models and eventually results in a MIMO system. The frequency and terminal voltage are chosen as control objectives according to the quality of power. For the control strategy, A PI controller coupled with droop control is implemented for the frequency, and a decentralized controller with stabilizer is applied to terminal voltage control. The interactions of these two controllers has been simulated and analyzed. Reasonable and acceptable simulation results are presented and discussed.**

Keywords- Hydropower; Mechanical power; Electrical power; Dynamic modelling; Hydraulic PDEs ; Decentralized control; Droop control.

I. INTRODUCTION

Hydropower is a renewable and safe energy compared to thermal and nuclear power. In Norway, hydropower covers close to 100% of the electricity production, so it is of interest to utilize this energy as efficiency as possible. With this purpose, it is important to develop models for hydropower system in a way suitable for developing modern control strategy.

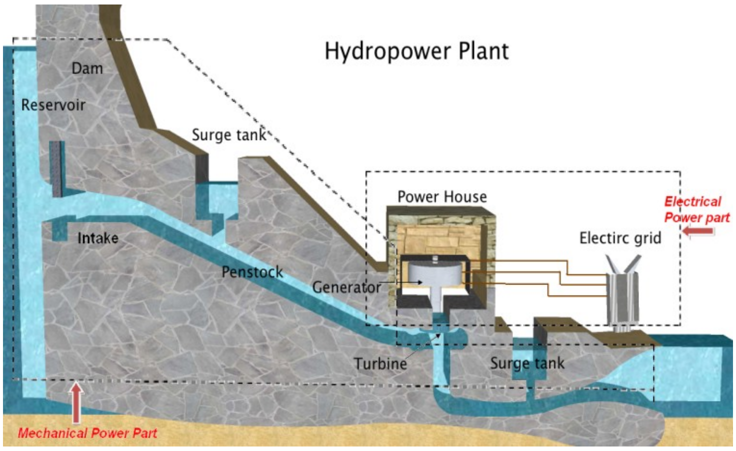

Figure1. Overview of a typical hydropower plant.

The plant in Figure 1 is a typical high head hydropower plant in Norway and the modelling and control object of this paper. A dam is built after the water reservoir to accumulate water. At the intake, there always is a gate to control water flows from the reservoir to tunnel, which is assumed opened with a constant value and ignored in modeling. The function of surge tank is briefly to reduce water hammer pressure variations and keep the mass oscillations, caused by load changes, within acceptable limits and decrease the oscillations to stable operation as soon as possible [1]. There is an upstream surge tank and a downstream surge tank included in this paper. A tunnel after dam and a penstock after upstream surge tank guide the water flow to the hydraulic turbine. Different tunnels, water conduits and penstocks are constructed for different purposes. For example, penstocks are in concrete plug in underground, while penstocks above ground are mounted in concrete blocks where they may have thermal expansion. In this paper, the only difference for penstock, except its length and radius, considered is the friction no matter what type of penstock it is. Hydraulic turbine is the mechanical part that transfers water kinetic power to mechanical power. There are two main kinds of hydraulic turbine in Norway: Francis turbine and Pelton turbine. For the Francis turbine, the water flows into the runner of the turbine through a guide vane with adjustable opening to control the rotation speed of the runner. For Pelton turbine, the water flows into runner bucket as a jet from a nozzle. The rotation speed is controlled by needle opening. The system from reservoir to turbine supplies mechanical power. Modelling of it is called mechanical power modelling in this paper. Furthermore, there are several synchronous generators in the power house to transfer mechanical power to electrical power to the grid for peoples' everyday use. Modelling of them is called electrical power modelling. It is assumed the electrical part is a single machine connected to infinite bus (SMIB) model.

II. MODELLING OF MECHANICAL POWER

One purpose of this paper is to develop an effective mathematical model of a hydropower plant. The modeling process of mechanical power is shown in Figure 2, which is achieved by several ordinary differential equations and two partial differential equations of penstock. There are two reservoirs and two surge tanks, which are located in upstream and downstream respectively, a hydraulic turbine, several tunnels, water conduits and penstocks connecting the other elements. The mechanical power is the output from this system, and the gate opening is the manipulated variable. The gate here is an effective opening of guide vane of Francis turbine or nozzles of Pelton turbine. It is assumed that the water head of the reservoirs are constant.

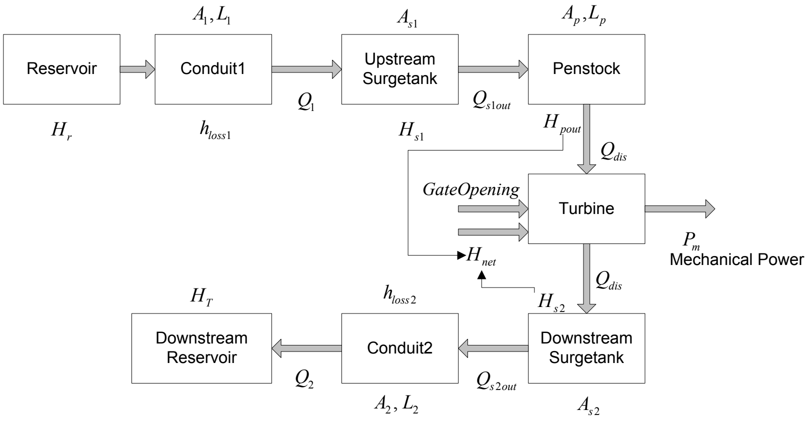

Figure 2. Flowchart for modeling mechanical power of a hydropower plant.

In this paper, tunnels and water conduits are modeling with same ODEs, but with different friction. The friction term of fluid expressed as a head loss which is derived from Darcy–Weisbach equation with considering the direction of the flow can be inversed, and then it is described as:

$$h_{loss} = \cdot \frac{}{D} \cdot \frac{\bar{v}\cdot}{2g} = \cdot \frac{}{D} \cdot \frac{}{2g\cdot} \tag{1}$$

Where:

A	Cross sectional area of pipe, m^2	L	Length of the pipe, m
D	Internal diameter of pipe, m	$\bar{Q}$	Average volume flow, m^3/s
f_r	Friction factor, *dimensionless*	$\bar{v}$	Average fluid flow, m/s
g	Gravity acceleration, m^2/s		

A. Reservoir

Upstream reservoir is the water resource for the hydropower. After producing electrical power, the water is gathered in downstream reservoir or directly flow into river. It is assumed here the water levels of reservoirs are constant. In other words, Hr and HT in Figure 2 are constant.

B. Tunnels and Conduits

Applying Newton's second law

$$F = \cdot = \cdot \frac{}{dt} \tag{2}$$

$$\Delta \cdot = \cdot \frac{}{dt} \tag{3}$$

The Δp is the pressure difference between inlet and outlet.

Take the conduit1 as example, its inlet pressure is from the reservoir, and its outlet pressure is roughly equal to pressure from surge tank. Considering the head loss along the penstock, the Equation (3) can be developed as:

$$m \cdot \frac{d}{dt} = \cdot \cdot - - \cdot \qquad \text{With } m = \cdot \cdot, \; v = \frac{}{A_1} \tag{4}$$

Then the dynamic ODE for conduit1 now can be stated as:

$$\frac{dQ_1}{dt} = \frac{}{L_1} - - \tag{5}$$

Correspondingly, the dynamic equation for the conduit2, which joins the downstream surge tank to the tail reservoir, is:

$$\frac{dQ_2}{dt} = \frac{}{L_2} - - \tag{6}$$

Where:

$A_{1,2}$	Cross sectional area of conduit 1,2, m^2	H_{s2}	Water head of downstream surge tank, *m*
$h_{loss1,2}$	Head loss along conduit 1, 2, *m*	H_T	Water head of downstream reservoir, *m*
H_r	Water head of upstream reservoir, *m*	$L_{1,2}$	Length of the conduit1, 2, *m*
H_{s1}	Water head of upstream surge tank, *m*	$Q_{1,2}$	Average volume flow along conduit1, 2 *m3/s*

C. Penstock

The water heads and volume flow can be analyzed and calculated with hydraulic PDEs for pipes which is consisted of a continuity equation and a motion equation [2]. It is only applied and simulated for the penstock connecting turbine and surge tank during modelling hydraulic system.

Continuity equation:

$$\frac{a^2\partial}{gA\partial} \perp \frac{}{\partial} \perp \frac{}{\partial} \perp = \tag{7}$$

Motion equation:

$$g\frac{\partial}{\partial} \perp \frac{}{\partial} \perp \frac{}{\partial} \perp = \tag{8}$$

Where:

a	penstock wave velocity, *m;*	θ	penstock slope, *rad*

D. Surgetank

The surge tanks are open volume in this paper which means there is no air compressed when water level increases inside it. The surge tank equations are derived from the continuity equation of flow at the two junctions, and where the hydraulic losses at orifices of surge tank are neglected [3].

For upstream surge tank:

$$A_{s1} \cdot \frac{}{dt} = - \tag{9}$$

For downstream surge tank:

$$A_{s2} \cdot \frac{}{dt} = - \tag{10}$$

Where:

A_{S1}	Cross sectional area of upstream surge tank, m^2	Q_{s2out}	Outlet flow of upstream surge tank, m^3/s
A_{s2}	Cross sectional area of downstream surge tank, m^2	Q_{s2in}	Inlet flow of downstream surge tank, m^3/s
Q_{s1in}	Inlet flow of upstream surge tank, m^3/s	Q_{s2out}	Outlet flow of downstream surge tank, m^3/s

E. Hydraulic Turbine

The general mechanical power from water, no matter which kind of the turbine it is, can be stated as:

$$P_m = \cdot \cdot \cdot \cdot \tag{11}$$

The net head is identical to the difference of output water head of penstock and the head of downstream surge and subtract the head loss between them:

$$H_{net} = - - \tag{12}$$

The discharged flow into the turbine is related the type of turbine. It is all generally applied as an effective gate opening *OP* [4]. Then, the discharged flow is formulated as

$$Q_{dis} = \cdot \quad \cdot \tag{13}$$

The flow before the turbine and after turbine is assumed identical. The gate opening OP is simulated as a linear function depends on time, every second the gate can move 1% of full opening.

$$\Delta \quad = \cdot \Delta \tag{14}$$

Where:

H_{net}	Net head to Turbine, *m*	P_m	Mechanical power, *MW*
H_{pout}	Outlet head of penstock, *m*	Q_{dis}	Discharge flow to Turbine, m^3/s
k	Gate constant, *dimensionless*	η	Turbine efficiency
OP	Effective gate Opening, *%*		

F. Model testing

A gate close situation was simulated. At time 100sec, the gate was closed from 50% to 35%, and sequentially, the mechanical power was reduced shown in Figure 3. This is because discharge flow reduced due to a smaller passage, but in opposite, the head or pressure in the penstock was raised up shown in Figure 6. This pressure should not be too high that it may damage the turbine or penstock. Therefore, the gate should be controlled very carefully.

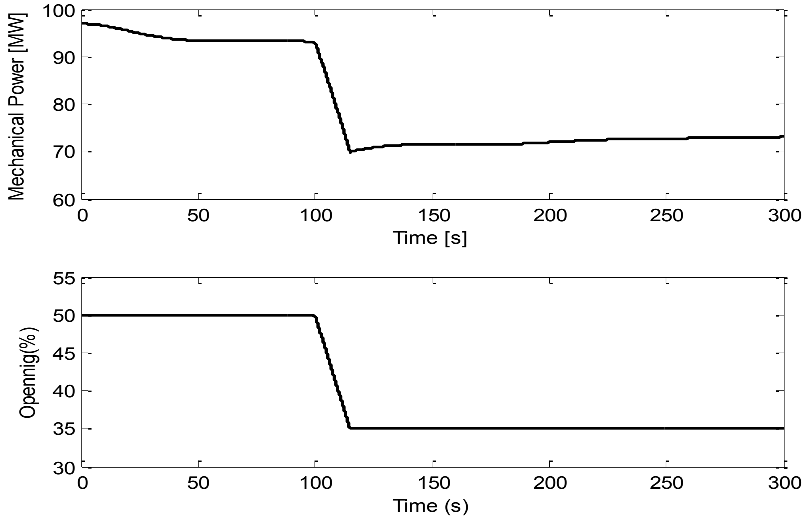

Figure 3. Simulated mechanical power when gate is closed.

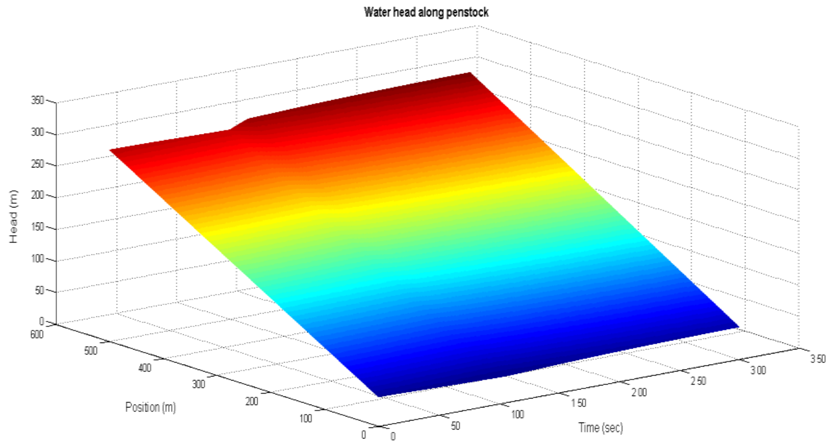

Figure 4. 3D plot of head along penstock (x axis is time, y axis is position in penstock, z axis is water head).

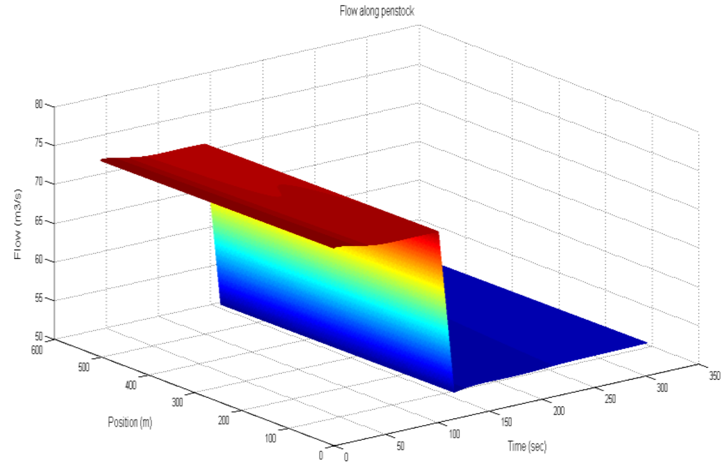

Figure 5. 3D plot of flow along penstock (x axis is time, y axis is position in penstock, z axis is flow)

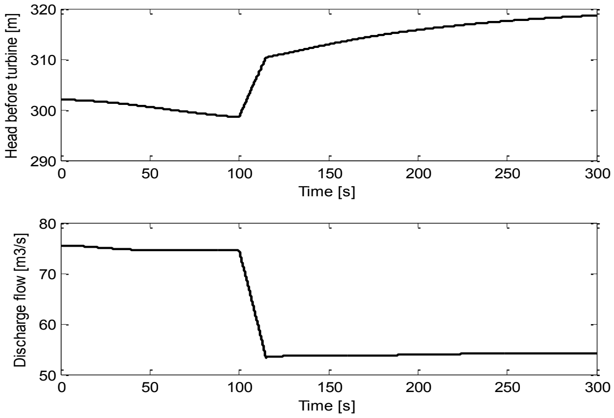

Figure 6. Head and flow at the end of penstock, before turbine.

I. MODELING OF ELECTRICAL POWER

In this paper, a practical synchronous generator model is presented with simplification of Park transformation [5]. But it is enough to analyze the electrical transient. A fourth order model [6] with ODEs is as below:

Electrical equations:

$$T'_{d0}\cdot\frac{dE'}{dt} = \; - \; \cdot \; - \; + \tag{15}$$

$$T'_{q0}\cdot\frac{dE'}{dt} = \; - \; \cdot \; - \tag{16}$$

Terminal equations:

$$U_{td} = \; - \; \cdot \; - \; \cdot \tag{17}$$

$$U_{tq} = \; - \; \cdot \; + \; \cdot \tag{18}$$

$$P_e = \; \cdot \; + \; \cdot \; + \; - \; \cdot \; \cdot \tag{19}$$

$$U_t = \sqrt{\; + \;} \tag{20}$$

Rotor motion equations:

$$\frac{d\delta}{dt} = \; \cdot \; - \tag{21}$$

$$M \cdot \frac{}{dt} = \quad - \quad - \quad \cdot \frac{}{dt} \tag{22}$$

With δ and consider the voltage from infinite bus, the terminal voltages can also be stated as [7]:

$$U_{td} = - \quad \cdot \quad + \quad \cdot \quad + \quad \cdot \tag{23}$$

$$U_{tq} = \quad \cdot \quad + \quad \cdot \quad - \quad \cdot \tag{24}$$

The exciter here is simply treated as a second order [8] dynamic model:

$$T_E \cdot \frac{}{dt} = \quad \cdot \quad - \quad - \quad - \tag{25}$$

$$T_{FE} \frac{dU_s}{dt} = \quad \cdot \frac{}{dt} \quad - \tag{26}$$

Where:

D_p	Damping coefficient, *dimensionless*	U_t	Generator terminal voltage, *p.u*
E_d'	d axis transient voltage, *p.u*	U_{td}	d axis component of terminal voltage, *p.u*
E_{fd}	d axis field voltage, *p.u*	U_{tq}	q axis component of terminal voltage, *p.u*
E_q'	q axis transient voltage, *p.u*	U_r	Reference voltage, *p.u*
I_d	d axis armature current, *p.u*	U_s	Stabilizer voltage, *p.u*
I_q	q axis armature current, *p.u*	V_0	Infinitive bus voltage, *p.u*
J	Generator inertia constant, *dimensionless*	X_d	Synchronous reactance, *p.u*
K_E	Exciter gain, *dimensionless*	X_d'	d axis transient reactance, *p.u*
K_F	Stabilizer gain, *dimensionless*	X_q	q axis synchronous reactance, *p.u*
R_a	Armature resistance, *p.u*	X_q'	q aixs transient reactance, *p.u*
R_e	Equivalent resistance of transmission lines, *p.u*	X_e	Equivalent reactance of transient line, *p.u*
T_{d0}'	d axis open circuit time constant, *second*	T_E	Exciter time constant
T_{q0}'	q axis open circuit time constant, *second*	T_{FE}	Stabilizer circuit time constant

A. Model Testing

For testing this model,, a short circuit error situation has been simulated at 1.1s and recovered at 1.2s, results shown in Figure 7 and Figure 8. The terminal voltage was suddenly set to zero because of the short circuit, and it leaded to an unexpected rise of electrical power. In the meanwhile, the excitation voltage, the manipulated variable for terminal voltage, went up to increase the voltage to reference point again. Apparently, the change of terminal voltage has caused oscillation of electrical power. From Equation (22), it will result in oscillation of angular speed, equilibrium to oscillation of frequency which should be as stable as possible. To obtain a constant frequency normally requires control actions for mechanical power. Therefore, the voltage control and frequency control sequentially should be implemented simultaneously for the hydropower system. This paper only presented a decentralized control with two separate controllers, which is discussed in next section. The generated electrical power is assumed thoroughly consumed. There is no mathematical modeling of detailed electric grid with load included.

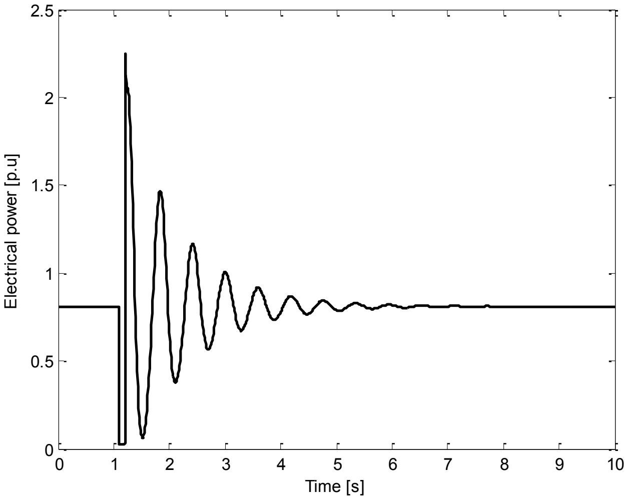

Figure 7. Simulated electrical power with a short circuit error

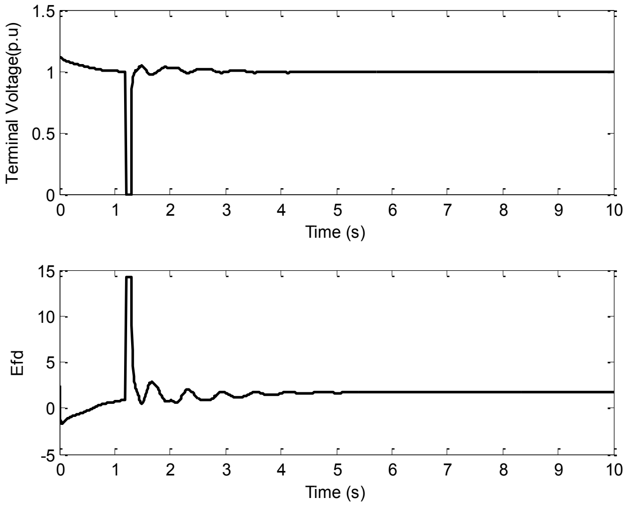

Figure 8. Simulated terminal voltage of excitation voltage with a short circuit error

I. CONTROL STRUCTURE

The modeling results in a MIMO system, the mechanical power from water and excitation voltage of synchronous machine are the inputs, while the frequency and voltage are the outputs in this system. A traditional PI controller was implemented with this model for frequency and another controller for voltage. This interacted and controlled MIMO system working process is shown in Figure 9.

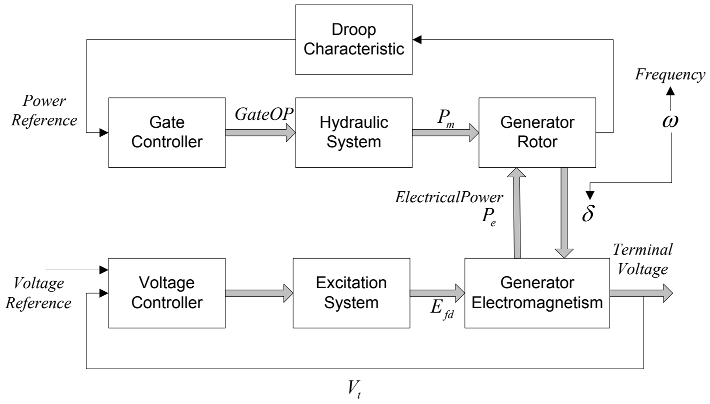

Figure 9. Flow chart of controlled MIMO hydropower system

A. Frequency Control

The frequency control is equal to the speed control or the active power control of the hydraulic turbine. The controller should comply with the main purpose, which is to keep the rotational speed stable and constant of the turbine-generator unit at any grid load. In other words, it should respond to any change of electrical power.

Considering a real hydropower plant is connected to electric grid, if a load decrease, the excess power will accelerate the rotation of the turbine. Then the controller should reduce turbine speed that means deceleration of the water masses in the penstock and corresponding pressure rise and oscillations. However, to avoiding it approach to bearable pressure for turbine and penstock, closing rate should be considerable. To fulfill and balance these two opposite demands, a PI controller has been designed. Since the SMIB assumption, this PI controller is implemented with a droop characteristic for active power and frequency. The droop relationship [9] for them is described as:

$$\frac{\Delta}{f_{nom}} = - \quad \cdot \frac{\hat{}}{P_{nom}} \tag{27}$$

Every simulation control interval, the function of droop characteristic will calculate deviation of frequency and obtain a reference power for PI controller as

$$P_{new} = -\frac{1}{Dr} \cdot \frac{f}{f_{nom}} \quad \cdot \quad + \quad = \tag{28}$$

B. Voltage Control

The voltage control will be carried out with the generator excitation control using a controller with a stabilizer. The controller is embedded in the second order model of exciter, which was shown in Equation (25), (26). The purpose of the controller is to hold the terminal voltage magnitude of a synchronous generator at a specific value. An increase in reactive power load of the generator should be accompanied by a drop in the terminal voltage magnitude. This voltage is rectified and compared to a set point signal. The difference of them input into a controller and then it controls the exciter field and increases the exciter voltage. Thus, the generator field current is increased, which results in an increase in the generated voltage [8].

II. RESULTS AND DISCUSSION

From the Figure 9, it can be seen the mechanical power, electrical power, frequency, phase angle and terminal voltage are interacting with each other. These mutual effects also can be seen from Equation (15) to Equation (24). Even with these effects, two decentralized controllers still perform working effectively with disturbances. Because this paper presumed it is a single machine to infinite bus system, the disturbances are simulated as a suddenly frequency change and voltage change in the bus. These changes are added to process compulsorily, not simulated as mathematical functions. That means during simulation process, when these disturbances happen, they will last for several seconds. In such a period, in order the see the results more obviously, although controllers output the manipulated signals, they cannot make the objective variables back to reference value immediately till the disturbances are erased. Because partial differential equations are time consuming with MATLAB, when developing controllers, only the head and flow in last element of penstock are considered.

In all simulations with controllers, what should mention are the oscillations in the beginning, it is because when combining the electrical power model and mechanical power part, the system needs some while to be initialized. The physical meaning of this situation can be explained like what happens when a generator injects to the electrical grid.

- Simulation results with a frequency disturbance of 51Hz at period 25s to 35s.

As it can be seen in Figure 10, when the system got this disturbance, the gate moved towards to a smaller opening with its characteristic steps to get a smaller mechanical to reduce the rotation speed. In other words, it is to reduce the frequency. After the disturbance ends, the gate moved back to its working point when it is at 50 Hz. And consequently, the mechanical power was also raised up. After the gate finished its movements, the mechanical power got some waves. It is because of the dynamics of the hydraulic system.

When the disturbance happened, there was a small effect to terminal voltage what is shown in Figure 11. The exciter maintained the terminal voltage quite well. It reduced to a smaller value, when got a disturbance. This is because the when the mechanical power is decreased, the electrical power will correspondingly should be reduced. Then, the excitation voltage should be smaller to keep the terminal voltage at its reference point. The control response of exciter can be seen rather faster than the gate. This is reasonable and practical. A turbine is mechanical equipment bearing high pressure and tons of water. Its movement is certainly slower than the electrical voltage generation with several windings.

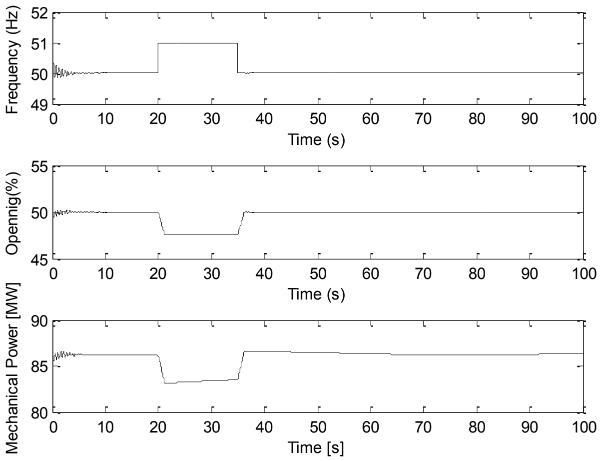

Figure 10. Simulation results of gate controller with a frequency disturbance

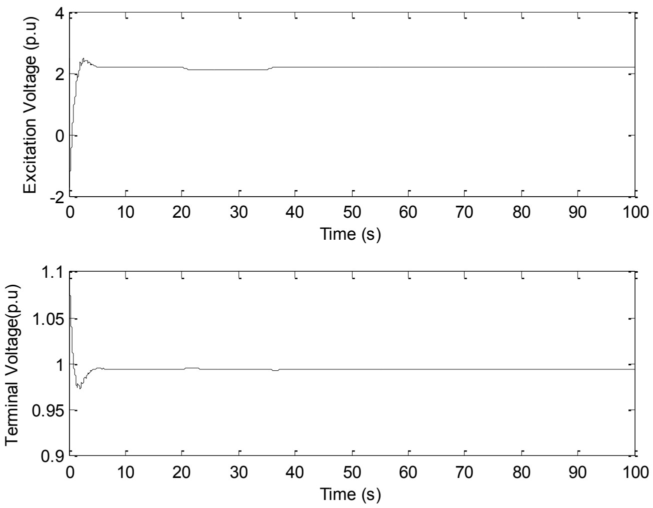

Figure 11. Simulation results of voltage controller with a frequency disturbance

- Simulation results with a voltage disturbance of 0.9 p.u at period 20s to 25s.

Firstly, the controlling of excitation voltage can be seen in Figure 12. It responded very quickly, and it went up to get a higher terminal voltage. Due to its quick response, the voltage also got a deviation after the disturbance ended. But again, the exciter forced it back to its nominal working point. This control result is acceptable.

However, when there is voltage disturbance, it caused some oscillations on the mechanical power control that can be seen in Figure 13. Terminal voltage changed means the electrical power changed, which is also presented in Figure 6; the frequency sequentially got a deviation from its nominal point. To control the frequency, the gate started to act. But because of interactions between electrical power control and mechanical power control, it caused some oscillations till the system went stable again. But those oscillations are just with 0.2 Hz and 0.2 MW deviations, which seems can be acceptable.

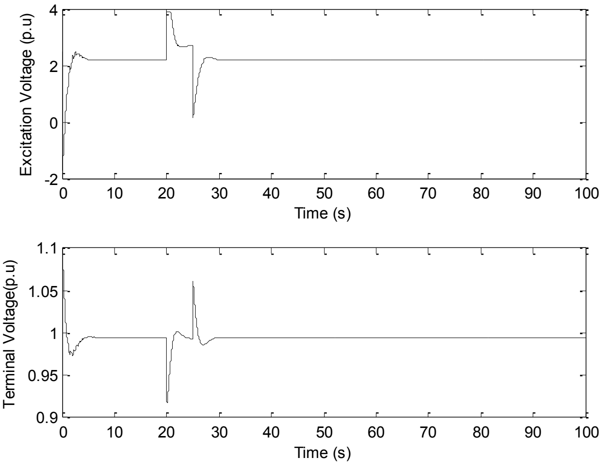

Figure12.Simulation results of voltage controller with a voltage disturbance

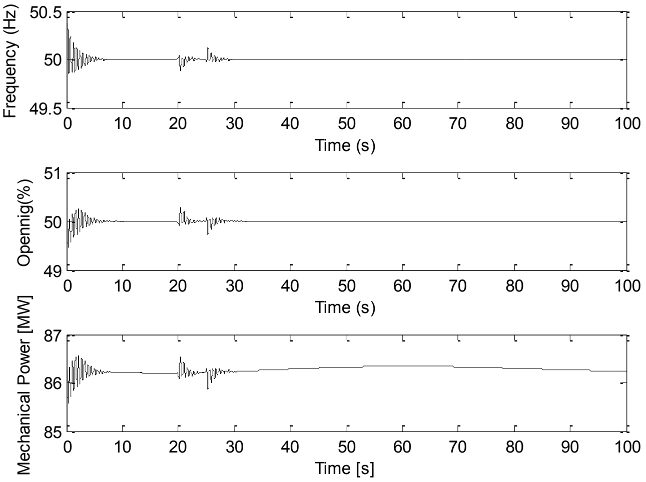

Figure13. Simulation results of gate controller with a voltage disturbance

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